on the asymmetry of the magnetic field and is determined mainly by the effective value of the parameter χ on the drift shell.

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Effect of cutoff radius of Coulomb scattering cross section on the distribution function and on the conductivity of nonequilibrium electrons

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The distribution function and the mean values are obtained for a system of noninteracting electrons situated in a strong electric field, whose momentum is scattered by ionized impurities (in semiconductors) or ions (in a plasma) and whose energy is dissipated as a result of extreme inelastic scattering at $\varepsilon > \varepsilon_0 \gg T$, ε_c (where ε_c is the characteristic energy of the electron-ion interaction and depends on the model and on the cutoff radius r_c of the Coulomb scattering cross section). It is shown that the dependence of the averaged quantities (the conductivity σ, the average energy $\bar{\varepsilon}$, and others) on E and on the parameters of the material is determined by the value of ε_c and by the form of the "Coulomb logarithm" $\Lambda(\varepsilon/\varepsilon_c)$. For a quite realistic form of $\Lambda(\varepsilon/\varepsilon_c)$ and not too small ε_c, the conductivity and the average energy have power-law dependences on $\varepsilon_c, \delta \propto \varepsilon_c^{1/2}$ and $\bar{\varepsilon} \propto \varepsilon_c^{1/2}$, with both $\sigma(E)$ and $\bar{\varepsilon}(E)$ constant. For some cutoff models it is shown that the dependence of the conductivity of the nonequilibrium electrons on the ion concentration N and on the longitudinal magnetic field intensity H can differ noticeably from the standard relations. The relations obtained for $\sigma(E, N, H)$ are in satisfactory agreement with the available experimental data.

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1. INTRODUCTION. FORMULATION OF PROBLEM

It is known that the calculation of the transport scattering cross section $\sigma_{tr}(\varepsilon)$ and of the pair-collision frequency $\tau_i^{-1}(\varepsilon)$ in the case of Coulomb scattering of electrons by ions (in a plasma) or by randomly distributed ionized centers (in semiconductors) encounters a characteristic difficulty, namely the logarithmic divergence of $\sigma_{tr}(\varepsilon)$ and $\tau_i^{-1}(\varepsilon)$ at small scattering angles. This difficulty is avoided by assuming that the Coulomb potential acts only up to distances $r < r_c$, so that it is possible to introduce a minimal scattering angle $\theta_{min}(\varepsilon, r_c)$ that depends on r_c and on the energy ε. This yields

$$\sigma_{tr}(\varepsilon) = \frac{\pi}{2}\left(\frac{e^2}{\varkappa\varepsilon}\right)^2 \ln\left(1+\operatorname{ctg}^2\frac{\theta_{min}}{2}\right)$$

($\varkappa$ is the permittivity). This relation is valid in both classical and quantum theory, except for the different connection between θ_{min} and r_c (Ref. 1). Different values are chosen for the Coulomb-potential cutoff radius r_c frequently on the basis of intuitive physical considerations, e.g., the Debye radius λ_D, the Larmor radius r_L (in the presence of an external magnetic field), or half the average distance between the ions $\frac{1}{2}N^{-1/3}$ (N is the ion concentration).[1-5] In all the foregoing cases the logarithmic factor (hereafter designated $\Lambda(\varepsilon)$) which enters in $\sigma_{tr}(\varepsilon)$ and $\tau_i^{-1}(\varepsilon)$ can be represented at $\varepsilon \gtrsim \varepsilon_c$ in the form

$$\Lambda(\varepsilon) = \ln\left[1+(\varepsilon/\varepsilon_c)^\nu\right] + \text{const},$$

where $\nu \gg 1$ and ε_c is a quantity on the order of the en-

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ergy of the electron-ion interaction $\varepsilon_c = \varepsilon_c(r_c)$, and is different for the different models. (Thus, in the case of classical scattering we have $\varepsilon_c = e^2/\varkappa r_c$, and in the case of quantum scattering $\varepsilon_c = \hbar^2/mr_c^2$, see Ref. 1.) However, in theories that describe the kinentic properties of the quasi-equilibrium electrons (see, e.g., Refs. 1 – 3 and 5 – 7), it customarily assumed that the particular choice of the cutoff model, r_c or ε_c, is of no special significance, and it can be assumed that $\Lambda(\varepsilon)$ on the premise that the electron gas is nearly ideal, i.e., that most electrons have energies $\varepsilon \sim \varepsilon_m$ greatly exceeding the energy ε_c of the electron-electron or electron-ion interaction[1]:

$$\varepsilon_m \gg n^{1/3}e^2/\varkappa, \quad \varepsilon_m \gg N^{1/3}e^2/\varkappa \tag{1}$$

(n is the electron concentration). The inequalities (1) are sufficient (but apparently not necessary, see Sec. 3) conditions for the applicability of the Boltzmann equation. In addition, it is customarily assumed that satisfaction of the inequalities (1) as $E \to 0$, when $\varepsilon_m = T/2$, $\bar{\varepsilon} = \frac{3}{2}T$, $\varepsilon_m \sim \bar{\varepsilon}$, guarantees their satisfaction in a strong field, since ε_m and $\bar{\varepsilon}$ customarily increase with increasing field and are close in order of magnitude ($\bar{\varepsilon}$ is the average energy, T is the temperature in energy units, and E is the field intensity). This is apparently why the choice of r_c, ε_c, and $\Lambda(\varepsilon)$ was not discussed when nonequilibrium electrons were considered in the case $T \gg \varepsilon_c$ or $\bar{\varepsilon}(E) \gg \varepsilon_c$, and it was assumed that $\Lambda(\varepsilon) = \text{const}$. Moreover, even in the study of electron-runaway effects and of the influence of inelastic scattering on the distribution function,[6,8,9] when the form of $\tau_i(\varepsilon)$ (and hence of $\Lambda(\varepsilon)$) is particularly important (see Refs. 8 and 9), it was also assumed that $\Lambda(\varepsilon) = \text{const}$.

