curve is also smeared by the electron phonon interaction. An estimate shows that allowance for this interaction worsens (48) by at most a factor of two.

The broadening of the absorption of the absorption lines on account of the interaction of the electron with neutral and ionized impurities can be estimated by using the appropriate expressions obtained in Refs. 12 and 13 for the relaxation times. In our case this broadening is small compared with the phonon broadening up to an impurity concentration on the order of 10^{16} cm^{-3}.

Third, the intensity of the absorption on bound state is proportional to the concentration of the electrons captured by neutral impurities. To facilitate the observation of atomic CR by such impurities it is expedient to use weakky compensated semiconductors, for which there exists a temperature region in which the donors have not yet been ionized, but the electrons are already uniformly distributed over the impurity band[3,14]; this increases the number of electrons captured by the neutral impurities.

In conclusion, I am grateful to Yu. A. Gurvich and I. B. Levinson for useful discussions in the course of the work and for a detailed evaluation of the result, and to M. A. Kozhushner for an evaluation of the results.

[1)] It is assumed that there are no bound states in the potential $U(\vec{r})$ in the absence of a magnetic field.

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Dispersion law and structural transitions in crystalline films

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An investigation has made it possible to explain the existence of an experimentally observed minimum of the resistance in size-quantizing films. Methods are proposed that permit investigations of the Fermi-line geometry. The question of quantization of the acoustic spectrum is examined. It is shown that the speed of sound oscillates with changing film thickness.

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We investigate in this paper the question of the dispersion law in size-quantizing crystal films. It is known that size quantization can be observed in thin crystal film. This effect is highly sensitive to the film quality. The best conditions for its observation are realized in semimetal films, where a number of factors (such as the low electron density) cause the de Broglie wavelength to exceed greatly the atomic dimensions, and it is this which makes the surface in fact specular. This is why the size-quantization effect was in fact discovered first in semi-metallic Bi and Sb films. The observation of this effect in thin metallic films (of Sn, Al, Pb, Mg, Au, and Ag) is possible, although more complicated.

We confine ourselves hereafter to semimetallic films in which the size quantization effect is most pronounced. In size-quantizing films, the energy $\varepsilon(\varkappa,n)$ is determined by the longitudinal two-dimensional quasimomentum and by the transverse quantum number n. Instead of the Fermi surface we have a group of two-dimensional subbands. Strictly speaking, the momentum projection k_z perpendicular to the plane of the film is not defined. It can be approximately assumed, however, that the different subbands have different values of $k_z = \pi n/L$ (see, e.g., Ref. 1).

The most interesting situation occurs in the case when the concentration satisfies the condition $n \lesssim L^{-3}$. In this case, only the lowest subband is filled. The film, which remains a three-dimensional system in coordinate space ($L \gg a$, where a is the lattice period), becomes a two-dimensional system in momentum space. This situation is realized, for example, in Bi films at $L \lesssim 5 \times 10^2$ Å. The electrons are characterized in this case not by a Fermi surface, but by the Fermi line

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$\varepsilon(\varkappa)=\varepsilon_F$.

Komnik *et al.*[2] observed in an experimental study of thin quantizing Bi films a low-temperature resistance minimum. In the present paper (see Sec. 1) we consider the cause of this minimum.

A perfectly realistic situation (see below) is one in which the Fermi line has linear sections. This leads to a logarithmic instability that corresponds to a restructuring of the spectrum, and to the possibility of a structural transition. The experimentally observed[2] minimum of the resistance of a thin Bi film is apparently due to such a transition.

In the investigation of the singularities of the electron dispersion law in thin films one encounters two related questions, although each of them is also of independent interest. The first is the study of the phenomena due to the presence of linear sections on the Fermi line. The second is connected with the possibility of a detailed analysis of the Fermi line. It is possible to formulate the question rigorously in a manner analogous to the known procedure in electron theory of metals. We have in mind the reconstruction of the Fermi line from experimental data.

