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Emission of γ rays by channeled particles

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A theory is developed for the emission of γ rays by electrons and positrons in planar channeling in the case when the recoil on radiation and the interaction of the particle spin with the radiation field become important. Analytic expressions are obtained for the spectral and angular densities of the probability of radiation for two models of the planar potential. It is shown that recoil on radiation is the cause of buildup of transverse oscillations of particles in the channel.

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INTRODUCTION

A relativistic particle channeled in a crystal moves on the average in a straight line along planes or strings of atoms of the crystal (for example, see the review by Gemmell[1]). The motion of a particle in the transverse direction is finite and consequently the particle energy associated with the transverse motion takes on discrete values. Thus, a channeled particle is a model of a one-dimensional or two-dimensional (for channeling along a string) atom uniformly moving in a crystal with a relativistic velocity. From this point of view, electromagnetic radiation occurs in spontaneous transition of the particle from the initial state of the transverse motion to the final state. This phenomenon has been discussed by Vorobiev *et al.*[2] However, the energy of the observed photon, generally speaking, does not coincide with the difference in the energy levels of the transverse motion. As a result of the Doppler effect, the photon energy depends on the angle θ between the direction of the particle's longitudinal velocity and the direction of observation. For this reason, as has been shown by Kumakhov,[3,4] the maximum of the spectral density of radiation by relativistic particles is shifted toward the x-ray frequency region and increases with increasing particle energy—in contradiction to the conclusions of Ref. 2. A detailed analysis of the errors contained in Ref. 2 has been given Kumakhov.[5]

The radiation arising in spontaneous transitions between the levels of the transverse motion can be called the "characteristic" radiation of channeled particles. As noted by Kumakhov,[4] and also as studied in more detail by Bazylev and the present author,[6] the spectrum of this radiation is determined to a significant degree by the form of the interplanar potential.

Another type of radiation (analogous to radiation in the radiative recombination of ions) arises in capture of a particle from the energy continuum to a level of the transverse motion.[7] A similar question has been discussed also by Fedorov and Smirnov[8] in a discussion of radiation by an electron diffracted in a single crystal.

After Kumakhov's work,[3] a number of articles by other authors appeared in which the theory of electromagnetic radiation in channeling was discussed. Baryshevskiĭ and Dubovskaya[9] discuss the general formulation of the problem of radiation by channeled electrons in a single crystal of finite thickness and the possibility of complex and anomalous Doppler effects. A. A. Vorob'ev and his colleagues[10] and also Terhune and Pantell[11] estimate the spectral distribution of the probability of radiation of relatively soft photons by electrons in axial channeling on the basis of the well known results of the theory of synchrotron radiation (for example, see Ref. 12). Akhiezer *et al.*[13] generalized to the case of an arbitrary potential the results of the classical calculation by Kumakhov[3] of the intensity of dipole radiation in planar channeling in a parabolic potential.

Bazylev and Zhevago[6] carried out a quantum-mechanical calculation of the spectral and angular distributions of the probability of radiation of relatively soft photons ($\hbar\omega \ll E$) in planar channeling for an arbitrary form of interplanar potential with inclusions of the effect of frequency and spatial dispersion of the electromagnetic

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wave on the radiation process.[1] In particular, it was shown that beginning with a particle energy $E_1=(mc^2)^2/2U_0$, where U_0 is the depth of the potential well in which the transverse motion of the channeled particle occurs, the radiation is no longer dipole. As a consequence of this, even in a parabolic potential well, in contrast to the case discussed by Kumakhov,[3,4] the probability of transition of a particle to the bottom of the well becomes relatively high. As a consequence of the Doppler effect, in such a transition a photon with energy $\hbar\omega \approx 2U_0(E_1/mc^2)^2=E_1$ is radiated. Thus, at sufficiently high energies[2)] of channeled positrons or electrons, the upper end point of the characteristic radiation spectrum reaches the initial energy of the particle. In this case in calculation of the spectral distribution of photons it is necessary to take into account the quantum-mechanical recoil on radiation and also the interaction of the particle spin with the effective radiation field. The present work is devoted to this question.

1. MOTION OF ELECTRON OR POSITRON IN THE FIELD OF CRYSTAL PLANES

The solution of the Dirac equation for an electron in an external field can be represented in the form

$$\Psi(x)=(\gamma^\mu P_\mu+1)\tilde{\Psi}(x), \tag{1}$$

where $P_\mu=-i\partial/\partial x_\mu-eA_\mu$ ($\hbar=m=c=1$) and γ^μ are the Dirac matrices. In a coordinate system with axis x_1 perpendicular to the channeling planes, only the scalar potential $\Phi(x_1)$ is nonzero and the function $\tilde{\Psi}(x_1)$ satisfies the equation

$$\left[\left(i\frac{\partial}{\partial t}-e\Phi(x_1)\right)^2+\Delta-1+ie\alpha_1\frac{d\Phi(x_1)}{dx_1}\right]\tilde{\Psi}(x)=0.$$

