Fiz. **68**, 1883 (1975) [Sov. Phys. JETP **41**, 944 (1975)].
[3]W. Maier and A. Saupe, Z. Naturforsch. **a13**, 564 (1958); **a14**, 882 (1959); **a15**, 287 (1960).
[4]O. Klein, Medd. Vetenskapsakad. Nobelinst. **5**, No. 6 (1919).
[5]T. H. Berlin and E. W. Montroll, J. Chem. Phys. **20**, 75 (1952).
[6]M. J. Stephen and J. P. Straley, Rev. Mod. Phys. **46**, 617 (1974).
[7]I. N. Makarenko, V. A. Ivanov, and S. M. Stishov, Prib. Tekh. Eksp. No. 3, 202 (1974) [Instrum. Exp. Tech. **17**, No. 3, 862 (1974)].
[8]E. McLaughlin, M. A. Shakespeare, and A. R. Ubbelohde, Trans. Faraday Soc. **60**, 25 (1964).
[9]B. Bahadur and S. Chandra, J. Phys. C **9**, 5 (1976).
[10]S. M. Stishov, V. A. Ivanov, and V. N. Kachinskiĭ, Pis'ma Zh. Eksp. Teor. Fiz. **24**, 329 (1976) [JETP Lett. **24**, 297 (1976)].
[11]J. R. McColl, Phys. Lett. **38A**, 55 (1972).
[12]S. M. Stishov, Usp. Fiz. Nauk **114**, 3 (1974) [Sov. Phys. Usp. **17**, 625 (1975)].
[13]J. O. Hirschfelder, C. F. Curtiss, and R. B. Bird, Molecular Theory of Gases and Liquids, Wiley, 1954 (Russ. transl., ILI, 1961).

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Theory of longitudinal magnetoresistance in weak magnetic fields

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Longitudinal magnetoresistance depends on the ratio of the distance between the Landau levels to the level width due to the interaction of the electron with the phonons and the impurities. If the level width exceeds the distance between the levels, resonance effects are impossible and the corresponding terms are exponentially small. The quantum corrections that determine the longitudinal magnetoresistance turn out to be analytically (quadratically) dependent on the magnetic field. The method of translationally invariant kinetic equation makes it possible to calculate these corrections for different scattering mechanisms. In scattering by ionized impurities, owing to the slow decrease of the Coulomb potential with distance, the order of magnitude of the magnetoresistance increases and it is determined by the classical theory, while its sign turns out to be negative.

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According to ordinary kinetic theory, application of a magnetic field does not change the resistance along the direction of the magnetic field. The magnetic field is regarded to be applied along a symmetry axis of the crystal, so that no resistance change due to crystalline anisotropy takes place. The magnetoresistance is zero because the action of the magnetic field on the electron system is taken into account only classically—via the Lorentz force. According to quantum mechanics, a system of Landau levels is produced in a magnetic field and the longitudinal magnetoresistance of the electrons that have a discrete spectrum does not vanish. It was investigated for the case of extremely strong (quantizing) magnetic fields by Adams and Argyres.[1] They have noted that when electrons are scattered by ionized impurities, the longitudinal magnetoresistance can be negative in a definite range of magnetic fields. In the limit of nonquantizing magnetic fields, allowance for the quantum corrections yields a nonzero longitudinal magnetroesistance. Argyres,[2] considering the scattering of electrons by acoustic phonons, reached the conclusion that a region of negative magnetoresistance exists. However, the order of magnitude and the dependence on the magnetic field, which were determined in Ref. 2, turned out to be incorrect, as was demonstrated by Dubinskaya.[3]

According to Argyres[2] and Dubinskaya,[3] the magnitude and sign of the magnetoresistance are determined by the resonant behavior of the state density and of the collision frequency in the vicinity of the Landau levels. This point of view implies tacitly that the system of Landau levels is well defined, i.e., the distance between the levels, Ω, which is equal to the frequency of revolution of the electron in the magnetic field, exceeds their smearing, which is of the same order as the reciprocal relaxation time $1/\tau_p$. It can be assumed that at $\Omega < 1/\tau_p$ the resonance effects are strongly smeared out. Thus, despite the fact that the considered effect is longitudinal, it depends essentially on $\Omega\tau_p$. A theory of magnetoresistance for a small $\Omega\tau_p$ was developed next.

