Superconducting filament near a dislocation

V. M. Nabutovskiĭ and V. Ya. Shapiro

Institute of Inorganic Chemistry, Siberian Division, USSR Academy of Sciences
(Submitted 18 February 1978)
Zh. Eksp. Teor. Fiz. **75**, 948–959 (September 1978)

It is shown on the basis of an analysis of the nonlinear Ginzburg–Landau equation that a superconducting "filament" capable of carrying a superconducting current is produced near an edge dislocation at a temperature $T_0 > T_c$ (T_c is the critical temperature of the homogeneous superconductor). Estimates for V_3Ge and Nb_3Sn yield a value $I_c \sim 10^{-5}$–10^{-6} A, where I_c is the critical pair-breaking current (per dislocation). The state is localized in a region with a transverse dimension $l \gg \xi_0$, where ξ_0 is the coherence length. In a magnetic field $H_c > H_{c2}$, where H_{c2} is the upper critical field of a homogeneous superconductor, a superconducting-phase plate appears near the dislocation and precedes the appearance of the mixed state. The function $T_0(H_c)$ is anisotropic in the regions of strong and weak fields. Estimates are presented for the diameter of the filament, for the shift of the temperature transition, as well as for the magnetic-field interval in which a superconducting plate exists in V_3Ge and Nb_3Sn. The effects for dislocation clusters are considered.

PACS numbers: 74.60.Ec, 74.60.Jg, 74.70.Lp, 61.70. – r

INTRODUCTION

A dislocation having a stress field $u_{ik}(\mathbf{r})$ interacts via this field with a system of superconducting electrons. Below the transition temperature T_c the dislocation stress field leads to a certain change in the order parameter, and this in turn affects the plastic properties of the metals on going through the superconducting state. A substantial number of studies, both theoretical and experimental, have been devoted to this phenomenon.[1] The change of the order parameter can be calculated by perturbation theory at all temperatures away from the vicinity of T_c.

Near T_c, however, the change of the order parameter cannot be calculated by perturbation theory.[2] Since, owing to the anisotropy of the stress field, the neighborhood of the dislocation contains regions that are more favorable to the onset of superconductivity, one might ask whether it is possible for a localized state of the order parameter, i.e., for a filament of the superconducting phase, to be produced at $T > T_c$ in the case when there is still no superconductivity in the volume. We show below that such states indeed exist because of the slow decrease, $u_{ik} \propto 1/r$, where r is the distance to the dislocation axis.

The current that can flow along the superconducting filaments is sufficient, according to estimates, to make individual filaments observable in experiments.[3] If the dislocations are located parallel to one another at a distance exceeding the dislocation radius l (it will be shown below that $l \sim \xi_0^2/a$, where ξ_0 is the coherence length and a is the distance between the atoms), then the effect due to all the filaments is simply additive. In the opposite limiting case, the dislocation cluster can be regarded as one "superdislocation."

We emphasize once more that a localized state cannot be obtained by perturbation theory: we must solve the nonlinear Ginsburg-Landau (GL) equation and take into account the stress field of the dislocation.

1. FUNDAMENTAL EQUATIONS

Recognizing that the effects of interest to us appear near T_c at

$$\tau = (T - T_c)/T_c \ll 1, \tag{1}$$

we express the free energy F in the form of a GL functional (without a magnetic field)[2]:

$$F = \int d^2\mathbf{r}\, dz \left\{ C(\mathbf{r}) |\nabla \eta(\mathbf{R})|^2 + \alpha(\mathbf{r}) |\eta(\mathbf{R})|^2 + \frac{\beta(\mathbf{r})}{2} |\eta(\mathbf{R})|^4 \right\}, \tag{2}$$

where C, α, and β are parameters in the GL expansion and depend on the strain tensor u_{ik}, $\mathbf{R} = \mathbf{r} + \mathbf{z}$, $\eta(\mathbf{R})$ is the order parameter, $\mathbf{z} = z\mathbf{n}$, and $\mathbf{n}$ is a unit vector in the direction of the dislocation axis.

For this dislocation to be valid it is necessary, besides (1), that the characteristic change of the order parameter occur over distances $l \gg \xi_0$. We shall show below that this does indeed take place. Recognizing that $l \gg a$, we expand the coefficients in (7) up to terms linear in u_{ik}. Finally, it is easy to verify that under the condition (1) we can neglect the dependence on $\mathbf{r}$ in all the coefficients with the exception of $\alpha(\mathbf{r})$.