Let E be the energy of the relativistic particle and U_0 the characteristic potential energy of the particle in the field. We neglect the terms $(e\Phi(x_1))^2$ and $ie\alpha_1 d\Phi/dx_1$ (the latter corresponds to interaction of the particle spin with the field of the crystal planes). The order of magnitude of these terms is equal to the order of $U_0^2 \ll E^2$. With this accuracy the solution of the equation was obtained in Ref. 6. Substitution of $\tilde{\Psi}(x_1)$ into Eq. (1) leads to the following result for the initial wave function of the electron in the interplanar field $\Phi(x_1)$:

$$\Psi_i(x)=\left(\frac{E_i^{\parallel}+1}{2E_i^{\parallel}}\right)^{1/2}\begin{pmatrix}\varphi_i\\ \sigma\mathbf{p}\varphi_i/(E_i+1)\end{pmatrix}\exp(i\mathbf{p}_i^{\parallel}\boldsymbol{\rho})\psi_i(x_1)\exp(-iE_it). \tag{2}$$

The initial values of the wave function $\psi_i(x_1)$ and the energy level ε_i which correspond to transverse motion are determined in this case by an equation of the Schrödinger type:

$$\left[-\frac{1}{2E_i^{\parallel}}\frac{d^2}{dx_1^2}-e\Phi(x_1)\right]\psi_i(x_1)=\varepsilon_i\psi_i(x_1). \tag{3}$$

Here we have introduced the following designations: $E_i^{\parallel}=[(\mathrm{p}_i^{\parallel})^2+1]^{1/2}$ is the initial energy of the longitudinal motion, $\mathrm{p}_i^{\parallel}$ is the projection of the initial electron momentum on the channeling plane, ρ is the radius vector lying in the channeling plane, φ_i is a spinor describing the initial spin state of the electron, σ are the Pauli matrices, $\mathrm{p}=-i\nabla$ is the particle momentum operator, and $E_i=E_i^{\parallel}+\varepsilon_i$.

The final-state wave functions of the particle are obtained from Eq. (2) by formal replacement of the initial quantum numbers by final ones. It should be noted that the Hamiltonian of the transverse motion corresponding to Eq. (3) depends parametrically on the energy of the longitudinal motion. Therefore the final wave functions of the transverse motion $\chi_f(x_1)$ form an orthonormal basis which, generally speaking, is different from $\{\psi_i(x_1)\}$.

The nonorthogonality of the wave functions of the initial and final states of the transverse motion creates certain complications in specific calculations. However, for a number of important forms of planar potential $\Phi(x_1)$ the problem can nevertheless be solved in analytic form (see below). On the other hand, analytic solution of the exact equation is impossible even in the case of a potential of the simplest form (for example, $\Phi(x_1)\propto x_1^2$).

2. CHARACTERISTIC RADIATION BY CHANNELED ELECTRONS OR POSITRONS

For sufficiently high energies of the particle and photon it is possible to neglect the effect of the frequency dispersion of the electromagnetic wave in the crystal on the probability of characteristic radiation of γ rays by channeled particles.[6] We shall also not discuss more complicated radiation processes involving the effect of spatial dispersion. Then the spectral and angular density of the probability of radiation of a photon with polarization $\mathbf{e}$ per unit time can be represented in the form (see Eq. (4) from Ref. 6)

$$\frac{d^2w}{d\omega d\Omega}=\frac{e^2\omega}{2\pi^2}\mathrm{Re}\int_0^\infty \mathscr{L}_{ik}(\tau,\omega)e_ie_k d\tau, \tag{4}$$

where

$$\mathscr{L}_{ik}(\tau,\omega)=e^{-i\omega\tau}\sum_F [j_i(x)e^{i\mathbf{kr}}]_{IF}[j_k(x)e^{-i\mathbf{kr}}]_{FI}, \tag{5}$$

$\mathbf{k}=\mathbf{n}\omega$, $\mathbf{n}$ is a unit vector in the direction of radiation, and i, $\mathrm{k}=1,2,3$; F is the set of quantum numbers of the final state of the particle.

For particles with spin $\frac{1}{2}$ the matrix elements of the transition current have the form

$$[\mathbf{j}(x)e^{i\mathbf{kr}}]_{IF}=\int\bar{\Psi}_I(x)\boldsymbol{\alpha}\Psi_F(x)e^{i\mathbf{kr}}d^3r. \tag{6}$$

Calculation of Eq. (6) with the wave functions (2) leads to the result

$$[\mathbf{j}(x)e^{i\mathbf{kr}}]_{IF}=\exp\{i(E_i-E_f)t\}\cdot\varphi_i^+(\mathbf{A}+i[\boldsymbol{\sigma}\mathbf{B}])\varphi_f(2\pi)^2\delta(\mathbf{p}_f^{\parallel}+\mathbf{k}^{\parallel}-\mathbf{p}_i^{\parallel}),$$

