Neutrino mechanism of propagation of thermonuclear burning in degenerate cores of stars

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An approximate model is constructed, with neglect of motion of matter, of the propagation of the thermonuclear burning front in degenerate carbon cores of stars. It is shown that the propagation of carbon burning ignited by neutrinos reaches an infinite velocity in a finite time (the sharpening time). The sharpening time, t_f, is compared with the characteristic hydrodynamic time t_H of the star for central density $\rho_c = 3\cdot 10^{10}$ g/cm^3. The equality $t_f = t_H$ is regarded as a sufficient condition for the occurrence of the supersonic regime of thermonuclear carbon burning. If the central density satisfies $\rho_c < 3\cdot 10^{10}$ g/cm^3, then $t_f > t_H$, supersonic burning does not occur, and the process takes place in the subsonic regime.

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The investigation of thermonuclear burning in degenerate carbon cores of stars has recently become the most promising direction of development in the theory of supernova explosions and the formation of neutron stars. In the study of this problem, the most important question relates to the nature of the propagation of thermonuclear burning and, in particular, to which of the different combustion regimes—deflagration or detonation—is realized.

In the hydrodynamic theory of burning with a plane stationary front, two main regimes are distinguished.[1,2] On the one hand, there is the subsonic regime (deflagration), in which the pressure and the density behind the burning front are lower than their initial values. The ignition mechanism in this regime is heat conduction or some other process of energy transfer which raises the temperature until the onset of the intense reaction. On the other hand, there is the supersonic burning regime (detonation), when the pressure and the density increase behind the front. In the special case of Chapman-Jouguet detonation, new layers of matter are ignited by a shock wave, behind the front of which the reaction zone is situated (the shock wave and the reaction zone together form the detonation front). In supersonic burning, a more rapid ignition than in Chapman-Jouguet detonation is realized by the heating of matter in front of the burning front, rather than by a shock wave. Because of the supersonic propagation velocity, the density and pressure increase in the reaction, though to a lesser degree.[3] As can be seen from the calculations of Ref. 4, the thermonuclear burning of carbon in the degenerate core of a star begins in the deflagration regime.

The propagation of the burning leads to the separation of the matter of the star into two regions: the central heated part (core) and the as yet unheated shell. For $3\cdot 10^9 \leqslant \rho \leqslant 3\cdot 10^{10}$ g/cm^3, the Fermi energy of the electrons is below the threshold of carbon neutronization, though it exceeds the threshold for the neutronization of the products of carbon burning (elements of the iron peak). Therefore, the core becomes a source of neutrino radiation. The neutrino flux is scattered on the degenerate electrons of the shell and gives up some of its energy to them. The low specific heat of degenerate electrons, the ready supply of thermonuclear fuel, and the increase in the neutrino scattering cross section in a relativistic degenerate Fermi gas enhance the effect of the heating of the matter in the shell to temperatures at which the thermonuclear reactions proceed rapidly.[5] This process becomes an effective mechanism of neutrino firing of thermonuclear burning in degenerate cores of stars.[1)]

Strictly speaking, a plane stationary burning front is not formed since the neutrino heating extends to all the outer layers of the star up to the surface. Moreover, as numerical calculations show, deflagration near the surface of the star goes over in a number of cases into detonation, and this leads to a very important effect—the ejection of a shell.[6]

In general, the problem of the occurrence of detonation is very complicated and does not have a smiple unique solution.[3] However, in the present case of thermonuclear burning of a degenerate carbon core of a star, one can comparatively easily obtain a criterion for the development of the supersonic burning regime, and one can also take into account the nonstationarity of the process. This criterion is useful when one is interpreting the results of numerical calculations that take into account the complete complex of hydrodynamic, nuclear, neutrino, and thermodynamic processes. The present paper is devoted to a qualitative investigation of the nature of propagation of thermonuclear burning and the conditions under which detonation occurs.

