Cascade capture in a quantizing magnetic field

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A theory is constructed of cascade capture of carriers by attracting centers in a quantizing magnetic field. The case of an extremely strong magnetic field ($\hbar\Omega > kT$, where Ω is the Larmor frequency) is considered in detail. It is shown that the lifetime decreases with increasing field like $\tau \sim H^{-2}$, and in the region of strong magnetic fields, when the inequality $(\hbar\Omega ms^2)^{1/2} > kT$ is satisfied (m is the effective mass and s is the speed of sound) its plot flattens out. In this case the lifetime τ_H in magnetic field turns out to be shorter than the lifetime τ in the absence of a magnetic field, with $\tau_H/\tau \sim (ms^2/kT)^3$. At very low temperatures, when $kT < ms^2$, the lifetime in a quantizing field, on the contrary, increases, so that in strong fields we have $\tau_H/\tau \sim ms^2/kT$. The theory is generalized to include the case of multivalley semiconductors such as Ge and Si, in which the equal-energy surfaces are ellipsoids. It is shown that the lifetimes of the electrons in the individual valleys depend on the orientation of the valleys relative to the magnetic field and differ greatly for different valleys.

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INTRODUCTION

In this paper we generalize the theory of cascade capture of carriers by attracting centers[1] to the case of capture in quantizing magnetic fields. We consider the case of an extremely strong magnetic field H, when only the zeroth Landau level need to be taken into account, i.e., when

$$\hbar\Omega \gg kT, \tag{1}$$

where $\Omega = eH/mc$ is the Larmor frequency and m is the effective mass. We direct the external magnetic field along the z axis. Then the motion of the electrons (holes) in the xy plane is quantized. If condition (1) is satisfied, the kinetic energy of the electrons is of the order of the cyclotron rotation energy $\hbar\Omega/2$. Therefore the energy of the characteristic phonon emitted by the electrons in the field is of the order of $(\hbar\Omega ms^2)^{1/2}$, which is much higher than the energy of the characteristic phonon in the absence of the field $(kTms^2)^{1/2}$ (s is the speed of sound).

The process of capture in a magnetic field depends substantially on whether the thermal energy of the carriers kT is larger or smaller than the energy of the characteristic phonon $(\hbar\Omega ms^2)^{1/2}$.

Let $kT \gg (\hbar\Omega ms^2)^{1/2}$, then the carrier loses energy in small batches, and the capture process can be described as a continuous drop of carrier energy from positive to negative values. The carrier is in practice captured if it drops below the level $E = -kT$. With increasing magnetic field, the electron emits phonons of ever increasing energy, the rate of its energy of relaxation increases, and accordingly the lifetime of the electron with respect to capture by an attracting center decreases. This case is considered in Sec. 1.

Assume now that the magnetic field is so strong that $(\hbar\Omega ms^2)^{1/2} \gg kT$. Then the interaction of the electrons with the phonons becomes essentially inelastic. The electron is now able to emit a strong acoustic phonon with energy larger than kT, but this process can be realized only sufficiently close to the center, where the electron of energy $\sim kT$ above the zeroth Landau level (assumed to be the bottom of the conduction band in the magnetic field) goes over, as it emits an acoustic phonon, into a bound state with energy $\sim(\hbar\Omega ms^2)^{1/2}$. However, since the electron is captured in practice if it drops below a level with binding energy kT, it does not have to emit so large a phonon and need not come very close to the center. The principal role in the capture is played by transitions with emission of thermal phonons. Therefore in the limit $kT \ll (\hbar\Omega ms^2)^{1/2}$ the lifetime ceases to depend on the magnetic field. This case is considered in detail in Sec. 2.

Thus, in a quantizing magnetic field in the limit $\hbar\Omega \gg kT$ the lifetime with respect to capture by attracting centers first decreases with increasing field, as shown below, like $\tau \sim H^{-2}$, but with further increase of the field, in the region $(\hbar\Omega ms^2)^{1/2} > kT$, it ceases to depend on the field.

We consider now capture at very low temperatures, when $kT \ll ms^2$. In the absence of a magnetic field the thermal electrons cannot in principle emit phonons far from the center. After it is accelerated in the attracting-center field to an energy $\sim ms^2$, the electron is in a bound state immediately after it emits the phonon. Thus, the only electrons that can be captured are those that land in a sphere of radius r_s ($r_s = e^2/\varkappa ms^2$ for a Coulomb center). The flux of such particles to the center is $\sim \pi p^2$, where p is the impact distance corresponding to the closest-approach radius r_s. Since the trajectories are curved, it turns out that p is substantially larger than r_s ($p^2 = ms^2/kTr_s^2$). In quantizing magnetic fields, the motion is uniform because $p = r_s$. Therefore in strong magnetic fields ($\hbar\Omega \gg kT$) at very low temperatures ($kT \ll ms^2$) the lifetime with respect to capture by attracting centers is larger than the lifetime in the absence of the field, by a factor ms^2/kT. This case is also considered in detail in Sec. 2.

The theory developed in Secs. 1 and 2 is generalized in Sec. 3 to the case of an ellipsoidal equal-energy surface. It is shown that in a quantizing magnetic field the levels should become non-uniformly populated, for example in photoexcitation, because the carrier lifetime depends on the orientation of the valleys relative to the

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direction of the magnetic field.

The results and the approximation employed in the paper are discussed in Sec. 4.

We note that capture by attracting centers in quantizing magnetic fields ($\hbar\Omega \gg kT$) was first considered in Refs. 2 and 3. The calculation was based on a different model. In particular, it was assumed that when an electron is captured it emits a single acoustic phonon and goes to a ground state on an impurity center. Under this assumption the authors found that the lifetime increases with increasing magnetic field. It is known, however, that the cross section for direct capture in the ground state is much smaller than that obtained from the cascade model of capture.