In the first part of the present paper we determine, in the Landau-number representation, the magnetoresistance in the case of elastic scattering of electrons by acoustic phonons. It is shown that the terms that might be resonant, are small for small $\Omega\tau_p$ like $\exp(-\pi/\Omega\tau_p)$, and the density of states and the relaxation time can be regarded as smooth functions of Ω. At a small ratio of the distance between the energy levels to the average energy, the relaxation time, and the state density, meaning therefore also the magneto-

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resistance, all depend quadratically on the magnetic field. In the second part are considered other scattering mechanisms, by a procedure made possible by the use of the method of translationally invariant kinetic equation, developed by Levinson.[4] It turns out that at small $\Omega\tau_p$ all the scattering mechanisms lead to a quadratic dependence of the magnetoresistance on the magnetic field. Scattering by ionized impurities is special because of the rapid increase of the matrix element of the potential with decreasing momentum transfer. The total scattering cross section diverges in this case and turns out to be finite only when screening is taken into account. Since the magnetoresistance depends precisely on the total scattering cross section rather than on the transport cross section, the order of magnitude of the magnetoresistance increases. The Planck constant is cancelled out in this expression, i.e., the magnetoresistance is determined by classical theory and its sign is negative.

1. Consider the self-energy part of the retarded Green's function $\Sigma(\varepsilon, n, p_\parallel)$ in the Landau-number representation:

$$G^{ret}=[\varepsilon-\hbar\Omega(n+{}^1/_2)-p_\parallel^2/2m-\Sigma(\varepsilon, n, p_\parallel)]^{-1}, \tag{1}$$

m is the isotropic effective mass, the symbols $\parallel$ and $\perp$ indicate the projections of the momentum on the direction of the magnetic field and on the perpendicular direction. In the lowest-order approximation in the coupling constant we have

$$\Sigma(\varepsilon, n, p_\parallel) = \sum_{\mathbf{q}, n'} 2|C_\mathbf{q}|^2 N_\mathbf{q} \frac{M_{nn'}(\mathbf{p}_\perp, \mathbf{p}_\perp-\hbar\mathbf{q}_\perp) M_{n'n}(\mathbf{p}_\perp-\hbar\mathbf{q}_\perp, \mathbf{p}_\perp)}{\varepsilon-\hbar\Omega(n'+{}^1/_2)-(p_\parallel-\hbar q_\parallel)^2/2m-\Sigma(\varepsilon, n', p_\parallel-\hbar q_\parallel)}, \tag{2}$$

$M_{nn'}(\mathbf{p}_\perp, \mathbf{p}_\perp')$ are known matrix elements of the exponential in the Landau-number representation. In expression (2) we substituted the total Green's function, i.e., all the non-intersecting Feynman diagrams were summed, so that formula (2) is an integral equation. Here $C_\mathbf{q}$ is the matrix element of the deformation interaction of the electrons with acoustic phonons. The quantity $|C_\mathbf{q}|^2$ is proportional to the wave vector $\mathbf{q}$, whereas for the phonon number N we substitute the Rayleigh-Jeans distribution function, which is inversely proportional to the phonon frequency. Consequently, in the elastic high-temperature limit, $V_0|C_\mathbf{q}|^2 N_\mathbf{q} = R$ does not depend on $\mathbf{q}$, making it possible to simplify (2) significantly. Replacement of summation with respect to the longitudinal component $\mathbf{q}$ shows that Σ does not depend on $p_\parallel$, and the summation over the perpendicular components reveals that Σ is likewise not dependent on the number n, and is a function of only the energy variable ε. Since the latter is not a variable of the summation in (2), this summation becomes possible.

For the Green's function in (2) we use the integral representation

$$\frac{1}{x+iy} = (-i)\int_0^\infty e^{+i(x+iy)t}dt, \tag{3}$$

after which expression (2) is transormed into

$$\Sigma(\varepsilon) = \frac{(2m)^{3/2}\Omega R}{2\pi^{1/2}\hbar^2} e^{-3i\pi/4} \int_C \frac{1}{z^{1/2}} \frac{d}{dz}\left[\frac{z\exp\{i(\varepsilon-\Sigma(\varepsilon))z\}}{e^{i\hbar\Omega z/2}-e^{-i\hbar\Omega z/2}}\right] dz. \tag{4}$$

