achieved suggests that the theoretical relationships used in the present work will be suitable for the description of cryogenic afterglow. On the other hand, experiment confirms the validity of the coefficients β and $\alpha(T_e)$ used in the theoretical analysis of the decay process.

An important feature of decay in cryogenic plasma is that hot electrons reaching the discharge-chamber walls nevertheless occupy the energy level $\varepsilon \approx kT_e$ within the volume, i.e., they contribute to the overall cold-electron density balance. This ensures that the density balance between n and M remains constant in a broad range of pressures. The relaxation of electron temperature is very dependent on the density ratio of two groups of hot electrons, i.e., electrons reaching the walls and electrons succeeding in relaxing within the volume of the chamber. The internal magnetic field controls this density ratio and, hence, the electron temperature. The magnetic field has its greatest effect when the ratio of the linear dimensions of the discharge chamber is $R/L \ll 1$, and the pressures are chosen so that $\tau_d/\tau_{ea} \ll 1$.

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1) We note that, if we follow Deloche *et al.*[8] and assume that $\alpha_2 \lesssim 10^{-9}$ cm$^3\cdot$s^{-1}, this inequality is satisfied for much lower pressures.

2) A simple proof consists of solving (2) or (4) on the assumption that the particle and energy sources are strictly constant.

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Boundary relaxation of a vibrationally excited gas

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The relaxation of a nonequilibrium, vibrationally excited gas, consisting of symmetric diatomic molecules, due to collision with the wall enclosing the gas, is considered. Collisions with the walls remove the vibrational excitation of the molecules. The vibrational energy is distributed among the kinetic degrees of freedom of the molecule and the surface, which can be simulated by a set of harmonic oscillators. An equation is derived for the time variation of the temperature T of the kinetic degrees of freedom of the gas. When T is close to the temperature of the surface T_s and to the population temperature of the vibrational state T_v, this equation reduces to the relaxation type. The characteristic time τ of boundary relaxation depends on the parameters of the gas and of the surface. It is shown that in a certain range of the parameters, boundary relaxation may be more important than the usual relaxation due to pair collisions.

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The total energy of a molecule consists of the energy of the translational, rotational, vibrational and electronic motions. Under conditions of thermal equilibrium, the energy distribution of the molecules with respect to these degrees of freedom is characterized by a single quantity—the temperature. It is not possible to describe a noneqiulibrium molecular gas by a single temperature. The comparatively weak coupling between the different degrees of freedom and the essentially quantum character of the vibrational states of the molecule (even in the region of rather high temperatures) leads to the result that the energy relaxation times between different degrees of freedom significantly exceed the relaxation time within each of them. This allows us to introduce the temperature of the population of states of each of the indicated degrees of freedom. If the non-equilibrium gas is left to itself, then an equilization of these temperatures takes place in it as the gas approaches the state of thermal equilibrium.

One usually starts out from the representation that at comparatively high gas concentrations the relaxation takes place as a result of collisions of the particles with one another (volume relaxation). The effect of the surface bounding the gas is neglected in this case. It can be shown that at not very high pressures, the particle traverses a path that is comparable with the characteristic dimensions of typical gas-discharge systems within the relaxation time of the vibration. In this case, it is necessary to take into consideration the relaxation processes that occur as a result of collisions of the gas molecules with the walls of the container (boundary relaxation). Actually, according to Ref. 1, 10^5 collisions are required for the vibrational relaxation of oxygen at $T=900$ K and a gas concentration of $n\sim 10^{16}$ cm^{-3}. The average relaxation time turns out to be of the order of $\tau\sim 10^{-1}$ sec. The molecule is displaced a distance $(\bar{v}l\tau)^{1/2}\sim 50$ cm in this time ($\bar{v}$ is the mean thermal speed of the molecule, l is the free path length).