This paper consists of two sections and an appendix. In Sec. 1 we investigate the instability of the phonon spectrum and the structural transition. In Sec. 2 we consider the question of reconstructing the dispersion law in quantizing films. The Appendix discusses the quantization of phonons; it is shown that electron-phonon interaction leads to an oscillatory dependence of the speed of sound on the film thickness.

1. INSTABILITY OF PHONON SPECTRUM. STRUCTURAL TRANSITION

Phonon spectrum. We write down the equations that determine the phonon dispersion law in a thin quantizing film. The fact that the film is bounded makes it possible also to quantize the phonon spectrum. We consider this general case. The phonon dispersion law is a function of $\omega_\nu(\mathbf{q})$ ($\mathbf{q}$ is the two-dimensional longitudinal phonon momentum and ν is the transverse quantum number). Thus, there are many phonon branches corresponding to different values of the transverse number ν.

We shall consider the case when the Fermi line is characterized by the presence of a linear section. That this situation is realistic follows from an examination of the Fermi surface of bismuth, which comprises an aggregate of ellipsoid with a very strong effective-mass anisotropy. The films are evaporated in such a way that the z axis, which is perpendicular to the plane of the film, is directed along the axis corresponding to $m_z \approx 0.01 m_0$. The Fermi line can be regarded approximately as the intersection of the ellipsoid with a plane, i.e., an ellipse, and the values of the Fermi momenta in the principal directions are $p_1 \approx 7.5\times 10^{-22}$ g-cm/sec and $p_2 \approx 9\times 10^{-21}$ g-cm/sec, i.e., they differ by an order of magnitude. The curvature radius

$$R=(p_2/p_1)^2[1-(p_x/p_2)^2]^{3/2}p_1$$

takes on values in the interval $(0.8-1.3)\times 10^{-19}$ g-cm/sec when p_x changes from zero to $p_2/2$, i.e., the curvature radius is larger than the dimensions of the Fermi line itself and even the dimensions of the Brillouin zone ($\sim 0.5\times 10^{-9}$ g-cm/sec for bismuth).[3] Thus, these sections can be regarded as straight lines with high degree of accuracy.

We proceed now to investigate the phonon spectrum. We write down (in symbolic form) the Dyson equations that define the phonon Green's function with electron-phonon interaction taken into account (see, e.g., Ref. 4):

$$D^{-1}=D_0^{-1}-g\Pi g^e,\qquad g^e=\varepsilon^{-1}g,\quad \varepsilon=1+V\Pi. \tag{1}$$

We change to the momentum representation. To this end we write

$$G(x,x')=\sum_\nu\int\frac{d^2\varkappa\,d\varepsilon}{(2\pi)^3}G_\nu(\varkappa,\varepsilon)\exp[i\varkappa(\rho-\rho')-i\varepsilon(t-t')]\varphi_\nu(z)\varphi_\nu^*(z').$$

Here $\varkappa$ is the longitudinal two-dimensional momentum and ρ is the longitudinal coordinate; we have obtained for the single-particle functions a representation in which the Green's function is diagonal (see, e.g., Ref. 5). We do not need the concrete form of the functions $\varphi_\nu(z)$. Analogous expressions can be written for the functions D and D_0. The Green's function of the free phonon is equal to $D_{0\nu}(\mathbf{k},\omega)=\omega_{0\nu}/(\omega^2-\omega_{0\nu}^2)$.

We change over next to the momentum representation in (1). This produces in the right-hand side the sum $\sum_{\mu\mu'}\Gamma^\nu_{\mu\mu'}\Pi_{\mu\mu'}(\varkappa,\omega)$, which contains the quantities $\Pi_{\mu\mu'}(\varkappa,\omega)$ that depend (at definite values of the transverse quantum numbers μ and μ') on the two-dimensional momentum. In fact, we write (1) in the form

$$\sum_\nu\int d\varkappa'\,d\omega D_\nu^{-1}(\varkappa',\omega)\exp[i\varkappa'(\rho-\rho')]\exp[-i\omega(t-t')]\xi_\nu(z)\xi_\nu^*(z')$$
$$=\sum_\mu\int dp_1\,d\omega\,D_{0\mu}^{-1}(p_1,\omega)\exp[ip_1(\rho-\rho')]\exp[-i\omega(t-t')]\xi_{0\mu}(z)\xi_{0\mu}^*(z')+P(x,x'),$$