It is shown in the present article that for non-interacting electrons that dissipate (at $\varepsilon < \varepsilon_0$) the momentum on the ions and experience at $\varepsilon = \varepsilon_0 \gg \varepsilon_c$ extreme inelastic scattering, there exists a range of fields $E_{\min} \ll E \ll E_{\max}$ in which the electron distribution

$$n(\varepsilon) = \rho(\varepsilon)F_0(\varepsilon) \tag{2}$$

takes on a maximum value at $\varepsilon = \varepsilon_m \lesssim \varepsilon_c$, and the σ and $\bar{\varepsilon}$ have a nearly power-law dependence on ε_c, namely $\sigma \propto \varepsilon_c^{1/2}$ and $\bar{\varepsilon} \propto \varepsilon_c^{1/2}$, in contrast to the frequently encountered logarithmic dependences. In this case $\bar{\varepsilon} \gg \varepsilon_m$, ε_c ($F_0(\varepsilon)$ is the symmetrical part of the momentum distribution function, $\rho(\varepsilon) = \rho_0\varepsilon^{1/2}$ is the state density on the energy axis, $\rho_0 = VM^{3/2}/2^{1/2}\pi^2\hbar^3$, and V is the volume). In this case the electron gas in fields $E_{\min} \ll E \ll E_{\max}$ is less "ideal" when $\varepsilon_c \ll T$ than as $E \to 0$. The condition of the validity of the approximation $\Lambda(\varepsilon) = \text{const}$ in fields $E_{\min} \ll E \ll E_{\max}$ is found and turns out to be more stringent than at $E = 0$. It is shown that the difference between the relations obtained for $\sigma(E)$ and $\bar{\varepsilon}(E)$ in Refs. 9 and in my earlier paper[10] in this range of fields is due only to the different choice of $\Lambda(\varepsilon)$. By way of example, the dependences of σ on the material parameters and on the external electric and longitudinal magnetic (H) fields are obtained for several known cutoff models. A comparison of the calculated values of $\sigma(\varepsilon_c, E, H)$ with the experimental ones makes it apparently possible, in principle, to assess the properties of the weakly nonideal electron gas, as well as to estimate ε_c and r_c. Conditions are discussed for the applicability of the kinetic equations in fields $E_{\min} \ll E \ll E_{\max}$.

2. PRINCIPAL APPROXIMATIONS AND EQUATIONS

1. The equation used below for $\tau_i(\varepsilon)$ has been obtained for a random disposition of the ions in the one-center scattering approximation[11]:

$$\tau_i(\varepsilon) = \frac{2^{1/2}}{\pi}\left(\frac{\varkappa}{e^2}\right)\frac{1}{N}\varepsilon^{1/2}\Lambda^{-1}(\varepsilon). \tag{3}$$

The form of $\Lambda(\varepsilon)$ in (3) depends on the cutoff model. Therefore all we assume everywhere, except in the examples, is that $\Lambda(\varepsilon)$ a function of the ratio $\varepsilon/\varepsilon_c$ and that

$$\Lambda(\varepsilon) \propto (\varepsilon/\varepsilon_c)^k \quad (k > 1/2) \quad \varepsilon \ll \varepsilon_c, \tag{4a}$$

$$\Lambda(\varepsilon) \propto \ln(\varepsilon/\varepsilon_c) + \text{const}, \quad \varepsilon \gg \varepsilon_c. \tag{4b}$$

The reasons for the choice of the value $k > \frac{1}{2}$ will be made clear below. In estimates of $\Lambda(\varepsilon)$ it will be assumed that $\Lambda(\varepsilon_c) \approx 1$. We note that at $k > \frac{3}{2}$ the quantity $\tau_i(\varepsilon)$ (3) has a minimum at $\varepsilon \sim \varepsilon_c$.

We make one remark to clarify (4a). The function $\Lambda(\varepsilon)$ at $\varepsilon \ll \varepsilon_c$ can be established for the following quite realistic model. Assume that: a) the ion potential $U(r)$ is Coulomb-like only up to distances $r < r_c$ from the given ion, but at $r > r_c$ the value of $U(r)$ falls off more rapidly than r^{-3}; b) r_c is independent of the electron energy; c) ε_c can be regarded as the "characteristic depth of the potential well"(see Ref. 12). For the cross section σ_p of the scattering of the "slow" particles with $\varepsilon \ll \varepsilon_c$ we obtain according to Ref. 12 $\sigma_p = \text{const}$, $\tau_i(\varepsilon) \propto \varepsilon^{-1/2}$, i.e., $\Lambda(\varepsilon) \propto \varepsilon^2$. We note that a smiilar $\Lambda(\varepsilon)$ dependence is obtained at $\varepsilon \ll \varepsilon_c$ in two frequently employed models—with exponential (Debye) screening[2,3,5] and with a sharp cutoff at $r_c = \frac{1}{2}N^{-1/3}$ (Refs. 2 and 4).

2. We now determine (using an n-type semiconductor as an example) the electric-field range in which the singularities of $\tau_i(\varepsilon)$ and $\Lambda(\varepsilon)$ should manifest themselves most strongly at $\varepsilon \lesssim \varepsilon_c$. Assume that

$$T \ll \varepsilon_0, \tag{5}$$

the scattering of electrons with $\varepsilon \geqslant \varepsilon_0$, characterized by a frequency $\tau_{in}^{-1}(\varepsilon)$ is inelastic and is determined, for example, by spontaneous emission of longitudinal phonons of frequency ω_0, while the scattering of the electrons with $\varepsilon < \varepsilon_0$ is quasi-elastic and is mainly from the ionized impurities. We assume furthermore that

$$\varepsilon_c \ll \varepsilon_0, \tag{6}$$

and let the ratio of ε_c to T be arbitrary.