(the Russian notation of square brackets indicating a vector product is used), and in the ultrarelativistic limit ($E_i^{\parallel}\gg 1$, $E_f^{\parallel}\gg 1$), the vectors $\mathbf{A}$ and $\mathbf{B}$ are determined by the equalities[3)]

$$\begin{gathered}2A_1=I_{if}^{(2)}-I_{if}^{(3)},\quad 2\mathbf{A}^{\parallel}=I_{if}^{(1)}E_f^{-1}[(E_i+E_f)\mathbf{v}_i^{\parallel}-\mathbf{k}^{\parallel}],\\ 2B_1=I_{if}^{(2)}+I_{if}^{(3)},\quad 2\mathbf{B}^{\parallel}=I_{if}^{(1)}E_f^{-1}[(E_i-E_f)\mathbf{v}_i^{\parallel}-(1+E_i^{-1})\mathbf{k}^{\parallel}],\\ I_{if}^{(1)}=\int e^{ik_1x_1}\psi_i^*\chi_f dx_1,\quad I_{if}^{(2)}=\frac{i}{E_i}\int e^{ik_1x_1}\frac{\partial\psi_i^*}{dx_1}\chi_f dx_1,\\ I_{if}^{(3)}=\frac{i}{E_f}\int e^{ik_1x_1}\psi_i^*\frac{d\chi_f}{dx_1}dx_1.\end{gathered} \tag{7}$$

Here $\mathbf{v}_i^{\parallel}=\mathbf{p}_i^{\parallel}/E_i^{\parallel}$ is the projection of the initial velocity of the particle on the channeling plane; k_1 and $\mathbf{k}^{\parallel}$ are the photon momentum components perpendicular and parallel to the channeling plane. There exists between the integrals in Eq. (7) the relation

$$-k_1 I_{if}^{(1)}+E_i I_{if}^{(2)}+E_f I_{if}^{(3)}=0, \tag{8}$$

which is a consequence of the equality

$$\int \frac{d}{dx_1}\left(e^{ik_1x_1}\psi_i^*\chi_f\right)=0.$$

We eliminate from A_1 and B_1 by means of Eq. (8) the quantity $I_{if}^{(3)}$. The further calculations can be simplified if we note that the vector $\mathbf{A}$ is multiplied in Eq. (4) by the photon polarization vector and if we take into account the condition that the photon be transverse, $\mathbf{k}^{\parallel}\mathbf{e}^{\parallel}+k_1e_1=0$. Then we can assume that

$$A_1=\frac{E_i+E_f}{2E_f}I_{if}^{(2)},\quad \mathbf{A}^{\parallel}=\frac{E_i+E_f}{2E_f}I_{if}^{(1)}\mathbf{v}_i^{\parallel}.$$

Taking into account energy conservation ($E_i-E_f=\omega$, see Eq. (11)) we represent the vector $\mathbf{B}$ in the form

$$B_1=\frac{\omega}{2E_f}\left(n_1I_{if}^{(1)}-I_{if}^{(2)}\right),\quad \mathbf{B}^{\parallel}=\frac{\omega}{2E_f}I_{if}^{(1)}\left[\mathbf{v}_i^{\parallel}-(1+E_i^{-1})\mathbf{n}^{\parallel}\right].$$

The further calculations we shall carry out for unpolarized particles. Summation over the final polarizations and averaging over the initial polarizations of the electron in Eq. (5) is equivalent to calculation of the trace

$$\tfrac{1}{2}\mathrm{Sp}\left(A_i+i[\boldsymbol{\sigma}\mathbf{B}]_i\right)\left(A_k^*-i[\boldsymbol{\sigma}\mathbf{B}^*]_k\right)=A_iA_k^*+|\mathbf{B}|^2\delta_{ik}-B_iB_k^*. \tag{9}$$

Summation over the photon polarizations reduces to replacement of the tensor $e_ie_k^*$ in Eq. (4) by the tensor $\delta_{ik}-n_in_k$. After folding this tensor with expression (9) it is necessary to calculate the quantity

$$|M_{if}|^2=|\mathbf{A}|^2+|\mathbf{B}|^2-|\mathbf{An}|^2+|\mathbf{Bn}|^2.$$

We choose a spherical system of coordinates with polar axis along the initial direction of the electron momentum projection $\mathbf{p}_i^{\parallel}$. The azimuthal angle φ is measured from the direction of the x_1 axis. In the ultrarelativistic case radiation occurs mainly at small polar angles $\theta\ll 1$, and therefore $|M_{if}|^2$ can be approximately represented in the form

$$|M_{if}|^2\approx(1+u+u^2/2)\left[|I_{if}^{(1)}|^2\theta^2+|I_{if}^{(2)}|^2-2\,\mathrm{Re}\,I_{if}^{(1)}I_{if}^{(2)}\theta\cos\varphi\right]+(u^2/2E_i^2)|I_{if}^{(1)}|^2\quad(u=\omega/E_f). \tag{10}$$

Further calculation shows that for scalar particles the terms proportional to u^2 in Eq. (10) vanish. Thus, the terms proportional to u in Eq. (10) are due to the quantum-mechanical recoil on radiation, and the terms proportional to u^2 are due to interaction of the particle spin with the effective radiation field.