Our model consists of a core of radius R, which is a source of neutrinos because of the neutronization of the elements of the iron peak, and a carbon shell with temperature profile $T = T_{ig}R^2/r^2$, $r \geqslant R$. The neutrino ignition is realized under conditions of almost complete transparency of the matter for neutrinos, and it is therefore necessary to take into account the neutrino radiation of the complete volume of the core. Absorption of only a small fraction (~0.1–1%) of the neutrino energy flux is sufficient[5,6] to heat the matter of the shell to the temperatures T_{ig} (the subscript *ig* stands for ignition) corresponding to burning of carbon in a

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time appreciably shorter than the characteristic hydrodynamic time. We shall not consider the kinetics of the carbon burning. Once the temperature in the layer of the shell next to the core has reached the critical value T_{ig}, we shall simply assume that this layer becomes part of the core. Thus, the boundary R of the core moves through the matter, which is at rest. We shall assume that the detonation regime is realized if the derivative dR/dt becomes infinite (a sharpening regime is established[2)]) in a characteristic time t_f that is shorter than the corresponding hydrodynamic time. Formally, R is then also infinite, but for t near t_f the velocity dR/dt is already very large (greater than the velocity of sound) at R less than the radius of the star. We have good reason to assume that a detonation regime is established since under these conditions the matter cannot expand appreciably and reduce the temperature below T_{ig}. In determining the change in the radius R of the core, we ignore hydrodynamic effects.

In the framework of the above model, we now obtain a solution of the problem of the velocity of the burning front R of the matter. Our point of departure is the equation for the rate of transfer of energy to a layer of the shell of arbitrary radius $r (r \geqslant R)$:

$$\frac{\partial E(r,t)}{\partial t}=n_e(r)\int_0^\infty d\varepsilon_\nu\left\langle\sigma\frac{\Delta\varepsilon_e}{\varepsilon_\nu}\right\rangle\frac{L_\nu(\varepsilon_\nu,t)}{4\pi r^2}. \tag{1}$$

where L_ν is the neutrino luminosity of the core, and $\langle\sigma\Delta\varepsilon_e/\varepsilon_\nu\rangle$ is the effective transfer of energy by the neutrinos to the electrons. In degenerate matter $\Delta E = c_V\Delta T$, where c_V is the specific heat of the carbon nuclei. Assuming that the density distribution satisfies the polytropic law under the condition $r^2/2R_*^2 \ll 1$(R_* is the outer radius of the shell), and that the Fermi energy of the electrons appreciably exceeds the energy threshold for neutronization of the nuclei, we obtain the simple equation for the temperature

$$\frac{\partial T}{\partial t}=A\rho_c^{11/3}\frac{R^2}{r^2}, \tag{2}$$

where ρ_c is the central density and $A = 1.5\cdot 10^{-35}$ cm^{10} $\cdot$g$^{-11/3}\cdot$deg$\cdot$sec^{-1}.

Using the expressions in the Appendix, we obtain from (2) an equation for the velocity of propagation of the burning front:

$$\frac{dR}{dt}=\frac{A\rho_c^{11/3}R^2}{2T_{ig}} \tag{3}$$

The solution of Eq. (3) has the form (see the Appendix)

$$R=R_0[1-t/t_f]^{-1}. \tag{4}$$

It can be seen from the expressions (3) and (4) that $dR/dt\to\infty$ in the characteristic sharpening time t_f, which is equal to

$$t_f=\frac{2T_{ig}}{A\rho_c^{11/3}R_0}=\frac{2T_{ig8}\cdot 10^3}{1.5\rho_{c9}^{11/3}R_{07}}\ [\mathrm{sec}], \tag{5}$$

where

$$T_{ig8}=T_{ig}/(10^8\,\mathrm{K}),\quad \rho_{c9}=\rho_c/(10^9\,\mathrm{g/cm^3}),\quad R_{07}=R_0/(10^7\,\mathrm{cm}).$$

In Eqs. (4) and (5), the value of R_0 cannot exceed R_* and may be less than some minimal value $R_{0\,\min}$ ($R_{0\,\min}<R_0<R_*$). This value of $R_{0\,\min}$ corresponds to the minimal radius of the core for which radiation of the complete energy released by the thermonuclear burning in the form of neutrinos is still capable of raising the temperature in the adjoining layer of the shell to the value T_{ig}: $R_{0\,\min 7}\approx 1/\rho_{10}^{5/3}$. By their meaning, $R_{0\,\min}<R_*$, and we therefore conclude that ignition is only possible at central densities $\rho_c>2.5\cdot 10^9$ g/sec^3.