For simplicity we consider an edge dislocation whose axis is proportional to a fourfold or sixfold symmetry axis of the crystal. In this case the deformation interaction is $\sim u_{ii}$

$$u_{ii} = \frac{b}{2\pi} \frac{1-2\sigma}{1-\sigma} \frac{\sin\varphi}{r}, \tag{3}$$

where $\mathbf{b}$ is the Burgers vector, σ is the Poisson coefficient, and φ is the azimuthal angle.[5] As a result we obtain for F the expression

$$F = \int d^2\mathbf{r}\, dz \left\{ C|\nabla\eta|^2 + \left(-\alpha_0\tau - \alpha_0 B \frac{\sin\varphi}{r} \right) |\eta|^2 + \frac{\beta}{2} |\eta|^4 \right\}. \tag{4}$$

Here

$$B = \frac{b}{2\pi} \frac{1-2\sigma}{1-2\sigma} \gamma; \quad C = \frac{\hbar^2}{2m}, \quad \alpha_0 = -1.83 \frac{\hbar^2}{2m\xi_0^2},$$
$$\beta = 0.35 \frac{(\hbar^2/2m\xi_0^2)^2}{N(0)(kT_c)^2}, \quad \gamma = \frac{\partial \ln T_c}{\partial \ln V},$$

where $N(0)$ is the state density on the Fermi surface, $\hbar$

 0038-5646/78/090480-06$02.40

is Planck's constant, m is the effictive mass of the electron, V is the specific volume, and k is the Boltzmann constant.[6]

For convenience we rewrite (4) in dimensionless form, introducing the dimensionless quantities defined by

$$\rho=\mathbf{r}/l,\quad \zeta=z/l,\quad \psi=-\eta(\beta C)^{1/2}/\alpha_0 B,\quad E=\tau C/\alpha_0 B^2,$$
$$U_1(\rho)=-\sin\varphi/\rho,\quad l=-C/\alpha_0 B,\quad F_0=|\alpha_0|BC/\beta. \tag{5}$$

In this case (4) takes the form

$$F=F_0\int d^2\rho\, d\zeta\left\{|\nabla\psi|^2+(U_1(\rho)-E)|\psi|^2+\frac{1}{2}|\psi|^4\right\}. \tag{6}$$

(Here and henceforth the differentiation in the operations of the type ∇, ∇^2, etc. is with respect to the dimensionless variables ρ, φ, ζ, x, and y, where x and y are the components of ρ along the axes x and y.)

Minimization of (6) with respect to ψ leads to a nonlinear GL equation with a potential

$$-\Delta\psi+[(U_1(\rho)-E)+|\psi|^2]\psi=0. \tag{7}$$

The boundary conditions are the absence of singularity at $\rho=0$ and a finite limit as $\rho\to\infty$.

We shall show that there exists a certain $E_0<0$ (i.e., $T_0>T_c$) such that Eq. (7) has a nontrivial solution at $E>E_0(T<T_0)$. We shall calculate E_0 and obtain the asymptotically exact solution $\psi_E(\rho)$ of Eq. (7) as $E\to E_0$.

2. TEMPERATURE OF APPEARANCE OF SUPERCONDUCTING FILAMENT

To determine T_0 we must find the value of E_0 at which a solution of (7) first appears. This value (the end point of the spectrum) can be determined from the solution of the linearized equation (7), as the lowest level of this equation. Indeed, the "bound" state $\psi_E(\rho)$ exists if the functional $F\{\psi_E\}<0$. Obviously, for the functional F without the term ψ^4 this is impossible if $E<E_0$, while the term ψ^4 can only increase the functional (6), so that there are no bound states at $E<E_0$. On the other hand, at $E>E_0$, substituting in (6) the eigenfunction $\varepsilon\psi_{E0}$ of the linearized equation (7), corresponding to $E=E_0$, and choosing the constant ε to be sufficiently small, we certainly obtain $F\{\varepsilon\psi_{E0}\}<0$, this being obvious for a functional without the terms ψ^4, while the term ψ^4 can be neglected if $\varepsilon\to 0$. Thus, E_0 is indeed the end point of the "spectrum" of Eq. (7).

Equation (7) without the nonlinear term takes in cylindrical coordinates the form

$$-\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial\psi}{\partial\rho}\right)-\frac{1}{\rho^2}\frac{\partial^2\psi}{\partial\varphi^2}-\frac{\partial^2\psi}{\partial\zeta^2}+(U_1(\rho)-E)\psi=0. \tag{8}$$

The variables in (8) do not separate, and we have therefore solved (8) by expanding ψ in terms of an appropriate system of functions. To choose this system, as well as for a qualitative investigation of a number of the problem, we have considered, besides U_1, also U_2 and U_3, where

$$U_2(\rho)=-\frac{1}{\pi\rho},\quad U_3(\rho)=\begin{cases}2U_2, & 0<\varphi<\pi,\\ \infty, & \pi<\varphi<2\pi.\end{cases} \tag{9}$$

The potentials U_2 and U_3 have the same behavior as U_1 as $\rho\to 0$ and $\rho\to\infty$. The potential U_2 does not take into account the anisotropy with respect to φ. On the other hand, the potential U_3 is more anisotropic than U_1 (the hump is infinite). These potentials are "normalized" in such a way that the angle-averaged values for a given ρ coincide. It can be assumed (as will be proved later on) that $E_0^{(1)}$ lies between $E_0^{(2)}$ and $E_0^{(3)}$ ($E_0^{(i)}$ is the lowest level for the potential U_i). The (unrenormalized) solutions of (8), with U_1 replaced by πU_2, take the form[7]

$$\psi^{cs}_{nmk_\zeta}(\rho)=\exp(ik_\zeta\zeta)\begin{Bmatrix}\cos m\varphi\\ \sin m\varphi\end{Bmatrix}\{\rho/(n+m+1/2)\}^m$$
$$\times\exp[-\rho/2(n+m+1/2)]\Phi(-n,2m+1;\rho/(n+m+1/2)), \tag{10}$$

where $\Phi(a;b;x)$ is a confluent hypergeometric function. For U_2 we have

$$E_{nmk_\zeta}=-[\pi(2n+2m+1)]^{-2}+k_\zeta^2,\quad E_{000}=-1/\pi^2. \tag{11}$$

