[10]B. L. Gel'mont, Fiz. Tverd. Tela (Leningrad) 11, 1096 (1969) [Sov. Phys. Solid State 11, 894 (1969)]; L. Liu and M. Tan, Phys. Rev. B 9, 632 (1974).
[11]Y. Guldner, C. Rigaux, M. Grynberg, and A. Mycielski, Phys. Rev. B 8, 3875 (1973).
[12]S. D. Beneslavskiĭ, V. I. Ivanov-Omskiĭ, B. T. Kolomiets, and V. A. Smirnov, Fiz. Tverd. Tela (Leningrad) 16, 1620 (1974) [Sov. Phys. Solid State 16, 1058 (1974)].
[13]R. Grisar, H. Wachering, G. Bauer, and W. Zawadzki, Infrared Phys. 16, 149 (1976).
[14]J. D. Wiley, in: Semiconductors and Semimetals (ed. by R. K. Willardson and A. C. Beer), Vol. 10, Transport Phenomena, Academic Press, New York (1975), p. 91.
[15]P. S. Zyryanov and M. I. Klinger, Kvantovaya teoriya yavleniĭ élektronnogo perenosa v kristallicheskikh poluprovodnikakh (Quantum Theory of Electron Transport Phenomena in Crystalline Semiconductors), Nauka, M., 1976.

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Dislocation damping by electrons in the ultraquantum limit

A. M. Grishin, L. N. Gumen, and É. P. Fel'dman

Donetsk Physicotechnical Institute, Ukrainian Academy of Sciences and Institute of Radio Physics and Electronics, Ukrainian Academy of Sciences
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The damping of dislocations by conduction electrons in a strong magnetic field corresponding to the ultraquantum limit is investigated. A substantial increase of the "electron friction" with increasing magnetic field ($\propto H^{7/2}$) is observed. The strongest slowing down is experienced by dislocations oriented at small angles Φ to the **H** direction. For these dislocations, the dependence of the damping force on the velocity and on the angle Φ can have a nonmonotonic character.

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1. INTRODUCTION

1. The plastic properties of metals at low temperatures are determined to a considerable degree by the damping of the dislocations by the conduction electrons. The energy of a moving dislocation is consumed in excitation of the electrons, i.e., in the raising of the electrons from states with lower energy to states with higher energy, and is then converted into heat when the electron system relaxes. This is how a fraction of the energy of the external loads that cause the dislocation motion is dissipated.

We are interested in the energy Q absorbed by the electrons per unit time. It is determined by two factors. First, Q depends on the parameters of the field of the dislocation strains; second, the dissipated energy is expressed in terms of characteristics of the electron system. An external magnetic field **H** alters the wave functions and the chemical potential of the electrons, i.e., the problem arises of determining the dependence of Q on **H**. The influence of a magnetic field on the dislocation damping force was investigated in Refs. 1–4 with a simple single-band model of electrons with a quadratic isotropic dispersion law as an example. In Refs. 1 and 3 they considered a special situation, where a linear dislocation was oriented parallel to **H**. This problem was solved for an arbitrary geometry in Ref. 4. One of the principal results of the latter reference is the conclusion in most cases that the damping force depends little on the magnetic field. The point is that the magnetic field alters the character of the electron motion only in a plane perpendicular to the vector **H**. Along the magnetic field, the electrons move as free particles. It is precisely this drift motion which plays the decisive role in the absorption of the dislocation energy. Owing to the drift along **H**, the center of the electron orbit manages, during the cyclotron period, to cross many times the fronts of the elastic wave generated by the dislocation. Therefore the absorption of such a dislocation phonon has a collisionless character and does not depend on **H**. This result is valid if the magnetic field is not too strong, when the distance $\hbar\Omega$ between the Landau levels ($\Omega \equiv eH/mc$ is the cyclotron frequency and e is the absolute value of the electron charge) is much less than the Fermi energy ε_F.

In very strong magnetic fields, in semimetals and semiconductors with low carrier density, the inverse limiting case

$$\hbar\Omega/\varepsilon_F \gtrsim 1 \qquad (1.1)$$

may be realized, and is called "ultraquantum." Here the cyclotron period $2\pi/\Omega$ becomes so small that during this time the electron moves in a field of practically homogeneous strains. Therefore in the case (1.1) one should expect a strong effect of the magnetic field on the damping force.[1)] An analysis of this question is the subject of the present paper.

2. Just as in Ref. 4, we confine ourselves to an electron gas with quadratic isotropic dispersion. The state of the electrons in the external constant and homogeneous magnetic field is classified with the aid of four quantum numbers. Three of them, namely, the momentum projection p_H on the direction of the magnetic field, the principal quantum number n, and the

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component of the spin σ along H, determine the electron energy[6]:

$$E_a = p_H^2/2m + (n+1/2)\hbar\Omega - g\mu_B\sigma H; \quad (1.2)$$

a is the aggregate of all the quantum numbers corresponding to the given state. With respect to the fourth quantum number (which can be chosen to be one of the coordinates of the electron-orbit center) we have a degeneracy of multiplicity

$$K = eH/2\pi\hbar c$$

(normalized to unit area). The continuous quantum number p_H can assume all values from $-\infty$ to $+\infty$, $n = 0, 1, 2, \ldots$, the projection of the spin σ is equal to $+1/2$ or $-1/2$. The spin term of (1.2) contains the Bohr magneton $\mu_B = e\hbar/2m_0c$ multiplied by the g-factor. Their product determines the effective magnetic moment of the electron, which depends on the character of the interaction of the electron with the crystal field (see, e.g., Ref. 7). In particular, for a free electron we have $g = -2$. It is convenient to introduce a dimensionless parameter $\gamma \equiv gm/2m_0$ (m_0 is the mass of the free electron and m is the effective mass). Then

$$E_a = p_H^2/2m + \hbar\Omega(n+1/2+\gamma\sigma). \quad (1.3)$$

As seen from (1.3), the minimum energy

$$E_{min} = \hbar\Omega(1-|\gamma|)/2 \quad (1.4)$$

is possessed by an electron in a state with $p_H = 0$, $n = 0$, and a magnetic moment directed along H. Relation (1.4) for E_{min} describes the displacement of the bottom of the energy band in the magnetic field.