We consider qualitatively the electron behavior in fields such that the energy relaxation of the system is the result of inelastic scattering at $\varepsilon \geqslant \varepsilon_0$, and the momentum relaxation is due only to elastic scattering by ionized impurities. The corresponding range of fields can be obtained by solving the Boltzmann equation and will be given below (see also Refs. 9 and 10).

We assume for simplicity that the scattering at $\varepsilon \geqslant \varepsilon_0$

is inelastic to the limit, i.e., $\tau_{in}^{-1}(\varepsilon) \to \infty$ at $\varepsilon = \varepsilon_0$.[1)] Then the electrons, having acquired an energy ε_0, lose this energy instantaneously and land at the origin of the energy axis, i.e., there is a "source" of electrons at $\varepsilon = 0$. The electrons can move from the source ($\varepsilon = 0$) into the region of higher energies either under the influence of the field or upon absorption of deformation acoustic[2)] (DA) or piezoacoustic (PA) phonons. [Absorption of optical phonons is neglected, in view of (5)]. We consider for the sake of clarity two simple cases.

Let $\varepsilon_c \gg T$ and $k > \frac{3}{2}$ (see (4)). Then in the region $\varepsilon \ll \varepsilon_c$ we have $\tau_i(\varepsilon) \propto \varepsilon^{3/2-k}$ and is large, the field "drags out" the electrons rapidly from this region. Most electrons should be at $\varepsilon \approx \varepsilon_c$, where $\tau_i(\varepsilon)$ has a minimum. One should expect a noticeable dependence of the distribution of the electrons (2) and of the mean values on ε_c.

Let now $\varepsilon_c \to 0$ and $\Lambda(\varepsilon) = \text{const}$ as $\varepsilon \to 0$. Then as $\varepsilon \to 0$ we have $\tau_i(\varepsilon) \propto \varepsilon^{3/2}$ and small, and the field near the "source" has little effect. In this case the energies land in the region of higher energies most readily as a result of the absorption of DA or PA phonons. After reaching values $\varepsilon = \varepsilon_E$ such that $\tau_i(\varepsilon_E)$ is large and the absorption of the energy from the field is substantial, the electrons are picked up by the field and move towards the sink $\varepsilon = \varepsilon_0$. One should expect the average quantities to depend on the characteristic energy ε_E, which in turn is determined, obviously, both by the field and by the interaction with the acoustic phonons.

Thus, with the electrons frequently "produced" at the point $\varepsilon = 0$ and then scattered by the ions, the distribution (2) and the average quantities should be sensitive to the form of $\tau_i(\varepsilon)$ and $\Lambda(\varepsilon)$ at low energies $\varepsilon \ll \varepsilon_0$.

It was shown[10] that for the considered scattering mechanism the relaxation of the momentum of the system is by scattering by ionized impurities in fields

$$E \ll E_{max}, \quad E_{max} = \frac{(2m\varepsilon_0)^{1/2}}{e\tau_i(\varepsilon_0)}. \tag{7}$$

For such fields, the Boltzmann equation can be solved in the diffusion approximation, so that we can obtain for $\varepsilon \leqslant \varepsilon_0$ the continuity equation

$$j_E(\varepsilon) = j_a^{sp}(\varepsilon) + j_a^{ind}(\varepsilon) + j_{in}(\varepsilon). \tag{8}$$

In this formula

$$j_E(\varepsilon) = -\frac{2}{3}\rho_0 \frac{(eE)^2}{m} \varepsilon^{1/2}\tau_i(\varepsilon)\frac{dF_0}{d\varepsilon}$$

is the field-dependent electron flux along the energy axis, and $j_{in}(\varepsilon) = \text{const}$ is the energy-independent flux of electrons due to inelastic scattering from the region $\varepsilon > \varepsilon_0$, while the terms

$$j_a^{sp}(\varepsilon) = \rho_0 \varepsilon^{1/2}\tau_a^{-1}(\varepsilon)\delta F_0(\varepsilon), \quad j_a^{ind}(\varepsilon) = \rho_0 \varepsilon^{1/2}\tau_a^{-1}(\varepsilon)\delta T\frac{dF_0}{d\varepsilon}$$

correspond to quasielastic acoustic scattering, with $j_a^{sp}(\varepsilon)$ corresponding to spontaneous emission of acoustic phonon, and $j_a^{ind}(\varepsilon)$ to induced interaction with them (δ and $\tau_a(\varepsilon)$ are the inelasticity parameter and the time of momentum relaxation for acoustic scattering). We note that $j_E(\varepsilon)$ and $j_a^{ind}(\varepsilon)$ are directed from the lower to the higher energies, while $j_{in}(\varepsilon)$ and $j_a^{sp}(\varepsilon)$ from the higher to the lower. It was shown[10] that in fields E satisfying the condition

$$\left(\frac{E}{E_{min}}\right)^2 = p \gg 1, \quad E_{min} = \left(\frac{3}{2}\frac{m\delta T}{e^2\tau_a(T)\tau_i(T)}\right)^{1/2}, \tag{9}$$

the field flux $j_E(\varepsilon) \approx j_{in}(\varepsilon)$ and for the greater part of the region form 0 to ε_0 the value of $j_E(\varepsilon)$ is appreciably larger than $j_a^{sp}(\varepsilon)$. The principal energy-loss mechanism in the fields (9) is inelastic scattering. Thus, the fields that satisfy the inequalities (7) and (9) satisfy the conditions formulated above. We point out that E_{min} is proportional to the electron–phonon interaction constant.