We sum in Eq. (5) over all projections $\mathbf{p}_f^{\parallel}$ of the final momentum of the particle on the channeling plane and then integrate in Eq. (4) over the time variable τ. We obtain

$$\frac{d^2w}{d\omega d\Omega}=\frac{e^2\omega}{2\pi}\sum_f|M_{if}|^2\delta\left[\omega-\omega_{if}-(E_p^{\parallel}-E_{p-k}^{\parallel})\right]. \tag{11}$$

Here $\omega_{if}=\varepsilon_i-\varepsilon_f$ is the difference of the energy levels of the transverse motion, and the summation in (11) is carried out over all quantum numbers f of the transverse motion of the particle with longitudinal energy $E_f^{\parallel}\equiv E_{p-k}^{\parallel}$. In the limit of small polarization angles $\theta\ll 1$ and ultrarelativistic energies ($E_p^{\parallel}\gg 1, E_{p-k}^{\parallel}\gg 1$) the change of the longitudinal energy on radiation has the form

$$E_p^{\parallel}-E_{p-k}^{\parallel}\approx\omega-\frac{\omega}{2(E_i-\omega)}\left[(\theta^2+E_i^{-2})E_i-\omega\theta^2\cos^2\varphi\right]. \tag{12}$$

We now reduce the quantity $|M_{if}|^2$ to a form more convenient for specific calculations. We represent the wave function $\chi_f(x_1)$ of the final state of the transverse motion in the form of a super-position of the wave functions of the basis of the initial states:

$$\chi_f=\sum_{f'}C_{ff'}\psi_{f'}.$$

The coefficient $C_{ff'}=\int\chi_f\psi_{f'}^*dx_1$ can be interpreted as the probability amplitude of a transition from the state χ_f to the state $\psi_{f'}$ for a certain change of the relativistic mass of the particle. Then the integrals $I_{if}^{(1)}$ and $I_{if}^{(2)}$ in Eq. (7) can be represented in the form of sums of similar integrals $J_{if}^{(1)}$ and $J_{if}^{(2)}$ which are calculated with wave functions ψ_i^* and $\psi_{f'}$ belonging to the same basis $\{\psi_i(x_1)\}$. Using the Schrödinger equation for ψ_i^* and $\psi_{f'}$, we can show that the following relation exists between $J_{if'}^{(1)}$ and $J_{if'}^{(2)}$:

$$J_{if'}^{(2)}=\left(\frac{\omega_{if'}}{k_1}+\frac{k_1}{2E_i}\right)J_{if'}^{(1)}; \tag{13}$$

here $\omega_{if'}=\varepsilon_i(E_i)-\varepsilon_{f'}(E_i)$.

Thus, the spectral and angular density of the probability of characteristic radiation per unit time by an electron or positron with initial energy E and initial transverse energy $\varepsilon_i(E)$ takes the form

$$\frac{d^2w}{d\omega\,d\Omega}=\frac{e^2\omega}{(2\pi)}\sum_{f,f'}|C_{ff'}|^2|J_{if'}^{(1)}(k_1)|^2\times\left\{\left(1+u+\frac{u^2}{2}\right)\left[\theta^2+\left(\frac{\omega_{if'}}{k_1}+\frac{k_1}{2E}\right)^2-\frac{2\omega_{if'}}{\omega}-\frac{k_1^2}{\omega E}\right]+\frac{u^2}{2E^2}\right\}\delta\left(\frac{u}{2}\left[(E-\omega\cos^2\varphi)\theta^2+\frac{1}{E}\right]-\omega_{if}\right). \tag{14}$$

Here

$$u=\omega/(E-\omega),\quad \omega_{if'}=\varepsilon_i-\varepsilon_{f'},\quad \omega_{if}=\varepsilon_i-\varepsilon_f,\quad k_1=\omega\theta\cos\varphi,$$
$$\varepsilon_i=\varepsilon_i(E),\quad \varepsilon_{f'}=\varepsilon_{f'}(E),\quad \varepsilon_f=\varepsilon_f(E-\omega),$$
$$C_{ff'}=\int\psi_{f'}^*(x_1)\chi_f(x_1)dx_1,\quad J_{if'}^{(1)}(k_1)=\int e^{ik_1x_1}\psi_i^*(x_1)\psi_{f'}(x_1)dx_1. \tag{15}$$

The functions ψ_i and $\psi_{f'}$ satisfy Eq. (3), where $E_i^{\parallel}\approx E$, and χ_f satisfies the equation which is obtained from (3) by replacing $E_i^{\parallel}$ by $E_f^{\parallel}\approx E-\omega$.

For photons with energy $\omega\ll E$ we obtain $u\approx\omega/E\ll 1$, $C_{ff'}\approx\delta_{ff'}$, where $\delta_{ff'}$ is the Kronecker symbol and expression (14) coincides in this case with the previously calculated spectral and angular distribution (see Eq. (15) from Ref. 6) in the limits of small polar angles and ultrarelativistic energies.

Thus, the problem reduces to solution of Eq. (3) and calculation by means of the transverse-motion wave functions of the matrix elements of the radiative transition $J_{if'}^{(1)}$ and of the quantities $C_{ff'}$ (or $I_{if}^{(1)}$ and $I_{if}^{(2)}$; see Eqs. (7) and (10)).