where

$$P(x,x')=g\Pi(x,x')g^e(x,x'),$$

and the functions $\xi_\nu(z)$ and $\xi_{0\mu}(z)$ realize the diagonal representations of D and D_0, respectively. Multiplying both sides by $\exp[-i\varkappa(\rho-\rho')]\xi_s(z)\xi_s^*(z')$ and integrating with respect to z, z', and $\rho-\rho'$, we get

$$D_s^{-1}(\varkappa,\omega)=\sum_\mu A_{\mu s}D_{0\mu}^{-1}(\varkappa,\omega)+P_{ss}(\varkappa,\omega), \tag{2}$$

where

$$P_{ss}(\varkappa,\omega)=\int P(x,x')\exp[-i\varkappa(\rho-\rho')]\xi_s(z)\xi_s^*(z')\,d\rho\,d\rho'\,dz\,dz'.$$

Recognizing that $\Pi(x,x')=2iG(x,x')G(x',x)$ we arrive, after simple transformation, at the expression

$$P_{ss}(\varkappa,\omega)=\sum_{\mu\mu'}\Gamma^s_{\mu\mu'}\Pi_{\mu\mu'}(\varkappa,\omega),$$
$$\Gamma^s_{\mu\mu'}=g\int dz\,\varphi_\mu(z)\varphi_{\mu'}^*(z)\xi_s(z)$$
$$\times\int d\rho'\,dz'\,d\rho_1\,dz_1\exp[i\varkappa(\rho'-\rho_1)]\varphi_\mu(z_1)\varphi_{\mu'}^*(z_1)g^e(x_1,x')\xi_s(z').$$

Thus, the right-hand sides of (1) and (2) actually contain the sum $\sum_{\mu\mu'}\Gamma^{\nu}_{\mu\mu'}\Pi_{\mu\mu'}(\varkappa,\omega)$. We investigate next, as already noted, the most interesting case, when one lowest subband is filled (the generalization to the filling of several subbands is not difficult). We consider in this connection the quantity

$$\Pi_1(\varkappa,\omega)=2i\int\frac{d^3p}{(2\pi)^3}G_1\left(p+\frac{q}{2}\right)G_1\left(p-\frac{q}{2}\right), \tag{3}$$

where $q=(\varkappa,\omega)$, $\mathbf{k}$, and $\varkappa$ are two-dimensional vectors. We consider furthermore the case of interest to us, when the Fermi line has a linear section. We assume in this connection that the dispersion law takes the form

$$\varepsilon_p=\begin{cases}p_y^2/2m_2, & -q_0<p_x<q_0,\\ (p_x-q_0)^2/2m_1'+p_y^2/2m_2, & p_x>q_0,\\ (p_x+q_0)^2/2m_1'+p_y^2/2m_2, & p_x<-q_0,\end{cases} \tag{4}$$

$m_1'=(p_{x0}-q_0)^2/2\varepsilon_F$ (p_{x0} is the Fermi momentum in a direction parallel to the linear section), so that a linear section exists. The ellipse $\varepsilon_p=p_x^2/2m_1+p_y^2/2m_2$, which is characterized by a strong mass anisotropy (see below), can be approximated to high accuracy by Eq. (4). Calculation of the integral (3) leads in this case to the expression

$$\Pi_1(\varkappa,\omega)=\frac{2m_2q_0}{\pi^2\varkappa}\ln\frac{8\varepsilon_F}{i\omega}. \tag{5}$$

Thus, the presence of the linear section leads to a logarithmic singularity in the polarization operator. This case is analogous to the situation of Ref. 6, where the Fermi surface is characterized in the three-dimensional case by the presence of a planar section (see the review[7]). In our case, when one lowest subband is filled, the principal role in the formula that determines $D^{-1}(\varkappa,\omega)$ is played by the considered quantity $\Pi_1(\varkappa,\omega)$. For the thicknesses $L\lesssim 5\times10^2$ Å under consideration, we have $\Pi_{\mu\mu'}\sim\Delta^{-1}_{\mu\mu'}$ ($\Delta_{\mu\mu'}$ is the difference between the transverse levels; for Bi, e.g., $\Delta_{\mu\mu'}$ ~0.01 eV), and therefore the quantities $\Pi_{\mu\mu'}$ do not contain any logarithmic singularities.

