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Long-wave secondary radiation in polar semiconductors

A. V. Gol'tsev, I. G. Lang, S. T. Pavlov, and G. Yu. Yashin

A. F. Ioffe Physico-technical Institute, USSR Academy of Sciences
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A new type of secondary radiation is predicted upon excitation of a semiconductor with light of frequency ω_l in the region of intrinsic absorption. The radiation is the result of resonant Raman scattering of light, wherein an electron-hole pair and a certain number of longitudinal optical phonons are present in the final state of the crystal. Since the kinetic energies of the electron and hole in the final state can be different, the secondary radiation spans the band $0 \leq \omega_s \leq \omega_{s\max}$ in the long-wave band. The scattering cross section is calculated with allowance for the dispersion of the LO phonons. It is shown that the dependence of the cross section of the process in question on the frequencies ω_l and ω_s has a steplike character. Each higher step corresponds to turning on a process in which the number of emitted LO phonons increases by unity. It is shown that the scattering cross section is a quantity of zero order in the Fröhlich coupling constants of the electrons or holes with the LO phonons. The reason is that the secondary radiation is the result of a sequence of real transitions, each of which is accompanied by the emission of one LO phonon.

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1. INTRODUCTION

With the appearance of high-power laser sources of light, and with increasing receiver sensitivity, the capabilities of modern experiments on registration of secondary radiation of condensed media have greatly increased. This progress gives grounds for hoping that study of secondary radiation will become a reliable method of measuring a number of parameters of solids. The experimental research in this field has stimulated theoretical investigations (see, e.g., Refs. 1 and 2).

In this paper we predict a new type of secondary radiation of polar semiconductors. The radiation is the result of the following set of real transitions: primary radiation with frequency ω_l produces an electron-hole pair (EHP), the electron (or hole) emits in succession k longitudinal optical (LO) phonons (we neglect interactions with acoustic phonons), and finally the electron (or hole) emits a quantum $\hbar\omega_s$ of secondary light, accompanied by still one more optical phonon. No annihilation of the electron and hole takes place; the electron and hole, having lost an energy $\hbar\omega_s + (k+1)\hbar\omega_{LO}$, can retain part of their kinetic energy. Indirect emission of the quantum $\hbar\omega_s$ accompanied by the LO phonon, by the electron, is a process inverse to indirect absorption of light by free carriers that interact with LO phonons.[3]

It follows from the energy conservation law that when an electron emits $k+1$ LO phonons the secondary radiation spans the frequency band

$$0<\omega_s<(1+m_e/m_h)^{-1}(\omega_l-E_g/\hbar)-(k+1)\omega_{LO}, \tag{1.1}$$

where $m_e(m_h)$ is the effective mass of the electron (hole), and E_g is the width of the forbidden band. In fact, if the kinetic energy of the electron after emission of secondary radiation is equal to zero, then

$$\omega_s=\omega_l-E_g/\hbar-(k+1)\omega_{LO}-E_{0h}/\hbar, \tag{1.2}$$

where E_{0h} is the kinetic energy of the hole produced by light ω_l. Since the equality

$$E_{oh}=(\hbar\omega_l-E_g)(1+m_h/m_e)^{-1} \tag{1.3}$$

is satisfied in direct production of EHP, it follows that by substituting (1.3) in (1.2) we obtain the upper bound from (1.1). The lower bound $\omega_s=0$ corresponds to the maximum residual kinetic energy of the electron. If secondary radiation with participation of $k+1$ LO phonons is to take place at all, it is necessary that the right-hand side of (1.2) be larger than zero, i.e., that the following condition hold:

$$\omega_l>E_g/\hbar+(1+m_e/m_h)(k+1)\omega_{LO}. \tag{1.4}$$

If a hole rather than an electron participates in the radiation, then the mass m_e in (1.1)–(1.4) must be replaced by m_h, and vice versa. It follows from (1.4) that the frequency ω_l should lie in the region of the intrinsic absorption of the semiconductor.

An examination of the electron energy distribution

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function readily reveals a few other features of features of the secondary radiation. At low constants of the coupling of the electrons with the LO phonons (but much larger than the constants of the coupling with acoustic phonons and with light), the most probable processes are those in which one LO phonon is discarded. The electron then "cascades" over the band and forms (in the case of nondispersing LO phonons) a distribution that differs from zero only at the points where the electron kinetic energy equals

$$E_k=(\hbar\omega_l-E_g)\,(1+m_e/m_h)^{-1}-k\hbar\omega_{LO},\quad k=0,\,1,\,2,\ldots\,.$$

Of course, the energy region $E<\hbar\omega_{LO}$ is excluded from consideration, since the energy relaxation of the electron or hole is determined by the interaction with the acoustic phonons, impurity centers, recombination, etc. It should be noted that the number of electrons with energy E_k has no parametric smallness compared with the number of electrons of energy $E_k+\hbar\omega_{LO}$ in the steady state, since the departure time and the arrival time are determined by the same electron—LO-phonon coupling constant. Thus, owing to the initial isotropy of the momentum distribution, the electron distribution function differs from zero in thin spherical layers in K space. The same holds also for the hole distribution function. If we include in the consideration the interaction of the electrons with light, it must be recognized that the contribution of the processes with photon participation to the distribution function is very small and can be discarded. As a result, radiation processes must be studied with the distribution function described above.