The magnetic field not only restructures the states of the electrons, but can also change the population of the levels, inasmuch as the chemical potential μ of the electron gas changes in the field. The dependence of μ on H and on the temperature T is determined in the usual manner from the condition that the electron density N_e is fixed:

$$\int_{\varepsilon_{min}}^{\infty} dE\, \nu_H(E) f_0[E-\mu(H,T)] = N_e. \quad (1.5)$$

Here $\nu_H(E)$ is the density of the electronic states (normalized to unit volume) in the field H; f_0 is the Fermi distribution function. Knowing the spectrum (1.3) and the degeneracy multiplicity K, we can easily obtain the function $\nu_H(E)$ (Ref. 6). Formula (1.5) yields implicitly the function $\mu(H, T)$. We confine ourselves to a summary of the principal results for the case $T = 0$. In the quasiclassical region ($\hbar\Omega \ll \varepsilon_F$), the change of μ is insignificant:

$$\frac{|\mu(H)-\mu(0)|}{\mu(0)} \approx \left(\frac{\hbar\Omega}{\varepsilon_F}\right)^{2/3},$$

where $\varepsilon_F \equiv \mu(0) \propto N_e^{2/3}$.

In a strong magnetic field (1.1), the chemical potential depends substantially on H (See, e.g., Ref. 8):

$$\Delta = \frac{\mu(H)-E_{min}}{\hbar\Omega} = \frac{16}{9}\left(\frac{\varepsilon_F}{\hbar\Omega}\right)^3. \quad (1.6)$$

It is seen that with increasing H the chemical-potential level approaches the minimum electron energy (1.4). If the inequality (1.1) is made stronger, then the parameter Δ is quite small. In other words, what is populated is a narrow energy interval from E_{min} to $\mu(H)$. Only states belonging to the zeroth Landau band

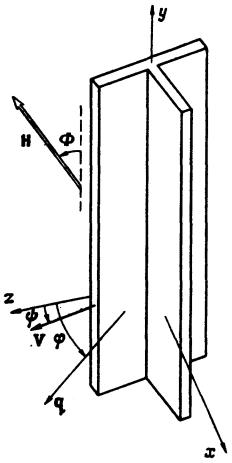

FIG. 1.

are occupied below the chemical-potential level. Their transverse-motion energy is $E_{min} \sim \hbar\Omega$ and the longitudinal energy takes on a value $0 \leq p_H^2/2m \leq \Delta\hbar\Omega$. The ratio of the longitudinal energy to the transverse one is on the order of $\Delta \lesssim 1$, i.e., in the ultraquantum case the fraction of the transverse energy increases noticeably.

3. In the ultraquantum limit, the premise that the electron drift motion plays a predominant role in the damping of the dislocations is no longer tenable. In fact, the maximum electron displacement across the wave front during the cyclotron period is $u = 2\pi v_H \sin\Phi/\Omega$. Here $v_H = p_H/m$ is the projection of the electron velocity on the H direction, and Φ is the angle between the dislocation axis and the magnetic field (see Fig. 1). Using formula (1.6), this displacement can be rewritten in the form

$$u = (16\pi/3)\sin\Phi(\hbar/p_F)(\varepsilon_F/\hbar\Omega)^2.$$

This quantity must be compared with a characteristic wavelength $2\pi q^{-1}$ of those dislocation phonons which interact effectively with the electrons.

The matrix element $\langle b|e^{iqr}|a\rangle$ of the phonon between states with one and the same $n = 0$ decreases exponentially at $q \gg 1/r_H$, where $r_H = (\hbar c/eH)^{1/2}$ is the magnetic length. This follows directly from the law of conservation of the H-component of the angular momentum in the course of absorption of a dislocation phonon by an electron. Thus, $M_H \sim p_\perp r_H \sim \hbar$. The change of M_H due to absorption of a phonon with a momentum $\hbar q_\perp$ transverse to H is equal to $\hbar q_\perp r_H$. It must not exceed a value of the order of M_H, i.e., $\hbar q_\perp r_H \leq \hbar$, whence in fact stems the restriction imposed above on q.

Comparison of u with $2\pi q^{-1}$ leads to the condition

$$qu \sim \sin\Phi(\varepsilon_F/\hbar\Omega)^{1/2} \leq 1.$$

Even this rough estimate shows that the electron is in a field of practically homogeneous strains. This leads to a strong dependence of the damping force on the magnetic field.

4. The functional relation $F(H)$ follows qualitatively from a simple consideration of the initial formula

$$\mathbf{F}\mathbf{V}W = \pi \sum_{a,b;\mathbf{q}} \mathbf{q}\mathbf{V}|U(\mathbf{q})|^2 |\langle b|e^{i\mathbf{q}\mathbf{r}}|a\rangle|^2 (f_a - f_b)\delta(E_b - E_a - \hbar\mathbf{q}\mathbf{V}), \quad (1.7)$$

in which $\mathbf{V}$ is the dislocation velocity and $\mathbf{q}$ is the two-dimensional wave vector of the dislocation phonon (see

Fig. 1); $U(\mathbf{q})$ is the Fourier transform of the energy of the interaction of the dislocation with the electron and is proportional to $1/q$; $f_a \equiv f_0(E_a - \mu)$ is the Fermi distribution function, and W is the volume of the crystal.