The presence of a logarithmic singularity in Π leads, as usual (see, e.g., Ref. 7), to the appearance of an imaginary pole in the D function and to lattice instability.

This instability can be most clearly investigated with the aid of the usual model, when the transverse-quantization subbands correspond to different values of the transverse momentum. The dispersion equation $D^{-1}=0$ is then easily seen to be of the form

$$\omega^2(\mathbf{k},k_z)=\omega_0^2-\omega_0g^2(\mathbf{k},k_z)\Pi(\mathbf{k},k_z)/\varepsilon(\mathbf{k},k_z). \tag{6}$$

It is seen directly from (6) that at $k\approx2p_{y0}$ the frequency corresponding to the lower acoustic branch (at $k_z=\pi/L$; see the Appendix) vanishes. In fact, the expression for the frequency of this branch contains the diagonal polarization operator, which has a logarithmic singularity. The instability is therefore connected with the lowest acoustic phonon mode. The temperature of the corresponding structural transition, T_p (given by expression (7) below) is characterized by the vanishing of the phonon frequency $\omega_\nu(\mathbf{q})$ and by the appearance of a static-deformation wave.

Interaction of the electron with the static-deformation wave leads to the appearance of a gap in the electron spectrum, corresponding to the linear sections of the Fermi surface. In fact, the instability considered above is described by introducing anomalous phonon mean values $\langle\Phi_q\rangle$ with momentum[7] $\mathbf{q}$ and correspondingly anomalous mean values $\langle a^+_{\varkappa+q}a_\varkappa\rangle$. This, naturally, produces a gap $\Delta=g\langle\Phi_q\rangle$.

The appearance of the gap can be described with a canonical transformation. The interaction of the static-deformation wave with electrons belonging to the linear sections can not be described by ordinary perturbation theory. Introducing therefore the wave function $\Psi_p=u_p\psi_p+v_p\Phi_p$ (ψ_p is an unperturbed wave function and $\Phi_p=\sum_k\alpha_{p-k}\psi_k$), and writing down the Schrödinger wave function $(H_0+V)\Psi_p=\varepsilon_p\Psi_p$, we obtain after simple transformations the spectrum $\varepsilon_p=(\xi_p+|\Delta_p|^2)^{1/2}$ (ξ_p is the energy of the ordinary electron, and $\mathbf{p}$ runs through the values pertaining to the linear sections).

Structural transition. To obtain an expression for the temperature T_p, we write down (3) with the aid of the temperature technique. Putting next $\omega_n=0$, we obtain the transition temperature

$$T_p\approx\varepsilon_Fe^{-1/\lambda}. \tag{7}$$

A similar formula is obtained also from the requirement that the parameter Δ vanish.

At temperature s lower than T_p the film is characterized by the appearance of a gap corresponding to the linear sections of the Fermi line. For example, for Bi films the gap is produced on the greater part (~0.9) of the Fermi line. Such a transition is accompanied by a sharp decrease of conductivity. A minimum of the resistance should be observed here.