As already noted, the most probable process is emission of a quantum $\hbar\omega_s$ accompanied by an LO phonon. It follows from (1.2) that the emission of a quantum $\hbar\omega_s$ is possible only from electrons having a kinetic energy

$$E\geqslant E_s=\hbar\omega_s+\hbar\omega_{LO}.$$

With increasing frequency ω_l of the incident light, the radius of the spherical layers increases. When the radius of the largest spherical layer reaches a value $(2m_eE_s)^{1/2}/\hbar$, observation of secondary light of frequency ω_s first becomes possible. When the next spherical layers pass through the same threshold, they also "come into play" and increase the scattering cross section for the given frequency ω_s. This explains the appearance of threshold singularities in the scattering cross section as a function of ω_l (see Fig. 4 below). At constant ω_l, a different number of spherical layers contribute to radiation with different ω_s, and this number depends on the value of ω_s. This is the reason for the presence of threshold singularities in the ω_s dependence of the cross section (see Fig. 3 below).

The existence of LO-phonon dispersion blurs somewhat the picture outlined above, but calculations show that all the singularities of the distribution remain well defined.

Examination of (1.2) and (1.3) shows that the secondary radiation can be treated as a sort of Raman scattering wherein $k+1$ LO phonons and an EHP are present in the crystal in the final state.

The cascade of successive real transitions wherein the electron emits a certain number of LO phonons was first considered by Martin and Varma[4] for a description multiphonon resonant Raman scattering (MRRS) of light, i.e., scattering with frequency $\omega_s=\omega_l-n\omega_{LO}$. However, the application of the cascade model to the MRRS is made complicated by the fact that annihilation of the EHP, accompanied by secondary radiation, takes place during the last stage of the process, and this annihilation is hindered by the fact that the electron and hole are separated in space. It is shown in Refs. 5–7 that the cross section for MRRS with participation of EHP has some additional smallness in terms of the electron-phonon interaction parameter compared with the results of Ref. 4. Owing to this smallness of the EHP contribution, it can compete in the MRRS cross section with the contribution of the hot excitons.[7,8]

The scattering considered in the present paper differs from MRRS in that the electron or hole emits a quantum $\hbar\omega_s$ without annihilation, so that this scattering can be described as a pure cascade process, i.e., as a sequence of known real transitions. In this respect it is similar to MRRS by hot excitons. It is important that in any cascade process with LO-phonon participation, in polar semiconductors an increase in the number of emitted phonons does not lead to an additional smallness with respect to the electron-phonon interaction constant.

The cross section was calculated by us by two methods. The first is the one proposed in Refs. 7 and 9 and based on the use, for the scattering tensor of rank 4, of a general formula that describes in principle any type of secondary radiation with allowance for all interactions in the crystal in arbitrary sequence. The formula was determined with the aid of a diagram technique. The second method is based on solving the balance equation for the distribution function of the electrons (holes). It turned out that both methods yield the same result, as might be expected for a cascade process of pure type. We present below only the calculations by the second method, and in addition show, by way of example, one of the diagrams (see Fig. 1), which corresponds to emission of two LO phonons and a quantum $\hbar\omega_s$ by an electron. In the diagram of Fig. 1, the pairs of the cuts 1–1′ and 2–2′ correspond to transitions through real states, the pair of cuts 3–3′ corresponds to a transition through a virtual state. The cut i corresponds to the initial state, when there is one photon of incident light with frequency ω_l. In the final state (cut f) there are two LO phonons, a secondary-radiation quantum $\hbar\omega_s$, an electron, and a hole.

Thus, assume a direct-band polar semiconductor with $m_e<m_h$. When an EHP is directly produced by the light, the hole kinetic energy is given by (1.3), and the kinetic energy of the electron at a light frequency ω_l is

$$E_{0e}=(1+m_e/m_h)^{-1}(\hbar\omega_l-E_g).\qquad(1.5)$$

The total secondary-radiation cross section consists of the cross sections that describe the processes of

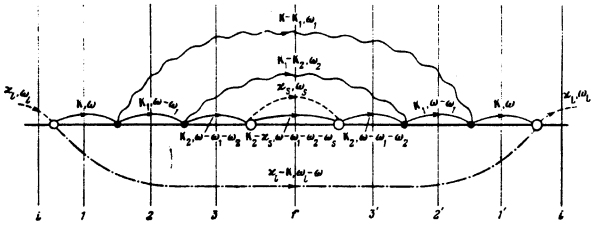

FIG. 1. One of the diagrams describing resonant Raman scattering of light with emission of two LO phonons without annihilation of an electron-hole pair.

photon emission by the electron and hole separately. Since these cross sections are functionally identical, it suffices to find the cross section for secondary radiation from the electrons.