The integration contour goes from zero to below the real axis in a positive direction, and circles the singular points of the integrand on the real axis from below. It is convenient next to transform the integral along such a contour into an integral along the imaginary axis in the upper $\Sigma_I(\varepsilon)$ half-plane and into a sum of residues on the real axis $\Sigma_{II}(\varepsilon)$:

$$\Sigma(\varepsilon) = B(\varepsilon) - iA(\varepsilon), \tag{5}$$

$$A_I(\varepsilon) = -\frac{(2m)^{3/2}\Omega R}{4\pi^{3/2}\hbar^2}\int_0^\infty \frac{1}{t^{1/2}}\frac{d}{dt}\left[\frac{t}{\mathrm{sh}(\hbar\Omega t/2)} e^{-(\varepsilon-B(\varepsilon))t}\cos A(\varepsilon)t\right]dt, \tag{6}$$

$$B_I(\varepsilon) = \frac{(2m)^{3/2}\Omega R}{4\pi^{3/2}\hbar^2}\int_0^\infty \frac{1}{t^{1/2}}\frac{d}{dt}\left[\frac{t}{\mathrm{sh}(\hbar\Omega t/2)} e^{-(\varepsilon-B(\varepsilon))t}\sin A(\varepsilon)t\right]dt. \tag{7}$$

The energy shift $B_I(\varepsilon)$ is proportional to the highest power of the coupling constant, so that this term can be neglected. The level width $A_I(\varepsilon)$ is a smooth function of the energy variable, and $\Sigma(\varepsilon)$ can be taken to be zero in its calculation. We present an expression for the level width in the quasiclassical limit $\varepsilon \gg \hbar\Omega$:

$$A_I(\varepsilon) = \frac{m^{1/2}R(2\varepsilon)^{1/2}}{\pi\hbar^3}\left[1+\frac{1}{24}\left(\frac{\hbar\Omega}{2\varepsilon}\right)^2\right]. \tag{8}$$

We turn now to the contribution from the sum of the residues

$$A_{II}(\varepsilon) = \frac{m^{1/2}R\sqrt{\Omega}}{\pi\hbar^{3/2}}\sum_{k=1}^\infty \frac{(-1)^k}{k^{1/2}} e^{-2\pi k A(\varepsilon)/\hbar\Omega} \times\cos\left[\frac{2\pi k}{\hbar\Omega}(\varepsilon-B(\varepsilon))-\frac{\pi}{4}\right]. \tag{9}$$

if we let $\Sigma(\varepsilon)$ tend to zero in this expression, then we can see that at $\varepsilon^{(n)} = \hbar\Omega(n+\frac{1}{2})$ this expression diverges like $(\varepsilon-\varepsilon^{(n)})^{-1/2}$, a fact that determined the result in Refs. 2 and 3. What distinguishes formula (9) from this resonant expression is the exponential cutoff factor. For medium electron energies $\varepsilon \gg \hbar\Omega$ the argument of the exponential turns out to have $\pi/\Omega\tau_p$, so that at $\Omega\tau_p \ll 1$ each term and the entire series as a whole are exponentially small. Comparison of (9) with (5) shows the difference between the parameters of the expansion of the smooth and resonant parts of $A(\varepsilon)$ and the possibility of taking into account the small quantum correction in $A_I(\varepsilon)$ when $A_{II}(\varepsilon)$ is exponentially small. At low energies $\varepsilon \sim \hbar\Omega$ both parts of $A(\varepsilon)$ are of the same order, and this complicates the analysis of the quantum corrections to the conductivity in the region of classically strong magnetic fields.

The use of the self-consistent equation (2) calls for estimates of the discarded terms corresponding to other Feynman diagrams. We calculate the expression corresponding to a diagram with intersecting phonon lines, in which the Green's functions (1) and (2) are substituted:

$$\Sigma^{(2)}(\varepsilon, n, p_\parallel) = i\frac{m^3(\Omega R)^2(-1)^n}{8\pi\hbar^4}\int\int\int \frac{\mathscr{E}C_1}{SC_2}dx_1dx_2dx_3, \tag{10}$$

where

$$\mathscr{E} = \exp\left\{-i(\varepsilon-B(\varepsilon)-iA(\varepsilon))(x_1+x_2+x_3)-i\frac{p_\parallel^2}{2m}\left(\frac{1}{x_1}+\frac{1}{x_2}+\frac{1}{x_3}\right)^{-1}\right\}$$