When U_1 is replaced by U_3 in (8), the boundary conditions are such that the only remaining functions of the system (10) are those containing $\sin m\varphi$, so that for the ground state we obtain

$$\psi^{(3)}_{E_0}=\vartheta(\varphi)\rho e^{-\rho/3}\sin\varphi,\quad E_0^{(3)}=-4/9\pi^2\approx-0.044, \tag{12}$$

where

$$\vartheta(\varphi)=0 \text{ at } \pi<\varphi<2\pi,\quad \vartheta(\varphi)=1 \text{ at } 0<\varphi<\pi.$$

As the appropriate system of functions for the solution of (7) we chose the system (10). The energy eigenvalues and eigenfunctions were obtained from the system of equations

$$\sum_j \mathcal{H}_{ij}a_j=Ea,\quad \psi=\sum_p a_p\psi_p,\quad \mathcal{H}=-\Delta_\rho+U_1(\rho), \tag{13}$$
$$\mathcal{H}_{ij}=\langle\psi_i\mathcal{H}\psi_j\rangle/\langle\psi_i^2\rangle,\quad \langle\psi_iK\psi_j\rangle=\int\psi_iK\psi_jd^2\rho,\quad p=\{n,m,k_\zeta\}.$$

The system (13) was solved with a computer. In view of the rapid convergence, the functions $\psi^c_{000}, \psi^s_{010}, \psi^c_{100}, \psi_{100}$ are sufficient to obtain $E_0^{(1)}$ with 1% accuracy. In this case

$$E_0^{(1)}\approx-0{,}089,\quad a_1\approx-0{,}27,\quad a_2\approx-0{,}62,\quad a_3\approx0{,}74,\quad a_4\approx0{,}05. \tag{14}$$

With the aid of (15) we obtained the connection between T_0 and $E_0^{(1)}$:

$$\tau_0=-1{,}83E_0^{(1)}B^2/\xi_0^2,\quad \tau_0=(T_0-T_c)/T_c. \tag{15}$$

This temperature lies in the region $\tau\sim(B/\xi_0)^2$. Recognizing that $b\sim a$ and $B\sim b\gamma$ for a single dislocation, we see that the inequality (1) is satisfied at not too large values of γ.

3. STRUCTURE OF LOCALIZED STATE AND FREE ENERGY

To calculate the thermodynamic parameters of the produced localized state, such as the free energy, the critical current, etc., it is necessary to know the solution of the nonlinear equation (7) at $E>E_0$. We can obtain an asymptotically exact solution in the temperature region satisfying the condition

$$\Delta E=E-E_0\ll E_1-E_0, \tag{16}$$

where E is the "level" closest to E_0, i.e., near the temperature T_0, in the linear equation (7) (at $k_\zeta=0$).

The point E_0 is the bifurcation point for the nonlinear equation (7), since appears one additional solution $\psi E_0^{(1)}$ appears at this point, besides the solution $\psi_E=0$. A theorem for the existence of a solution of an equation of this type was proved by Berger.[8] The solution of the nonlinear GL equation under the condition (16) is obtained by us in the Appendix, where it is shown that

$\psi_E(\rho)$ is given by

$$\psi_E(\rho)=g(\Delta E)^{1/2}\psi_{E_0}(\rho)+O((\Delta E)^{3/2}),$$

$$g^2=\langle|\psi_{E_0}^{(1)}|^2\rangle/\langle|\psi_{E_0}^{(1)}|^4\rangle. \tag{17}$$

We emphasize that the solution (17) cannot be obtained by perturbation theory, as is evident by the very character of the solution. For the potential U_1 we have $g^2 \approx 4.2$.

Substituting (17) in (6), we obtain an expression for the free energy of the filament (per unit length):

$$F\approx -83F_0(\Delta E)^2/l. \tag{18}$$

In concluding this section, we know that the order parameter $\psi_E(\rho)$ changes substantially over distances $\rho\sim 1$ or $r\sim l$. It follows from (5) that $l\sim \xi_0^2/B\gg\xi_0$, so that the GL equation can be used.

4. CRITICAL FIELD FOR LOCALIZED STATES

Superconducting states localized near a dislocation appear not only at $T>T_c$, but also in a magnetic field H exceeding the critical field H_{c2}. The phase diagrams on the $T-H$ plane is shown schematically in Fig. 1. Curve 1 is the line of appearance of the bulk phase, while curves 2 and 3 are the lines of appearance of localized states at different orientations of the field relative to the dislocation axis.