3. RADIATION SPECTRUM FOR VARIOUS MODELS OF PLANAR POTENTIAL

The potential $\Phi(x_1)$ in which the channeled particle is moving is in the general case a periodic function with a period d equal to the distance between neighboring crystal planes:

$$\Phi(x_1)=\sum_{n=-\infty}^{\infty}V(x_1-nd). \qquad (16)$$

The potential produced by an individual plane $V(x_1)$ can be represented in analytic form in the Molière approximation for the potential of an atom and with inclusion of isotropic thermal vibrations.[14]

It turns out that in most cases in analysis of the states of negatively charged channeled particles it is sufficient in Eq. (16) to consider only the potential of one plane, and for positively charged particles—the potential of two neighboring planes (see for example Section 2.4 of Ref. 1). Diffraction effects associated with the periodicity of the potential can be appreciable only in channeling of light particles (electrons) with energies of several MeV.[15] With increasing energy of the particles, diffraction effects become less and less important. This is due to the relatively low transmission of the potential barrier separating neighboring channels. At high energies, where the number of levels in an isolated potential well is large (see below), it is possible to use a quasiclassical estimate for the ratio of the splitting $\Delta\varepsilon$ of the transverse energy level ε to the distance ω_0 between neighboring levels (see for example Ref. 16, §50, problem 3):

$$\frac{\Delta\varepsilon}{\omega_0}=\frac{1}{\pi}\exp\left\{-\int_{-a}^{a}(2E[U(x)-\varepsilon])^{1/2}dx\right\}. \qquad (17)$$

Here the integration is carried out over the classically forbidden region of transverse motion of the particle. According to Lindhard (see for example Ref. 1), channeling is stable if a positively charged particle does not approach the plane closer than the Thomas-Fermi radius ($a \gtrsim a_{TF}$). Using the explicit form of the silicon (100) plane potential,[14] we obtain from Eq. (17) $\Delta\varepsilon/\omega_0 \approx 6\cdot 10^{-5}$ for $2EU_0=1$, where $U_0\approx 23$ eV is the depth of the potential well. At the same time the radiative width Γ of this energy level, calculated by means of the results of Refs. 3–6, for a parabolic potential has the form

$$\Gamma\approx\omega_0 e^2(2U_0E)^{1/2}.$$

For $2U_0E=1$ this value is three orders of magnitude greater than the splitting of the level as the result of tunneling. Thus, at sufficiently high particle energies if we limit ourselves to states of the transverse motion not too close in energy to the barrier height, we can neglect the band nature of the transverse energy spectrum in the first approximation and solve the problem for an isolated potential well. Kumakhov and Wedell[17] reached the same conclusion on the basis of somewhat different calculations. The existence of relatively narrow bands in the transverse energy spectrum may turn out to be important only in study of such questions as induced radiation by channeled particles of not too high energy,[18] when it is necessary to know the spectral width of the radiation at a definite angle θ.

In the present work we discuss mainly effects arising at rather high particle energies where the states of the transverse motion are with sufficient accuracy determined by the isolated potential well. In the general case the band nature of the transverse energy spectrum can be taken into account subsequently by the strong-coupling method (for example, see Ref. 19).

The potential of an isolated well has a rather complicated form.[14] Analytic solution of Eq. (3) and calculation of $J_{if'}^{(1)}$ and $C_{ff'}$ is possible only for certain very simple models. Thus, in the case when the effective potential is a rectangular well of width d with infinitely high walls,[4)] the wave functions do not depend on the relativistic mass of the particle (see for example §22 of Ref. 16) and the energy values are quantized according to the equation

$$\varepsilon_i(E)=\pi^2 i^2/2Ed^2 \quad (i=1,\ 2,\ 3,\ldots). \qquad (18)$$

In this case the result of the calculations has a particularly simple form:[5)]

$$|J_{if'}^{(1)}(k_1)|^2=4^3y^2\left|\frac{\pi^2 if'\sin\{[y-\pi(i-f')]/2\}}{[y^2-\pi^2(i-f')^2][y^2-\pi^2(i+f')^2]}\right|^2, \quad C_{ff'}=\delta_{ff'}, \qquad (19)$$

where $y=k_1d$ and $\delta_{ff'}$ is the Kronecker symbol.

For planar channeling of positively charged particles the potential is fitted with sufficient accuracy by a parabolic well (see Fig. 9 of Ref. 1):

$$e\Phi(x_1)=(4U_0/d^2)x_1^2.$$

Here the energy spectrum is equidistant:

$$\varepsilon_i(E)=\frac{2}{d}\left(\frac{2U_0}{E}\right)^{1/2}\left(i+\frac{1}{2}\right),\quad i=0,1,2,\ldots. \qquad (20)$$

The quantity $|J_{if'}^{(1)}|^2$ was calculated in Ref. 6 and has the form

$$|J_{if'}^{(1)}(k_1)|^2=2^{i-f'}\frac{f'!}{i!}e^{-2\xi}\xi^{i-f'}|L_{i}^{i-f'}(2\xi)|^2, \qquad (21)$$

where $\xi=k_1^2d/[8(2U_0E)^{1/2}]$; $L_n^\alpha(z)$ is the Laguerre polynomial.[20]