3. ELECTRON DISTRIBUTION AND MEAN VALUES. GENERAL FORMULAS

1. We seek the solution of (8) satisfying the boundary condition $F_0(\varepsilon_0) = 0$. We introduce the variable $x = \varepsilon/T$ and the notation

$$\Lambda_1 = \Lambda(\varepsilon = T), \quad u = \varepsilon_0/T, \quad p_1 = p\Lambda_1 \tag{10}$$

and for future convenience represent $\tau_a(\varepsilon)$ in the form

$$\tau_a(\varepsilon) = \tau_a(T)(\varepsilon/T)^{-3/2+\alpha}, \tag{11}$$

where $\alpha = 1$ and $\alpha = 2$ for DA and PA scattering, respectively. We then get from (8)

$$F_0(x) = C\int_x^u \frac{dx'\Lambda(x')}{x'^3(1+\Lambda(x')p_1^{-1}x'^{-\alpha})}. \tag{12}$$

The second term in the denominator of (12) is due to allowance for $j_a^{ind}(\varepsilon)$. The constant C is determined from the condition of normalization to the total concentration:

$$C = n(\rho_0 T^{3/2} I_{in})^{-1},$$

where

$$I_{in} = \int_0^u x^{1/2}dx \int_x^u \frac{dx'}{x'^3(1+\Lambda(x')p_1^{-1}x'^{-\alpha})}. \tag{13}$$

We consider now a simplified solution of (12), $F_0^{simp}(x)$, obtained by neglecting the x dependence of $\Lambda(x)$ and the term $\Lambda(x')(p_1 x'^{\alpha})^{-1}$ in (12). Physically this corresponds to the assumption that the energy of the electron-ion interaction ε_c and the electron–phonon interaction constant tend to zero. The function $F_0^{simp}(x)$ diverges as $x \to 0$, $F_0^{simp}(x) \propto (x^{-2} - u^{-2})$, and the normalization integral (13) tends formally to infinity, so that it is impossible to calculate the average quantities (i.e., the electrons "accumulate" in the region $\varepsilon \approx 0$). It is therefore necessary to take into account the differences between the true function $F_0(x)$ and the simplified $F_0^{simp}(x)$; these differences are significant either where the dependence of $\Lambda(x)$ on x can not be neglected, or where the term $\Lambda(x)p_1^{-1}x^{-\alpha}$ in (12) is substantial.

2. We assume $\varepsilon_c > 0$ and an exponent $k > \frac{1}{2}$ (see (4)). Then the divergence of $F_0(x)$ is weaker than $x^{-3/2}$ as $x \to 0$, and (13) can be integrated by parts. This yields

$$I_{in} = \frac{2}{3}\int_0^u \frac{dx\,\Lambda(x)}{x^{1/2}(1+\Lambda(x)p_1^{-1}x^{-\alpha})}. \tag{13a}$$

Let us find the conductivity σ and the average energy $\bar{\varepsilon}$ of the system of nonequilibrium electrons. From

(12) and (13), integrating by parts, we get

$$\sigma=\frac{2}{3}\frac{ne^2\tau_t(T)}{mI_{in}}\int_0^u\frac{dx}{1+\Lambda(x)p_1^{-1}x^{-\alpha}}=\frac{2}{3}\frac{ne^2\tau_t(T)}{m}\frac{I_\sigma}{I_{in}}, \tag{14}$$

$$\varepsilon=\frac{2}{5}TI_{in}^{-1}\int_0^u\frac{\Lambda(x)dx}{x^{1/2}(1+\Lambda(x)p_1^{-1}x^{-\alpha})}=\frac{2}{5}T\frac{I_\varepsilon}{I_{in}}. \tag{15}$$

In the calculation of the integrals I_σ and I_ε defined by formulas (14) and (15), the main contribution is made by the region $x\sim u\gg 1$ and these integrals are not very sensitive to the choice of the parameters r_c, ε_c, and α. Thus, for example, $I_\sigma=u$, and the partial contribution to the conductivity of electrons of all energies is the same. The situation is entirely different when it comes to calculate the point of the peak (or the largest value) of x_m of the electron distribution n_x (see (2)) and the integral I_{in} (13). Both quantities depend substantially on ε_c (i.e., on r_c) and on α.

3. We consider first the case when

$$\varepsilon_c\gg\varepsilon_E,\quad x_c\gg x_E, \tag{16}$$

where $\varepsilon_E=Tp_1^{-1/\alpha}$. From the definitions (9) and (10) of p and p_1 it follows that (16) can be realized at a high energy of the electron–ion interaction, at low temperatures, and at a small electron–phonon interaction constant. The second term in the denominator of (12) is in this case always smaller than the first and can be neglected. Then $F_0(x)$ and I_{in} do not depend on the field:

$$F_0(x)\approx\frac{n}{\rho_0T^{1/2}I_{in}}\int_x^u\frac{dx'\Lambda(x')}{x'^3}\propto\int_x^u\frac{dx'}{x'^{1/2}\tau_c(x')}. \tag{17}$$

From (17) follow the results obtained earlier from simple considerations, namely, at $u\gg x\gg x_c$ the function $F_0(x)\propto x^{-2}$, and at $x<x_c$ we have $F_0(x)\propto x^{-2+k}$ at $k\neq 2$ or $\propto\ln(1/x)$ at $k=2$; at $k>\frac{3}{2}$ the electron distribution $n(x)$ has a maximum, with $x_m\approx x_c$; at $k<\frac{3}{2}$ the distribution $n(x)$ increases with decreasing x in proportion at $x^{-3/2+k}$ and assumes the largest value at $x\sim x_E\ll x_c$. The resultant position of the point x_m, namely $x_m\lesssim x_c$, is quite interesting. We recall that we assumed that $\varepsilon_c\ll T$ and $x_c\ll 1$. The conclusion $x_m\lesssim x_c$ means that the maximum of the distribution in a strong field is in a region of energies lower than at $E=0$ ($x_m(E=0)=\frac{1}{2}$), i.e., the electron gas becomes less ideal in a strong field.