To describe the superconducting states in the general case, we use the GL equations with allowance for the magnetic field:

$$(i\nabla+\mathbf{a})^2\psi+[(U_1(\rho)-E)+|\psi|^2]\psi=0, \tag{19}$$

$$\operatorname{rot}\mathbf{v}=\frac{i}{2\varkappa^2}(\psi\nabla\psi^*-\psi^*\nabla\psi)-\frac{|\psi|^2}{\varkappa^2}\mathbf{a}, \tag{20}$$

where

$$\mathbf{a}=2\pi l\mathbf{A}/\Phi_0,\quad \mathbf{v}=\pi l^2\mathbf{H}/\Phi_0,\quad \mathbf{H}=\operatorname{rot}\mathbf{A},\quad \Phi_0=hc/2e,$$
$$\mathbf{v}=\tfrac{1}{2}\operatorname{rot}\mathbf{a},\quad \varkappa=1.83\lambda/\xi_0,\quad \lambda^2=|mc^2\beta/16\pi e^2\alpha_0|, \tag{21}$$

λ is the depth of penetration of the field in the superconductor at $T=0$, c is the speed of light, and e is the electron charge.

Equations (20) and (21) describe the dependence of the dimensionless critical field ν_c on the temperature (E_0) as the limit of the set E and ν at which (20) has a nontrivial solution. This limit can be obtained in the form of the function $E_0(\nu)$, where E_0, just as before, is the lowest level in Eq. (19) without the nonlinear terms, i.e., it is necessary to find the lowest eigenvalue of E_0 as a function of ν. An asymptotically exact solution can be obtained in two limiting cases: at $\nu\ll 1$ and in the opposite limiting case (regions A and B in Fig. 1, respectively).

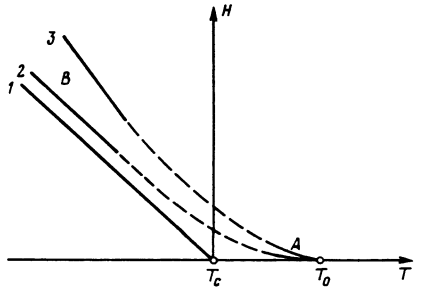

FIG. 1.

In the case $\nu\ll 1$ we can use perturbation theory in ν, taking the zeroth-approximation function to be the solution of Eq. (19) without the field. It is then easy to verify that, accurate to $1/\varkappa^2$, we can neglect the right-hand side of (20), i.e., we can assume the field is the region of the filament to be equal to the external field. We shall assume henceforth that $\varkappa\gg 1$.

For the model potentials U_2 and U_3 can be solved exactly (in this approximation). We put

$$\mathbf{a}=(-v_z y,\ v_z x,\ 2(v_z y-v_y x)). \tag{22}$$

The perturbation linear in ν is given by

$$2iv_z\frac{\partial}{\partial\varphi}. \tag{23}$$

The matrix elements (23) between the functions $\psi E_0^{(1)}$ and $\psi_p^{(1)}$ are equal to zero, inasmuch as for the potential U_2 the function $\psi E_0^{(2)}$ does not depend on φ, and for U_3 all the functions contain only $\sin m\varphi$. Thus, the second-order correction in ν in E is only the result of the terms

$$v_z^2(x^2+y^2)+4(v_x^2y^2+v_y^2x^2). \tag{24}$$

Averaging (23) over $\psi E_0^{(2)}$ and $\psi E_0^{(3)}$, we obtain for the potentials U_2 and U_3 respectively

$$E-E_0^{(2)}\approx 15\nu^2(1+\sin^2\theta),$$
$$E-E_0^{(3)}\approx 112\nu^2(1+2\sin^2\theta\sin^2\varphi), \tag{25}$$

where θ is the angle between the direction of the magnetic field and the dislocation axis.

The eigenfunctions, of the dislocation potential U_1, generally speaking, contain $\sin m\varphi$ and $\cos m\varphi$, so that the off-diagonal matrix elements of the term linear in the field in (23) between the functions $\psi E_0^{(1)}$ and $\psi E^{\prime(1)}$ become different from zero (the diagonal terms are equal to zero for all $\varkappa$), but the functions obtained by us by the variational method do not contain, with good accuracy, terms $\propto\cos m\varphi$. We therefore confine ourselves in the calculations of the corrections $\propto\nu^2$ only to the diagonal matrix elements of the terms $\propto\nu^2$, so that the angular dependence takes the form

$$E-E_0\approx 58\nu^2 f(\theta,\varphi),\quad f(\theta,\varphi)\approx[1+0.6\sin^2\theta(1+1.4\cos^2\varphi)]. \tag{26}$$

At not too large $\varkappa$ ($\varkappa\sim 1$), the constants in formulas (25) and (26) change, but, as before, the increment to E_0 is $\sim\nu^2 f_1(\theta,\varphi)$, $f_1(\theta,\varphi)\sim 1$. In the opposite limiting case the situation is somewhat more complicated. In the absence of dislocations, an Abrikosov structure[9] should appear on curve 1 of Fig. 1, i.e., at $H=H_{c2}$ ($\nu=\nu_{c2}$). This structure results from superposition of wave functions with different centers of the orbits y_0, of the form $\psi\sim\exp\{-2i\nu x y_0-2\nu(y-y_0)^2\}$. In the homogeneous case the energy degeneracy with respect to the centers of the orbit is partically lifted by the nonlinear terms, and this leads to the formation of the periodic structure.