We note that the difference $f_a - f_b$ can as a rule be replaced by $\hbar\mathbf{q}\cdot\mathbf{V}\delta(E_a - \mu)$. From this and from the estimate of the matrix elements it follows that at $q \lesssim 1/r_H$ the coefficient of the δ function with the energy conservation law does not depend on q. The summation over the spin and over the continuous quantum numbers that enter in b is obviated by the δ symbols. The summation of $\delta(E_a - \mu)$ over the initial states a yields, by definition, the state density $\nu_H(\mu)$:

$$\nu_H(\mu) = \sum_a \delta(E_a - \mu) = \frac{WK}{\pi\hbar}\sum_{n,\sigma} |v_H|^{-1}. \tag{1.8}$$

The sum over $\mathbf{q}$ is in fact an integral with respect to $\mathbf{q}$ and is proportional to q^2. The integration over the directions of the vector $\mathbf{q}$ singles out characteristic orientations near $\mathbf{q} \perp \mathbf{H}$. In fact, the drift of the electrons in the $\mathbf{H}$ direction does not lead to the intersection of the wave fronts of those phonons for which $\mathbf{q} \parallel \mathbf{x}$ and $\varphi = \pi/2$ (see Fig. 1). The discrimination of the azimuthal angles φ is via the energy conservation law. The corresponding δ function in (1.7) is proportional to the characteristic time $\tau_{eff} \sim (q_H \sin\Phi)^{-1}$ of the interaction of the electron with the wave. It agrees in order of magnitude with the time of travel of the electron between neighboring wave fronts.

Thus, the damping force f turns out to be proportional to the product

$$F \propto \nu_H(\mu) q^2 \tau_{eff}.$$

From (1.6) we get

$$v_H \leqslant v_F(1/3)(\varepsilon_F/\hbar\Omega) \propto H^{-1}.$$

Recognizing that $q \approx 2\pi/r_H \sim H^{1/2}$, we get $\tau_{eff} \sim H^{1/2}/\sin\Phi$. Finally, using the expressions for v_H and K from (1.8), we get $\nu_H(\mu) \sim H^2$. Consequently

$$F \propto H^{7/2}/\sin\Phi,$$

which is the main result of the present paper. In addition to the strong field dependence, the damping force in the ultraquantum limit exhibits a curious nonmonotonic dependence on the velocity V and on the angle Φ.

2. CALCULATION OF THE DISLOCATION DAMPING FORCE IN THE ULTRAQUANTUM LIMIT

1. To find the dislocation damping force in explicit form, we indicate first the concrete form of all the factors that enter in the "golden rule" (1.7). Just as in Ref. 4, to simplify the form of the potential $U(q)$ we assume that we are dealing with a screw dislocation.

A most important question is the allowance for the temperature and for the scattering of the electrons by the thermal phonons, by defects, etc. Strictly speaking, the presence of collisions changes the classification of the electronic states. The matrix elements and the energy levels E_a and E_b also change in correspondence with the perturbation of the stationary states. The procedure for taking into account the changes due to the collisions consists of classifying the states in the same manner as before (so that the matrix elements do not change their form), but now they are quasistationary with a finite lifetime $\tau \equiv \nu^{-1}$, so that $E \to E + i\hbar\nu/2$. The quantity τ can be interpreted as the time interval between the collisions, and $\nu(E)$ can be interpreted as the frequency of the collisions of the electron (with energy E) and the scatterers. In accordance with the described concept, the δ function in (1.7) must be (see Ref. 9) "smeared out" by an amount $\hbar\nu$. As for the temperature, it enters the expression for the force only via the Fermi distribution functions. Taking the foregoing arguments, we get from (1.7)

$$F\mathbf{V} = \frac{B\pi^2\hbar}{4mp_F}\int\frac{d\mathbf{q}}{(2\pi)^2}\mathbf{q}\mathbf{V}\frac{(\zeta_{xy}q_z - \zeta_{yz}q_x)^2}{q^4}$$
$$\times\sum_{\sigma=\pm 1/2}\sum_{s=-\infty}^{\infty}\sum_{n=0}^{\infty} M_{n+s,n}^2\left(\frac{\hbar q_\perp^2}{2m\Omega}\right)\int_{-\infty}^{\infty} dp_H\left\{f_0\left[\frac{p_H^2}{2m}+\hbar\Omega\left(n+\frac{1}{2}+\gamma\sigma\right)\right.\right.$$
$$\left.\left.-\mu(H)\right]-f_0\left[\frac{p_H^2}{2m}+\hbar\Omega\left(n+\frac{1}{2}+\gamma\sigma\right)+\hbar\mathbf{q}\mathbf{V}-\mu(H)\right]\right\}$$
$$\times\frac{1}{\pi}\frac{\nu/\Omega}{(\nu/\Omega)^2+(s+p_H q_H/m\Omega+\hbar q_H^2/2m\Omega-\mathbf{q}\mathbf{V}/\Omega)^2}. \tag{2.1}$$

Here $B \equiv 3N_e b^2 m\varepsilon_F/2\pi\hbar$ is a constant that depends on the concentration of the electrons and on the square of the Burgers vector, $p_F = (2m\varepsilon_F)^{1/2}$, $\zeta_{ik} \equiv \Lambda_{ik}/\varepsilon_F$ are dimensionless constants of the deformation potential, $q_H = q_z \sin\Phi$ is the projection of the vector $\mathbf{q}$ on the $\mathbf{H}$ direction

$$q_\perp = (q^2 - q_H^2)^{1/2}, \quad s = n_b - n_a;$$
$$M_{n+s,n}(t) = t^{s/2}L_n^s(t)e^{-t/2}, \tag{2.2}$$

$L_n^s(t)$ is a generalized Laguerre polynomial normalized to unity. Formula (2.1) takes into account the fact that the initial matrix elements in (1.7) contain a definite aggregate of δ symbols.