Detonation occurs if t_f is shorter than the characteristic hydrodynamic time $t_H\approx 10^3\,\mathrm{sec}/\sqrt{\rho}$, i.e., the expansion of matter cannot quench the propagation of burning. Then for $T_{ig\,8}=5$ and $R_{0\,7}=5$ we obtain from the equation

$$t_f=t_H \tag{6}$$

the value $\rho_{cr}=3\cdot 10^{10}$ g/cm^3 of the critical density. This value is equal to the maximal possible initial density of the carbon cores of stars (the Fermi energy of the electrons in this case becomes equal to the threshold, 12 MeV, for the capture of electrons by carbon nuclei).

The condition (6) can be somewhat weakened since the transition to detonation may still be realized if near the outer surface of the shell a supersonic velocity of burning propagation is attained. From Eqs. (3) and (4) with velocity of sound near the outer surface of the shell $\sim R_*/t_H$, we obtain instead of (6) a weaker condition for transition to the supersonic regime:

$$t_f\frac{R_0}{R_*}=\frac{2T_{ig}}{A\rho_c^{11/3}R_0}\frac{R_0}{R_*}\leqslant t_H \tag{7}$$

or

$$t_{f\,\min}=\frac{2T_{ig}}{A\rho_c^{11/3}R_{0\,\max}}=\frac{2T_{ig}}{A\rho_c^{11/3}R_*}\leqslant t_H.$$

Hence, $\rho_c\geqslant 2.5\cdot 10^{10}$ g/cm^3.

These estimates show that detonation cannot occur because of ignition near the center at central densities $\rho_c\leqslant 3\cdot 10^{10}$ g/cm^3. From this point of view, the postulate that the detonation regime is realized from the very beginning of the development of the carbon flash[8,9] appears physically quite unjustified. Therefore, the calculations of,[6,10] in which the deflagration burning regime was adopted in the initial phase, appear to give the most reasonable solution of the problem of thermonuclear burning in degenerate cores of stars.

The occurrence of detonation in self-consistent hydrodynamic calculations[6] is due to the preceding considerable contraction of the central part of the star. The possibility of such a contraction is also indicated by the estimate (A.8) obtained in the Appendix for the time required for the establishment of a temperature profile with $T=T_{ig}$ for $r=R$. The results of the numerical calculations of Gershteĭn *et al.*[6] agree qualitatively with the criteria (6) and (7) for the transition to detonation when the central density has such an increased value as a result of the preceding contraction.

The occurrence of detonation does not yet mean that Chapman-Jouguet detonation is realized. Indeed, under conditions of neutrino ignition it is natural to expect a rapid transition to the regime of supersonic burning, which is more rapid than Chapman-Jouguet burning and takes place without a shock wave;[3)] for self-

Table I.

t, sec	T_1	T_2	p_1	p_2	ρ_1	ρ_2	$\dot{r}_{bf}\cdot 10^{-9}$, cm/sec	$R\cdot 10^{-8}$, cm	$u\cdot 10^{-9}$, cm/sec	j_b	$D\cdot 10^{-9}$ cm/sec
	10^9 K		10^{26} bar		10^9 g/cm^3						
4.97103	0.93	8.3	6.52	10.03	1.24	1.43	0.90	0,43	0,22	85	1,12
4.97549	0.98	8,3	3.93	9.56	0.85	1.37	0.96	0.47	0,25	92	1.21
4.97819	0.85	8.1	1,92	6,29	0.50	0.99	0.90	0,50	0.26	96	1.16
4.98006	0.87	8,2	1.41	5,95	0.39	0.94	0.91	0,52	0.27	98	1.18

consistent thermodynamic calculations with finite rate of the nuclear reaction show that, on the one hand, the burning propagation velocity in the detonation regime appreciably exceeds the Chapman-Jouguet velocity D_{CJ}, and, on the other, there are no indications that a shock wave is formed in front of the burning zone. In Table I, we give the values of the pressure (p), the temperature (T), and the density (ρ) in the last unignited zone (subscript 1) and in the zone behind the burning front, in which the maximum of the density is attained (subscript 2). In Table I we also give the velocities of propagation of the burning front ($\dot{r}_{bf}$), the front radii (R), and the velocity of the oncoming flow (u), so that the velocity of the burning front relative to the unheated matter is $D = u + \dot{r}_{bf}$. The times (t) and the numbers (j_b) of the burnt zones are also given. The data given in the table correspond to a calculation made for the initial values $\rho_c = 1.4\cdot 10^{10}$ g/cm^3 and $T_c = 0.3\cdot 10^9$ °K, the result of which was collapse of the carbon core.[6] The Chapman-Jouguet velocity D_{CJ} can be estimated as $D_{CJ} = [2(\gamma^2 - 1)q]^{1/2}$, where γ is the adiabatic exponent and q is the caloricity of the fuel. Taking $q = 3\cdot 10^{17}$ erg/g, which corresponds to the results of the calculation in [6], [4)] and $\gamma = 4/3$, we obtain $D_{CJ} = 0.65\cdot 10^9$ cm/sec. The velocity D in Table I is close to $1.2\cdot 10^9$ cm/sec, so that this velocity would be equal to the Chapman-Jouguet velocity only in the case of complete energy release $q = 9\cdot 10^{17}$ erg/g or for $\gamma = 5/3$. It can also be seen from Table I that the inequalities $p_1 < p_2$ and $\rho_1 < \rho_2$ for the detonation regime are satisfied, and that the temperature $T_1 \sim 10^9$ °K already ensures ignition of carbon without the assistance of a shock wave.