$$S=\left[x_1x_2x_3\left(\frac{1}{x_1}+\frac{1}{x_2}+\frac{1}{x_3}\right)\right]^{1/2}\sin\frac{\hbar\Omega x_1}{2}\sin\frac{\hbar\Omega x_2}{2}\sin\frac{\hbar\Omega x_3}{2},$$

$$C_1=\left(1+\operatorname{ctg}\frac{\hbar\Omega x_1}{2}+\operatorname{ctg}\frac{\hbar\Omega x_2}{2}+\operatorname{ctg}\frac{\hbar\Omega x_3}{2}\right)^n,$$

$$C_2=\left(1-\operatorname{ctg}\frac{\hbar\Omega x_1}{2}-\operatorname{ctg}\frac{\hbar\Omega x_2}{2}-\operatorname{ctg}\frac{\hbar\Omega x_3}{2}\right)^{n+1}.$$

The main contribution is made by the points where the integrand is maximal: the maximum at the origin determines the smooth part analogous to (6), whereas the maxima at $x^{(k)}=2\pi(k+\frac{1}{2})/\hbar\Omega$ make a contribution similar to (9). The smooth part again turns out to be a function of $\hbar\Omega/\varepsilon$, whereas the resonant part contains an exponential dependence on $A(\varepsilon)/\hbar\Omega$. Thus, at $\Omega\tau_p\ll 1$ all the terms of the perturbation theory have the exponential smallness of the resonant terms. On the other hand, the proportionality to a higher degree of the coupling constant makes it possible to disregard the corrections to (2).

For electron scattering by acoustic phonons it is easy to calculate the longitudinal conductivity, since the arrival terms in the collision operator vanish. We do not take into account anywhere the spin of the electron, and assume the spin phenomena to be completely separated from the orbital phenomena; therefore

$$\sigma=\frac{e^2\Omega}{m\hbar}\sum_{n=0}^{\infty}\int p_\parallel^2dp_\parallel\int\left(-\frac{df_0(\varepsilon)}{d\varepsilon}\right)\times\frac{1}{[\varepsilon-\hbar\Omega(n+1/2)-p_\parallel^2/2m]^2+A^2(\varepsilon)}\frac{d\varepsilon}{(2\pi)^3},\tag{11}$$

$f_0(\varepsilon)$ is an equilibrium electron distribution function. In the derivation of formula (11) we have assumed that the action of the electric field on the electron reduces to the Lorentz force in the kinetic equation, and have neglected the influence of the electric field on the electron-phonon collision act, since the latter is small for nonquantizing magnetic fields.

We separate in (11) the analog of the state density

$$\chi(\varepsilon)=\hbar\Omega\sum_{n=0}^{\infty}\int p_\parallel^2\delta\left(\varepsilon-\hbar\Omega\left(n+\frac{1}{2}\right)-\frac{p_\parallel^2}{2m}\right)dp_\parallel,\tag{12}$$

$$\sigma=\frac{e^2}{m\hbar^2}\iint\left(-\frac{df_0(\varepsilon)}{d\varepsilon}\right)\frac{\chi(\varepsilon')}{(\varepsilon-\varepsilon')^2+A^2(\varepsilon)}\frac{d\varepsilon d\varepsilon'}{(2\pi)^3}.\tag{13}$$

In the state density we can separate the smooth and oscillating functions of the energy. The latter is equal to

$$\chi_{\mathrm{II}}(\varepsilon)=\frac{(m\hbar\Omega)^{3/2}}{\pi}\sum_{k=1}^{\infty}\frac{(-1)^{k-1}}{k^{3/2}}\cos\left(\frac{2\pi\varepsilon}{\hbar\Omega}k+\frac{\pi}{4}\right).\tag{14}$$

The expression for the smooth part is given here in the quasiclassical limit:

$$\chi_{\mathrm{I}}(\varepsilon)=\frac{2(2m\varepsilon)^{1/2}}{3}\left[1-\frac{1}{8}\left(\frac{\hbar\Omega}{2\varepsilon}\right)^2\right].\tag{15}$$

Integrating with respect to ε' in (13), we arrive at the usual expression for the conductivity in the form of an integral of the relaxation time with respect to all the quasiparticle energies. The main contribution is made by the average energies $\bar{\varepsilon}$: the temperature T (in the case of Boltzmann statistics), and the chemical potential μ (in the case of Fermi statistics). Here

$$\tau_p(\varepsilon)=\left[\frac{2A(\varepsilon)}{\hbar}\right]^{-1},\quad \tau_p=\tau_p(\bar{\varepsilon}).\tag{16}$$

Just as in (9), the oscillating part of the state density gives here an exponentially small contribution at $\Omega\tau_p\ll 1$. Consequently, we are left with only the Ω-dependent quantum corrections to both the relaxation time and to the state density.