In calculation of $|C_{ff'}|^2$ we shall use the results of Popov and Perelomov,[21] who solved the problem of parametric excitation of a quantum oscillator with variable mass. For a sudden change of mass the coefficients $|C_{ff'}|^2$ are nonzero if the difference of the quantum numbers $f-f'$ is even, and in this case the coefficients have the form

$$|C_{ff'}|^2=\frac{f_<!}{f_>!}\sqrt{1-\rho}\,|P_{(f+f')/2}^{|f-f'|/2}(\sqrt{1-\rho})|^2, \qquad (22)$$

where $f_<=\min\{f,f'\}$, $f_<=\max\{f,f'\}$,

$$\rho=\left[\frac{E^{-1/2}-(E-\omega)^{-1/2}}{E^{-1/2}+(E-\omega)^{-1/2}}\right]^2$$

and $P_n^m(z)$ is the associated Legendre function.[20]

We note also that in planar channeling of electrons a potential of the form $e\Phi(x_1)=U_0\cosh^{-2}\alpha x_1$ is closer to reality and permits analytic solution.

We now consider some general properties of photon emission in channeling of charged particles.

Conservation of energy and momentum in the radiation require vanishing of the argument of the δ function in Eq. (14). Thus in the transition of a particle from level i to level f the frequency of the radiation is strictly related to the direction of radiation:

$$\theta=\left[\frac{2\omega_{if}E^2-\omega(1+2\omega_{if}E)}{\omega E(E-\omega\cos^2\varphi)}\right]^{1/2}. \qquad (23)$$

The dependence on the azimuthal angle φ in Eq. (23) appears as a consequence of the effect of recoil in the radiation on the longitudinal motion of the particle. It must also be recalled that the quantities ω_{if} in the general case depend parametrically on the radiation frequency as the result of the recoil effect on the transverse motion. The maximum frequency $\omega_{max}^{(f)}$ is radiated at $\theta=0$. Using for a rectangular well (see Eq. (18)) the explicit form of the parametric dependence $\omega_{if}=\varepsilon_i(E)-\varepsilon_f(E-\omega)$ on the frequency ω, we obtain in this case

$$\omega_{max}^{(f)}=\frac{2\tilde{\omega}_{if}E^2}{1+2E\varepsilon_i(E)}, \qquad (24)$$

where $\tilde{\omega}_{if}\equiv\varepsilon_i(E)-\varepsilon_f(E)$. This result shows that at relatively low energies where $2E\varepsilon_i\ll 1$ the maximum frequency increases as $E^{3/2}$ (if $i-f\ll i$, then $\tilde{\omega}_{if}\sim E^{-1/2}$), which corresponds to the results of Kumakhov.[3,4] However, at high energies ($2E\varepsilon_i\gg 1$) the maximum frequency increases more slowly (as $E^{1/2}$) with increase of the energy. For a parabolic well we obtain a result similar to Eq. (24) but more cumbersome.

The absolute upper end point of the spectrum corresponds to transition of the particle to the bottom of the potential well ($\omega_{if}\approx\varepsilon_i(E)$). If as an estimate of $\varepsilon_i(E)$ we take the value $U_0\approx 10$ eV, we find that $2EU_0\approx 1$ at electron or positron energies $E\approx 10$ GeV and consequently in this case the particle can emit photons with energy $\omega\sim E$ (see Eq. (24)).

It should be noted that in a certain region near the angle $\theta=0$ the radiation has a dipole nature for any particle energies. In this case the arguments y in (19) and ξ in (21) turn out to be small and the radiative transition matrix elements are substantially simplified.[6] Therefore in evaluating the spectral density $dQ/d\omega$ of the radiated energy for $\omega=\omega_{max}^{(f)}$ we can use the dipole approximation formulas obtained in Refs. 3–6:

$$dQ/d\omega\sim\omega_{max}\tilde{\omega}_{if}^2|d_{if}|^2. \qquad (25)$$

Then, according to Eq. (24), the spectral density of energy at $\omega=\omega_{max}^{(f)}$ first rises as $E^{1/2}$ ($2E\varepsilon_i\ll 1$) and then ($2E\varepsilon_i\gg 1$) begins to fall as $E^{-1/2}$ with increasing energy of the particle. This important result shows that the maximum spectral density of the energy of characteristic radiation by channeled particles is reached at energies $E=(\frac{1}{2})U_0$ (~10 GeV for planar channeling).

The spectral distribution of radiated energy obtained by means of expressions (14), (15), and (18), (19), by numerical integration of (14) over the azimuthal angle φ, is given in the figure for an electron with energy 10 GeV which is channeled in the (100) plane of silicon.[6)] The potential was fitted by a rectangular well with parameters $U_0=18$ eV, $d=1.92$ Å. The initial state of the transverse motion corresponds to a quantum number $i=185$, at which the energy of the transverse motion ε_i is close to the value U_0. Sharp maxima in the spectrum arise at frequencies $\omega_{max}^{(f)}$ for even $f<i$. A small region of frequencies near these maxima is described by the dipole approximation. Smoother maxima correspond to the frequencies $\omega_{max}^{(f)}$ for odd f. These maxima appear as the result of explicit violation of the dipole approximation at high energies.