2. The quantum oscillations of the damping force in the quasiclassical situation $\varepsilon_F/\hbar\Omega \gg 1$ were considered in Refs. 2 and 4. Here we investigate the ultraquantum case (1.1) at $T = 0$. The results will be valid when the following degeneracy conditions are satisfied:

$$T \ll \mu(H) - E_{min} = \frac{16}{9}\hbar\Omega\left(\frac{\varepsilon_F}{\hbar\Omega}\right)^3.$$

The inequality (1.1) makes it possible to retain a single term in the triple sum over s, n, and σ and by the same token simplify greatly formula (2.1). To verify this, we estimate the order of magnitude of the energy terms in the arguments of the Fermi functions. If $n = 0$ and $\sigma = -\frac{1}{2}\,\mathrm{sgn}\gamma$, then the argument of the first of these functions is minimal:

$$p_H^2/2m + E_{min} - \mu(H) = p_H^2/2m - \hbar\Omega\Delta.$$

Furthermore, it can be either positive or negative. On the other hand, if $n \neq 0$ or if the spin σ is chosen to be different, then a large positive on the order of $\hbar\Omega$ is added to the argument. Since $\Delta \lesssim 1$ and $qV \ll \Omega$, the arguments of both Fermi "steps" are positive in this case, and both functions f_0 are equal to zero, i.e., the terms with $n \neq 0$ and $\sigma \neq -\frac{1}{2}\,\mathrm{sgn}\gamma$ vanish.

The index s enters the matrix elements (2.2) in such a way that the quantities $M_{s0}(n = 0)$ vanish identically for all $s < 0$. The physical reason for this selection

rule is that only the zeroth Landau band is located under the chemical-potential level. Likewise forbidden are electronic transitions with $s>0$. The point is that at $s>0$ the argument of the "smeared" δ function in (2.1) becomes positive, this means that terms with $s\neq 0$ are negligibly small ($\sim\nu/\Omega$). The positiveness of the arguments is established by the following chain of reasoning. The fourth term in the argument is much smaller than the first ($s\geqslant 1$) because of the small dislocation velocity, and can be neglected. The third term is larger than the second at $\hbar|q_H|>2|p_H|$. Since it is always positive, the entire argument is also positive in this case. In the opposite case at $\hbar|q_H|<2|p_H|$, the maximum of the modulus of the second term is

$$2p_{H_{max}}^2/m\hbar\Omega=4\Delta\leqslant 1.$$

Thus, for terms with $s\geqslant 1$ the argument does not vanish in this case, too.

As a result, all that is left of the entire triple sum is the term with $n=s=0$ and $\sigma=-\frac{1}{2}\operatorname{sgn}\gamma$. The matrix element M_{00} corresponding to this term turns into the exponential $\exp(-\hbar q_\perp^2/4m\Omega)$, which "cuts off" the integral at $q\leqslant r_H^{-1}$. Since the magnetic length r_H in realistically attainable magnetic fields is much larger than the Burgers vector, the integration with respect to q can be carried out not to b^{-1} but to infinity. The integral with respect to p_H is calculated in elementary fashion if the difference between the Fermi steps is replaced by

$$\hbar qV\delta(p_H^2/2m-\hbar\Omega\Delta).$$

This replacement is valid because the shift of the arguments is $\hbar qV\ll\hbar\Omega\Delta$. This inequality is satisfied because of the smallness of V for any of the values of q that are possible in this problem. Thus, the integration with respect to p_H reduces to the appearance of a factor $\hbar\mathbf{q}\cdot\mathbf{V}m/|p_H^*|$ and to replacement of p_H by $p_H^*=\pm(2m\hbar\Omega\Delta)^{1/2}$.

This reasoning should make it clear that to calculate the force F it remains only to integrate with respect to the two-dimensional vector $\mathbf{q}$. This integration is conveniently carried out in the cylindrical coordinates q and φ (see Fig. 1). The quantity q must be made "dimensionless" by introducing the variable $x=qr_H/\sqrt{2}$. We arrive ultimately at the following formula for the dislocation damping force in the ultraquantum limit:

$$F=\frac{9}{2^9}BV\left(\frac{\hbar\Omega}{\varepsilon_F}\right)^{7/3}\sum_{\pm}\int_0^\infty x\,dx\oint d\varphi\cos^2(\varphi-\psi)(\zeta_{xy}\cos\varphi-\zeta_{yz}\sin\varphi)^2D(xd_\pm)\exp[-x^2(1-\sin^2\Phi\cos^2\varphi)], \tag{2.3}$$

$$D(t)=\frac{1}{\pi}\frac{\Gamma}{\Gamma^2+t^2},\quad \Gamma=\frac{3}{8}\left(\frac{\hbar\Omega}{\varepsilon_F}\right)^{1/3}\frac{\nu}{\Omega};$$

$$d_\pm(x,\varphi)=\frac{3}{4}\frac{\hbar\Omega}{\varepsilon_F}\frac{V}{v_F}\cos(\varphi-\psi)\pm\cos\varphi\sin\Phi-\frac{3}{8}\left(\frac{\hbar\Omega}{\varepsilon_F}\right)^{1/3}x\cos^2\varphi\sin^2\Phi. \tag{2.4}$$

It is impossible to integrate with respect to x and φ in (2.3) in explicit form. It may seem therefore that we must find the asymptotic forms of F with respect to the separate parameters in (2.3). However, in view of the large number of these parameters (Γ, $\hbar\Omega/\varepsilon_F$, V/v_F, and the angles ψ and Φ) this manner of using formula (2.3) is not constructive. We shall show that it is possible to combine these parameters into a single one (albeit of rather cumbersome structure) and write down asymptotic expressions for F at small and large values of this parameter.