This leads to a simple expression for the magnetoresistance. Its dependence on the magnetic field is quite different than in Refs. 2 and 3—namely, quadratic. For Boltzmann statistics, the integral with respect to energy at Ω squared diverges logarithmically at the lower limit. The cutoff of the integral at low energies is connected with two circumstances: at energies on the order of $\hbar\Omega$, the quasiclassical expansions of $A(\varepsilon)$ and $\chi(\varepsilon)$ are not valid, and at energies on the order of $\hbar/\tau_p(\varepsilon)$ the level width becomes comparable with the energy, a fact that must be taken into account when integrating with respect to ε' in (13). Which of the circumstances manifests itself depends on the value of the magnetic field and on the ratio of two small quantities: $\Omega\tau_p$ and the kinetic-equation parameter $\hbar/\tau_pT$. At $1\gg\hbar/\tau_pT\gg\Omega\tau_p$ we have

$$\frac{\Delta\sigma}{\sigma}=-\frac{(\hbar\Omega)^2}{24T^2}\left[\ln\left(\frac{\tau_pT}{\hbar}\right)^2-C\right],\tag{17}$$

C is the Euler constant. At $1\gg\Omega\tau_p\gg\hbar/\tau_pT$ we have

$$\frac{\Delta\sigma}{\sigma}=-\frac{(\hbar\Omega)^2}{24T^2}\left[\ln\left(\frac{T}{\hbar\Omega}\right)+a\right],\tag{18}$$

where a is a constant that could not be determined analytically. If no attention is paid to the logarithm, then the dependence on the magnetic field is analytic. This is natural, since the dependence on the magnetic-field intensity vector should in fact enter in the form of the square of the absolute value. The origin of the logarithm will be illustrated by the following nonrigorous arguments. Imagine that the integrand in the usual classical formula

$$\sigma=\frac{e^2(2m)^{1/2}}{3\pi^2\hbar^3}\int\varepsilon^{3/2}\tau_p(\varepsilon)\left(-\frac{df_0(\varepsilon)}{d\varepsilon}\right)d\varepsilon\tag{19}$$

depends on the changed energy

$$\tilde{\varepsilon}=[\varepsilon^2+(b\hbar\Omega)^2]^{1/2}\tag{20}$$

(b is a constant); we assume that this change takes into account effectively the discrete character of the levels. Since $\tau_p(\varepsilon)$ decreases with energy like $\varepsilon^{-1/2}$, expansion in powers of $\hbar\Omega$ results in a logarithmically diverging integral. It can be assumed that for another scattering mechanism with a different $\tau_p(\varepsilon)$ dependence there is no logarithm, since the quadratic dependence on the magnetic field at $\Omega\tau_p\ll 1$ is a general phenomenon. This will be proved later on. On the other hand, for scattering by acoustic phonons the theory, as we see, gives a positive magnetoresistance.

2. It is of interest to consider other scattering mechanisms, but this is difficult to do in the Landau-number representation. We shall use the method of the translationally invariant kinetic equation for the density matrix in the Wigner representation.[4,5] The density matrix f_p depends on the kinematic momentum p and is a generalization of the distribution function:

$$\left\{eE+\frac{e}{c}\left[\frac{\partial\varepsilon_p}{\partial p}\times H\right]\right\}\frac{\partial f_p}{\partial p}=\mathrm{Re}\sum_q\frac{NU_q^2}{\hbar}\left\{\left(s+i\frac{\varepsilon_p-\varepsilon_{p-\hbar q}}{\hbar}\right.\right.$$
$$+eE\frac{\partial}{\partial p}+\frac{e}{c}\left[\frac{\partial\varepsilon_{p-\hbar q/2}}{\partial p}\times H\right]\frac{\partial}{\partial p}\Big)^{-1}[f_{p-\hbar q}-f_p]$$
$$+\left(s+i\frac{\varepsilon_{p+\hbar q}-\varepsilon_p}{\hbar}+eE\frac{\partial}{\partial p}+\frac{e}{c}\left[\frac{\partial\varepsilon_{p+\hbar q/2}}{\partial p}\times H\right]\frac{\partial}{\partial p}\right)^{-1}\cdot$$
$$\left.\times(f_{p+\hbar q}-f_p)\right\},\quad s\to 0, \tag{21}$$

