[1]The question of the applicability of the Fermi-liquid approximation to our system remains open from the theoretical point of view. This approximation does not hold for ordinary ordered one-dimensional systems of fermions with spins, since a charge-density-wave instability and a superconducting instability or a spin-density-wave instability alter the ground state of the system.[8,9] In a system of spinless fermions there is no spin-density-wave instability or a superconducting instability with momentum $q=0$. In a strongly disordered system we have also no charge-density-wave instability or a superconducting instability with momenta $q\neq 0$. Thus, a system of spinless interacting Fermi particles, equivalent to the Heisenberg Hamiltonian of spin-1/2 particles with random interaction, constitutes a normal Fermi liquid. In the case of a normal three-dimensional homogeneous Fermi liquid the damping of the quasiparticles near the Fermi level is small because of the constraints imposed on the decay by the energy and momentum conservation laws (see Note Added in Proof).

[2]We know now of magnetic crystals that can be described within the framework of a one-dimensional model of spins with $n=1$, 2, and 3.[11] It is not clear as yet, however, whether exchange interaction with sufficiently strong disorder can be realized in real compounds in the situations $n=1$ and 2.

[3]The conclusion that χ increases as $T\to 0$ in the Heisenberg model as $\alpha=0$ is cited in Ref. 2. It follows from (11) that the growth of χ as $T\to 0$ in the continuous models ($n=2$, 3) occurs when $f_J(x)$ decreases as $x\to 0$, but not faster than logarithmically, i.e., not faster than $|\ln x|^{-\beta}$ with $1>\beta>0$.

[4]If we consider two neighboring clusters with strong interaction I_0 within the clusters and weak interaction $J_k\ll I_0$ for spins k and $k+1$ on their boundary, then the energy required to destroy the antiferromagnetic order for the spins k and $k+1$ is proportional to J_k only if each cluster has an odd number of spins, and furthermore in this case the proportionality coefficient depends on the number of spins in the clusters. In all the remaining situations the interaction energy of the clusters is proportional to J_k^2. Under these conditions all the excitations of the quantum chain can be delocalized, as is the case in the quantum XY model (see Ref. 7). The cluster interpretation cannot be used to describe excitations of the delocalized type.

[5]This result is connected with the fact that in a one-dimensional spin-1/2 system there is no order even at $T=0$.[14]

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Scattering of light in smectic *A*

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It is shown that because of an anomalous momentum dependence, fluctuations of the deviation of layers (the Landau-Peierls mode) lead to strong fluctuations of the modulus of the order parameter. This produces additional scattering of light, which can be observed at small scattering angles when there is zero momentum transfer in the plane of a layer.

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A smectic liquid crystal of type A is a system with one-dimensional periodicity. In it, as in a nematic, the molecules are oriented along a certain axis; and in addition, there is a density wave along this axis because of ordering of the centers of mass of the molecules. A smectic *A* is usually represented as a system of layers with a thickness of the order of the length of a molecule, in each of which elongated, rod-shaped molecules are arranged with their long axes along the normal to the layer. Fluctuations of the displacement of the layers in such a system, with one-dimensional periodicity, were considered by Landau and Peierls.[1] Let the layers be perpendicular to the z axis, and let u be the displacement of a layer from the equilibrium position. Then the free energy F of such a system, in the approximation quadratic with respect to u, has the

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form

$$F=\int dr\left\{B\left(\frac{\partial u}{\partial z}\right)^2+k\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)^2\right\}. \quad (1)$$

Here k is a quantity of the order of the Frank constants,[2] $k\sim 5\cdot 10^{-7}$ dyn; B and k are connected by the relation $B=q_1 k$, where q_1 is a momentum of the order of the reverse momentum q_0 of the structure. For CBOOA, for example, $q_1\sim 0.31$ Å^{-1} at 78 °C,[3] $q_0\sim 0.18$ Å^{-1}.[4] In consequence of the symmetry with respect to replacement of z by $-z$, the expansion (1) contains no term of the form

$$\frac{\partial u}{\partial z}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right).$$