4. INFLUENCE OF POPULATION OF STATES ON THE RADIATION SPECTRUM

Expressions (10), (11) and (14), (15) for the spectral and angular density of the probability of radiation were obtained on the assumption that the particle before radiation is in a definite state i of the transverse motion. In real situations the initial state of channeled particles is characterized by some distribution in the transverse energy levels. This distribution is formed on entry of the particles into the crystal and, generally speaking, changes as the beam of particles moves into the interior of the crystal as the result of scattering of the particles by the electrons of the medium, lattice defects, impurities, and other factors. It is important, however, that with increase of the particle energy the radiative lifetime of the levels turns out to be shorter than the lifetime due to inelastic scattering processes.[18] Thus, in the first approximation we can assume that the population of the levels in the radiation process coincides with this value at the moment of entry into the crystal.

Let the wave function of transverse motion of the particle before entering the crystal be a plane wave

$$\psi_{p_x}(x_1)=\exp(ip_x x_1),$$

which corresponds to a particle traveling at a definite angle $\theta=p_x/E$ relative to the crystal planes. Near the crystal boundary the plane wave is reorganized into a superposition of the wave functions of the transverse motion $\psi_i(x_1)$. As shown by Kagan and Kononets[22] the time l_1 of this reorganization turns out to be significantly shorter than the characteristic time l_0 of the transverse motion of the particle. Therefore according to the theory of sudden perturbations the relative probability P_n of capture into a level n of the transverse motion is determined by the square of the corresponding coefficient of expansion of the plane wave in the wave functions $\psi_i(x_1)$. For a rectangular well this probability has the form

$$P_n(p_x)=C_1\frac{4\pi n^2 d}{[(\pi n)^2-(p_x d)^2]^2}\sin^2\frac{\pi n+p_x d}{2}. \qquad (26)$$

For a parabolic potential (see also Ref. 17)

$$P_n(p_x)=\frac{C_2}{2^n n!(\pi E\omega_0)^{1/2}}\exp\left\{-\frac{p_x^2}{E\omega_0}\right\}H_n^2\left(\frac{p_x}{(E\omega_0)^{1/2}}\right), \qquad (27)$$

where $\omega_0^2=8U_0/d^2E$; C_1 and C_2 are normalization constants and the $H_n(z)$ are Hermite polynomials.

Analysis of expressions (26) and (27) shows that the capture probability has a distinct maximum for a small number of levels whose energy is close to the value $p_x^2/2E$ (see also Fig. 4 of Ref. 22). Hence it follows in particular that the critical entry angle at which discrete states in a potential well of depth U_0 are still effectively populated is determined by the relation

$$\theta_0^{(cr)}\approx(2U_0/E)^{1/2}, \qquad (28)$$

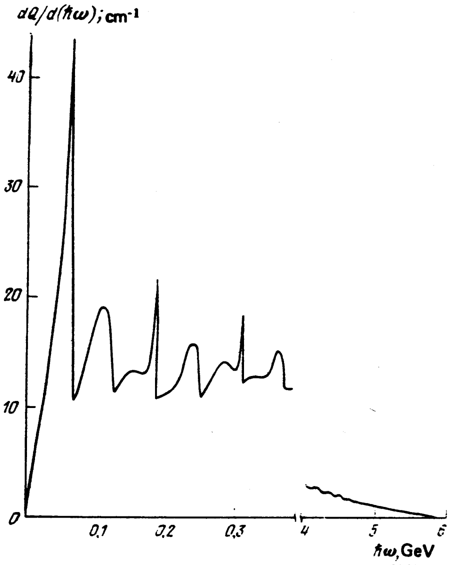

FIG. 1. Spectral density of radiated energy per unit path of an electron with energy 10 GeV channeled in the (110) plane of silicon, as a function of the frequency of the radiation.

which is in agreement with the classical estimates of Lindhard (see for example Ref. 1).

Thus, if the angular spread $\Delta\theta_0$ of the particles in the beam is small ($\Delta\theta_0 \ll \theta_0^{(cr)}$), it is possible by appropriate orientation of the crystal to populate a relatively small number of close-lying levels, and then the pattern of the radiation spectrum will be close to that shown in the figure. However, for high-energy particles ($\gtrsim$10 GeV) the critical channeling angle is so small ($\lesssim 10^{-4}$) that the angular divergence of real beams can be comparable with $\theta_0^{(cr)}$. In this case it is necessary to further average the probabilities $P_n(p_x)$ over the distribution of transverse momenta of the particles in the beam. Here the radiation spectrum can change substantially in shape. Changes in the spectrum due to averaging over a relatively broad distribution of initial states of the transverse motion is more important, the larger is the deviation from an equidistant spectrum[7)] of the frequencies ω_{if}.

