# Absolute negative conductivity of a relaxing weakly ionized gas

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(Submitted 6 May 1977; resubmitted 27 January 1978)
Zh. Eksp. Teor. Fiz. 75, 1315–1319 (October 1978)

The transport equation is used to calculate the time dependence of the conductivity of a weakly ionized gas relaxing after switching off the electric field used to heat the charged component of the system. This system is assumed to be spatially homogeneous. Conditions are found under which the conductivity becomes negative and the system can emit electromagnetic waves during some part of the relaxation process. A detailed calculation of the conductivity is carried out for argon as a function of time, frequency and initial conditions.

PACS numbers: 51.50. + v

We shall consider the electronic conductivity of a spatially homogeneous weakly ionized gas in the range of particle energies and concentrations such that the dominant process causing thermalization of an initially heated electron gas is the elastic or almost elastic[1] scattering of electrons by neutral molecules. In an earlier paper,[2] the present author found the condition for the appearance of a local inversion of the distribution function of the relaxing electrons, associated mainly with the behavior of the second derivative of the electron scattering cross section $\sigma$ with respect to the energy $\varepsilon$. We shall show here that, in the presence of sufficiently clear regions of rise or fall of $\sigma(\varepsilon)$, there may be a relative accumulation of intermediate-energy electrons so that some time from the beginning of the gas relaxation the electron distribution function becomes locally inverted. On the other hand, the dephasing of electrons by collisions gives rise to a dependence of the intensity of the interaction of electron gas with an electromagnetic wave field on the function $\sigma(\varepsilon)$. The rising and falling values of $\sigma(\varepsilon)$ can, under certain conditions, make the inversion region of the distribution function so important and the interaction with the electromagnetic field so suppressed in the normal distribution region (at high electron energies) that, at some suitable moment, the electron gas may transfer energy to the electromagnetic field: the conductivity of the gas becomes negative and remains so for a certain time interval which can naturally be reduced by the developing collective processes. If the distribution function of the relaxing electrons does not exhibit considerable anisotropy in the velocity space, the Langmuir oscillations cannot grow[3] and the emission of an electromagnetic wave pulse is the principal process.

We shall use models to study the characteristic features of $\sigma(\varepsilon)$ necessary for the occurrence of a negative conductivity and we shall establish that these features are exhibited by the cross sections for the scattering of electrons by the rare gas atoms, particularly argon atoms. We shall calculate the time dependence of the change in the conductivity of weakly ionized argon at various frequencies, which can then be used to find the characteristics of the emitted pulses.

We shall use the solution of the transport equation $f(v,t)$ obtained earlier[2] for a relaxaing electron gas whose effective temperature is much higher than the temperature of the gas molecules $T$. This allows us to write down the real part of the conductivity at a frequency $\omega$

$$K(\omega,t)=-\frac{ne^2}{3m}\int_0^\infty \frac{v^3\tau}{1+\omega^2\tau^2}\frac{\partial f(v,t)}{\partial v}dv\left(\int_0^\infty v^2 f dv\right)^{-1} \tag{1}$$

in the form

$$K(\omega,t)=\frac{ne^2}{3m}\int_0^\infty \frac{\partial}{u^2\partial u}\left[\frac{u^4\lambda(u)}{u^2+\omega^2\lambda^2(u)}\right]v^2 f(v)dv\left[\int_0^\infty v^2 f(v)dv\right]^{-1} \tag{2}$$

The following notation is used for the various properties of electrons: $v$ is the velocity; $e$ is the charge; $m$ is the mass; $n$ is the density; $\tau(v)=(N\sigma v)^{-1}$ is the mean free time ($N$ is the concentration of the molecules); $\lambda(v)$ is the range. The velocity $u=u(v,t)$ is a functional governed by the equation

$$t=2\int_u^v \frac{\tau(v')dv'}{v'\chi(v')}, \tag{3}$$

where the time $t$ is measured from the beginning of the relaxation at $t=0$, which corresponds to the distribution function $f(v,0)\equiv f(v)$ of an electron gas preheated in some way (for example, by an electric field $E$ switched off at the moment $t=0$). In this case, we have

$$f(v)=\exp\left\{-\int_0^v [kT+2e^2E^2\tau^2/3m\chi(v)]^{-1}mvdv\right\}. \tag{4}$$