Komnik *et al.*[2] observed a resistance minimum in an investigation of the conductivity of thin Bi films. This minimum may be due precisely to the transition described above. It is noted in Ref. 2 that the minimum of the resistance is observed at relatively low temperatures. Thus, Bi films with $L\approx220$ Å are characterized by $T_{min}\approx5$ K. The question of the structural transition due to the phonon instability has by now been investigated for many systems (see, e.g., Ref. 7). This transition usually takes place at temperatures higher than the values of T_{min} of the investigated quantizing film. The reason why T_{min} is small compared with the usual case is above all the small value of ε_F ($\varepsilon_F\sim0.01$ eV in the films considered). The coupling constant in the exponential can be written in the usual form $\lambda\approx\nu_0v$, where $\nu_0\sim(p_0L)^{-1}$ is the state density; the matrix element V does not contain the additional small quantity $\sim a/L$ (this is easiest to see by expressing the electron wave functions in terms of[1] $\cos k_zz$). The Fermi momentum is $p_0\approx p_1\approx7.5\times10^5$ cm^{-1} at $L\approx10^{-6}$, so that $p_0L\approx1$. Equation (7) leads, taking the given values of T_p and ε_F into account, to $\lambda\approx0.2$, which is the usual value of the electron-phonon interaction constant (see, e.g., Ref. 8). Thus, it is the anomalously low value of ε_F in the size-quantizing films which leads to the low temperature T_p.

2. RECONSTRUCTION OF THE DISPERSION LAW IN THIN FILMS

It was postulated above that a situation wherein the Fermi line is characterized by the presence of a linear section is realistic. A more general and rigorous formulation of the problem of finding the dispersion law ε_p in a size-quantizing film is also possible. The Fermi line can be regarded approximately is the intersection of a three-dimensional Fermi surface with the plane $p_z = 0$. However, because of the specifics of the film state, brought about by the sputtering conditions (as well as because, strictly speaking, the quantity p_z is not defined for the considered bounded system) the Fermi line can differ from the usual section through a three-dimensional Fermi surface. This raises the question of finding a method for determining the electron dispersion law in a size-quantizing film. We regard as the most consistent, for this problem, a method analogous to the known procedure[9] developed in electron theory of ordinary metals. We have in mind here the reconstruction of the Fermi line by using corresponding experimental data.

The quantization of the transverse motion and the associated two-dimensional character of the problem makes it impossible for ordinary methods, say galvanomagnetic measurements, to yield the required information. We shall consider here some possible methods for the investigation of the geometry of the Fermi line, although a number of other methods also exist.

1. An effective way of deducing the Fermi line can be an investigation of sound absorption in thin films in a magnetic field. The magnetic field is perpendicular to the plane of the film. In this case the sputtered film and the substrate must have close elastic constants, so that the phonons in the film can be regarded as the same as in the bulk sample. At a definite ratio of the scales of the trajectory in coordinate spectrum (this trajectory, in analogy with the usual situation, is obtained by rotating the Fermi line through $\pi/2$ and changing it by a factor c/eH times) to the scale of the wavelength, a Pippard resonance sets in. The resonance condition is $c(\Delta p_1^F)_{extr}/eH = (n+1/2)\lambda_{ac}$ so that the absorption is an oscillating function of H^{-1}.

A more rigorous analysis, similar to that of Ref. 10, also leads to this result. The solution of the kinetic equation

$$\frac{df}{dt}+\frac{f-f_0}{\tau}=0,$$
$$\frac{d}{dt}=\frac{\partial}{\partial t}+\mathbf{v}\frac{\partial}{\partial \mathbf{r}}+\frac{d\mathbf{p}}{dt}\frac{\partial}{\partial \mathbf{p}}+\Omega\frac{\partial}{\partial\varphi}, \tag{8}$$

(the conditions $\Omega\tau \gg 1$ and $\Omega \gg \omega_{ac}$, where Ω is the Larmor frequency, are satisfied), followed by calculation of the absorption coefficient

$$\Gamma = T\left\langle \frac{dS}{dt}\right\rangle (2\langle \varepsilon_s\rangle)^{-1}$$

where S is the entropy of the electron gas,

$$\langle \varepsilon_s\rangle = {}^1/_2\langle(\partial \mathbf{u}/\partial t)^2\rangle = \rho_f\ \omega_{ac}^2 u_0^2, \quad \mathbf{u}=\mathbf{u}_0\exp[i(\mathbf{q}_{ac}\mathbf{r}-\omega_{ac}t)]$$

lead to the expression

$$\Gamma = \Gamma_0 + \Gamma_1\cos\Delta, \tag{9}$$
$$\Delta = \frac{c}{e\lambda_{ac}H}D_\perp, \tag{10}$$