CONCLUSION

In the theory of the radiation by channeled particles it is possible to separate a dimensionless parameter $\beta = (2U_0E)^{1/2}$ which represents the ratio of the critical channeling angle to the effective angle of the radiation. On the other hand, this parameter also determines the number of levels $n_{max} \sim \beta d$ in the potential well of the transverse motion, as well as the relativistic effects in the transverse motion of the channeled particles in the coordinate system where there is no longitudinal motion.

The characteristics of the radiation turn out to be completely different, depending on whether the parameter β is small or large. The main features in radiation by high-energy particles ($\beta \gtrsim 1$) are as follows:

1. With increase of β the number of levels in the potential well increases and reaches a value ~10^2 for $\beta \sim 1$, while the distance between neighboring levels decreases. As a result the quantum-mechanical recoil in radiation of even a relatively soft photon ($\omega \ll E$) becomes important.

2. Multipole expansion of the radiation field, generally speaking, becomes inapplicable. An exception is in relatively narrow frequency regions corresponding to angles $\theta \approx 0$. The width of these regions decreases with increase of β.

3. The factors enumerated above lead in turn to the start of a drop in the size of the peaks in the spectral density of the radiated energy with increasing energy of the particle E, and not a rise[3-5] as in the case of relatively low energies ($\beta \ll 1$). The positions of these peaks in the spectrum shift toward higher frequencies more slowly with increase of the particle energy than in the case of low energies.

4. The spectrum of the frequencies ω of the radiated photons advances into the region $\omega \sim E$ where the radiation has essentially a quantum-mechanical nature. Under these conditions the radiation damping of the classical amplitude of transverse oscillations of particles in the channel is partially compensated by a parametric buildup of the oscillations (see Eq. (22)). The cause of the buildup is the recoil which the particle undergoes in radiating a sufficiently energetic photon.

5. Since the number of levels increases with increase of β, the transverse motion of a particle in the field of the planes for most states becomes more and more quasiclassical. The quantum nature of the transverse motion appears in the radiation only for transitions of a particle to the very lowest levels f such that $(f-i) \sim i$, i.e., in the most energetic portion of the radiated spectrum. In the relatively soft part of the spectrum which is determined by transitions between the closest levels with large quantum numbers, the spectrum of the radiation by channeled particles is similar to the spectrum of radiation in macroscopic external fields (undulator radiation) (see for example Ref. 23). The analogy of the spectra arises as a consequence of the similarity of the particle trajectories in the two cases. Only the causes producing periodic motion of the particles are different.

The features considered in the radiation of high-energy particles in planar channeling apply also to axial channeling, where they should be observed at somewhat lower energies (~1 GeV), since the characteristic potential of an atomic string is as a rule higher than the potential of a plane.

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1) In the studies cited, except for Ref. 9, the estimates of the radiation intensity agree with the results of Kumakhov.[3-5]

2) For positrons channeled along the (110) plane in silicon, E_1 = 10 GeV.

3) In calculation of **A** and **B** it is not necessary also to distin-

guish between the total energies $E_i (E_f)$ and the longitudinal energies $E_i^{\parallel} (E_f^{\parallel})$.

4) The specific barrier height turns out to be unimportant for the levels considered.

5) In Ref. 6 the quantity $J_{if}^{(1)}$ is given erroneously for this case.

6) Integration over θ is carried out by elementary means in view of the presence of the δ function in Eq. (14).

7) At high energies ($2E\varepsilon_i \gtrsim 1$) this spectrum is non-equidistant even for a parabolic well.

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Spin–orbit interaction in an excitonic dielectric

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The influence of spin–orbit interaction on the character of electron–hole pairing in a two-band model is considered. It is shown that the classification by the possible types of the ground state, given by Halperin and Rice [Solid State Physics, **21**, 115 (1968)], remains in force in this case. The degeneracy between the charge-current and spin-current states, which exists in the absence of spin–orbit interaction, is lifted. If the spin–orbit interaction is strong enough, the current state may turn out to be the ground state even in the absence of impurity scattering.

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1. INTRODUCTION

It is known that, depending on the phase of the order parameter and on its spin structure, four types of anomalous mean values are possible in electron–hole pairing.[1] It is shown in Ref. 2 that if no account is taken of the spin degree of freedom the system goes over into a state $\langle n(\mathbf{r})\rangle$, with a charge-density wave (ChDW) if the order parameter is real, and into a state $\langle \mathbf{j}(\mathbf{r})\rangle$ with a current-density wave (CuDW) if the order parameter is imaginary. If account is taken of the spin and of the associated choice of the sign of the order parameter Δ for opposite spin directions, then it can be seen that, depending on this choice, we get also a spin-density wave (SDW) $\langle \mathbf{S}(\mathbf{r})\rangle$, and for an imaginary order parameter we get a spin flux density wave (SFDW) $\langle \mathbf{S}(\mathbf{r})\mathbf{j}(\mathbf{r})\rangle$ for an imaginary order parameter. Each of these order parameters is characterized by its own effective coupling constant. Following Ref. 3, we can show that in the scheme of an isotropic semimetal, the effective coupling constants

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