Collisions of an excited molecule with the surface leads to transitions between its vibrational levels with a certain probability, with a further redistribution of the energy of transition between the degrees of freedom of the surface, translational and rotational motions of the molecule. Excitation of vibrations of the molecule is also possible in scattering at the surface. However, if the surface temperature T_s is less than the temperature T_v of the population density of the vibrational states, then the collisions with the surface will be accompanied by preferential de-excitation of the molecule with subsequent "heating" both of the surface and of the translational degrees of freedom of the particle. Such a mechanism of relaxation of the states of diatomic, symmetric molecules is considered in the present work.

A model was proposed by us earlier[2] for the interaction of a symmetric diatomic molecule with a surface. The model was based on the assumption that the characteristic time of collision of the molecule with the surface $a/\bar{v}$ is less than the characteristic time of intramolecular motions $1/\omega_0$ (a is the effective length of the interaction, ω_0 is the frequency of vibration of the molecule). This enables us to consider the collision as an instantaneous blow on the solid wall, without specifying the form of the potential of interaction with the surface. A physical model of the surface was represented by a set of harmonic oscillators. The total differential probability of transition into states of the molecule accompanied by absorption of a quantum $\hbar\omega$ by the surface can be calculated within the framework of the method of sudden perturbations. It is equal to

$$dw(nlv\rightarrow n'l'v')=\frac{4}{3}\frac{vv'}{\pi^2\gamma\hbar c^3}\frac{\omega\,d\omega}{\exp(\hbar\omega/T_{\text{п}})-1}w(nl\rightarrow n'l'). \tag{1a}$$

The probability of a transition with excitation of a vibrational state of the molecule and with a decrease of the surface energy by

$$dw(nlv\rightarrow n'l'v')=\frac{4}{3}\frac{vv'}{\pi^2\gamma c^3\hbar}\frac{\omega\,d\omega}{1-\exp(-\hbar\omega/T_{\text{п}})}w(nl\rightarrow n'l'), \tag{1b}$$

is

$$w(nl\rightarrow n'l')=\frac{(2l+1)(2l'+1)}{q^2\rho_0^2}\frac{n'!}{n!}x^{n-n'}$$

$$\times\, e^{-x}[L_n^{n-n'}(x)]^2\Delta(ll',nn'), \tag{2}$$

where

$$x=\frac{q^2\hbar}{2\mu\omega_0},\qquad \Delta(ll',nn')=1-\exp\{i\pi(l+l'+n-n')\}\cos 2q\rho_0.$$

Here n and l are the vibrational and translational quantum numbers of the molecule, ρ_0 is the equilibrium distance between the nuclei of the molecule, $q=M(v+v')/4\hbar$, M is the mass of the molecule, v and v' are its velocity before and after the collision, $L_n^\alpha(x)$ is a Laguerre polynomial, γ is the density of the wall material, c is the sound velocity in this material, and $\mu=M/2$ is the reduced mass of the molecule. These expressions are valid for the dispersion law of acoustical phonons $\omega=kc$ and for an equilibrium distribution of their energies.

To obtain the total transition probability averaged over the states of the surface it is necessary to integrate the expression (1) with respect to $d\omega$ with account of the law of conservation of energy of the molecule and the surface:

$$\hbar\omega=\hbar\omega_0+\frac{M}{2}(v^2-v'^2)+\frac{\hbar^2}{2I}[l(l+1)-l'(l'+1)]=\hbar\omega_0+\varepsilon-\varepsilon',$$

I is the moment of inertia of the molecule. At not very high temperature, $T_i, T_v, T_s\ll\hbar\omega_0$ the principal fraction of the excited molecules is located in the first vibrational level. Therefore, the principal contribution to the probability (1) is made by single-quantum transitions with change in the vibrational energy of the molecule by $\hbar\omega_0$, which is distributed among the surface oscillators and the translational and rotational degrees of freedom of the molecule. The energy $\hbar(\omega-\omega_0)$ is left for the kinetic degrees of freedom of the molecule.