$D_\perp$ is the projection of the extremal diameter on the direction perpendicular to the sound-wave propagation direction. Thus $\Gamma = \Gamma_0 + \Gamma_1\cos(cD_\perp H^{-1}/e\lambda_{ac})$ and the absorption coefficient contains an oscillating part. Using (8), we can directly write down an expression for the amplitude Γ_1. It is seen that the absorption coefficient oscillates as a function of H^{-1}, with a period $\delta = e\lambda_{ac}/c(2p_\perp)_{extr}$ ($2p_\perp$ is the extremal diameter of the Fermi line in the sound-wave propagation direction).

2. A highly effective method in the investigation of the electron spectrum in thin film can be the study of the absorption of an electromagnetic field.[1)] It is important that the film thickness is less than the depth of the skin layer. It is therefore possible to study directly the absorption process (the film can be placed in a waveguide). In addition, the film is situated in an external constant magnetic field directed along the z axis (the z axis is chosen to be perpendicular to the film plane). The electric vector and the propagation direction lie in the xy plane. The situation is similar to that considered in the preceding section (the sound damping is also connected, as is well known, with the appearance of deformation fields) and it is possible to observe the corresponding geometric resonance. Writing down the kinetic equation in analogy with (8) and calculating the absorption coefficient, we find that $\Gamma = \Gamma_0 + \Gamma_1\cos\Delta$, $\Delta = cD_\perp/e\lambda_e H$ ($D_\perp$ is the projection of the extremal diameter of the Fermi line on the y axis). Thus, Γ is an oscillating function of H^{-1}.

3. The character of the Fermi line can be deduced by a method analogous to the observation of the Gantmakher effect in ordinary metals.[11] We consider the behavior of a thin film of thickness L in a longitudinal electric field $E_x = E_0 e^{-i\omega t}$ and a magnetic field H perpendicular to the plane of the film. The field E_x can be regarded as homogeneous, since $\delta \gg L$ (δ is the skin-layer depth). We consider a situation wherein one of the logitudinal dimensions of the film, d, is comparable with the Larmor radius. We note also that it is possible to obtain a film that is monocrystalline in the xy plane. We write next the kinetic equation

$$\frac{\partial f}{\partial t}+\frac{\partial f}{\partial \varepsilon}v_x eE_x+\frac{\partial f}{\partial\varphi}\omega_c = -\frac{f-f_0}{\tau}.$$

We assume satisfaction of the conditions $\tau_1 \gg \tau_2$ (τ_1 is the relaxation time in scattering by volume defects, and τ_2 is connected with the scattering by the boundaries) and $\omega_c\tau_1 \gg 1$. Calculating next the conductivity and the energy absorption coefficient $\eta = 4\pi\omega^2 c^{-3}\mathrm{Re}\sigma$, we get

$$\eta \sim \frac{\omega^2 e^2}{L}\frac{\tau}{1+(\omega-\omega_c)^2\tau^2}. \tag{11}$$

The absorption is characterized by a maximum at $\omega_{max} = (1+\omega_c^2\tau^2)/\omega_c\tau^2$. If $d > r_H$, then $\tau \approx \tau_1$. The absorption is then characterized by a sharp maximum at $\omega = \omega_c$. If the field satisfies the condition $r_H > d$, then the principal role is assumed by scattering by the boundary. In this case, as can be easily seen from the

equations given for η and ω_{max}, the maximum is much weaker than in the former case, so that $\eta_{max\,2}/\eta_{max\,1} \ll 1$.

Thus, an increase of the external magnetic field leads to satisfaction of the condition $r_H = d$ at a certain value of H. A maximum appears then at $\omega = \omega_c$ and η_{max} changes jumpwise. By varying the value of d in various directions, we can determine the main parameters of the Fermi line.

In conclusion, we are sincerely grateful to V. F. Gantmakher and V. N. Lutskiĭ for interesting discussions concerning the experimental situation, and A. L Larkin for a useful discussion.