1. CALCULATION OF THE LIFETIME UNDER CONDITIONS OF QUASICONTINUOUS ENERGY LOSS

We consider the case of a sufficiently low concentration of the capture centers, when the total recombination flux per unit volume $I = n/\tau$, where n is the concentration of the free carriers and τ is the carrier lifetime, can be represented in the form

$$I = Nj. \tag{2}$$

Here j is the recombination flux per attracting center and N is the concentration of the capture centers. The lifetime τ is connected with the recombination flux to an individual isolated center by the relation[1)]

$$\tau^{-1} = (N/n)j. \tag{3}$$

We calculate the recombination flux j by generalizing the procedure of Pitaevskiĭ.[2)4,1]

The electron distribution function in the region of negative total energy is defined by

$$B(E)\left[f(E) + kT\frac{df(E)}{dE}\right] = j, \tag{4}$$

where the diffusion coefficient $kTB(E)$ in energy space must be calculated with account taken of the presence of the quantizing magnetic field. The solution of (4) with the boundary condition

$$f(-E_1) = 0, \quad E_1 \gg kT \tag{5}$$

is of the form

$$f(E) = \frac{j}{kT} e^{-E/kT} \int_{-E_1}^{E} \frac{e^{E'/kT}}{B(E')} dE'. \tag{6}$$

The recombination flux j is determined from the condition of matching together the solution of (6) with the Boltzmann distribution at the point $E = 0$. The normalization of the Boltzmann function

$$f(E) = Ane^{-E/kT}, \quad A = \left(\frac{2\pi}{m}\right)^{3/2} \frac{\hbar^3}{\Omega (kT)^{1/2}} \tag{7}$$

corresponds to the ultraquantum limit, when all the carriers are at the zeroth Landau level, which is taken to be the zeroth level in the total energy space.[3)] Thus, the lifetime τ (just as in the absence of a magnetic field) is defined by

$$\frac{1}{\tau} = NAkT \Big/ \int_{-\infty}^{0} \exp(E'/kT) B^{-1}(E')\, dE'. \tag{8}$$

In (8) we replaced the lower integration limit $-E_1$ by $-\infty$. This can be done because the main contribution to the integral is made by the region of energy values $|E| \sim kT$ (it will be shown below that the coefficient $B(E)$ is a power-law function).

The coefficient $B(E)$ is expressed in terms of the mean-squared energy loss $\overline{(\Delta E)^2}$.[4,7] For a bound electron in a quantizing magnetic field, the coefficient $B(E)$ is given by

$$B(E) = (2VkT)^{-1} \int d^3r \sum_{i,j} w_{ji} (\varepsilon_i - \varepsilon_j)^2 \delta(E - \varepsilon_i - u(r)). \tag{9}$$

Here V is the volume of the crystal and the subscripts i and j stand for sets of three quantum numbers of the electron (n, k_x, k_z), (n', k_x', k_z'), since the state of the electron in a magnetic field, as is well known, is described by the function

$$\psi_{nk_xk_z} = \exp(i[k_x x + k_z z])\, \chi_n(y - y_0), \quad y_0 = -\frac{\hbar c}{eH} k_x, \tag{10}$$

where $\chi_n(y)$ is a Hermite polynomial. We shall need subsequently to know only

$$\chi_0(y) = \pi^{-1/4} a_H^{-1/2} \exp\left\{-\frac{(y-y_0)^2}{2a_H^2}\right\}, \quad a_H^2 = \hbar c/eH. \tag{11}$$

The probability w_{ji} of the transition from the i-th quantum state into the j-th state when an electron interacts with acoustic phonons is determined in the ultraquantum limit ($n = n' = 0$) by the expression[8]

$$w_{ji} = \frac{2\pi}{\hbar} \sum_{q} \frac{E_c^2 q \hbar}{2\rho V s} \delta_{k_x', k_x + q_x} \delta_{k_z', k_z + q_z} \times \exp\left\{-\frac{(q_x^2 + q_y^2) a_H^2}{2}\right\} \left[N_q \delta\left(\frac{\hbar^2 (k_z + q_z)^2}{2m} - \frac{\hbar^2 k_z'^2}{2m} - \hbar q s\right) + (N_q + 1)\, \delta\left(\frac{\hbar^2 (k_z - q_z)^2}{2m} - \frac{\hbar^2 k_z'^2}{2m} + \hbar q s\right)\right]. \tag{12}$$

Here E_c is the constant of the deformation potential, ρ is the density of the crystal, q is the wave vector of the phonon, and N_q is the equilibrium distribution function of the phonons.

Substituting (12) in (9) and integrating, we obtain ultimately for the coefficient $B(E)$ in the case of a Coulomb attracting center with charge eZ

$$B(E) = \frac{(\hbar\Omega)^3 E_c^2 m^4}{\pi^2 \hbar^7 \rho} \left(\frac{e^2 Z}{\varkappa}\right)^3 \frac{1}{|E|^4} \ln \frac{|E|}{(3ms^2\hbar\Omega)^{1/2}}. \tag{13}$$

For the lifetime in a magnetic field, using (8), we obtain correspondingly

$$\frac{1}{\tau_H} = \frac{1}{12} \left(\frac{\pi}{2}\right)^{1/2} \frac{(\hbar\Omega)^2 N}{kT (kTm)^{1/2} l_0} \left(\frac{e^2 Z}{\varkappa kT}\right)^3 \ln \frac{4kT}{(3ms^2\hbar\Omega)^{1/2}}, \qquad l_0 = \pi\hbar^7 \rho / 2m^3 E_c^2. \tag{14}$$