The change in the temperature T of these degrees of freedom per unit time (the translational and rotational motions of the molecule are in equilibrium and are practically always classical) can be obtained by integrating over $d\omega$ product of the molecular current by the sum of the probability (1a) multiplied by $\hbar(\omega_0-\omega)$ and multiplied by the probability (1b) $\hbar(\omega-\omega_0)$,

$$\frac{N}{V}\left(\frac{T}{2M}\right)^{1/2}\frac{M\hbar^2}{2IT^2}\exp\left[-\frac{Mv^2}{2T}-\frac{\hbar^2l(l+1)}{2IT}\right]v\,dv$$

over the entire surface of the wall S. We replace integration over $d\omega$ by integration over v, v' and summation over l, l'. Since the rotational quantum $\hbar^2/2IT\ll 1$, summation over l, l' reduces to integration. The law of energy conservation determines the limits of the integration.

The following inequality holds in the range of temperatures considered:

$$x=M(v+v')^2/16\hbar\omega_0\sim T/\hbar\omega_0\ll 1,$$

and enables us to simplify the expressions for the transition probabilities (1a) and (1b), representing them in the form of a power series in x. Limiting ourselves to terms that are linear in x, we obtain

$$\frac{5}{2}\frac{dT}{dt}=\frac{2^{7/2}}{3\pi^{5/2}}\frac{S}{V}\frac{M^{1/2}T^{11/2}}{\gamma c^3\hbar^4\omega_0}\frac{\exp\{-\hbar\omega_0/2T_\kappa\}}{\operatorname{sh}\{\hbar\omega_0/2T_\kappa\}}$$
$$\times\int_0^\infty dx\int_0^\infty dy\int_0^X dx'\int_0^Y dy'(xx')^{1/2}\frac{\hbar^2\omega(\omega_0-\omega)}{T}e^{-(x+y)}$$
$$\times\frac{1-\exp\{-\alpha\hbar\omega_0/T+(\alpha-\beta)\hbar\omega_0/T\}}{1-\exp(-\hbar\omega/T_s)}, \tag{3}$$

where

$$\alpha=T/T_s-1,\quad \beta=T/T_v-1,\quad x=Mv^2/2T,\quad x'=Mv'^2/2T,$$
$$y=\frac{\hbar^2 l(l+1)}{2IT},\quad y'=\frac{\hbar^2 l'(l'+1)}{2IT},\quad \frac{\hbar\omega}{T}=\frac{\hbar\omega_0}{T}+x+y-x'-y',$$
$$X=\hbar\omega_0/T+x+y,\quad Y=\hbar\omega_0/T+x+y-x',$$

N is the total number of particles in the system, V is its volume, and the rapidly oscillating terms in the expression $\Delta(ll',\ nn')$ disappear upon integration.

Calculation of the integral (3) at arbitrary values of α and β can be carried out by numerical methods. Here we shall use approximate analytic integration. We isolate the region of integration into two regions, in which $\hbar\omega/T>1$ and $\hbar\omega/T<1$. In these regions, the integration can be carried out analytically. It was shown earlier[2] that the region of large energy transfer $\hbar\omega/T>1$ makes the principal contribution to (3). Taking this into account, we neglect in the integrand the exponentials with the indices $-\hbar\omega/T$, $-\hbar\omega/T_s$. After integration, we obtain an equation describing the change in the kinetic temperature of the gas T:

$$\frac{dT}{dt}=0.04\frac{S}{V}\frac{MT^4}{\pi^{3/2}\gamma\hbar^4\omega_0c^3}\left(\frac{T}{2\pi M}\right)^{1/2}\left(\frac{\hbar\omega_0}{T}\right)^{9/2}\frac{\exp\{-\hbar\omega_0/2T_v\}}{\operatorname{sh}\{\hbar\omega_0/2T_v\}}$$
$$\times\left(1+\frac{25}{4}\frac{T}{\hbar\omega_0}\right)\left(1-\exp\left[(\alpha-\beta)\frac{\hbar\omega_0}{T}\right]\right). \tag{4}$$