Note that in the variants of the calculations made in Refs. 4, 6, and 10 for lower initial central densities, the detonation velocity D is much closer to the value D_{CJ} and there are other indications of Chapman-Jouguet detonation. This is explained by the fact that the principal part in the burning process is here played by the increase in the temperature resulting from the increase in the pressure rather than from neutrino heating, which merely provides a preliminary heating of the matter. The detonation occurs during the strong pulsation of the star during the stage of secondary contraction at densities appreciably lower than the initial density, and therefore at a lower degree of degeneracy of the matter. Therefore, in this situation all the usual conditions for the occurrence of Chapman-Jouguet detonation are satisfied.

Such conditions were established for thermonuclear carbon burning in Ref. 11, in which an energy criterion for self-sustaining Chapman-Jouguet detonation was derived. It was shown that such detonation is realized for $\rho \lesssim 5\cdot 10^7$ g/cm^3, in good agreement with our calculations. Nevertheless, the possibility of detonation without shock wave discussed above was not considered in Ref. 11.

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APPENDIX

We find the solution of the equation

$$\frac{\partial T(r,t)}{\partial t} = \frac{a(r)}{r^2} b(R) \tag{A.1}$$

for the initial temperature profile $T(r,0) \equiv T(r, t=0)$:

$$T(r,0) = \begin{cases} T_0, & r<R \\ T_{ig}\dfrac{a(r)}{r^2}\dfrac{R^2}{a(R)}, & r\geqslant R \end{cases}, \tag{A.2}$$

where T_0 is the temperature of the heated matter, T_{ig} is the ignition temperature, and $T_0 \gg T_{ig}$. Substituting the profile (A.2) in Eq. (A.1), we obtain the equation for the propagation velocity of the burning front:

$$\frac{dR}{dt} = \frac{a(R)\,b(R)}{T_{ig}R[2 - d\ln a/d\ln R]}. \tag{A.3}$$

For $a(R) = a_0R^{-m}$, $m > 0$ and $b(R) = b_0R^n$, $n > 0$, Eq. (A.3) has the form

$$\frac{dR}{dt} = \frac{a_0b_0R^{n-m-1}}{(2+m)T_{ig}}. \tag{A.4}$$

The solution of Eq. (A.4) for $m + 2 \neq n$ is

$$R = R_0\left[1 + \frac{(m-n+2)a_0b_0t}{(2+m)T_{ig}R_0^{m+2-n}}\right]^{1/(m+2-n)}. \tag{A.5}$$

The solution (A.5) corresponds to a sharpening regime for $m + 2 - n < 0$, and the characteristic sharpening time is given by

$$t_f = \frac{(2+m)T_{ig}}{(n-m-2)a_0b_0R_0^{n-m-2}}. \tag{A.6}$$

Note that if the initial temperature profile has the form $T = T_0$ for $r \leqslant R_0$ and $T = T_1$ for $r > R_0$, $T_1 \ll T_{ig}$, then because of Eq. (A.1) the profile (A.2) is established in the time

$$t_* = \frac{(T_{ig}-T_1)R_0^2}{a(R_0)b(R_0)} \approx \frac{T_{ig}R_0^2}{a(R_0)b(R_0)}. \tag{A.7}$$

For $m = 0$, $n = 3$ (see Eq. (2))

$$t_* = t_f/2. \tag{A.8}$$

At real initial central densities of degenerate carbon cores $\rho_c < 3\cdot 10^{10}$ g/cm^3, the relation $t_* > t_H$ holds, and therefore hydrodynamic effects change the initial configuration and the criterion obtained in the present paper should be used for this new modified configuration.