We determine now I_{in}, σ, and $\bar{\varepsilon}$. Recognizing that $\Lambda(\varepsilon)$ is a function of the dimensionless quantity $\varepsilon/\varepsilon_c$ and $\Lambda(\varepsilon)\varepsilon^{-1/2}\to 0$ as $\varepsilon\to 0$ (see (4)), we integrate (13a) by parts. We have

$$I_{in}=\frac{4}{3}x_c^{-1/2}\left[I(\infty)-\int_{u/x_c}^{\infty}\frac{d\Lambda}{dy}\frac{dy}{y^{1/2}}-\left(\frac{x_c}{u}\right)^{1/2}\Lambda\left(\frac{u}{x_c}\right)\right] \tag{18}$$

where

$$I(\infty)\equiv\int_0^\infty\frac{d\Lambda}{dy}\frac{dy}{y^{1/2}}$$

is a numerical quantity that does not depend on x_c. According to (6) and (4) we have $u/x_c\gg 1$, and $\Lambda(x)$ is a slow function at $x\gg x_c$. The first term of (18) is then the principal one, and from (14), (15), and (18) we get

$$\sigma=\frac{1}{2}\frac{ne^2\tau_t(T)}{m}\Lambda_1I^{-1}(\infty)\frac{\varepsilon_0}{T^{3/2}}\varepsilon_c^{1/2}, \tag{19}$$

$$\varepsilon=\frac{3}{5}T^{1/2}u^{1/2}\frac{\Lambda(u)}{I(\infty)}\varepsilon_c^{1/2}. \tag{20}$$

It follows from (19) and (20) that in the considered field range the conductivity and $\bar{\varepsilon}$ are independent of the field but depend on the cutoff energy ε_c in power-law fashion.[3)] In addition, for the distribution function (17) the average energy $\bar{\varepsilon}$ and the maximum point ε_m of the distribution differ parametrically, $\bar{\varepsilon}/\varepsilon_m\gtrsim u^{1/2}\gg 1$ (see (5) and (10)). We recall that $\bar{\varepsilon}\sim\varepsilon_m$ for exponential distribution functions.

4. We determine now the conditions under which the approximation $\Lambda(\varepsilon)=\text{const}$ is valid in the range of fields (7) and (9). In this case, to obtain a finite value of I_{in} (13), we must take into account in (12) the term $\Lambda(x)p_1^{-1}x^{-\alpha}$, which is appreciable at $x\leqslant p^{-1/\alpha}$, $\varepsilon\leqslant Tp^{-1/\alpha}$. Since $\Lambda(\varepsilon)$ is constant only at $\varepsilon\gg\varepsilon_c$, allowance for the term of (12) presupposes in essence satisfaction of an inequality opposite to (16):

$$\varepsilon_c\ll Tp^{-1/\alpha}. \tag{21}$$

When (9) and (11) are taken into account, it follows from (21) that for nonequilibrium electrons the requirement $\Lambda(\varepsilon)=\text{const}$ is more stringent than for equilibrium electrons.

It is easy to obtain from (12) the values of $F_0(x)$ for DA ($\alpha=1$) and PA ($\alpha=2$) interactions; at $x>p^{-1/\alpha}$ these functions behave in analogous fashion: $F_0^{DA}(x)\propto F_0^{PA}(x)\propto x^{-2}$. A study of the electron distribution (2) for $F_0^{DA}(x)$ and $F_0^{PA}(x)$ shows readily that for the DA interaction $n(x)$ has no peak and its largest value is at $x\to 0$, while for the PA interaction we obtain approximately $x_m\approx p^{-1/2}e^{-2}\ll 1$. Consequently, at $\varepsilon_c\ll Tp^{-1/\alpha}$ the maximum of the electron distribution in the fields (7) and (9) shifts from its equilibrium value $\varepsilon_m(E=0)=T/2$ to the left, towards lower energies,[4)] i.e., in this case, too, the electrons are not heated but cooled in the fields (7) and (9). The physical reason for this was explained in Sec. 2.

Thus, the position of the peak (or the largest value) ε_m of the electron distribution $n(\varepsilon)$ in fields (7) and (9) is determined by the larger of the quantities, ε_c on $Tp^{-1/\alpha}$, i.e., by the maximum energy up to which a substantial role is played by the differences between the real function $F_0(x)$ and the "simplified" one (see (12) ff). From (13) we can determine the normalization integrals I_n^{DA} and I_n^{PA} for DA and PA scattering. The main contribution to I_n^{DA} is made by the region $x\sim p^{-1}$, and to I_n^{PA} by the region $x\sim p^{-1/2}$; in this case

$$I_{in}^{DA}\approx\frac{2}{3}\pi p^{1/2}\propto E,\quad I_{in}^{PA}\approx\frac{2^{1/2}}{3}\pi p^{1/2}\propto E^{1/2}.$$

The formal analogy between these expressions and (19) is readily discerned.