In the presence of a dislocation potential, the degeneracy with respect to the orbit centers is lifted already by the dislocation potential itself, so that to determine the end point $\nu_c(E)$, as well as the structure of the state near this point, we can use as before the linearized equation (19). (The nonlinear term determines only the

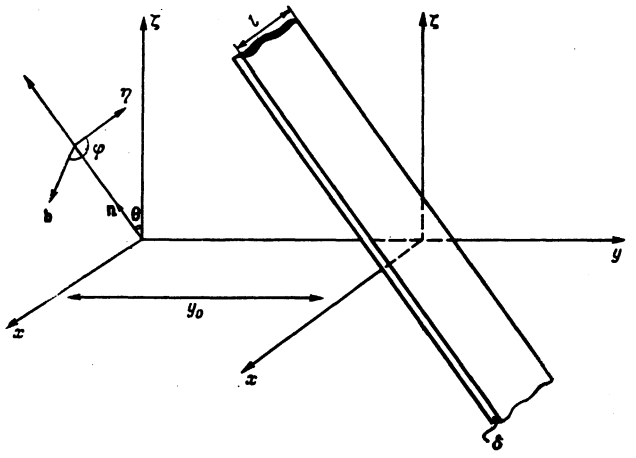

FIG. 2.

amplitude of this state.)

For a formal solution of the problem, we write down (19) and (20) without the nonlinear terms

$$\left(-i\frac{\partial}{\partial x}+2\nu y\right)^2\psi-\frac{\partial^2\psi}{\partial\xi^2}-\frac{\partial^2\psi}{\partial y^2}+(U(x,y,\xi)-E)\psi=0, \tag{27}$$

$$U(x,y,\xi)=-\frac{y\cos\varphi+\eta\sin\varphi}{\eta^2+y^2}, \quad \eta=x\cos\theta-\xi\sin\theta. \tag{28}$$

The choice of the coordinate system for this case and the location of the dislocations relative to the magnetic field and the resultant superconducting state are shown in Fig. 2. The magnetic field is directed along the ζ axis. Just as before, the angle φ characterizes the orientation of the Burgers vector **b** in a plane perpendicular to the dislocation axis, and θ is the angle between the dislocation axis **n** and the magnetic field.

The dislocation potential cannot be taken into account by perturbation theory, because of the degeneracy of the ground state. However, as is clear from (27), by virtue of the condition $\nu\gg1$, the localization of the wave function ψ in the direction of the y axis is determined as before by the magnetic field, and $U(x,y,\zeta)$ is inessential for this motion. The characteristic localization distance with respect to y is $\rho_0\sim\nu^{-1/2}$, where $\rho_0\sim R/l$ and R is the Larmor radius of the lower level, given by

$$R=(\Phi_0/\pi H)^{1/2}=l/(2\nu)^{1/2}, \tag{29}$$

so that we can use, as before, the concept of "orbit center." We shall show that an approximate separation of the variables is practically always possible in the system (27) and (28).

We consider first the case $y_0\gg\rho_0$. In the potential (28) we can then replace y by y_0, since $|y-y_0|\sim\rho_0$, and the relative change of the potential (28) over such distances is small ($\sim\rho_0/y_0$). The variables can then be separated:

$$\frac{\partial^2\psi'}{\partial y^2}+[E'+4\nu^2(y-y_0)^2]\psi'=0, \tag{30}$$

$$\frac{\partial^2\psi''}{\partial t^2}+\left[\varepsilon''+\frac{p\cos\varphi+t\sin\varphi}{p^2+t^2}\right]\psi''=0. \tag{31}$$

Here

$$\psi=\psi'(y)\psi''(t)\exp(-2i\nu xy_0),$$
$$p=y_0/\sin^2\theta, \quad \varepsilon''=E''\sin^2\theta, \quad E=E'+E'', \quad t=\eta/\sin^2\theta.$$

We investigate now the function $E_0''(p)$. It is obvious that at $p\gg1$ the potential in (31) has many levels, and the lowest one lies near the bottom of the well, so that $E_0''\sim-1/y_0$. At $\rho_0/\sin^2\theta\ll p\ll1$ the solution can be obtained by joining together the exact solutions in the regions $t\gg p$ and $t\sim p$. It can be shown that in this case

$$E''\approx-f_2(\varphi)/\sin^2\theta, \quad f_2(\varphi)\approx c(1+\sin2\varphi), \quad c\approx1. \tag{32}$$

It is important that E'' does not depend on y_0 in this case.

If $y_0\lesssim\rho_0$, asymptotically exact formulas are obtained at $|\varphi-\pi/2|\ll1$. Indeed, if this inequality is satisfied, we can neglect the term $y\cos\varphi$ relative to $\eta\sin\varphi$ in the numerator of (28), and then discard also the term $y\sim\rho_0\ll1$ in the denominator, inasmuch as in the potential in the form $1/\eta$, obtained after discarding y, the contribution to the energy is due to $\eta\sim1$, since there is no falling to the center. Thus, the variables separate in this case, too, and formulas (32) remain in force.