1) It can be called for brevity "neutrino ignition" or ignitation.

2) A theory of sharpening regimes for burning problems is constructed in[7].

3) This fact was noted by Ya. B. Zel'dovich.

4) With allowance for the distribution of the elements produced in accordance with nuclear statistical equilibrium, the energy release is approximately halved compared with the

transition $C^{12} \to Ni^{56}$ ($q = 9.10^{17}$ erg/g). Note that the subsequent neutronization is a secondary endothermal process and cannot lead to a reduction in the detonation velocity.[2]

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Qualitative isotropic cosmology with cosmological constant and with allowance for dissipation

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New types of evolution that arise in isotropic cosmological Friedmann models with cosmological constant when allowance is made for bulk viscosity are decribed. A family of solutions is obtained for a closed model without singularities of the metric and the energy density. The stability of static solutions in the presence of viscosity is investigated. The coefficient of viscosity is assumed to be a function of the energy density that has power-law asymptotic behaviors at small and large values of the argument.

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In the investigation of isotropic cosmological models with allowance for the λ term, attention has been mainly concentrated on the stability of static solutions and the description of the evolution of various types of solution. On the other hand, the equations that describe the evolution of models constitute a dynamical system and it is of interest to investigate the behavior of the integral curves on the phase plane. This makes it possible to give a perspicuous classification of the possible types of evolution.

For the case $\lambda = 0$, this was done in[1], in which one can find the necessary details that are omitted here.

The behavior of the solutions in the region of large values of the Hubble "constant" $H = \dot{R}/R$ or energy density are the same as in[1]. The most interesting effects occur near the static de Sitter and Einstein solutions.

We write the Friedmann metrics in the form

$$-ds^2 = -dt^2 + \frac{R^2(t)\,(dx^2+dy^2+dz^2)}{[1+{}^1/_4 k(x^2+y^2+z^2)]^2}, \tag{1}$$

where $k = +1$, -1 and 0 correspond to closed, open, and flat models, respectively. In an isotropic cosmological evolution, the shear (first) viscosity is not manifested and one need only consider the bulk viscosity, whose coefficient is ζ.

The energy-momentum tensor has the form

$$T_k^i = (\varepsilon + p')u^i u_k + p'\delta_k^i, \quad p' = p - \zeta u^k_{;k}. \tag{2}$$

For simplicity, we take the equation of state in the form $p = (\gamma - 1)\varepsilon$. A comoving frame can be chosen.

We introduce $H = (\ln R)^{\cdot} = \dot{R}/R$, the Hubble "constant". Then the Einstein equations $R_k^i + \lambda\delta_k^i = T_k^i - \frac{1}{2}\delta_k^i T$ and the hydrodynamic equations $T^i_{k;i} = 0$ reduce to the three equations

$$\dot{\varepsilon} = 3H(3\zeta H - w), \tag{3}$$

$$\dot{H} = {}^1/_3(\lambda+\varepsilon-3H^2) + {}^1/_2(3\zeta H - w), \tag{4}$$

$$\lambda + \varepsilon - 3H^2 = 3kR^{-2}, \tag{5}$$

In the variables (H, ε), the system of equations (3)–(5) does not depend on k. The parabola $\lambda + \varepsilon - 3H^2 = 0$ separates the open and closed models: The integral curves of the closed model lie within it; those of the open model, outside it.

As in[1], we assume that $\zeta(\varepsilon) = \alpha\varepsilon^{a_2}$, as $\varepsilon \to 0$ and $\zeta(\varepsilon) = \beta\varepsilon^{b_2}$, as $\varepsilon \to \infty$ ($a_2 \geqslant 1$, $b_2 < \frac{1}{2}$), since unphysical effects do not occur for such exponents.

The singular points of the system (3)–(5) lie on the parabola $\lambda + \varepsilon = 3H^2$ and on the straight line $H = 0$.

We consider first the case $\zeta = 0$. The system (3)–(5) can be integrated, and the equation of the integral phase curves is

$$\lambda + \varepsilon - 3H^2 = 3k(\varepsilon/\varepsilon_0)^{2/3\gamma}, \tag{6}$$