Here l_0 is the characteristic length and is connected with the mean free path l for scattering by acoustic phonons by the relation $l_0 = lkT/2ms^2$. It is easily seen that in a magnetic field the lifetime decreases in proportion to H^2, accurate to a logarithmic factor. Indeed,

$$\tau_H = \tau \left(\frac{8kT}{\hbar\Omega}\right)^2 \left[\ln \frac{4kT}{(3ms^2\hbar\Omega)^{1/2}}\right]^{-1}, \tag{15}$$

where the electron lifetime τ in the absence of a magnetic field is described by the formula[1]

$$\tau=\frac{3}{16}\left(\frac{2}{\pi}\right)^{1/2}\left(\frac{\varkappa kT}{e^2Z}\right)^2\frac{l_0}{Ns}\left(\frac{ms^2}{kT}\right)^{1/2}. \tag{16}$$

2. CALCULATION OF THE LIFETIME AT $kT \ll (\hbar\Omega ms^2)^{1/2}$

When the temperature is lowered or the magnetic field increased, so that the mean thermal energy of the electron becomes less than the energy of the characteristic phonon, the interaction of the electron with the acoustic phonons becomes essentially inelastic. To calculate the lifetime it is then convenient to use the Lax procedure.[9,1] The recombination flux per unit center j is determined by the expression

$$j=\int d^3\mathbf{r}\int d\varepsilon\int d\varepsilon' F(\varepsilon,r)\rho(\varepsilon)w^{(s)}(\varepsilon,\varepsilon')[N(\varepsilon-\varepsilon')+1]P(\varepsilon',u(r)). \tag{17}$$

Here ε and ε' are the kinetic energies of the electron before and after the capture, respectively, $\rho(\varepsilon)$ is the density of states of the free carriers in quantizing magnetic fields, and the equilibrium distribution function of the phonons is $N(\varepsilon-\varepsilon')=[\exp((\varepsilon-\varepsilon')/kT)-1]^{-1}$. The probability $w^{(s)}(\varepsilon,\varepsilon')$ is the probability of the transition, per unit time, with spontaneous emission of an acoustic phonon, while $P(\varepsilon,u(r))$ is the sticking probability introduced by Lax. The function $F(\varepsilon,r)$ is the distribution function of electrons with kinetic energy ε at a distance r from the attracting center. In the case of thermalized carriers, this is the Boltzmann distribution, while for the ultraquantum case it is defined by (7) and the total energy is $E=\varepsilon+u(r)-\hbar\Omega/2$ (the last term is due to the fact that the zeroth level of the total energy coincides by definition with the zeroth Landau level). In this section, just as above, the calculations are carried out for a Coulomb attracting center $(u(r)=-e^2Z/\varkappa r)$.

The probability $w^{(s)}(\varepsilon,\varepsilon')$ is connected with the probability of the transition between the states with given quantum numbers $i=(n,k_x,k_z)$ and $j=(n',k_x',k_z')$ upon spontaneous emission of an acoustic phonon $w_{ji}^{(s)}$ by the relation

$$w^{(s)}(\varepsilon,\varepsilon')=\frac{1}{\rho(\varepsilon)}\sum_{i,j}\delta(\varepsilon'-\varepsilon_j)\delta(\varepsilon-\varepsilon_i)w_{ji}^{(s)}. \tag{18}$$

In the ultraquantum limit $(n=n'=0)$ the probability $w_{ji}^{(s)}$ is defined by[7,8]

$$w_{ji}^{(s)}=\frac{2\pi}{\hbar}\sum_{\mathbf{q}}\frac{E_c^2\hbar^2q}{2\rho Vs}\delta_{k_x,k_x'+q_x}\delta_{k_z,k_z'+q_z}$$
$$\times|I_{00}|^2\delta\left(\frac{\hbar^2k_z^2}{2m}-\frac{\hbar^2k_z'^2}{2m}-\hbar qs\right), \tag{19}$$

where

$$|I_{00}|^2=\left|\int\chi_0(y)\chi_0(y+a_H^2(q_x^2+q_y^2)^{1/2})dy\right|^2. \tag{20}$$

Performing the calculations and introducing the transition probability $w_0(\varepsilon,\varepsilon')$ in the absence of a magnetic field and the state density $\rho_0(\varepsilon)$ in the absence of a magnetic field, we obtain for the product $\rho(\varepsilon)w^{(s)}(\varepsilon,\varepsilon')$ in the ultraquantum limit the expression

$$\rho(\varepsilon)w^{(s)}(\varepsilon,\varepsilon')=w_0(\varepsilon,\varepsilon')\rho_0(\varepsilon)\frac{\hbar^2\exp\{-q_\perp^2a_H^2/2\}}{2ma_H^2(\varepsilon-{}^1/_2\hbar\Omega)^{1/2}(\varepsilon'-{}^1/_2\hbar\Omega)^{1/2}}, \tag{21}$$

where

$$q_\perp^2=\left(\frac{\varepsilon-\varepsilon'}{\hbar s}\right)^2-\frac{2m}{\hbar^2}\left[\left(\varepsilon-\frac{1}{2}\hbar\Omega\right)^{1/2}-\left(\varepsilon'-\frac{1}{2}\hbar\Omega\right)^{1/2}\right],$$

and in accordance with Ref. 1

$$\rho_0(\varepsilon)w_0(\varepsilon,\varepsilon')=(8\pi^2\hbar^3ms^4l_0)^{-1}(\varepsilon-\varepsilon')^2. \tag{22}$$

From the condition that $q_\perp^2$ in (21) is positive it follows that $w^{(s)}(\varepsilon,\varepsilon')$ differs from zero under the condition

$$(\varepsilon-{}^1/_2\hbar\Omega)^{1/2}+(\varepsilon'-{}^1/_2\hbar\Omega)^{1/2}\geqslant(2ms^2)^{1/2}. \tag{23}$$