The quantity $\chi(v)$ which occurs in Eqs. (3) and (4) is the average value of the fraction of the electron energy lost in one collision. Its dependence on the velocity may give rise to various effects, including the inversion of the distribution function.[2] Throughout our treatment, we shall assume that $\chi=\text{const}$.

For a normal population of the energy states, we have $\partial f/\partial v<0$ and, consequently, it follows from Eq. (1) that $K>0$. The occurrence of regions with a positive derivative $\partial f/\partial v$ along the energy axis is a necessary but insufficient condition for negative conductivity. The inversion of the distribution can be described as follows. The rate of thermalization, i.e., the rate at which a group of electrons with a given energy travels along the energy axis to its origin, is governed by the collision frequency and increases on increase in the cross section $\sigma(\varepsilon)$. According to Eq. (4), the initial distribution of electrons in a certain range of $\varepsilon$ is approximately uniform and the electrons have a tendency

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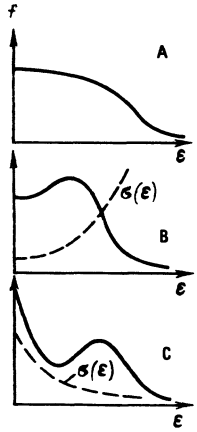


FIG. 1. Evolution of the distribution function under relaxation conditions: A corresponds to $t=0$, whereas B and C correspond to $t>0$.

to accumulate in the vicinity of small values of $\sigma(\varepsilon)$, rapidly leaving those parts of the energy axis where $\sigma(\varepsilon)$ is large. Figure 1 shows schematically how the distribution A changes after some time to a distribution B or C, depending on the nature of the behavior of the cross section $\sigma(\varepsilon)$, shown by a dashed curve.

When the energy $\varepsilon = mv^2/2$ is high, it is clear that $\partial f/\partial v < 0$ and the influence of this region and of the vicinity of the point $v=0$ on the integral (1) may be reduced by the factor $v^3\tau(v)/[1+\omega^2\tau^2(v)]$, which describes dephasing, as a result of collisions with the gas molecules, of the electrons interacting with an electromagnetic field. At low frequencies, close to $\omega\approx 0$, a negative conductivity is favored by a reduction in $\tau$ on increase in energy, i.e., by a rising $\sigma(\varepsilon)$ curve (Fig. 1b); on the other hand, at high frequencies, a fall in $\tau$ is the favorable influence (Fig. 1C).

An analysis of Eq. (2) by means of Eq. (3) and (4), carried out for three simple models of the cross section

$$\sigma_{\rm I}=\begin{cases}\sigma_1, & 0\leqslant v\leqslant v_1;\\ \sigma_2, & v_1<v\end{cases}\quad \sigma_{\rm II}=\begin{cases}\sigma_1, & 0\leqslant v\leqslant v_1\\ \sigma_2, & v_1<v\leqslant v_2:\\ \infty, & v_2<v\end{cases}\quad \sigma_{\rm III}=\left(1+\frac{\varepsilon}{\varepsilon_0}\right)^p\sigma_0,$$

shows that a negative conductivity can be realized under conditions which are not stringent:

I) $\sigma_1/\sigma_2<0{,}36$ $(\omega=0)$;
II) $\sigma_1/\sigma_2<0{,}41$ $(\omega=0)$, $\sigma_1/\sigma_2>1{,}77$ $(\omega\neq 0)$;
III) $p>1$ $(\omega=0)$, $p<-2$ $(\omega\neq 0)$.