At $|\varphi-\pi/2|\sim1$ the variables, generally speaking, do not separate, since the contribution made to the energy by the term $y\cos\varphi$ in (28) turns out to be of the same order as the contribution of the term $\eta\sin\varphi$. It is obvious that (32) is of the right order of magnitude, but $f_2(\varphi)$ is now an arbitrary function of φ such that $f_2(\varphi)\sim1$. Comparing the expressions for E'' at different values of y_0, we find that the minimum of E'' is reached at $y_0\lesssim\rho_0$ and is determined by the formula (32), where $f_2(\varphi)\sim1$.

The final forms of the energy and of the wave function are

$$E_0\approx2\nu-f_2(\varphi)/\sin^2\theta, \tag{33}$$

$$\psi\sim\exp[-2\nu(y-y_0)^2-|\eta|/\sin^2\theta-2i\nu xy_0], \tag{34}$$

i.e., the produced state takes the form of a plate of width 1 in the direction of the η axis and thickness ρ_0 along the y axis.

Let us estimate the region of small angles θ at which our arguments become incorrect. An important factor for the separation of the variables in (27) and (28) is that the characteristic distance over which the wave function changes in the direction of the η axis exceeds substantially the characteristic length of localization in the direction of the y axis. Therefore our analysis does not hold at angles $\theta<\theta_c$. The quantity θ_c and the value of the energy E_0 at $\theta=\theta_c$ are determined from the condition $\theta_c^2\sim\rho_0$ and take the form

$$\theta_c\sim\nu^{-1/4}, \quad E_0\approx2\nu-\nu^{1/2}. \tag{35}$$

At $\theta=0$ we can estimate the corrections to E_0' by perturbation theory, for in this case there is no localiza-

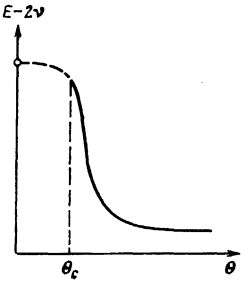

FIG. 3.

tion in the direction of the magnetic field. The estimate leads to the value $E_0'' \sim -\nu^{1/2}$, i.e., $E_0''(0) \sim E_0''(\theta_c)$ so that at $0 < \theta < \theta_c$ we have $E_0''(\theta) \sim E_0''(\theta_c)$.

The general form of the angular dependence of $E - 2\nu$ (i.e., of the magnetic-field interval in which localized states exist but there is still no Abrikosov structure) is shown in Fig. 3. (The dashed lines mark the region in which our analysis is only qualitative in character.)

5. CRITICAL CURRENT

We now estimate the current that can be made to flow through a single superconducting filament in a zero external field ($\nu = 0$). It is easy to show that in the case $\varkappa \gg 1$ considered by us we have $\lambda \gg l$, so that the critical current, just as in the case of a thin plate,[10] is the pair-breaking current.

To solve the problem, we use Eqs. (19)–(21) and neglect the magnetic field produced by the current. In this case the vector potential takes the form

$$\mathbf{a} = (0, 0, a_\zeta). \tag{36}$$

We seek the order parameter in the form

$$\psi = q \exp(ik_\zeta \zeta)\chi(x, y), \quad \int |\chi|^2 d^2\rho = 1. \tag{37}$$

The right-hand side of (20) is the dimensionless current density. Integrating both halves of (20) over the (x, y) plane we obtain the connection between the dimensionless total current $\mathbf{J}$, the vector potential $\mathbf{a}$, and q:

$$J_\zeta = -\frac{(-k_\zeta + a_\zeta)}{\varkappa^2} q^2, \quad \mathbf{J} = \frac{4\pi^2 l}{c\Phi_0}\mathbf{I}, \tag{38}$$

where $\mathbf{I}$ is the total current.

It follows from (38) and (19) that χ satisfies the equation

$$-\Delta\chi + [U_1(\boldsymbol{\rho}) + J^2\varkappa^4/q^4 - E]\chi + q^2\chi^3 = 0. \tag{39}$$

This equation is formally equivalent to (7). Therefore at $\tilde{E} - E_0 \ll E_0$ we obtain $\chi = \chi_0$, and consequently

$$q^2 = [(E - E_0) - J^2\varkappa^4/q^4]\langle|\chi_0|^4\rangle^{-1}, \quad \langle|\chi_0|^4\rangle^{-1} \approx 167, \tag{40}$$

$\tilde{E} = E - J^2\varkappa^4/q^4$, and χ_0 is the normalized solution, of lowest energy, of the linearized equation (29).

Equation (40) connects the current with q^2. The critical value of the current is the maximum value of J at which (40) has a solution. At this value of J, the derivatives of both halves of (40) with respect to q^2 are equal. As a result we have

$$I_c = I_{c0}(\Delta E)^{3/2}, \quad I_{c0} \approx 0.9 c\Phi_0 B/\lambda^2. \tag{41}$$

Formula (41) is asymptotically exact at $\Delta E \ll E_0$. At $\Delta E \sim E_0$ we get the estimate $I_c \sim 0.03 I_{c0}$.

CONCLUSION

Thus, a superconducting filament with diameter l is produced near the dislocation at $T > T_c$ in the absence of a magnetic field, and it follows from (4), (5), (14), and (15) that

$$\Delta T = T_0 - T_c = 0.16 B^2/\xi_0^2, \quad l \approx 0.55\xi_0^2/B. \tag{42}$$

At $T < T_0$ the order parameter and the energy of the localized state per unit length are described by formulas (18) and (19), which are obtained as a result of the solution of the nonlinear GL equation.