$\varepsilon_p = p^2/2m$ is the dispersion law of the electron, E and H are the intensities of the external electric and magnetic fields, U_q is the Fourier component of the potential of the randomly disposed impurities, and N is the total number of impurities. For scattering by phonons, in the elastic approximation and neglecting the spontaneous emission of phonons by electrons, NU_q^2 must be replaced by $2|C_q|^2N_q$.

According to Refs. 4 and 5, Eq. (21) is valid at any external-field strength. It is seen, however, that it does not take into account the finite width of the levels. This may turn out to be a shortcoming for problems in which an important role is played by the resonant behavior of the collision operator in the vicinity of the Landau levels. As already shown, when the longitudinal magnetoresistance in weak fields is calculated, such terms yield an exponentially small contribution, so that Eq. (21) is necessary and sufficient for the problem in question.

In the classical limit we can neglect the influence of the external field on the collision act, after which (21) takes the form of an ordinary kinetic equation. Being interested in the response to a weak electric field, we linearize (21) in the electric field. In nonquantizing magnetic fields, the action of the electric field on the collision can be disregarded, so that the electric field remains only in the left-hand side of the equation. This term appears as an inhomogeneity, since the density matrix in it is replaced by the equilibrium value, which itself depends on the magnetic field. For Boltzmann statistics, according to Ref. 4, it is equal to

$$f_p^{(0)}=\frac{1}{\mathrm{ch}(\hbar\Omega/2T)}\exp\left[\frac{\mu}{T}-\frac{p_\parallel^2}{2mT}-\frac{p_\perp^2}{2mT_\perp(\Omega)}\right],$$
$$p=p_\parallel+p_\perp\quad\Omega=\frac{eH}{mc},\quad T_\perp(\Omega)=\frac{\hbar\Omega}{2}\mathrm{cth}\frac{\hbar\Omega}{2T}. \tag{22}$$

Integration of the function (22) over all the momenta gives the total concentration n_0 of the conduction electrons. For Fermi statistics, in accord with Ref. 5, we have

$$f_p^{(0)}=\sum_{k=1}^{\infty}\frac{(-1)^{k-1}}{\mathrm{ch}(k\hbar\Omega/2T)}\exp\left[\frac{k(\mu-p_\parallel^2/2m)}{T}-\frac{kp_\perp^2}{2mT_\perp(k\Omega)}\right]. \tag{23}$$

The magnetic-field corrections of interest to us are now obtained by expanding the simplified equation (21) in powers of H. The change of the density matrix consists in this case of two additive parts: first, of the solution of the ordinary kinetic equation, in whose homogeneous term allowance is made for the change of the equilibrium density matrix in the magnetic field; second, of the solution of a kinetic equation in which the inhomogeneity is due to the change of the collision integral in the magnetic field. Accordingly, two increments are added to the electric conductivity. $\sigma^{(0)}$, namely $\Delta\sigma^{(1)}$ and $\Delta\sigma^{(2)}$.

Expressions (22) and (23) show that the first increment to the distribution function is proportional to $(\hbar\Omega/\bar{\varepsilon})^2$. The corresponding increment to the electric conductivity is obtained in the same manner as the expression for the electric conductivity in the absence of a magnetic field. The quantity $\Delta\sigma^{(1)}$, just as $\sigma^{(0)}$, depends on the scattering mechanism, a fact that we take into account by assuming the relaxation time to be proportional to the s-th power of the electron energy. In the case of scattering by acoustic phonons (16) we have $s = -1/2$, in the case of scattering by optical phonons and phonons that interact piezoelectrically with the electrons we have $s = 1/2$, while in the case of scattering by ionized impurities we have $s = 3/2$. Thus

$$\frac{\Delta\sigma^{(1)}}{\sigma^{(0)}}=\frac{s}{30}\left(\frac{\hbar\Omega}{T}\right)^2\ (\mathrm{B}),\quad\frac{\Delta\sigma^{(1)}}{\sigma^{(0)}}$$
$$=\frac{s(s+3/2)(s-3/4)}{30}\left(\frac{\hbar\Omega}{\mu}\right)^2\ (\mathrm{F}). \tag{24}$$

The relative change of the longitudinal resistance is equal in magnitude but opposite in sign to the relative change of the electric conductivity.