3. We turn to formula (2.3) and attempt to integrate with respect to φ. This has helped to a considerable degree by two circumstances. The first, which can be directly verified, is that the third term of the function $d_\pm$ of (2.4) can in practice always be neglected. The second important aspect reduces to the use of the smallness of the parameter Γ in strong magnetic fields. The function D is then similar to a δ function. It reaches a maximum at $d_\pm=0$, i.e., at the points $\varphi=\varphi_\pm$ defined by the relation

$$\operatorname{tg}\varphi_\pm=-\frac{\Phi_{cr}\cos\psi\pm\sin\Phi}{\Phi_{cr}\sin\psi}\qquad|\varphi_\pm|<\frac{\pi}{2}. \tag{2.5}$$

In the derivation of this formula we have neglected the third term in the function $d_\pm$ (2.4) and have put

$$\Phi_{cr}=\frac{3}{4}\frac{\hbar\Omega}{\varepsilon_F}\frac{V}{v_F}.$$

The condition under which the function D in (2.3) can be replaced by a δ function is the satisfaction of the relation $x|\partial d_\pm/\partial\varphi|\gg\Gamma$ at the points $\varphi_\pm$. With the aid of (2.5) this relation can be rewritten in the form

$$G_\pm=\frac{8}{3}\left(\frac{\varepsilon_F}{\hbar\Omega}\right)^{1/3}\frac{\Omega}{\nu}[\Phi_{cr}^2\sin^2\psi+(\Phi_k\cos\psi\pm\sin\Phi)^2]^{1/3}\gg\frac{1}{x}. \tag{2.6}$$

The quantity $G_\pm$ is in fact the aforementioned single parameter of the asymptotic form. It is easy to verify that after making the substitution

$$D(xd_\pm)\to\delta(xd_\pm)$$

in formula (2.3) only the last exponential factor is integrated with respect to x. The integral of this factor converges at $x\lesssim 1$ and is equal to

$$\pi^{1/2}/2(1-\sin^2\Phi\cos^2\varphi)^{1/2}.$$

Thus, the final result for F takes the form

$$F=\frac{9\pi^{1/2}}{2^9}BV\left(\frac{\hbar\Omega}{\varepsilon_F}\right)^{7/3}\sin^2\Phi\sin^2\psi\times\sum_\pm\frac{(\zeta_{xy}\cos\varphi_\pm-\zeta_{yz}\sin\varphi_\pm)^2}{[\Phi_{cr}^2\sin^2\psi\cos^2\Phi+(\Phi_{cr}\cos\psi\pm\sin\Phi)^2]^{1/2}}\times[\Phi_{cr}^2\sin^2\psi+(\Phi_{cr}\cos\psi\pm\sin\Phi)^2]^{-1}. \tag{2.7}$$

The applicability of this formula is restricted by the condition that the third (proportional to x) term in the d-function (2.4) be much less than the sum of the two first terms. An analysis of (2.4) shows that if we disregard the third term we have

$$d_\pm^2=G_\pm^2\Gamma^2\sin^2(\varphi-\varphi_\pm). \tag{2.8}$$

This indicates directly that the third term can be left out if

$$G_\pm\Gamma\gg\frac{3}{8}\left(\frac{\hbar\Omega}{\varepsilon_F}\right)^{1/3}\cos^2\varphi_\pm\sin^2\Phi.$$

Combining this inequality with (2.6), we obtain a final criterion in the form

$$G_\pm\gg 1+\left\{\frac{\Omega}{\nu}\sin^2\Phi\left[\Phi_{cr}^2\sin^2\psi\Gamma^{-2}+\sin\Phi\left(\frac{\Omega}{\nu}\right)^{1/2}\right]\right\}^{1/3}. \tag{2.9}$$

In practice this always reduces to the condition (2.6).

Let us dwell on the inverse limiting case

$$G_\pm\ll 1. \tag{2.10}$$

It is easy to conclude from (2.6) that this situation is possible only at small angles $\Phi \lesssim \Phi_{cr}$. The smallness of the parameter $G_\pm$ means that in the vicinity of the points $\varphi_\pm$ both the derivative $\partial d_\pm/\partial\varphi$ and the function $d_\pm$ itself are close to zero. We use this circumstance and replace $D(xd_\pm)$ in (2.3) by the constant $1/\pi\Gamma$. Since $\Phi \ll 1$, the argument of the exponential is simply $-x^2$, the sum $\sum$ yields the factor 2, and elementary integration with respect to x and φ in (2.3) leads to

$$F=\frac{3}{2^6}BV\left(\frac{\hbar\Omega}{\varepsilon_F}\right)^2\frac{\Omega}{\nu}[\zeta_{xy}^2+\zeta_{yz}^2+2(\zeta_{xy}\cos\psi-\zeta_{yz}\sin\psi)^2]. \quad (2.11)$$

Comparison of the third term for $d_\pm$ in (2.4) with Γ leads to the inequality $\sin^2\Phi \ll \nu/\Omega$. Just as before, however, it imposes no additional restrictions on the angle Φ, since it is less stringent than the general criterion (2.10) for the validity of (2.11).

4. Expression (2.7) for F is due to the collisionless mechanism of the energy dissipation of the moving dislocation, and describes almost all the limiting cases. What is singled out, however, is a very small region of angles Φ and ψ (Φ is close to zero and ψ is close to zero or π), when the δ-function contribution written out in (2.7) becomes small. Mathematically this means that the residue of the function at the point $\varphi=\varphi_\pm$ is equal to zero and it is necessary to take into account the contributions of the remaining $\varphi \neq \varphi_\pm$, which are proportional to Γ.

To this end, we again neglect the third term in (2.4), and first integrate in (2.3) with respect to x with logarithmic accuracy:

$$I=\int_0^\infty dx\, xD(xd_\pm)e^{-Ax^2}\approx\frac{\Gamma}{2\pi d_\pm^2}\ln\left(1+\frac{d_\pm^2}{A\Gamma^2}\right). \quad (2.12)$$

Here $A\equiv 1-\sin^2\Phi\cos^2\varphi$. Then, prior to the integration with respect to φ, the intermediate formula for F takes the form

$$F=\frac{3^3}{\pi 2^{13}}BV\left(\frac{\hbar\Omega}{\varepsilon_F}\right)^5\frac{\nu}{\Omega}\sum_\pm\oint d\varphi(\zeta_{xy}\cos\varphi -\zeta_{yz}\sin\varphi)^2\frac{\cos^2(\varphi-\psi)}{d_\pm^2}\ln\left(1+\frac{d_\pm^2}{A\Gamma^2}\right). \quad (2.13)$$

In the case (2.10) we have $d_\pm^2/A\Gamma^2 \ll 1$ and after expanding the logarithm we obtain directly (2.11).