The correlator $\langle u(\mathbf{q}),\ u(-\mathbf{q})\rangle$ corresponding to (1) has the form

$$G_0=\langle u(\mathbf{q}),u(-\mathbf{q})\rangle=\frac{T}{Bq_z^2+kq_\perp^4}=\frac{T}{k}\frac{1}{q_1^2q_z^2+q_\perp^4}. \quad (2)$$

The correlator (2) has an anomalous dependence on $\mathbf{q}_\perp$, and this leads to the well-known logarithmic divergence of the fluctuations in a system with one-dimensional periodicity.[1] But in a smectic A, because of the small coefficient of the logarithm, it can be neglected in a real experimental situation.[2]

Recent precision x-ray measurements[5] have confirmed the anomalous dependence of the correlator (2) on $\mathbf{q}_\perp$. Furthermore, it was found that the density wave in a smectic A is an almost ideal sinusoid (to within accuracy 10^{-4} in the intensity, no Bragg peaks were detected at $q=2q_0$ or $q=3q_0^5$). This means that the dependence of the density on the coordinate z has the form

$$\rho=\rho_0+d_0\cos(q_0z+q_0u).$$

A smectic liquid crystal is degenerate with respect to a shift of this density wave along the z axis. This enables us to apply certain general results of the theory of degenerate systems[6] regarding the longitudinal and transverse susceptibilities. In the present case, the role of transverse coordinate (the phase $\delta\varphi$) is played by the layer displacement u multiplied by q_0 ($\delta\varphi=uq_0$), of longitudinal by the amplitude d_0 of the density wave. Therefore the transverse susceptibility (except for a factor q_0^2) will be the correlator (2), the longitudinal the correlator $\langle\delta d(\mathbf{q}),\ \delta d(-\mathbf{q})\rangle$. The principle of conservation of the modulus in a degenerate system[6] enables us to obtain the contribution of the massless transverse fluctuations to the longitudinal. As a result we have

$$\delta d=-\tfrac{1}{2}d_0q_0^2u^2. \quad (3)$$

Here δd and u are taken at a single point of coordinate space. From the relation (3) we get an expression for the correlator:

$$G_1=\langle\delta d(\mathbf{q}),\ \delta d(-\mathbf{q})\rangle$$

$$=\frac{T^2d_0^2q_0^4}{2k^2}\int\frac{dp_z\,d\mathbf{p}_\perp}{(2\pi)^3}\frac{1}{q_1^2(p_z+q_z)^2+(\mathbf{p}_\perp+\mathbf{q}_\perp)^4}\frac{1}{q_1^2p_z^2+p_\perp^4}. \quad (4)$$

The integral in (4) has a singularity at $\mathbf{p}\to 0$ and at $\mathbf{p}\to-\mathbf{q}$. We consider, for example, the first case. Let $p_z\ll q_z$, $|\mathbf{p}_\perp|\ll|\mathbf{q}_\perp|$; then the first factor in (4) takes the form $(q_1^2q_z^2+q_\perp^4)$ and may be taken out from under the integral sign. The integral of the second factor diverges logarithmically. This integral can be estimated as

$$(8\pi q_1)^{-1}\ln\{\max(q_z/q_{z0},\ q_\perp^2/q_{\perp0}^2)\},$$

where q_{z0} and $q_{\perp0}$ are cutoff momenta, connected for example with the specimen size. In a real experimental situation, we may neglect these logarithmic divergences and set the logarithms equal to unity. This simplification only decreases the effect under consideration. A completely analogous estimate can be made for the case $\mathbf{p}\to-\mathbf{q}$. Thus we have

$$\int\frac{dp_z\,d\mathbf{p}_\perp}{(2\pi)^3}\frac{1}{q_1^2(p_z+q_z)^2+(\mathbf{p}_\perp+\mathbf{q}_\perp)^4}\frac{1}{q_1^2p_z^2+p_\perp^4}\approx\frac{1}{4\pi q_1}\frac{1}{(q_1^2q_z^2+q_\perp^4)}. \quad (5)$$