Model II with $\sigma_1/\sigma_2>1$ describes approximately the Ramsauer scattering.

We shall concentrate our attention on the conductivity of a weakly ionized rare gas in which only elastic collissions occur up to electron energies of about 20 eV and which is characterized by $\chi = 2m/M$ ($M$ is the mass of a molecule). We shall carry out a calculation for argon and the results should be similar for xenon and krypton. The scattering of electrons by helium atoms is described by an exceptionally smooth function $\sigma(\varepsilon)$ so that, in this case, a negative conductivity is unlikely. We shall take the function $\sigma(\varepsilon)$ from the experimental data given by Mott and Massey.[4]

The results of calculations carried out on the basis of Eqs. (2)–(4) are plotted in Figs. 2 and 3. The various curves in Fig. 2 correspond to a set of different intensities of the electric field accelerating the electrons.

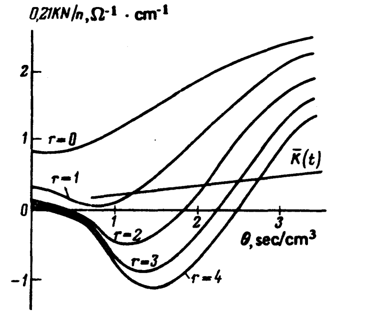


FIG. 2. Conductivity $K$ of weakly ionized argon plotted as a function of time $t$ from the end of application of an electric field $E=4.83\times 10^{-19}\times 3^r N$ V/cm ($\omega=0$); the abscissa gives the quantity $\theta = 1.6\times 10^{-11}\, Nt$ proportional to $t$; $r<5$ is a parameter representing the field intensity.

The negative conductivity regions are naturally more pronounced when the pump field is stronger. Since the recombination processes are ignored and the temperature of the neutral component of the gas is assumed to be zero, the dc conductivity rises in an unrestricted manner in the limit $t\to\infty$. On the basis of Eq. (2), it is found that, after a sufficiently long time $t$, the value of $u$ is small and almost independent of $v$ in those regions of integration of Eq. (2) which make a considerable contribution to the value of $K$. It means that, in the limit $t\to\infty$, when $2\lambda(0)/\chi t \ll 1$, we have $u\approx 2\lambda(0)/\chi t$ and obtain the asymptotes

$$\bar K(\omega,t) \underset{t\to\infty}{=} \frac{ne^2}{3m}\cdot\begin{cases}\lambda'(0)+\chi t & \text{for } \chi t\omega\ll 1\\ 8/\omega^2\chi t & \text{for } \chi t\omega\gg 1\end{cases},$$

which are approached by the curves in Fig. 2. To allow for recombination, one has to multiply all these curves by a positive monotonically falling function describing the loss of electrons from the system. We shall not do this because the rate of recombination at electron energies of ~1 eV is a second-order quantity with respect to the parameter $n/N$, assumed to be very small. Consequently, the time scale adopted in Fig. 2 is insufficient to reveal the reduction in conductivity due to recombination.

The curve for $r=4$ in Fig. 2 shows that, in the time interval corresponding to $1<\theta<1.5$, weakly ionized argon relaxing after switching off a pump field of about $4\times 10^{-17} N$ V/cm intensity is capable of emitting electromagnetic waves. The spectrum of these waves can be obtained if their amplitude is small enough by considering $K(\omega,t)$ as a function of frequency $\omega$. The graphs in Fig. 3, each of which corresponds to a specific moment, show that, immediately after the beginning of electromagnetic wave emission, the frequency

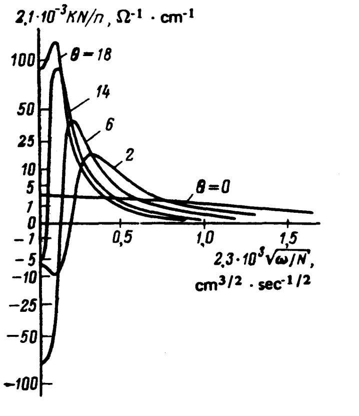


FIG. 3. Changes in the frequency dependence of the conductivity of weakly ionized argon during its relaxation; $r=4$, $\theta=1.6\times 10^{-11}\, Nt$.