In a weak magnetic field ($H \ll H_0 B^2/\xi_0^2, H_0 \equiv H_{c2}(T = 0)$) the temperature at which the filament appears is shifted by an amount δT, and in accordance with (26) and (21) we have

$$\delta T \approx 8(H\xi_0/H_0 B)^2 f(\theta, \varphi). \tag{43}$$

Below the transition point, at $\tau \gg B^2/\xi_0^2$, the fields close to critical are strong ($H \gg H_0 B^2/\xi_0^2$). In this case there exists a field interval $\Delta H = H_c(T) - H_{c2}(T)$ in which localized states exist near the dislocation that take in this case the form of plated of thickness $\delta \sim \xi_0/\tau^{1/2}$ and width $\sim l$, arranged parallel to the dislocation axes (Fig. 2).

The quantity ΔH has strong anisotropy and increases sharply with decreasing angle between the dislocation axis and the magnetic field. Since formulas (34) and (35) determine only the angular dependence on θ and the order of magnitude of the quantities, it is convenient down here an interpolation formula

$$\Delta H = \Delta H_0 \tau^{1/2}, \quad \Delta H_0 \approx H_0 \frac{B}{\xi_0} \frac{\theta_c^2}{\theta_c^2 + \sin^2\theta_c}$$
$$\theta_c = (B/\xi_0)^{1/2}\tau^{-1/4}. \tag{44}$$

The critical current that can be made to flow through a unit dislocation at $T_c < T < T_0$ is determined by formula (41). At $T \approx T_c$ its maximum is given by

$$I_0 \approx 0{,}1 c\Phi_0 B/\varkappa^2\xi_0^2. \tag{45}$$

From (42)–(45) it is clear that all the effects depend essentially on the derivative $\partial \ln T_c/\partial \ln V$, so that the effects are particularly strong for those substances which have a strong dependence of T_c on the pressure and a low value of ξ_0.

Some compounds of the A-15 type satisfy the foregoing criteria. We present some examples (for a single dislocation). For V_3Ge we have according to Refs. 11–14

$$\xi_0 \approx 70\ \text{Å}, \quad T_c \approx 6.3\ \text{K}, \quad a \approx 4.7\ \text{Å}, \quad \sigma \approx 0.35, \quad K \approx 3 \cdot 10^6\ \text{bar},$$

$\partial T_c/\partial p \sim 6 \times 10^{-5}$ K/bar. Here K^{-1} is the compressibility. Assuming also $\lambda \approx 2 \times 10^3$ Å and $b \sim a$, we obtain

$$B \approx 10\ \text{Å}, \quad l \approx 250\ \text{Å}, \quad \Delta T \sim 10^{-2}\ \text{K}, \quad \Delta H \sim 4 \cdot 10^3\ \text{G}, \quad \theta_c \sim 0{,}4, \quad I_0 \sim 5 \cdot 10^{-5}\ \text{A}.$$

For Nb_3Sn with

$$\xi_0 \approx 50\ \text{Å}, \quad T_c \approx 18\ \text{K}, \quad a \approx 5.3\ \text{Å}, \quad \sigma \approx 0.37, \quad K \approx 2.5 \cdot 10^6\ \text{bar},$$
$$\frac{\partial T_c}{\partial p} \sim -1.4 \cdot 10^{-5}\ \text{K/bar}$$

we obtain

$$B \approx 0.7\ \text{Å}, \quad l \sim 2 \cdot 10^3\ \text{Å}, \quad \Delta T \sim 10^{-3}\ \text{K}, \quad \Delta H \sim 4 \cdot 10^3\ \text{G}, \quad \theta_c \sim 0.1, \quad I_0 \sim 5 \cdot 10^{-6}\ \text{A}.$$

The quantity B is proportional to the Burgers vector and therefore increases in proportion to the number N of the dislocations contained in the cluster in the case when the distance d between these dislocations satisfies the condition $d \ll l$. Such a cluster can be regarded as one "superdislocation" with a Burgers vector $N\mathbf{b}$, and this leads to a sharp increase of all the effects.

The effects considered by us can be revealed by the following: 1) the sharp decrease of the resistance at the points where the dislocations of their clusters emerge to the surface of the sample or when the dislocation or system of dislocations is "connected" into the current circuit; 2) the sharp anisotropy of the smearing of the critical field (H_{c2}) in strongly case-hardened samples in fields with various orientations relative to

the dislocation axes (it is assumed that the dislocation axes are parallel). It can also be shown that in this case the critical pair-breaking current should be strongly anisotropic. For a two-dimensional periodic system of dislocations with distance d between them, the anisotropy factor (the ratio of the critical current perpendicular to the axes of the dislocation system to the critical current along the system) is proportional to the overlap of the wave functions of the individual filament, $\propto e^{-d/l}$. In addition, one can expect the appearance of localized states to lead to a change in the dislocation damping force, and consequently to a change in the plastic properties in the vicinities of T_c and H_{c2}.