We introduce for convenience new symbols

$$\tilde{\varepsilon}=\varepsilon-{}^1/_2\hbar\Omega,\quad \tilde{\varepsilon}'=\varepsilon'-{}^1/_2\hbar\Omega. \tag{24}$$

In terms of the new variables, the inequality (23) becomes

$$(\tilde{\varepsilon})^{1/2}+(\tilde{\varepsilon}')^{1/2}\geqslant(2ms^2)^{1/2}. \tag{25}$$

In the calculation of the recombination flux j by means of (17), we take into account the fact that $w^{(s)}(\varepsilon,\varepsilon')$ is the probability of the transition of free electrons (with total energy $E>0$) into a bound state (with total energy $E'<0$). We therefore have for $\tilde{\varepsilon}=E+|u(r)|$ and $\tilde{\varepsilon}'=E'+|u(r)|$ the inequalities

$$\begin{aligned}&|u(r)|<\tilde{\varepsilon},\quad \tilde{\varepsilon}>0,\\ &|u(r)|>\tilde{\varepsilon}',\quad \tilde{\varepsilon}'>0.\end{aligned} \tag{26}$$

From (25) and (26) it follows that the region of integration in (17) with respect to the variable $\tilde{\varepsilon}$ is limited by the condition $|u(r)|<\tilde{\varepsilon}<\infty$, and the limits of integration with respect to $\tilde{\varepsilon}'$ depend on the relation between the energy $\tilde{\varepsilon}$ and $2ms^2$, namely

$$0<\tilde{\varepsilon}'<|u(r)|,\quad \tilde{\varepsilon}>2ms^2, \tag{27a}$$
$$[(2ms^2)^{1/2}-\tilde{\varepsilon}^{1/2}]^2<\tilde{\varepsilon}'<|u(r)|,\quad \tilde{\varepsilon}<2ms^2. \tag{27b}$$

We consider the temperature region bounded by the inequalities $ms^2\ll kT\ll(2ms^2\hbar\Omega)^{1/2}$. The main contribution to the recombination flux is made here by electrons with energy $\sim kT$. The quantities $\sim ms^2$ can be neglected (since $ms^2\ll kT$); this makes it possible to express the recombination flux j in the form

$$j=M\int_0^\infty r^2dr\int_{|u(r)|}^\infty d\tilde{\varepsilon}\int_0^{|u(r)|}d\tilde{\varepsilon}'\frac{(\tilde{\varepsilon}-\tilde{\varepsilon}')^2}{\tilde{\varepsilon}^{1/2}\tilde{\varepsilon}'^{1/2}}$$
$$\times\frac{\exp\{-(\tilde{\varepsilon}-|u|)/kT\}}{1-\exp\{-(\tilde{\varepsilon}-\tilde{\varepsilon}')/kT\}}P\left(\frac{|u|-\tilde{\varepsilon}'}{kT}\right), \tag{28}$$

where

$$M=\left(\frac{\pi}{2}\right)^{1/2}n[m^{1/2}(kT)^{1/2}s^4l_0]^{-1}. \tag{29}$$

We have used in (28) expression (21) for $\rho(\varepsilon)w^{(s)}(\varepsilon,\varepsilon')$, and left out $\exp(-q_\perp^2a_H^2/2)$ because the main contribution in the integration is made by the region in which $q_\perp^2a_H^2\ll1$. In addition, we have used the assumption that the sticking probability depends only on the ratio of the binding energy to kT, so that

$$P(\tilde{\varepsilon}',|u|)=P\left(\frac{|u|-\tilde{\varepsilon}'}{kT}\right).$$

This assumption is discussed in detail in Sec. 4 and in the Appendix.

Introducing the dimensionless variables

$$x=\frac{\bar{\varepsilon}-|u|}{kT},\quad x'=\frac{|u|-\bar{\varepsilon}'}{kT},\quad R=\frac{r}{r_T}=\frac{\varkappa kTr}{e^2Z}, \tag{30}$$

we obtain ultimately

$$j=M\left(\frac{e^2Z}{\varkappa}\right)^3\gamma, \tag{31}$$

where γ is defined by an integral that contains the sticking probability

$$\gamma=\int_0^\infty dx\int_0^\infty\frac{dx'P(x')}{(e^x-e^{-x'})}\int_0^{1/x'}\frac{R^2\,dR}{(x+R^{-1})^{1/3}(R^{-1}-x')^{1/3}}. \tag{32}$$

The internal integral with respect to R in (32) can be evaluated and has a singularity of the type $(x')^{-7/2}$ at small x'. It will be shown in the Appendix that $P(x')\approx x'^{7/2}$ at $x'\ll 1$. Thus, the integrand in (32) has no singularity and γ is a number of the order of unity. We obtain ultimately for the lifetime in the temperature region $ms^2\ll kT\ll(ms^2\hbar\Omega)^{1/2}$ the expression

$$\tau_H^{-1}=\gamma\left(\frac{\pi}{2}\right)^{1/3}\frac{s}{l_0}\frac{(e^2Z/\varkappa)^3N}{(kT)^{1/3}(ms^2)^{1/3}}. \tag{33}$$

It is seen from this formula that even before the magnetic field becomes strong enough to make $\hbar\Omega\gg(kT)^2/ms^2$, the lifetime ceases to depend directly on the magnetic field. However, τ_H is shorter than in the absence of a magnetic field at the same temperature by a factor $(ms^2/kT)^2$.