spectrum occupies the range $0 \leq \omega \leq 10^{-8}N$ sec$^{-1}$. As the electron component cools, the intensity of the emitted radiation gradually rises and then falls; however, the width of the spectrum decreases monotonically, approaching $\omega = 0$. The application of a homogeneous magnetic field may shift the spectrum completely so that $\omega = 0$ coincides with the cyclotron frequency of the electrons and then all the other results (with the exception of some numerical factors) remain constant on condition that $\tau\omega \gg 1$.

We have ignored the exchange of energy between the electrons, which prevents inversion of the distribution function. In the case of argon at electron energies not too low compared with 1 eV, the criterion of validity of this approximation is $n/N < 10^{-6}$. In designing experiments, we have to bear in mind that an increase in the electron density $n$ first increases the negative conductivity, in accordance with Eq. (1), but, when $n$ is high, Eq. (2) is no longer valid and the effect disappears. The initial establishment of a distribution function $f(v)$ and the application of a probe field for the measurement of the conductivity should ensure the retention of a sufficient degree of spherical symmetry of the distribution function and, consequently, of the stability against plasma oscillations.

The author is grateful to É. I. Rashba for an extremely useful critique and encouraging interest in this investigation and is also very grateful to G. I. Gudzenko and V. I. Shaldyrvan for their help in the calculations.

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Translated by A. Tybulewicz

# Thermal conductivity of pure vanadium in normal, superconducting, and mixed states

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(Submitted 13 January 1978)
Zh. Eksp. Teor. Fiz. **75**, 1320–1329 (October 1978)

The thermal conductivity of very pure vanadium ($\rho_{273}/\rho_{4.2} = 1570$) was investigated in the normal, superconducting, and mixed states. A satisfactory agreement was obtained betwen the experimental and theoretical values of the thermal conductivity in the superconducting state and the half-width of the energy gap $\Delta_0 = 9.5$°K was determined. The results obtained demonstrated that vanadium is a superconductor with a weak electron–phonon coupling. An investigation of the thermal conductivity in the mixed state yielded the critical magnetic fields $H_{c1}$ and $H_{c2}$. A comparison was made of the theory and experiment and the upper limit of the effective electron-scattering width of Abrikosov filaments ($\sigma \lesssim 0.6 \times 10^{-6}$ cm was determined. A study of the electrical resistivity in a magnetic field at various temperatures T made it possible to deduce the temperature dependences of the critical magnetic fields $H_{c2}$ and $H_{c3}$.

PACS numbers: 74.30.Ek, 74.60.Ec, 74.70.Lp, 72.15.Eb

The physical essence of the superconducting state of a metal can be demonstrated by comparing the electrical and thermal conductivities above and below the superconducting transition temperature $T_c$. The characteristic relationship between these conductivities embodied in the Wiedemann–Franz relationship for the normal state of a metal changes greatly on transition to the superconducting state. The electrical conductivity rapidly tends to infinity on lowering of the temperature $T$ below $T_c$ but the thermal conductivity of a pure metal does not change abruptly but gradually decreases compared with that in the normal state, and in the limit $T \to 0$ it approaches a value typical of an insulator (which forms from the original metal by the gradual exclusion of electrons from the thermal balance of the crystal). This simple experimental picture of the comparative behavior of the electrical and thermal conductivities yields important conclusions on the physical nature of the superconducting state. Therefore, much experimental information has now been accumulated not only about the electrical conductivity but also about the thermal conductivity of superconducting pure metals and alloys.

The thermal conductivity of type II superconductors is of special interest because in this case it is possible to study not only the influence of the superconducting transition on the thermal conductivity but also on the

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