If we use now the formula (2.8) and the identity

$$\cos^2(\varphi-\psi)=\cos^2(\varphi_\pm-\psi)+\sin(\varphi_\pm-\varphi)\sin(\varphi+\varphi_\pm-2\psi),$$

then we can represent (2.13) in the form

$$F=\frac{3}{2^{13}\pi}BV\left(\frac{\hbar\Omega}{\varepsilon_F}\right)^5\frac{\nu}{\Omega}\sum_\pm[\Phi_{cr}^2\sin^2\psi+(\Phi_{cr}\cos\psi\pm\sin\Phi)^2]^{-1} \times\oint d\varphi(\zeta_{xy}\cos\varphi-\zeta_{yz}\sin\varphi) \times\ln\left(1+\frac{d_\pm^2}{A\Gamma^2}\right)\left\{\frac{\cos^2(\varphi_\pm-\psi)}{\sin^2(\varphi-\varphi_\pm)}+\frac{\sin(\varphi+\varphi_\pm-2\psi)}{\sin(\varphi_\pm-\varphi)}\right\}. \quad (2.14)$$

It is easy to verify that at large $G_\pm$ (2.9) the first term in the curly brackets of (2.14) yields, in order of magnitude, the δ-function contribution already written out by us in (2.7). Indeed, at $G^2 \gg 1$ we have

$$\int_{-\infty}^{\infty}\frac{dy}{y^2}\ln(1+G^2y^2)=2\pi G\int_{-\infty}^{\infty}dy\,\frac{1}{\pi}\frac{G^{-1}}{G^{-2}+y^2}\cong 2\pi G.$$

The approximate answer obtained in this manner differs from (2.7) by a factor $\pi^{1/2}/2$. The reason is that the calculation of the integral (2.12) is asymptotically exact only at small and large values of the parameter $d_\pm^2/A\Gamma^2$. If it is approximately equal to unity then the result (2.12) is valid only in order of magnitude. This is precisely the situation at $G \gg 1$. In the integral with respect to y written out above the important quantities are $y \propto G^{-1}$, i.e., the argument of the logarithm in (2.12) is indeed of the order of unity.

This remark does not pertain to the second fraction in the curly brackets of (2.14). It describes the sought damping-force increment due to the collision absorption of the dislocation phonons. Bearing inequality (2.9) in mind, we can neglect within the logarithmic accuracy used by us the unity term and the quantity $\sin^2(\varphi-\varphi_\pm)/A$ under the logarithm sign, and the final result for δ_F in the form

$$\delta F=\frac{3^3}{2^{11}}BV\left(\frac{\hbar\Omega}{\varepsilon_F}\right)^5\frac{\nu}{\Omega}\sum_\pm\frac{\ln G_\pm}{\Phi_{cr}^2\sin^2\psi+(\Phi_{cr}\cos\psi\pm\sin\Phi)^2}\mathcal{F}_\pm(\hat{\zeta}), \quad (2.15)$$

where

$$\mathcal{F}_\pm(\hat{\zeta})=\frac{1}{2\pi}\int_0^{2\pi}d\varphi(\zeta_{xy}\cos\varphi-\zeta_{yz}\sin\varphi)^2\frac{\sin(\varphi_\pm+\varphi-2\psi)}{\sin(\varphi_\pm-\varphi)}.$$

The explicit form of the factor $\mathcal{F}_\pm$ is not very important, since the main anisotropy $\delta F(\Phi, \psi)$ is described by the denominator of the first formula of (2.15). This is all the more valid since the angular dependence of $\mathcal{F}_\pm$ is determined mainly by the type of location (edge or screw) and by its orientation in the crystal. Therefore the actual formulas that describe the "smooth" anisotropy of $F(\psi)$ depend on the model and are of no interest.

It may seem at first glance that in those situations when F of (2.7) becomes smaller than δF of (2.15) it is necessary to retain, besides the δ_F term, also all the collision terms with $s>0$. Analysis shows, however, that the sth term in the sum (2.1) differs from the term with $s=0$ only by the value of the D-function, i.e., by a factor of the order of

$$(\varepsilon_F/\hbar\Omega)^s[\Phi_{cr}\sin^2\psi+(\Phi_{cr}\cos\psi\pm\sin\Phi)^2]^s \ll 1.$$

At the same time, the sum over s converges well because $D \sim 1/s^2$. Thus, with respect to the parameter (1.1) (and all the more at small $\Phi \lesssim \Phi_{cr}$) the electronic transitions with $s \neq 0$ result in a negligibly small increment.

5. In concluding this section, we consider the single case in which the third term of the $d_\pm$ function becomes the principal one. As seen from (2.9), it corresponds to the inequality

$$\left\{\frac{\Omega}{\nu}\sin^2\Phi\left[\Phi_{cr}^2\sin^2\psi\Gamma^{-2}+\sin\Phi\left(\frac{\Omega}{\nu}\right)^{1/2}\right]\right\}^{1/5} \gg G_\pm.$$

From the definition of $G_\pm$ in (2.6) it follows that this situation is realized in the geometry $\psi=0,\pi$ at small deviation angles $\Phi \sim \Phi_{cr}$ such that

$$|\Phi_{cr}-\Phi|^2 \lesssim \frac{9}{64}\Phi_{cr}^2\frac{\nu}{\Omega}\left(\frac{\hbar\Omega}{\varepsilon_F}\right)^3. \quad (2.16)$$

If the condition

$$\Phi_{cr}^2 \gg \nu/\Omega \quad (2.17)$$

is additionally satisfied, then we can replace the D

function in (2.3) by $\delta(x^2\Gamma\Omega\nu^{-1}\cos^2\varphi\sin^2\Phi)$, and elementary integration yields

$$F=\frac{3\pi}{2^6}BV\left(\frac{\hbar\Omega}{\varepsilon_F}\right)^2\frac{\zeta_{xy}^2+\zeta_{yz}^2}{\sin^2\Phi_{cr}}=\frac{\pi}{12}\frac{Bv_F^2}{V}(\zeta_{xy}^2+\zeta_{yz}^2). \quad (2.18)$$

Thus, formulas (2.7), (2.11), (2.15), and (2.18) constitute the solution of our problem.