We consider now the case when $kT\ll ms^2$ and assume that in this case $ms^2\ll(\hbar\Omega ms^2)^{1/2}$. When setting the integration limits in the calculation of the recombination flux in accordance with (17), it is then necessary to take into account the inequalities (27), since ms^2 is now large compared with the mean thermal energy of the carriers. The region of integration with respect to r breaks up in natural fashion into two regions, in one of which $|u|>2ms^2$ and in the other $|u|<2ms^2$. In the first region we have automatically $\bar{\varepsilon}>2ms^2$, and the limits of integration with respect to $\bar{\varepsilon}'$ are determined by (27a). In the second region $\bar{\varepsilon}$ can be either larger or smaller than $2ms^2$. Accordingly, the region of integration with respect to $\bar{\varepsilon}'$ is determined by (27a) or (27b). We note that if $|u|<2ms^2$ and $\bar{\varepsilon}<2ms^2$, then it follows from (27b) that $|u|>ms^2/2$. Taking the foregoing into account, we represent the recombination flux (17) as a sum of three terms, $j=j_1+j_2+j_3$. Each of these fluxes is described by (28), but the limits of integration with respect to the variables r, $\bar{\varepsilon}$, and $\bar{\varepsilon}'$ are determined by the following inequalities. For j_1 we have

$$0<r<\frac{e^2Z}{2\varkappa ms^2},\quad |u(r)|<\bar{\varepsilon}<\infty,\quad 0<\bar{\varepsilon}'<|u(r)|. \tag{34}$$

For j_2 we have

$$\frac{e^2Z}{2\varkappa ms^2}<r<\frac{2e^2Z}{\varkappa ms^2},\quad |u(r)|<\bar{\varepsilon}<2ms^2,$$
$$[(2ms^2)^{1/2}-\bar{\varepsilon}^{1/2}]^2<\bar{\varepsilon}'<|u|. \tag{35}$$

For j_3 we have

$$\frac{e^2Z}{2\varkappa ms^2}<r<\infty,\quad 2ms^2<\bar{\varepsilon}<\infty,\quad 0<\bar{\varepsilon}'<|u(r)|. \tag{36}$$

The flux j_3 is smaller than j_1 or j_2 by a factor on the order of (kT/ms^2), so that it can be neglected. In the calculation of the fluxes j_1 and j_2 we can assume the sticking probability $P((|u|-\bar{\varepsilon}')/kT)$ to be equal to unity, and the quantity $\exp\{-(\bar{\varepsilon}-\bar{\varepsilon}')/kT\}$ can be neglected compared with unity, since the main contribution to these fluxes is made by the region where all the characteristic energies are of the order of ms^2.

Carrying out the calculations we obtain for the lifetime, in the limit of very low temperatures and sufficiently strong magnetic fields $kT\ll ms^2\ll(\hbar\Omega ms^2)^{1/2}$,

$$\frac{1}{\tau_H}=\frac{33}{35}\left(\frac{\pi}{2}\right)^{1/3}\frac{s}{l_0}\left(\frac{kT}{ms^2}\right)^{1/3}\left(\frac{e^2Z}{\varkappa ms^2}\right)^3N. \tag{37}$$

For comparison we present the expression for the lifetime τ in the limit $kT\ll ms^2$ in the absence of a magnetic field[1]

$$\frac{1}{\tau}=\frac{32}{45}\left(\frac{\pi}{2}\right)^{1/3}\frac{s}{l_0}\left(\frac{kT}{ms^2}\right)^{1/3}\left(\frac{e^2Z}{\varkappa kT}\right)\left(\frac{e^2Z}{\varkappa ms^2}\right)^2N. \tag{38}$$

Thus, at $kT\ll ms^2$ and in the limit of strong fields $\hbar\Omega\gg ms^2$ the lifetime does not depend on the magnetic field, but is ms^2/kT longer than the lifetime in the abscene of a magnetic field.

3. GENERALIZATION OF THE THEORY TO THE CASE OF AN ELLIPSOIDAL EQUAL-ENERGY SURFACE OF THE CONDUCTION ELECTRONS

The cascade model of the capture was most convincingly confirmed by experiments on Ge and Si.[1] These are multivalley semiconductors. The equal-energy surface for the conduction electrons in each valley is an ellipsoid of revolution. Ge has four valleys whose axes are equivalent to the direction [1, 1, 1]. In Si, the conduction band consists of six equivalent valleys, and the axes of the ellipsoids correspond to the direction [1, 0, 0]. It is therefore natural to generalize the theory developed in Secs. 1 and 2 in such a way as to make it applicable directly to the capture of electrons in Ge and Si.

We consider one valley–an ellipsoid. The motion of a free electron in such a valley, following application of a quantizing magnetic field, was considered in Ref. 10. Let the magnetic field H be directed along the z axis and let it make an angle ϑ with the revolution axis of the ellipsoid. The kinetic energy ε of the electron in the valley is then given by

$$\varepsilon=\left(n+\frac{1}{2}\right)\hbar\Omega+\frac{\hbar^2k_z^2}{2m_z},\quad m_z=m_\perp\sin^2\vartheta+m_\parallel\cos^2\vartheta,$$
$$\Omega=\frac{eH}{cm_\Omega},\quad \frac{1}{m_\Omega^2}=\frac{\cos^2\vartheta}{m_\perp^2}+\frac{\sin^2\vartheta}{m_\parallel m_\perp}, \tag{39}$$

where $m_\parallel$ and $m_\perp$ are the effective masses of the electrons moving respectively along the revolution axis of the ellipsoid and in a plane perpendicular to the axis. The wave function of the electron is determined in this case by the expressions (10) and (11), but the length a_H must now be taken to be the quantity

$$a_H=(\hbar/m_\perp\Omega)^{1/2}. \tag{40}$$

In addition, the center of the oscillatory function χ_0 de-

pends not only on k_z but also on k_x, so that now y_0 is defined by

$$y_0=-\frac{\hbar c}{eH}\left[k_x-k_z\frac{(m_\parallel-m_\perp)\sin\vartheta\cos\vartheta}{m_\parallel\cos^2\vartheta+m_\perp\sin^2\vartheta}\right]. \quad (41)$$