Proceeding to the calculation of the second increment to the density matrix, we note that the distribution function, which is the solution of the zeroth approximation—the classical kinetic equation—is substituted into the inhomogeneity, which is the expansion of the collision operator in powers of the magnetic field. In the equation for this increment, the expansion of the operator denominators gives rise formally to terms of first order in the magnetic field. But these make no contribution to the longitudinal electric conductivity, and we must continue this expansion to the second order. It is easily seen that this increment to the distribution function is proportional to $(\hbar\Omega/\varepsilon_p)^2$. The calculations of $\Delta\sigma^{(2)}$ are simple in concept but exceedingly cumbersome. This change of the conductivity depends substantially on the nature of the scatterers, i.e., on the matrix element. It is expressed not in terms of the transport scattering cross section of the electrons, which enters in the relaxation time, but in terms of the total scattering cross section, which determines the "departure" time τ_{LT} of the electron.

In scattering by acoustic phonons, using the usual expression for the relaxation time (16), we get

$$\frac{\Delta\sigma^{(2)}}{\sigma^{(0)}}=-\frac{(\hbar\Omega)^2}{24M}\int\left(\frac{1}{\varepsilon}\frac{df_0}{d\varepsilon}-\frac{d^2f_0}{d\varepsilon^2}\right)d\varepsilon,$$
$$M=\int\varepsilon_1\frac{df_0}{d\varepsilon_1}d\varepsilon_1,\quad f_0=[e^{(\varepsilon-\mu)/T}+1]^{-1}. \tag{25}$$

For Fermi statistics, $\Delta\sigma^{(2)}$ is negative and exceeds $\Delta\sigma^{(1)}$ in absolute magnitude; the longitudinal magnetoresistance is positive. In the case of Boltzmann statistics, the integral in (25) diverges logarithmically. This divergence was discussed earlier. The results (17), (18), and (25), which were obtained by different methods, are in agreement.

The potentials of piezoelectric electron-phonon interaction and of the interaction of electrons with optical phonons have similar momentum dependences. There-

fore the relative change of the conductivity for these scattering mechanisms is the same. Our calculation is model dependent, since we have neglected the inelasticity, a step permissible for optical phonons only at high temperatures, and neglected also the anisotropy, a step difficult to apply to the piezoelectric interaction. For these interactions the lifetime, in contrast to the relaxation time, diverges logarithmically and it is necessary to take into account the screening of the electron-phonon interaction. The static screening cuts off the divergence at momenta at the order of $\hbar/r_D$, where r_D is the Debye radius. We have

$$r_D^2=\frac{T}{4\pi n_0 e^2}\ (\text{B}),\quad r_D^2=\frac{\mu}{6\pi n_0 e^2}\ (\text{F}). \tag{26}$$

In the calculation the Coulomb logarithm $L=\ln[2r_D(2m\varepsilon)^{1/2}/\hbar]$ was assumed to be large, and this made it possible to dispense with determining more exactly the number under the logarithm sign. It was also assumed that $r_D(2m\varepsilon)^{1/2}/\hbar\gg 1$, which holds true for semiconductors. In this case

$$\frac{\Delta\sigma^{(2)}}{\sigma^{(0)}}=\frac{3}{40}\left(\frac{\hbar\Omega}{T}\right)^2 L\ (\text{B}),\quad \frac{\Delta\sigma^{(2)}}{\sigma^{(0)}}=\frac{3}{20}\left(\frac{\hbar\Omega}{\mu}\right)^2 L\ (\text{F}). \tag{27}$$

The contribution of $\Delta\sigma^{(2)}$ to the magnetoresistance is negative, and one can see that the magnetoresistance has a tendency to be negative in the case of scattering mechanisms with $s>0$.