3. DISCUSSION OF RESULTS

1. We note first that $\sin\Phi \gg \Phi_{cr}$ almost always, since the dislocation velocity is much less than the Fermi velocity. This means that the principal parameter of the asymptotic form

$$G_{\pm}=\frac{8}{3}\left(\frac{\varepsilon_F}{\hbar\Omega}\right)^{1/2}\frac{\Omega}{\nu}\sin\Phi$$

is large at large values of Φ. Using (2.7), we obtain

$$F=\frac{9\pi^{1/2}}{2^6}BV\left(\frac{\hbar\Omega}{\varepsilon_F}\right)^{7/2}\frac{\zeta_{yz}^2\sin^2\psi}{\sin\Phi}. \quad (3.1)$$

In the ultraquantum limit this expression holds true in a very wide range of variation of both the angle parameters and the dislocation velocities. It is seen from (3.1) that in strong magnetic fields the damping force increases substantially, like $H^{7/2}$. No such behavior appears at moderately strong magnetic fields; on the other hand, in the ultraquantum limit (1.1) it is due mainly, as indicated in the introduction, to the monotonic growth of the state density at the level of the chemical potential and to the appreciable decrease of the role of the drift motion of the electron along the vector **H**.

The "dislocation friction" force remains linear with the velocity. Eq. (3.1) does not contain the electron frequency ν. This means that almost any geometry the absorption of the dislocation phonon is of the collisionless type, i.e., during the time between the collision the electron "manages" to interact effectively with the strain field. The quantity $\zeta_{yz}\equiv\Lambda_{yz}/\varepsilon_F$, which enters in (3.1), can be a rather large number because the Fermi energy decreases like $N_e^{2/3}$ for substances with low carrier density. A comparison of F from (3.1) with the deceleration force in the absence of a magnetic field (see, e.g., Ref. 4) yields

$$F(H)/F(0)\approx 0{,}1(\hbar\Omega/\varepsilon_F)^{7/2}.$$

2. Formula (3.1) does not work in two rare cases. The first is the region of relatively small angles: $\Phi \leqslant \Phi_{cr}$. The second is when $\sin\psi$ is small. Let us discuss these situations separately.

Let $\psi \neq 0, \pi$. It is seen from (3.1) that when Φ decreases the deceleration force increases like $1/\sin\Phi$. This variation of $F(\Phi)$ is described by the right-hand wing of the dashed line in Fig. 2a. It continues down to small angles $\Phi \approx \Phi_{cr}$. At smaller values of Φ it is necessary to use the more accurate formula (2.7). This formula reflects the fact that at $\Phi \approx \Phi_{cr}$ the $F(\Phi)$ curve has a smeared-out maximum. On its plot, F reaches a value

$$F_{max}\approx\frac{3\pi^{1/2}}{2^6}Bv_F\left(\frac{\hbar\Omega}{\varepsilon_F}\right)^{3/2}\frac{\zeta^2}{\sin\psi}. \quad (3.2)$$

With further decrease of Φ, formula (2.7) also ceases to be valid.

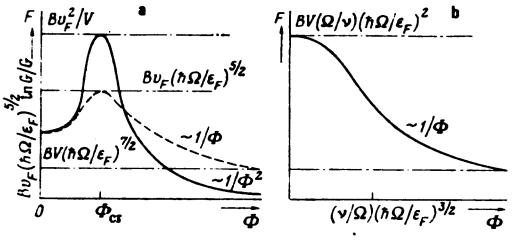

FIG. 2. Damping force vs the inclination angle Φ: a) $V/v_F \gg (\nu/\Omega)(\hbar\Omega/\varepsilon_F)^{1/2}$, dashed—arbitrary ψ, solid—geometry satisfying the condition (3.4); b) $V/v_F \ll (\nu/\Omega)(\hbar\Omega/\varepsilon_F)^{1/2}$.

The collisionless regime of phonon absorption gives way to a situation wherein an electron drifting along **H** does not manage to negotiate the distance between the neighboring "hills" of the inhomogeneous strains during the time between the collisions. Relation (2.15) now becomes valid and $F(\Phi)$ assumes a small constant value

$$F(0)=\frac{3}{2^6}B\frac{\hbar\nu}{mV}\left(\frac{\hbar\Omega}{\varepsilon_F}\right)^2\ln\left[\frac{8}{3}\left(\frac{\varepsilon_F}{\hbar\Omega}\right)^{1/2}\frac{\Omega}{\nu}\Phi_{cr}\right](\zeta_{xy}^2+\zeta_{yz}^2). \quad (3.3)$$

The transition from (2.7) to δF via (2.15) takes place at angles

$$\sin^2\Phi\sin^2\psi \lesssim \frac{3}{4\pi^{1/2}}\frac{\nu}{\Omega}\left(\frac{\hbar\Omega}{\varepsilon_F}\right)^{1/2}\Phi_{cr}. \quad (3.4)$$

This estimate follows directly from a comparison of F and δF.