It is known that in multivalley semiconductors the conduction electrons interact with both longitudinal and transverse phonons. It has been shown[11,12] that in the absence of a magnetic field the square of the modulus of the matrix element of the operator of the interaction of the electron with the acoustic phonons has the standard form, the only difference being that the deformation-potential constant E_c contains a combination made up of two independent constants Ξ_u and Ξ_d, which were introduced by Herring.[11] For the interaction with the longitudinal acoustic phonons, E_c^2 is replaced by the expression

$$\{\Xi_d+\Xi_u\cos^2\theta\}^2, \quad (42)$$

while for the interaction with the transverse phonons it is replaced by

$$\Xi_u^2\sin^2\theta\cos^2\theta. \quad (43)$$

In (42) and (43), θ is the angle between the wave vector q and the axes of the ellipsoid of revolution.

For simplicity, we carry out the subsequent calculation in the approximation of the averaged deformation potential, and in addition we neglect the anisotropy of the sound velocities and the difference between the velocities of the longitudinal and transverse sound, and use for $\langle E_c^2\rangle$ an expression obtained by summing the angle-averaged relations (42) and (43)

$$\langle E_c^2\rangle=\Xi_d^2+{}^2/_3\Xi_d\Xi_u+{}^1/_5\Xi_u^2. \quad (44)$$

The use of the approximation of the averaged deformation potential is reasonable, since all the qualitative consequences that follow from the anisotropy of the electron spectrum are obtained already within the framework of this approximation. In addition, in Ge the mass anisotropy is substantially larger than the anisotropy of the deformation constants.

The lifetime of conduction electrons with ellipsoidal equal-energy surfaces in a quantizing magnetic field is calculated by the procedures developed in Secs. 1 and 2.

If $kT\gg(\hbar\Omega m_\perp s^2)^{1/2}$, i.e., the energy is lost by the carriers quasicontinuously, then we have for the lifetime

$$\frac{1}{\tau_H}=\frac{1}{6(2\pi)^{1/2}}\left(\frac{m_z}{kT}\right)^{1/2}\frac{(m_\perp+m_0)m_\perp\langle E_c^2\rangle kT(\hbar\Omega)^2N}{(kT)^5\rho\hbar^4(\varkappa/e^2Z)^2}\times\left\{\ln\frac{4kT}{(3m_\perp s^2\hbar\Omega)^{1/2}}+\frac{4kT}{\hbar\Omega}\Phi(\vartheta)\right\}, \quad (45)$$

$$\Phi(\vartheta)=\frac{\sin^2\vartheta\cos^2\vartheta(m_\parallel-m_\perp)^2+m_z^2}{m_\perp[(m_\parallel m_z)^{1/2}+m_z]}.$$

The obtained lifetime is connected with the lifetime τ in the absence of a magnetic field, calculated in the same approximation, by the relation

$$\frac{1}{\tau_H}=\frac{1}{\tau}\left(\frac{\hbar\omega}{8kT}\right)^2\left[1+\left(\frac{m_z}{m_\parallel}\right)^{1/2}\right]\left\{\ln\frac{4kT}{(3m_\perp s^2\hbar\Omega)^{1/2}}+\frac{4kT}{\hbar\Omega}\Phi(\vartheta)\right\} \quad (46)$$

which is analogous to (15) of Sec. 1.

As seen from (46), the lifetime in the magnetic field depends substantially on the orientation of the valley relative to the magnetic field, since m_z and the cyclotron mass m_Ω, which enters in the definition of Ω, depend on the angle ϑ (see (39)). We note that the second term in the curly brackets can be significant even though $\hbar\Omega\gg kT$, because at small ϑ we have $\Phi(\vartheta)\approx(m_\parallel/m_\perp)^2\gg1$. In this case the lifetime τ_H decreases in proportion to H^{-1} and not H^{-2}.

In extremely strong magnetic fields, when the interaction with the acoustic phonons becomes essentially inelastic ($kT\ll(m_\perp s^2\hbar\Omega)^{1/2}$, the lifetime ceases to depend on the magnetic field, just as in the isotropic case. However, it still remains dependent on the direction of the magnetic field.

If $m_z s^2\ll kT\ll(\hbar\Omega m_\perp s^2)^{1/2}$, then the electrons interact mainly with phonons whose momentum is perpendicular to the direction of the magnetic field ($q_z/q\approx(m_z s^2/kT)^{1/2}\ll1$). In this case it is easy to carry out an exact averaging of the expression for the reciprocal lifetime, since $\cos\theta\approx\sin\vartheta\sin\varphi$, where φ is the azimuthal angle in the plane normal to the magnetic field. We obtain for the lifetime

$$\frac{1}{\tau_H}=\left(\frac{2}{\pi}\right)^{1/2}\gamma\left(\frac{m_z}{kT}\right)^{1/2}\frac{m_\perp}{m_0}\frac{\Xi^2}{\rho\hbar^4s^4}\left(\frac{e^2Z}{\varkappa}\right)^2N, \quad (47)$$

where

$$\Xi^2=\Xi_d^2\left\{1+\frac{\Xi_u^2}{\Xi_d^2}\left[\frac{1}{2}\sin^2\vartheta\left(1+\left(\frac{s}{s_t}\right)^4\right)-\frac{3}{8}\left(\frac{s}{s_t}\right)^4\sin^4\vartheta\right]\right\} \quad (48)$$

Here s and s_t are the velocities of the longitudinal and transverse sound.