We consider now scattering by ionized impurities. For these, it is necessary to take into account the screening of the Coulomb potential. The total scattering cross section, and with it also the lifetime, diverge in power-law fashion, and their finite values are due to screening. We have

$$(\tau_{\text{tr}})^{-1}=\frac{2\pi N}{\hbar V_0}\int\frac{d^3q}{(2\pi)^3}\left(\frac{4\pi e^2}{q^2+1/r_D^2}\right)^2\delta(\varepsilon_p-\varepsilon_{p-\hbar q})=\frac{2\pi(2m)^{1/2}Ne^4r_D^2}{V_0\varepsilon_p^{1/2}\hbar^2}. \tag{28}$$

For the change of conductivity, this quantity is multiplied, as before, by $(\hbar\Omega/\varepsilon_p)^2$. In this product, however, the Planck constant cancels out, thus demonstrating that this result should follow from the classical theory. In the classical limit, the collision term of the kinetic equation (21) can be simplified (cf. Ref. 6):

$$I^{\text{cr}}(f_p)=\sum_q NU_q^2 q\frac{\partial}{\partial\mathbf{p}}\left\{\left(s+i\mathbf{q}\frac{\partial\varepsilon_p}{\partial\mathbf{p}}+e\mathbf{E}\frac{\partial}{\partial\mathbf{p}}+\frac{e}{c}\left[\frac{\partial\varepsilon_p}{\partial\mathbf{p}}\times\mathbf{H}\right]\frac{\partial}{\partial\mathbf{p}}\right)^{-1}\mathbf{q}\frac{\partial}{\partial\mathbf{p}}f_p\right\}. \tag{29}$$

Allowance for the action of the magnetic field on the collisions leads to a nonzero longitudinal magnetoresistance, which is determined by the classical theory. In this theory, the magnetoresistance is a function of the ratio of the Debye radius r_D and of the Larmor radius $r_L=(2\varepsilon)^{1/2}/\Omega m^{1/2}$. When their ratio is small we have

$$\frac{\Delta\sigma^{(2)}}{\sigma^{(0)}}=\frac{11}{120}\left(\frac{r_D^2m\Omega^2}{T^2}\right)\frac{1}{L}\ (\text{B}),\quad \frac{\Delta\sigma^{(2)}}{\sigma^{(0)}}=\frac{11}{40}\left(\frac{r_D^2m\Omega^2}{\mu^2}\right)\frac{1}{L}\ (\text{F}) \tag{30}$$

This contribution exceeds parametrically the contribution of $\Delta\sigma^{(1)}$ if $\hbar\omega_{pl}/\varepsilon<1$ (ω_{pl} is the plasma frequency), as is usually the case in semiconductors. Expression (30) corresponds to negative magnetoresistance. When r_L becomes comparable with r_D, the explanation of Eqs. (21)–(29) is, of course, not valid and the change of the resistance becomes large in this sense. But when r_L becomes much smaller than r_D, the difference between $\sigma(H)$ and $\sigma^{(0)}$ is small—it consists in the fact that in the Coulomb logarithm, which enters in $\sigma^{(0)}$, the Debye radius is replaced by the Larmor radius.[7] This singles out a magnetic-field interval in which there exists a negative magnetoresistance determined by the classical theory. The presence of such an interval agrees qualitatively with the experimental data obtained with tellurium.[8]

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[1]E. N. Adams and P. N. Argyres, Phys. Rev. **104**, 900 (1956).
[2]P. N. Argyres, J. Phys. Chem. Solids **4**, 19 (1958).
[3]L. S. Dubinskaya, Fiz. Tverd. Tela (Leningrad) **7**, 2821 (1965) [Sov. Phys. Solid State **7**, 2280 (1966)].
[4]I. B. Levinson, Zh. Eksp. Teor. Fiz. **57**, 660 (1969) [Sov. Phys. JETP **30**, 362 (1970)].
[5]P. E. Zil'berman, Fiz. Tverd. Tela (Leningrad) **12**, 1697 (1970) [Sov. Phys. Solid State **12**, 1343 (1970)].
[6]S. V. Gantsevich, V. D. Kagan, and R. Katilyus, Zh. Eksp. Teor. Fiz. **67**, 1765 (1974) [Sov. Phys. JETP **40**, 878 (1975)].
[7]V. L. Gurevich and Yu. A. Firsov, Zh. Eksp. Teor. Fiz. **41**, 1151 (1961) [Sov. Phys. JETP **14**, 822 (1962)].
[8]L. S. Dubinskaya, Zh. Eksp. Teor. Fiz. **56**, 801 (1969) [Sov. Phys. JETP **29**, 436 (1969)].

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