It is seen from (5) that G_1 has the same momentum dependence as has G_0. We shall estimate the value of $\langle\delta d(\mathbf{q}),\ \delta d(-\mathbf{q})\rangle d_0^{-2}$ and compare it with the correlator of phase fluctuations $\langle\delta\varphi(\mathbf{q}),\ \delta\varphi(-\mathbf{q})\rangle=q_0^2G_0$. By use of expressions (2), (4), and (5) it is easy to see that

$$\frac{G_1}{d_0^2G_0q_0^2}\approx\frac{1}{8\pi}\frac{Tq_0^2}{kq_1}.$$

On substituting the data for CBOOA, $T\sim 350$ K and $k\sim 5\cdot 10^{-7}$ dyn, we find that the relative magnitude of the fluctuations is ~0.05; that is, the relative magnitude of the fluctuations of modulus is an order smaller than the magnitude of the fluctuations of phase.

We consider light scattering due to the modulus fluctuations described by the correlator.[5]1) For the differential extinction coefficient, we have in the case of scalar scattering[8]

$$dh=\frac{\omega^4}{6\pi c^4}\langle\delta\varepsilon(\mathbf{q}),\delta\varepsilon(-\mathbf{q})\rangle\frac{3}{4}(1+\cos^2\theta)\frac{do}{4\pi}, \quad (6)$$

where $q^2=4\omega^2c^{-2}\sin^2(\theta/2)$; θ is the scattering angle, ω the light frequency. The change of permittivity $\delta\varepsilon$ as a function of δd can be written in the form

$$\delta\varepsilon=\frac{\partial\varepsilon}{\partial\rho}\delta d\cos(q_0z+q_0u)+\frac{1}{4}\frac{\partial^2\varepsilon}{\partial\rho^2}(\delta d)^2(1+\cos 2(q_0z+q_0u))+\ldots$$

Since $q_0\gg\omega/c$, the contribution of the first term will be exponentially small because of the rapidly oscillating factor. The contribution of the second term to the correlator of the fluctuations of permittivity will be determined by the following expression:

$$\langle\delta\varepsilon(\mathbf{q}),\ \delta\varepsilon(-\mathbf{q})\rangle=\frac{1}{8}\left(\frac{\partial^2\varepsilon}{\partial\rho^2}\right)^2\int\frac{d\mathbf{p}}{(2\pi)^3}G_1(\mathbf{p}+\mathbf{q})G_1(\mathbf{p}). \quad (7)$$

The integral in the expression (7) is estimated in exactly the same way as was the correlator G_1. Here again, for simplicity, we may omit the weak logarithmic divergence. We finally get

$$\langle\delta\varepsilon(\mathbf{q}),\delta\varepsilon(-\mathbf{q})\rangle\approx\frac{1}{2^{11}\pi^3}\left(\frac{\partial^2\varepsilon}{\partial\rho^2}\right)^2\frac{T^4d_0^4q_0^8}{k^4q_1^3}\frac{1}{q_1^2q_z^2+q_\perp^4}. \quad (8)$$

In a smectic A, two scattering processes are usually considered. One of them is due to ordinary fluctuations of density, the other to fluctuations of the orientation of the director (normal to a layer). The correlator of the fluctuations of density has the form

$$\langle\delta\rho(\mathbf{q}),\ \delta\rho(-\mathbf{q})\rangle=T/A. \quad (9)$$

Here $A=s^2\rho^{-1}$, where s is the velocity of sound and ρ is the density. The deviation δn of the director from the z axis is determined by the condition for constancy of the distances between layers:

$$\delta\mathbf{n}=-\mathbf{i}\frac{\partial u}{\partial x}-\mathbf{j}\frac{\partial u}{\partial y}.$$