Finally, if the temperatures are so low that $kT\ll m_z s^2$ but the magnetic field is strong enough to make $(m_\perp s^2\hbar\Omega)^{1/2}\gg m_z s^2$, then in the approximation of the averaged deformation potential we obtain for the lifetime the expression

$$\frac{1}{\tau_H}=\frac{33}{35}\left(\frac{2}{\pi}\right)^{1/2}\frac{m_\perp(kT)^{1/2}\langle E_c^2\rangle}{m_z^{1/2}m_0\hbar^4\rho s^6}\left(\frac{e^2Z}{\varkappa}\right)^2N. \quad (49)$$

4. DISCUSSION OF RESULTS

The general character of the behavior of the lifetime with changing magnetic field is shown schematically in the figure, where the ordinates represent the capture coefficient $(N\tau)^{-1}$ (N is the concentration of the capture centers) and the abscissas represent the magnetic field H. The capture coefficient begins to increase at values of the magnetic field at which quantization sets in, and saturates when $(\hbar\Omega ms^2)^{1/2}$ is already larger than kT. Curves 1 and 2 correspond to the temperatures T_1 and T_2, with $T_1>T_2>ms^2$. The drop in the values of the capture coefficient increases with increasing temperature and according to estimates, it amounts to two orders of magnitude in Ge at $T\approx4$K. With decreasing temperature, in the region $kT<ms^2$, the dependence of the capture coefficient is reversed, as demonstrated by curve 3.

It should be noted that in a multivalley semiconductor, since the recombination rate depends strongly on the

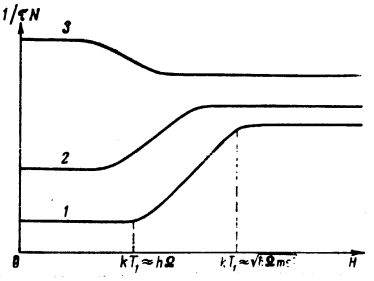

FIG. 1. General character of the behavior of the capture coefficient in a magnetic field (explanations in the text).

orientation of the valley axes relative to the direction of the magnetic field (see (46), (47), and (49)), the valleys can become unevenly populated, for example in the case of photoexcitation. Consequently the photoconductivity in Ge, in quantizing magnetic fields, should be anisotropic even when a weak electric field is superimposed. We know of no measurements of the lifetime in strong magnetic fields. It appears that the performance of such experiments entails no difficulties in principle. These measurements could serve as a good check on the cascade theory of capture and, in our opinion, would lead to interesting new effects.

It should be noted that in the present paper, just as in Ref. 1, we essentially used the fact that in the region of negative total energy the distribution function and the sticking probability depend only on the total energy E. For this approximation to be valid, it is necessary that the spatial mixing take place more rapidly than the energy diffusion. In the absence of a magnetic field, this condition is satisfied.[13] In quantizing magnetic fields, however, the spatial mixing across the magnetic field is hindered.

Indeed, during the time of energy diffusion over an interval of the order of kT, the bound electron negotiates in space, in a direction perpendicular to the magnetic field, a distance $\Delta = (D\tau_\varepsilon)^{1/2}$, where the diffusion coefficient in the transverse direction is $D = a_H^2/\tau_p$, while τ_ε and τ_p are the energy and momentum relaxation times of an electron with kinetic energy kT. For the length Δ we have

$$\Delta = a_H((kT)^2/\hbar\Omega ms^2)^{1/4}. \tag{50}$$

In order for spatial mixing to be possible, it is necessary to satisfy the inequality $\Delta \gg r_t$, and the characteristic capture radius is $r_T = e^2Z/\varkappa kT$.

On the other hand, our approach to the description of the capture in a quantizing magnetic field is valid if $a_H \ll r_T$. Estimates show that this inequality is usually well satisfied in quantizing magnetic fields. In this case, however, there is no sufficiently large region in which the inequality $r_T \ll \Delta$ could be simultaneously satisfied.

Thus, in quantizing magnetic fields the distribution function and the sticking probability depends, generally speaking, on the coordinates x and y. An exact calculation with such functions is much more difficult than our calculation, in which we used quantities averaged over the coordinates x and y. On the other hand, the result of the exact calculation can differ from ours only by a number of the order of unity, and not parametrically.

The absence of spatial mixing may affect the character of the behavior of the capture coefficient in a magnetic field only in the region of the start of quantization, and can lead here to a certain decrease of the capture coefficient.

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APPENDIX

The sticking probability $P(|E|)$ of an electron in a bound state with total energy $E<0$, averaged over the permissible values of the spatial coordinates, is defined by the integral equation

$$P(E) = \int_{-\infty}^{0} W(E,E')P(E')dE' \Big/ \int_{-\infty}^{+\infty} W(E,E')dE', \tag{A.1}$$

where $W(E, E')$ is the total probability of the transition from a state with energy E into a state with energy E'. It is connected with the probability $W^{(s)}(E, E')$ of the transition in spontaneous emission of a phonon and with the phonon numbers N_q and N_q+1 in the usual manner. The probability $W^{(s)}(E, E')$ is obtained, in turn, by averaging over the microcanonical distribution of the transition probability between states with fixed kinetic energy $w^{(s)}(\varepsilon, \varepsilon')$ (formula (18)).

$$W^{(s)}(E,E') = (\rho(E))^{-1}\int w^{(s)}(\varepsilon,\varepsilon')\delta(E-\varepsilon-u(r)) \times\delta(E'-\varepsilon'-u(r))\rho(\varepsilon)d\varepsilon\,\rho(\varepsilon')d\varepsilon'\,d^2\mathbf{r}. \tag{A.2}$$