From (14) we obtain for the conductivity the relations $\sigma^{DA}\propto E^{-1}$ and $\sigma^{PA}\propto E^{-1/2}$, which coincide with the results of Ref. 9; there, however, the electron distribution has not discussed, nor was the criterion (21) given for the applicability of the approximation $\Lambda(\varepsilon)=\text{const}$. The calculation of the averages in accord with (15)

yields results that agree with Ref. 9:

$$\bar{\varepsilon}^{DA}\approx\frac{6}{5\pi}Tu^{1/2}p^{-1/2},\quad \bar{\varepsilon}^{PA}\approx\frac{6\cdot 2^{1/2}}{5\pi}Tu^{1/2}p^{-1/2}.$$

Just as in the case $\varepsilon_c > Tp^{-1/\alpha}$, the values of $\bar{\varepsilon}^{DA}$ and $\bar{\varepsilon}^{PA}$ exceed significantly (by approximately $(up)^{1/2}$ and $(up^{1/2})^{1/2}$ times for DA and PA scattering, respectively) the energy values that make the main contribution to the integrals I_n^{DA} and I_n^{PA}. Therefore the different validity conditions (say, for the expressions for σ) should be verified not for electrons with energies $\varepsilon\sim\bar{\varepsilon}$, but for electrons with the energies that determine I_n^{DA} and I_n^{PA}. We call attention also to the fact that the field dependence of the conductivity in the region of the fields (7) and (9) is determined not only by the relation between ε_c and $Tp^{-1/\alpha}$, but also by which of the acoustic mechanisms of energy relaxation is significant at $\varepsilon\sim Tp^{-1/\alpha}\ll T$.

4. DISCUSSION OF THE VALIDITY OF THE APPROXIMATIONS AND OF THE RESULTS

1. In the foregoing determination of the electron distribution and of the conductivity in strong electric field, the classical kinetic equation was used. For this equation to be valid it suffices[11] to have the electron kinetic energy much higher than the energy of the interaction with the ions

$$\varepsilon\gg\varepsilon_c, \tag{22}$$

and the energy uncertainty of the electron must be small[14]:

$$\hbar/\varepsilon\tau(\varepsilon)\ll 1. \tag{23}$$

Scattering of electrons by ions was considered in the approximation of one-center scattering, which is valid (see Ref. 11) if

$$\lambda_B=\hbar/(2m\varepsilon)^{1/2}\ll N^{-1/3}. \tag{24}$$

In addition, it was assumed that the electron concentration and the frequency of the interelectron collisions are so low that the corresponding flux $j_{e-e}(\varepsilon)$ can be neglected in the continuity equation (8), and the electron scattering at $\varepsilon>\varepsilon_0$ is inelastic to the limit. The latter is equivalent to the assumption that the characteristic "depth of entry" $\Delta\varepsilon$ of the electron into the region $\varepsilon>\varepsilon_0$ is less than the energies ε_c and $Tp^{-1/\alpha}$, and can be readily satisfied in the fields (7) and (9) (see Ref. 10). More significant are the remaining assumptions. We shall now discuss their validity for the case $\varepsilon_c > Tp^{-1/\alpha}$.

One of the main results of the work is, in our opinion, that in the fields (7) and (9) the maximum of the electron distribution is in the energy region of the order of the interaction energy ε_c, i.e., $\varepsilon_m\lesssim\varepsilon_c$, with $\varepsilon_c\ll\bar{\varepsilon}$. Then (22) is violated for most electrons; in addition, (23) and (24) should be satisfied as applied to electrons with $\varepsilon\sim\varepsilon_m\sim\varepsilon_c$. We begin the discussion with the condition (22). When (22) is satisfied, the system of electrons and ions is an almost ideal plasma, and only for this plasma do we have theoretical proof of the validity of the Boltzmann equation (see [15]). It is known, however, that the application of the results of the solution of this equation to systems with a Maxwellian energy distribution

$$F_0(\varepsilon)\propto\exp(-\varepsilon/T^*)$$

and with a strong interaction (the ratio $\varepsilon_c/\varepsilon_m\sim\varepsilon_c/\bar{\varepsilon}\sim\varepsilon_c/T^*$ amounts to several units) provides a good qualitative and quantitative description[15,16] (T^* is the effective temperature, $T^*\geq T$). Therefore, according to Ref. 15, the inequality (22) is not necessary for system characterized by T^* (or for equilibrium systems, i.e., with $T^*=T$), and can be replaced by the much weaker condition $\varepsilon_c/\varepsilon_m\sim\varepsilon_c/\bar{\varepsilon}\leq 5$ (see Refs. 15 and 16). The question of the necessity of (22) for systems with power-law distribution functions (see (17)) and with $\varepsilon_m\sim\varepsilon_c\ll\bar{\varepsilon}$ has not been discussed in the literature. It appears that it can be answered by comparing the results of calculations (such as (17) and (19) with appropriate experimental data. One such comparison will be made below (see (2a) and Fig. 1), and it will be shown that the conductivity values obtained from (19) agree quantitatively with the experimental ones in the case $\varepsilon_c/\varepsilon_m\sim 4$. This allows us to conclude that for the situation considered in Secs. 2 and 3 the inequality (22) is likewise not necessary and can be replaced by a much less stringent one.

We now examine the restrictions that follow from (23) and (24) at $\varepsilon\sim\varepsilon_c$. We use for ε_c an expression obtained in the Conwell–Weiskopf (CW) model[4]

$$\varepsilon_c=\frac{e^2}{\varkappa r_c}=2\frac{e^2N^{1/3}}{\varkappa}.$$

It follows then from (3) that

$$\frac{\hbar}{\varepsilon_c\tau_i(\varepsilon_c)}\sim\frac{\lambda_B(\varepsilon_c)}{N^{-1/3}}\sim(Na^3)^{1/4}, \tag{25}$$

where $a=\varkappa\hbar^2/me^2$ is Bohr radius of the hydrogenlike impurity; the quantity Na^3 is frequently called the degree of doping. Thus, the CW formula can be used for $\tau_i(\varepsilon)$ in the range of fields (7) and (9) only in the case of weakly doped semiconductors.