APPENDIX

We consider here the behavior of the function $\omega(q)$ that describes the phonon dispersion law, at low values of $\mathbf{q}$. We obtain an expression for the speed of sound in size-quantizing films; in particular, it turns out that a consistent allowance for the electron-phonon interaction leads to an oscillatory dependence of the speed of sound on the film thickness L.

It is clear that at small $\mathbf{q}$ the reflection of the phonons from the surface is specular and the quantization of the phonon spectrum must be taken into account (for simplicity, we shall omit the polarization index). The dispersion law is determined in the course of the solution of the system (1). We use hereafter an approximation wherein the role of the transverse quantum number is played by the phonon transverse momentum q_z, which takes on a discrete series of values $q_z = \pi n/L$. A more rigorous analysis, in which the transverse quantum number ν is not set in direct correspondence with q_z, is more cumbersome but leads to analogous results.

The phonon spectrum $\omega(\mathbf{q})$ as $q_\rho \to 0$, which is of interest to us, is determined from the equation $D^{-1}(\omega) = 0$. From this equation and the system (1) we readily obtain

$$\omega^2 = \omega_0^2/(1 - V\Pi), \tag{A.1}$$

ω_0 is the plasma frequency, equal to $4\pi e^2 N_i/M$, $V = 4\pi e^2/(q_\rho^2 + q_z^2)$, and

$$\Pi(q_\rho q_z) = -\frac{2i}{L}\sum_{p_z}\int \frac{d^3p}{(2\pi)^3} G(p)G(p+q)$$

is the polarization operator. We consider, as above, the case when one electron subband is filled. Calculation of $\Pi(q_\rho, q_z)$ as $q_\rho \to 0$ yields

$$\Pi(0, q_z) = -2m/\pi L. \tag{A.2}$$

Taking (A.1) and (A.2) into account, we arrive at the following expression for the dispersion law of interest to us:

$$\omega^2 = \omega_{\perp\nu}^2 + u_\nu^2 q_\rho^2, \tag{A.3}$$

where

$$\omega_{\perp\nu}^2 = \frac{\omega_0^2 L q_z^2}{8me^2 + Lq_z^2}, \quad u^2 = \frac{8me^2\omega_0^2 L}{(8me^2 + Lq_z^2)^2} \tag{A.4}$$

We see thus that size quantization causes the phonon spectrum to break up into branches corresponding to different values of the transverse quantum number. The fact that the film is bounded leads to $\omega(q) \neq 0$ as $q_\rho \to 0$. In addition, the speed of sound, as seen from (A.4), depends on q_z and is thus not the same for different branches.

As L increases, an increasing number of subbands becomes gradually filled with electrons. If ν_0 is the number of subbands filled at a given L, then the expressions for $\omega_{\perp\nu}$ and u_ν take the form

$$\omega_{\perp\nu}^2 = \omega_0^2 q_z^2 L/(8me^2\nu_0 + Lq_z^2), \quad u_\nu^2 = 8me^2\omega_0^2 L\nu_0/(8me^2\nu_0 + Lq_z^2)^2. \tag{A.5}$$

As $L \to \infty$ (at small q_z), taking the equality $\nu_0 = p_0 L/2\pi$ into account, we obtain from (A.5) $\omega^2 = u^2(q_\rho^2 + q_z^2)$, where $u^2 = \pi\omega_0^2/4me^2 p_0 = \frac{1}{3}(m/M)v_F^2$, i.e., we arrive at the usual acoustic dispersion law (see, e.g., Ref. 12).

Formulas (A.4) were written for the case when one electron subband is filled. With increasing L, as noted above, the remaining subbands become gradually filled. As seen from (A.5), at the thicknesses corresponding to the filling of the new subband, the speed of sound changes jumpwise. These changes, which correspond to oscillations of the electron state density, cause thus the speed of sound to oscillate with changing film thickness L. The oscillation period is $\Delta L \approx (m\varepsilon_F)^{-1/2}$. For Bi, for example, $\Delta L \approx 300$ Å. The oscillations of the speed of sound are directly connected with the singularities of the electron-phonon interaction in size-quantizing thin films.

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