Of course, the absence of spatial mixing in a plane perpendicular to the magnetic field. To obtain the final form of the equation for the averaged sticking function we need to know the probability $W^{(s)}(E, E')$ at negative values of both energies E and E', as well as under the condition that $E<0$ but $E'>0$. Integrating (A.2) in the case $E<0$, $E'<0$, we have

$$\rho(E)W^{(s)}(E,E') = \frac{2\pi\hbar^2\beta kT}{ma_H^2}\left(\frac{e^2Z}{\varkappa kT}\right)^3 (x-x')^2 \times\exp\left(-\frac{(x-x')^2}{\Gamma^2}\right)\int\limits_{\max(x,x')}^{\infty}\frac{dt}{t^4(t-x)^{1/2}(t-x')^{1/2}}. \tag{A.3}$$

In the case $E<0$, $E'>0$ we have

$$\rho(E)W^{(s)}(E,E') = \frac{2\pi\hbar^2\beta kT}{ma_H^2}\left(\frac{e^2Z}{\varkappa kT}\right)^3 (x+x')^2 \times\exp\left(-\frac{(x+x')^2}{\Gamma^2}\right)\int\limits_{x}^{\infty}\frac{dt}{t^4(t+x')^{1/2}(t-x)^{1/2}}. \tag{A.4}$$

In (A.3) and (A.4)

$$x = \frac{|E|}{kT},\quad x' = \frac{|E'|}{kT},\quad \Gamma = \frac{(2ms^2\hbar\Omega)^{1/2}}{kT},\quad \beta = (8\pi^2\hbar^3ms^4l_0)^{-1}, \tag{A.5}$$

with $\Gamma \gg 1$. The final expression for the sticking function, in the dimensionless variables x, is

$$P(x) = \Big[\int_0^x P(x')\Big[\exp\Big\{\frac{(x-x')^2}{x-x'}\Big\}-1\Big]G\Big(\frac{x'}{x}\Big)dx' + \int_x^\infty P(x')\frac{(x-x')^2}{1-\exp[-(x'-x)]}\Big(\frac{x}{x'}\Big)^4 G\Big(\frac{x}{x'}\Big)dx'\Big] \times\Big[\int_{-\infty}^{x}\frac{(x-x')^2}{\exp[(x-x')]-1}G\Big(\frac{x'}{x}\Big)dx' + \int_x^\infty\frac{(x-x')^2}{1-\exp[-(x'-x)]}\Big(\frac{x}{x'}\Big)^4 G\Big(\frac{x}{x'}\Big)dx'\Big]^{-1}. \tag{A.6}$$

We have left out here the factors $\exp\{-(x-x')/\Gamma^2\}$ because $\Gamma \gg 1$, and the significant x and x' are of the order of unity. The function $G(u)$ is defined by

$$G(u)=\int_1^\infty \frac{dx}{x^{1/4}(x-1)^{1/4}(x-u)^{1/4}}, \quad u<1. \tag{A.7}$$

It is seen from (A.6) that the sticking probability $P(x)$ is in fact a function of the dimensionless argument x, i.e., according to (A.5) it depends only on the ratio of the binding energy to the temperature. The solution of the homogeneous integral equation (A.6) at large $x(x \gg 1)$ tends to $P(x) = \text{const}$. We are interested in a solution for which the constant is equal to unity, i.e., $P(x) \to 1$ at $x \gg 1$.

Let us investigate the behavior of $P(x)$ at $x \ll 1$. In this case the principal term in the denominator of (A.6) is I_1, and in this case

$$I_1=x^{1/4}\int_1^\infty \frac{dy}{y^{1/4}(y-1)^{1/4}}\int_0^\infty \frac{z^{1/4}}{e^z-1}\,dz=\frac{15\pi^{1/4}\zeta(5/2)}{64}x^{1/4}, \tag{A.8}$$

where $\zeta(5/2) \approx 1.34$ is the Riemann zeta function. The principal role in the numerator is played by the second term I_2, and in the limit $x \ll 1$ we have

$$I_2=x^{1/4}\int_0^\infty P(x')\frac{dx'}{x'^{1/2}(1-e^{-x'})}\int_1^\infty \frac{dy'}{y'^{1/2}(y-1)^{1/4}}=x^{1/4}\frac{96}{105}\int_0^\infty \frac{P(x')\,dx'}{x'^{1/2}(1-e^{-x'})}. \tag{A.9}$$

We ultimately get at $x \ll 1$

$$P(x)=Cx^{1/4}, \tag{A.10}$$

where the constant C is determined by the integral of the sticking function over the entire range of variation of the argument

$$C=\frac{2{,}91}{\pi^{1/4}}\int_0^\infty \frac{P(x')\,dx'}{x'^{1/2}(1-e^{-x'})} \tag{A.11}$$

An approximate calculation yields $C \approx 1$.

[1)]It is shown in Ref. 1 that the recombination process can be regarded as capture by individual isolated centers under the condition that the thermal energy kT of the electron exceeds the Coulomb interaction energy $e^2\varkappa^{-1}N^{-1/3}$ at the average distance between the capture centers, i.e., the inequality $kT \gg e^2\varkappa^{-1}N^{-1/3}$ is satisfied. Satisfaction of this condition means simultaneously that an equilibrium distribution in the electron system manages to be established during the lifetime. Otherwise, when the carriers are excited by light, the electrons have essentially a nonequilibrium distribution, and the lifetime is simply the time of energy relaxation to the percolation level. This question is discussed in detail in Refs. 1 and 5.

[2)]The authors are grateful to the referee for pointing out the paper by Belyaev and Budker,[6] who considered for the first time, using the classical kinetic equation, the transition of a free electron to the ground state of an atom in an ionized plasma via multiple collisions with other electrons and ions, as well as via multiple emission of small quanta.

[3)]We disregard in our calculation the spin state of the electron. For a simple band the change of the electron spin can be neglected in electron-phonon collisions, so that the presence of the spin does not come into play in the recombination and does not lead to a change of the lifetime.

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