The foregoing analysis is valid for small angles Φ if

$$G_{\pm}=\tfrac{8}{3}(\varepsilon_F/\hbar\Omega)^{1/2}(\Omega/\nu)\Phi_{cr}\gg 1.$$

In the opposite case of extremely slow dislocation motion,

$$\frac{V}{v_F}\ll\frac{1}{2}\left(\frac{\hbar\Omega}{\varepsilon_F}\right)^{1/2}\frac{\nu}{\Omega}, \quad (3.5)$$

the right-hand wing of $F(\Phi)$ from (3.1) assumes at

$$\sin\Phi\approx\frac{3}{8}\left(\frac{\hbar\Omega}{\varepsilon_F}\right)^{1/2}\frac{\nu}{\Omega}\gg\Phi_{cr}$$

a constant value determined by (2.11). The variation of $F(\Phi)$ is monotonic in this case (Fig. 2b).

3. The nonmonotonic angular dependence of the deceleration force is particularly emphasized in a special geometry wherein the dislocation moves in the yz plane and $\psi = 0, \pi$. In this case formula (2.7) is not suitable. The entire behavior of $F(\Phi)$ is described by relation (2.15) for δF. An exception is the vicinity of the maximum (2.16), where F coincides with (2.18). At $\Phi \leqslant \Phi_{cr}$ the deceleration force follows the dashed curve (3.3). At finite angles Φ the force F decreases much more steeply, $\propto \sin^{-2}\Phi$, to a value

$$F\approx\frac{3^3}{2^{11}}BV\left(\frac{\hbar\Omega}{\varepsilon_F}\right)^5\frac{\nu}{\Omega}\times\ln\left[\frac{8}{3}\left(\frac{\varepsilon_F}{\hbar\Omega}\right)^{1/2}\frac{\Omega}{\nu}\right]\times(\zeta_{xy}^2+\zeta_{yz}^2). \quad (3.6)$$

At large inclination angles (on the right-hand wing) the deceleration of the dislocation is much smaller than at $\Phi = 0$ [see (3.3)]. A plot of the function $F(\Phi)$ for this geometry is shown schematically by the solid line on Fig. 2a.

A common feature of all the curves shown in Fig. 2 is the substantial increase of the damping of the dis-

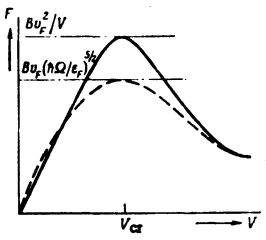

FIG. 3. Nonlinear dependence of damping force on the dislocation velocity. The notation is the same as in Fig. 2.

locations making small angles Φ with the external magnetic field **H**. This means that when curved dislocations move in a metal they may straighten out in the direction of **H**.

4. We discuss in conclusion the dependence of the damping force on the dislocation velocity. At large velocities

$$V \gg V_{cr} = v_F \frac{4}{3} \frac{\varepsilon_F}{\hbar\Omega} \sin\Phi$$

(this inequality is meaningful only at quite small values of $\sin\Phi$) we have $F(V) \propto 1/V$. This dependence is universal and is described by formula (3.3). The real situation wherein the dislocation moves slowly

$$V \ll V_{cr} \tag{3.7}$$

is characterized by the linear law $F \propto V$. The proportionality coefficient, however, depends on the geometry of the experiment. In the most general case ($\varphi \neq 0, \pi$) the function $F(V)$ varies in accord with relation (3.1), and its plot is shown dashed in Fig. 3. But if the dislocation moves in a plane that contains its axis and the magnetic field $H(\psi = 0, \pi)$, then the left-hand wing of the function $F(V)$ (the solid curve of Fig. 3) is given by the relation (3.6). Near $V = V_{cr}$ both curves go through a maximum. Since $V \ll v_F$, a nonlinear dependence of the force on the velocity exists only at small angles Φ.

Thus, an important feature of the ultraquantum limit is a noticeable strengthening of the metal, i.e., an appreciable increase of the resistance to the dislocation motion. Such a phenomenon can be observed under experimental conditions in semimetals and semiconductors with sufficiently low carrier density. For these substances, the ultraquantum limit (1.1) is realized in presently attainable magnetic fields

$$H(\text{kOe}) \geqslant 10^{-3} \frac{\pi^2 c\hbar}{2e} \left(\frac{3}{\pi}\right)^{1/3} N_e^{2/3} \approx 3.2 \cdot 10^{-10} N_e^{2/3}.$$

Another object of experimentation may be conductors under conditions of a phase transition of order $2\frac{1}{2}$, due, for example, to pressure.

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[1)] Absorption of ultrasound in this situation was investigated by Gokhfel'd.[5]

[1] V. Ya. Kravchenko, Pis'ma Zh. Eksp. Teor. Fiz. 12, 551 (1970) [JETP Lett. 12, 391 (1970)].
[2] G. Bellessa, Phys. Rev. Lett. 28, 668 (1972).
[3] V. D. Natsik and L. G. Potemina, Zh. Eksp. Teor. Fiz. 67, 240 (1974) [Sov. Phys. JETP 40, 121 (1975)].
[4] A. M. Grishin, É. A. Kaner, and É. P. Fel'dman, Zh. Eksp. Teor. Fiz. 70, 1445 (1976) [Sov. Phys. JETP 43, 753 (1976)].
[5] V. M. Gokhfel'd, Zh. Eksp. Teor. Fiz. 69, 1683 (1975) [Sov. Phys. JETP 42, 855 (1975)].
[6] L. Landau, Z. Phys. 64, 629 (1930).
[7] J. I. Pankove, Optical Processes in Semiconductors, Prentice-Hall, 1964.
[8] H. B. Brandt and S. M. Chudninov, Elektronnaya struktura metallov (Electronic Structure of Metals), Moscow Univ., 1973.
[9] É. A. Kaner and V. G. Skobov, Zh. Eksp. Teor. Fiz. 53, 375 (1967) [Sov. Phys. JETP 26, 251 (1968)].

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