The interelectron $e-e$ collisions in the range of fields (7) and (9) are important in two respects: they lead to an energy flux $j_{e-e}(\varepsilon)$, not accounted for in (8), from the region $\varepsilon<\varepsilon_0$ into the region $\varepsilon>\varepsilon_0$, where it is rapidly transferred to the lattice[17]; the frequent collis-

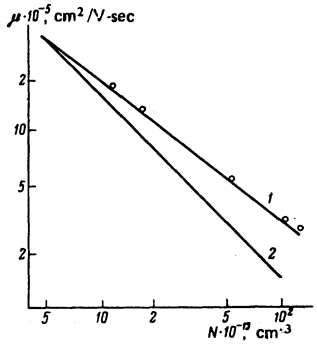

FIG. 1. Mobility of nonequilibrium electrons in InSb vs. the ion concentration: 1: $\mu^{theor}(N)$ calculated from formula (26), —experimental data[19,20]; 2—standard relation $\mu^{st}(N)=CN^{-1}$, $C=\mu^{theor}(N=5\times10^{13}\ cm^{-3})\cdot 5\times10^{13}\ cm^{-3}$.

ions of the electrons of the region of low energies $\varepsilon \sim \varepsilon_c \ll \varepsilon_0$ (or $\varepsilon \sim Tp^{-1/\alpha} \ll \varepsilon_0$) can lead to a change of $F_0(\varepsilon)$ in this region compared with the value obtained above.[5)] It appears that neglect of the $e-e$ collisions is valid only if $n \ll N$, i.e., for example in compensated semiconductors.

2. The singularities of the conductivity for the case $\varepsilon_c > Tp^{-1/\alpha}$ (see (19)) will be illustrated here with several concrete cutoff models. We note first that for any model the conductivity σ and the mobility $\mu = \sigma/en$ in the fields (7) and (9) do not depend on the electric field or the temperature. Such sections of the field E where $\mu(E,T) = \text{const}$ have already been observed in several experimental studies[19-22]; it was shown in Refs. 19 and 20 that the corresponding fields satisfy relations (7) and (9). It follows also from (19) that an appreciable dependence of σ on ε_c can alter the usual dependences of σ on the parameters of the material (and on the external longitudinal magnetic field $\mathbf{H} \uparrow\uparrow \mathbf{E}$, see below). This question, to our knowledge, has not been discussed in the literature. On the other hand, it appears that a comparison of the experimental values of σ with the calculated ones makes it possible in principle to determine ε_c and r_c, and to assess the applicability of the kinetic equation. We consider now several examples.

a) Let the electron scattering be classical in accord with the CW model,[4] and

$$r_c = {}^1/_2 N^{-1/3}, \quad \varepsilon_c = 2e^2 N^{1/3}/\varkappa.$$

Then $\Lambda(\varepsilon) = \ln[1 + (\varepsilon/\varepsilon_c)^2]$ and a numerical solution of the equation obtained for ε_m from (17) yields $\varepsilon_m \approx 0.25\varepsilon_c$. For μ^{CW} we get from (19)

$$\mu^{CW} \approx \frac{\mu(T)\Lambda_1}{4\pi} u^{3/2} \left(\frac{2e^2N^{1/3}}{\varkappa\varepsilon_0}\right)^{1/2} \times \left\{1 + 2\left(\frac{2e^2N^{1/3}}{\varkappa\varepsilon_0}\right)^{1/2} \ln\left[\left(\frac{\varkappa\varepsilon_0}{e^2N^{1/3}}\right)^{1/2} + 1\right]\right\}. \tag{26}$$

The $\mu^{CW}(N)$ relation differs from the usual one $\mu(N) \propto N^{-1}$ and turns out to be close to $\mu^{CW} \propto N^{-5/6}$. It is shown in Fig. 1. In the calculations we used the InSb electron parameters. For comparison, the figure shows the "standard" function $\mu^{st}(N) = CN^{-1}$ with a value of C such that at $N = 5 \times 10^{13}$ cm^{-3} the values of μ^{CW} and μ^{st} coincide.

The cited references 19 – 22, only the energy and momentum relaxation mechanisms were investigated and the $\mu(N)$ relation was not discussed; values of N are given only in Refs. 19 and 20. The measurements in Refs. 19 and 20 were made for electrons in n-InSb with $\varepsilon_c > T, Na^3 \ll 1, n = (0.2 - 0.4)N$. The values of μ in the field regions (7) and (9), where $\mu(E)$ is constant, are shown in Fig. 1 for several samples from Refs. 19 and 20. It is seen that they are close enough to the calculated curve (26). Consequently, under the conditions of Refs. 19 and 20, the characteristic cutoff energy can be taken to be $\varepsilon_c = 2e^2N^{1/3}/\varkappa$, and violation of the inequality (22) does not alter the expressions for σ.

b) In a strong magnetic field H ($\omega_c\tau_i \gg 1$, $\omega_c = eH/mc$) there appears one more characteristic length, namely the Larmor radius

$$r_L = (2\varepsilon/m)^{1/2}\omega_c^{-1},$$

and a sharp cutoff of the Coulomb potential at $r = r_L$ is frequently used in the calculation of τ_i and σ of equilibrium electrons if r_L is smaller than the other characteristic lengths. The dependences of σ and τ_i on H then turn out to be logarithmic (see, e.g., Refs. 1, 3, 24). Let us find $\sigma(H,N)$ in the region of the fields (7) and (9) for the case of a longitudinal magnetic field, $\mathbf{H} \| \mathbf{E}$. At $\mathbf{H} \| \mathbf{E}$ the expressions (17) – (19) obtained above are valid for $F_0(\varepsilon)$ and σ. At $r_c = r_L$ the characteristic energy ε_0 can be determined from the relation $\varepsilon_c = e^2/\varkappa r_L(\varepsilon_c)$, which yields $\varepsilon_c = (me^4\omega_c^2/2\varkappa^2)^{1/3}$, with $\Lambda(\varepsilon) = \ln[1 + (\varepsilon/\varepsilon_c)^3]$.