Thus the correlator of these fluctuations has the form

$$\langle\delta\mathbf{n}(\mathbf{q}),\delta\mathbf{n}(-\mathbf{q})\rangle=\frac{T}{k}\frac{q_\perp^2}{q_1^2q_z^2+q_\perp^4}.$$

Since the corresponding contribution to the light scattering is proportional to this correlator, this contribution can be eliminated by choosing the geometry of the experiment such that $\mathbf{q}_\perp=0$. Therefore in order to estimate the possibility of observing the effect under consideration, we must compare the intensity of the light scattered by the ordinary fluctuations of density and by the anomalous longitudinal fluctuations of the order parameter when $\mathbf{q}_\perp=0$. For the ratio of the differential extinction coefficients of scattering by anomalous fluctuations of modulus dh_d and by fluctuations of density dh_ρ we can, by use of formulas (6), (8), and (9), obtain the following expression:

$$\frac{dh_d}{dh_\rho}\approx\frac{1}{2^{11}\pi^3}\frac{(\partial^2\varepsilon/\partial\rho^2)^2}{(\partial\varepsilon/\partial\rho)^2}\frac{d_0^4s^2T^3q_0^8}{\rho k^4q_1^3q_z^2}. \quad (10)$$

In the expression (10), $\mathbf{q}_\perp$ has been set equal to zero. The value of $\partial^2\varepsilon/\partial\rho^2$ can be estimated as $\partial^2\varepsilon/\partial\rho^2 \approx\rho^{-1}\partial\varepsilon/\partial\rho$. For estimation of d_0, we note that since the constants A and B describe the compressibility and the correction to it due to the density wave, $B\approx Ad_0^2$. Hence we obtain the estimate $d_0^2\approx kq_1^2\rho s^{-2}$. On substituting these estimates in (10), we find

$$\frac{dh_d}{dh_\rho}\approx\frac{1}{2^{11}\pi^3}\frac{T^3\rho q_0^8}{k^2s^2q_1}\frac{1}{q_z^2}. \quad (11)$$

We substitute in (11) the following data (corresponding to CBOOA): $T\sim 350$ K, $k\sim 5\cdot 10^{-7}$ dyn, $s\sim 1.5\cdot 10^5$ cm/sec, $\rho\sim 1$ g/cm³, $q_0\sim 2\cdot 10^7$ cm^{-1}, $q_1\sim 3\cdot 10^7$ cm^{-1}; we find then that $dh_d\sim dh_\rho$ when $q_z^2\sim 3\cdot 10^8$ cm^{-2}. If the scattered light has wavelength 6000 Å, then for scattering angle $\theta\lesssim 10°$ the scattering on the anomalous fluctuations of modulus will predominate. A characteristic feature of the anomalous scattering is a quadratic dependence of the intensity on the frequency, whereas scattering by density fluctuations behaves as the fourth power of the frequency. In the expression (11), the reverse momentum of the structure enters to the eighth power; therefore the mechanism under consideration will be most pronounced in materials with the smallest possible distance between layers (the size of a molecule).

Thus in a smectic A there can occur a scattering mechanism additional to the generally accepted one. It is due to the anomalous behavior of the fluctuations of the modulus of the order parameter. These fluctuations arise in consequence of strong massless fluctuations of the shift of the layers. This mechanism is most pronounced when the momentum transfer in the plane of a layer is zero ($\mathbf{q}_\perp=0$). Then the anomalous scattering will predominate at small scattering angles. Its contribution can be separated, for example, by use of the variation of its intensity with q_z or with the light frequency.

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1) Light scattering caused by the contribution of the transverse fluctuations to the longitudinal was treated earlier by Kats and Pokrovskiĭ for the case of a nematic.[7]

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