To isolate the characteristic time of boundary relaxation, we assume that all the temperatures are of similar magnitude: $T\approx T_s\approx T_v$. Expanding the exponential in the last factor of (4), we obtain the relaxation equation

$$\frac{dT}{dt}=\frac{T_v+T_s-2T}{\tau}. \tag{5}$$

The quantity

$$\tau\approx 5\cdot 10^2\frac{V}{S}\gamma\left(\frac{c}{\omega_0}\right)^3\frac{\exp(\hbar\omega_0/T_v)}{(M\hbar\omega_0)^{1/2}} \tag{5a}$$

is equal to the mean time in which the vibrational excitation of the molecule is removed as a result of collisions with the surface. We shall call this the boundary relaxation time.

With the purpose of making clear the physical meaning of the parameters entering into τ, we consider the following elementary reasoning. The number of collisions ν_0 of the excited molecule with the wall per unit time is proportional to the surface of the wall, to the velocity of the molecule, and to the population of the corresponding vibrational level

$$\nu_0\sim\frac{S}{V}\left(\frac{T}{M}\right)^{1/2}\exp\left(-\frac{\hbar\omega_0}{T_v}\right).$$

In view of the large value of the vibrational quantum $\hbar\omega_0$, not every collision is accompanied by the removal of the vibrational excitation. The number of collisions where the energy of vibration of the molecule is realized can be estimated as

$$\nu\sim\nu_0 M/m,$$

where m is the mass of the particles involved in the vibrational motion of the surface.

We introduce the average frequency $\bar{\omega}$ of the vibrations of the surface, which are excited by the molecule in the collisions; then $m\sim\gamma(c/\bar{\omega})^3$, since $c/\bar{\omega}\sim\lambda$ is the wavelength of these vibrations. The condition $T\approx T_s\approx T_v$ enables us to assume that $T\approx\hbar\omega$. The boundary relaxation time is inversely proportional to ν; therefore,

$$\tau\sim\frac{S}{V}\gamma\left(\frac{c}{\bar{\omega}}\right)^3\frac{\exp(\hbar\omega_0/T_v)}{(M\hbar\bar{\omega})^{1/2}}.$$

The parameter $\bar{\omega}$ remains undetermined in this relation. It can be connected with the frequency of vibrations of the molecule ω_0 by comparison of the obtained expression with the exact (5a). As a result, we get $\bar{\omega}\approx\omega_0/6$.

We now compare the obtained expression for τ with the characteristic time of volume relaxation

$$\tau_{vol}\approx\eta/\sigma n v,$$

where η is the number of collisions necessary for vibrational-translational relaxation, σ is the gaskinetic cross section, n is the gas concentration, v is the thermal speed of the molecule. The boundary relaxation becomes significant at $\tau<\tau_{vol}$. This takes place when

$$n\frac{V}{S}<2\cdot 10^{-3}\frac{\eta M}{\gamma\sigma}\left(\frac{\hbar\omega_0}{T}\right)^{1/2}\left(\frac{\omega_0}{c}\right)^3\exp\left(-\frac{\hbar\omega_0}{T_v}\right).$$

As an example, we consider a cylindrical glass tube filled with oxygen at $T\approx T_v\sim 900$ K, when $\eta\sim 10^5$ (Ref. 1). The parameters characterizing the oxygen molecule and the surface are: $M=3\times 10^{-23}$g, $\hbar\omega_0=0.2$ eV, $\sigma=3.6\times 10^{-15}$ cm^2, $\gamma=3$g/cm^3, $c=5\times 10^5$ cm/sec. For these parameters, the boundary relaxation becomes important at $Vn/S<10^{19}$ (V/S is measured in centimeters, n is cm^{-3}). Typical dimensions of laboratory gas-discharge systems have $V/S\sim 10$ cm. Thus boundary relaxation should predominate in it right up to concentrations $n\sim 10^{18}$ cm^{-3}.