For the case $\varepsilon_c > Tp^{-1/\alpha}$ we obtain from (19)

$$\sigma(E,H,N) \approx \sigma(T)\Lambda_1 \frac{u^{1/2}}{8^{1/2}\pi} \left(\frac{e^6}{2\varkappa^2\varepsilon_0^3 mc^2}\right)^{1/6} H^{1/2}, \tag{27}$$

i.e., the conductivity should increase with the magnetic field in power-law fashion: $\sigma(H) \propto H^{1/3}$. A considerable increase of the conductivity of the electron with increasing magnetic field in the electric fields (7) and (9) was observed in Refs. 23, where n-InSb crystals with $NA^3 \ll 1$ were used at $T \sim 4$ K. It is seen from the plots of Ref. 23 that in fields $H = 1 - 4$ kOe the $\sigma(H)$ dependence is close to (27).

We point out, without presenting the calculations, two other consequences of the arguments given above and of (19). First, analysis of scattering in strongly doped semiconductors ($NA^3 \gg 1$) shows that the relations between θ_{min}, r_c and ε_c must be obtained by a quantum approach.[1] This leads to the function $\Lambda(\varepsilon) = \ln[1 + \varepsilon/\varepsilon_c]$, where $\varepsilon_c \approx (2N^{1/3}\hbar)^2/m$, and gives for μ and for the function $\mu(N)$ a formula different from (26), namely, $\mu(N) \propto N^{-2/3}$. The second example pertains to semiconductors with correlated disposition of impurities. Allowance for the correlations yields for $\tau_i(\varepsilon)$ formula (3), but with $\Lambda(\varepsilon)$ and ε_c that depend on the character and radius of the correlation. For the correlation model in compensated semiconductors,[25] with $\varepsilon_c = \hbar^2(\pi n)^{2/3}/2m$, it is easy obtain from (19) a conductivity $\sigma \sim n^{4/3}$ and a mobility $\mu \propto n^{1/3} \propto (1-K)^{1/3}$ (K is the degree of compensation).

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[1)] For plasma electrons (just as for a semiconductor), such a "threshold" character is possessed also by excitation of neutral atoms by electron impact.[13]

[2)] In a plasma this mechanism corresponds to elastic scattering by neutral atoms.

[3)] A formula similar to (19) was obtained earlier[10] for the cutoff model.[4] There, however, principal attention was paid to the independence of σ of E; the conditions for the validity of (19), for the dependence of σ and $\bar{\varepsilon}$ on ε_c, and the distribution of the electrons were not discussed in Ref. 10.

[4)] Consequently, at $\varepsilon_c \ll T$ the diffusion approximation should be valid for the fields (7) and (9) both at $\varepsilon \gtrsim T$ and at $\varepsilon \lesssim \max\{\varepsilon_c, Tp^{-1/\alpha}\}$, and this calls for sufficiently small values of δ.

[5)] It must be borne in mind, however, that $e-e$ collisions do

not necessarily bring $F_0(\varepsilon)$ closer to a Maxwellian function: it was shown recently[18] that the $e-e$ collision integral can be equal to zero also for power-law distribution functions that describe, just as (12), distribution with flows of particles and of energy from a "source" ($\varepsilon = 0$) to a "sink" ($\varepsilon = \varepsilon_0$).

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Three-band Kane model and Auger recombination

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A derivation is given of a dispersion equation from which the spectrum of electrons, light holes, and heavy holes can be obtained in the three-band Kane model in which the interaction between the three bands is included rigorously but the interaction with the other bands is allowed for by the $\mathbf{k}\cdot\mathbf{p}$ approximation. Overlap integrals governing the Auger recombination rate are calculated. The overlap integral between the conduction and heavy-hole valence bands is zero for threshold values of the particle momenta if the interaction with the higher bands is ignored. Consequently, the preexponential function in the expression for the Auger recombination rate has a different temperature dependence from that obtained in the case of simple parabolic bands. This theoretical calculation is in good agreement with the experimental recombination time reported for InSb at 300°K.

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1. INTRODUCTION

The Kane model[1] allows rigorously for the interaction between the s and p bands, whereas the interaction with higher bands is included by the $\mathbf{k}\cdot\mathbf{p}$ approximation. However, in the case of narrow-gap semiconductors, such as InSb or $Hg_{1-x}Cd_xTe$, the spin–orbit splitting is large compared with the band gap E_g and the spectrum of electrons and holes can be determined using just the three-band approximation, i.e., by making rigorous allowance for the interaction with the conduction band with the light- and heavy-hole valence bands. The interaction with the spin–orbit–split band can also be included within the framework of the $\mathbf{k}\cdot\mathbf{p}$ approximation. This makes it possible to allow simultaneously for the band nonparabolicity and the corrugated nature of the constant-energy surfaces.

Allowance for the band nonparabolicity is essential in the calculation of the rate of the Auger recombination of electrons and holes[2] within the framework of the Kane model. The expression for this rate includes overlap

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