We note also that although the theory described above is valid, strictly speaking, at $B \ll \xi_0$, localized states should exist also at $B \gtrsim \xi_0$. Moreover, for some substances the quantity B can be unusually large. In such a case we can have a situation wherein the superconductivity is caused by the existence of dislocations.

The authors thank V. L. Ginzburg and I. M. Lifshitz for interest in the work, I. O. Kulik, and A. I. Larkin for useful discussions, and N. E. Alekseevskii for a discussion of the possibilities of experimentally observing the described effects.

APPENDIX

The GL functional with a potential and the corresponding nonlinear equation and equation without the nonlinear terms take respectively the forms (6), (7), and (8). We obtain an asymptotically exact solution of Eq. (7) near the bifurcation point $E = E_0$ (E_0 is the smallest eigenvalue of Eq. (8)) under the condition (16), which presupposes that E_1 is separated from E_0 by a finite interval. We seek the solution of (7) in the form

$$\psi(\rho, \zeta) = A_0(\zeta)\psi_{E_0}(\rho) + \sum_{E'} B_{E'}(\zeta)\psi_{E'}(\rho). \tag{A1}$$

By summation over E' we mean summation over discrete values and integration over continuous values, $\psi_E(\rho)$ is the set of those eigenfunctions of (8) which do not depend on ζ; this set is assumed to be complete. Substituting (A1) in (6) we easily verify that the minimum of the functional (6) is reached for A_0 and $B_{E'}$ independent of ζ. Assuming that $B_{E'} \ll A_0$, we retain in the functional only the terms that are linear and quadratic in $B_{E'}$, after which we vary (6) with respect to A_0 and $B_{E'}$. As a result we get

$$A_0^2 = g^2\Delta E, \quad B_{E'} = g^3 g_{E'}(\Delta E)^{1/2}/(E' - E_0),$$

$$g_{E'} = \langle \psi_{E_0}^3 \psi_{E'} \rangle \Big/ \sum_{E''} \langle \psi_{E'}\psi_{E''} \rangle. \tag{A2}$$

It is clear from (A2) that $B_{E'}/A_0 \sim \Delta E$, and it is this which justifies the possibility of expansion with respect to $B_{E'}$. To be able to use the procedure described above, it is necessary also that the sum (integral) over E' in (A1) exists, i.e., the coefficients $B_{E'}$ must decrease rapidly enough. The corresponding coefficients calculated for U_2 decrease like $1/n^3$ for the discrete spectrum (E_{nm0}) and like $1/k^5$, for the continuous spectrum ($E' = k^2$) so that (17) is valid. $B_{E'}$ has a similar form for U_1 and U_3. Substituting (A.1) in (6), we obtain the expression (18) for the free energy.

[1]M. I. Kaganov, B. Ya. Kravchenko, and V. D. Natsik, Usp. Fiz. Nauk 111, 655 (1973) [Sov. Phys. Usp. 16, 878 (1974)].
[2]L. A. Manakova and V. M. Nabutovskiĭ, Abstracts of Papers of Twentieth All-Union Conf. on Low Temperatures, 1972, p. 98.
[3]V. M. Nabutovskiĭ and B. Ya. Shapiro, Pis'ma Zh. Eksp. Teor. Fiz. 26, 624 (1977) [JETP Lett. 26, 473 (1977)].
[4]R. Labusch, Phys. Rev. 170, 470 (1968).
[5]L. D. Landau and E. M. Lifshitz, Teoriya ugrugosti (Theory of Elasticity), Nauka, 1965. [Pergamon, 1968].
[6]L. P. Gor'kov, Zh. Eksp. Teor. Fiz. 36, 1918 (1959) Sov. Phys. JETP 9, 1364 (1959).
[7]V. L. Bonch-Bruevich, Fiz. Tverd. Tela (Leningrad) 3, 47 (1961) [Sov. Phys. Solid State 3, 34 (1962)].
[8]M. S. Berger, transl. in: Teoriya vetvleniya i nelineĭnye zadachi na sobstvennye znacheniya (Branching Theory and Nonlinear Eigenvalue Problems), Mir, 1974.
[9]A. A. Abrikosov, Zh. Eksp. Teor. Fiz. 32, 1442 (1957) [Sov. Phys. JETP 5 1174 (1957)].
[10]E. M. Savitskiĭ, Yu. V. Efimov, N. D. Kozlova, B. P. Mikhaĭlov, L. F. Myzenkova, and E. D. Doron'kin, Sverkhprovodyashshe materialy (Superconducting Materials), Metallurgiya, 1976, p. 85.
[11]V. L. Ginzburg, Dokl. Akad. Nauk SSSR 118, 464 (1958) [Sov. Phys. Dokl. 3, 102 (1958)].
[12]L. R. Testardi, Phys. Rev. B 3, 95 (1971).
[13]M. Rosen, H. Klimker, and M. Weger, Phys. Rev. 184, 466 (1969).
[14]E. Rehmald, M. Rayl. R. W. Cohen, and G. D. Cody, Phys. Rev. 6, 363 (1972).
[15]I. N. Khlyustikov and M. S. Khaĭkin, Zh. Eksp. Teor. Fiz. 75, 780 (1978) [Sov. Phys. JETP 48, 394 (1978)].

Translated by J. G. Adashko