In conclusion, we note that the characteristic time of boundary relaxation can be introduced only at $T\approx T_v$. In the case of a large difference between the vibrational temperature and the kinetic temperature of the gas, it is necessary to start from the general form of Eq. (4). However, even in this case, the equation obtained for τ can be used for the estimates.

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Bleaching of the region of electron cyclotron absorption in a plasma in an inhomogeneous magnetic field

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The results are given of an investigation of the increase in the transparency (bleaching) in the region of electron cyclotron absorption of an electromagnetic wave traveling in a plasma along an inhomogeneous magnetic field. It is shown that measurements of the absorption of electromagnetic waves in a plasma subjected to a magnetic field of known geometry can be used to determine the electron density.

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1. INTRODUCTION

An inhomogeneous magnetized plasma is characterized by the existence of zones (barriers) of total absorption or strong collisionless attenuation of waves traveling in it. It is interesting to create conditions in a plasma that facilitate the penetration of a wave field in the form of undamped oscillations (bleaching). The need for such a bleaching is frequently encountered when hf and microwave methods are used to heat a plasma in thermonuclear experiments.

There are various ways of bleaching which can be divided arbitrarily into local and nonlocal. The local effects are realized under the conditions of validity of the hydrodynamic approximation. One should mention here particularly the penetration of a large-amplitude field into an opaque medium and wave transformation.[1,2] Plasma bleaching occurs as a result of transformation of oscillations because the region of a plasma under discussion becomes transparent to the secondary wave formed as a result of such transformation.

The nonlocal effects are the echo-type kinetic phenomena in which plasma memory elements in one of the microscopic oscillations of the distribution function becomes manifest. In contrast to the homogeneous case, an inhomogeneous plasma may exhibit linear echo effects, such as nonlocal reflection of waves and their regeneration behind the wave barrier. We shall consider the bleaching effect in a region of electron cyclotron absorption as a result of interference of waves reflected from the cyclotron resonance points in a plasma subjected in an nonmonotonically varying magnetic field.

2. THEORY

We shall consider the profile of a plasma-confining magnetic field (Fig. 1) used in the experiments described below. An electromagnetic wave travels from the left to right and its frequency is ω varying from ω_{e0} to ω_{e1}, where ω_{e0} and ω_{e1} are the minimum and maximum electron cyclotron frequencies, respectively. Under these conditions there are two points of electron cyclotron resonance in the plasma, $\omega=\omega_e(z)$, separated by a distance l. The resonance at the first point can be treated in the same way as in the case of a linearly decreasing magnetic field when a wave travels in the direction of reduction in the field. It has been shown[3,4] that under these conditions there is partial absorption of a wave without reflection. At the second point the resonance occurs when a wave travels in the plasma in the direction of increase in the magnetic field. Under these conditions there is partial reflection of an incident electromagnetic wave. If the electron density in the plasma is sufficiently high, the wave is practically totally reflected. The phase of the wave changes by the angle $\Delta\varphi$ as a result of such reflection.

If there are two resonance points separated in space, a wave reflected from the point 2 reaches the point 1 and the latter is the reflection point for the wave because it now travels in the direction of increasing magnetic field. A characteristic capture of oscillations in-

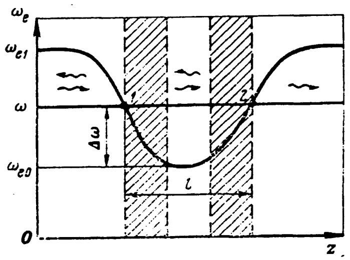

FIG. 1. Variation of the electron cyclotron frequency ω_e along the z axis of the system. The total absorption region is shown shaded.

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