The cross section of the process, averaged over the polarizations of the incident light and summed over the polarizations of the secondary radiation, is given by

$$\frac{d^2\sigma_e}{d\Omega d\omega_s}=\frac{V_0^2\omega_s^2 n(\omega_s)}{(2\pi)^3 c^4 n(\omega_l)}W(\omega_s), \tag{1.6}$$

where V_0 is the volume of the crystal, c is the speed of light in vacuum, $n(\omega)$ is the refractive index, $V(\omega_s)$ is the probability of emission of a secondary-radiation photon with frequency ω_s per unit time in a unit solid-angle interval, when the volume V_0 contains one incident-light quantum. By definition, the quantity $W(\omega_s)$ pertains to a medium with a unity refractive index. The probability $W(\omega_s)$ does not depend on the direction in which the secondary photon is emitted, and is given by

$$W(\omega_s)=\int_0^\infty P(E)W_1(E,\omega_s)dE, \tag{1.7}$$

where $W_1(E, \omega_s)$ is the probability, averaged over the angle between the electron and photon momentum directions, of emission, in a unit time, into a unit solid angle, of a photon of energy $\hbar\omega_s$ by an electron having a kinetic energy E; $P(E)dE$ is the number of electrons with energies in the interval from E to $E+dE$.

For the case when the momentum distribution of the electrons is isotropic, the balance equation takes the form

$$\frac{\partial P(E)}{\partial t}=\int_0^\infty P(E')W(E',E)dE'$$
$$-P(E)\int_0^\infty W(E,E')dE'+W_l\delta(E-E_{0e}), \tag{1.8}$$

where $W(E', E)dE$ is the probability of the transition of an electron from a state with energy E' to the energy interval from E to $E+dE$; W_l is the number of EHP produced in a unit time in a medium with unity refractive index if the volume V_0 contains one incident-light photon with frequency ω_l; E_{0e} is determined by formula (1.5). In polar semiconductors, the interaction of the electrons with the LO phonons is much stronger than all other types of interaction, so that for energies $E>\hbar\omega_{LO}$ we put $W(E', E)\approx W_{LO}(E', E)$, where $W_{LO}(E', E)dE$ is the probability of an electron transition from a state with kinetic energy E' into a state with energy in the interval from E to $E+dE$, with emission of an LO phonon. We shall consider effects at a temperature below the Debye temperature, when processes with LO-phonon absorption can be neglected. We shall solve Eq. (1.8) for the region $E>\hbar\omega_{LO}$.

2. CALCULATION OF THE TRANSITION PROBABILITIES

In the calculation of $W_{LO}(E, E')$ we choose the LO-phonon dispersion law in the form

$$\omega_{LO}(\mathbf{q})=\omega_{LO}-{}^1/_2\eta q^2, \tag{2.1}$$

where η is a dimensional dispersion parameter that can be either positive or negative; $\mathbf{q}$ is the wave vector of the LO phonon. The Fröhlich Hamiltonian of the interaction of the electrons with the LO phonons is

$$H_{el-ph}=\sum_{\mathbf{q}}(C_{\mathbf{q}}b_{\mathbf{q}}e^{i\mathbf{q}\mathbf{r}}+C_{\mathbf{q}}^*b_{\mathbf{q}}^+e^{-i\mathbf{q}\mathbf{r}}), \tag{2.2}$$

where

$$C_{\mathbf{q}}=-i\hbar\omega_{LO}\left(\frac{4\pi\alpha l^3}{V_0}\right)^{1/2}\frac{1}{lq},$$
$$l=(\hbar/2m_e\omega_{LO})^{1/2},\quad \alpha=\frac{e^2}{2l\hbar\omega_{LO}}(\varepsilon_\infty^{-1}-\varepsilon_0^{-1}),$$

$b_q(b_q^+)$ is the phonon annihilation (creation) phonon, ε_0 is the static dielectric constant, and ε_∞ is the high-frequency dielectric constant. The probability that an electron with kinetic energy E will emit one LO phonon is

$$W_{LO}(E)=\frac{2\pi}{\hbar}\sum_f|\langle f|H_{el-ph}|i\rangle|^2\delta(E_f-E_i). \tag{2.3}$$

The quantity $W_{LO}(E)$, calculated from (2.3) is equal to

$$W_{LO}(E)=\begin{cases}2\alpha\omega_{LO}\left(\dfrac{\hbar\omega_{LO}}{E}\right)^{1/2}\operatorname{arch}\left(\dfrac{E}{\hbar\omega_{LO}(1-\bar\eta)}\right)^{1/2}, & E\geqslant\hbar\omega_{LO}(1-\bar\eta)\\ 0, & E<\hbar\omega_{LO}(1-\bar\eta),\end{cases} \tag{2.4}$$

where

$$\bar\eta=\eta m_e/\hbar \tag{2.5}$$

is the dimensionless dispersion parameter.[1)]

We calculate the probability $W_{LO}(E, E')$ in first order in α. Recognizing that

$$W_{LO}(E)=\int W_{LO}(E,E')dE', \tag{2.6}$$

we get for $W_{LO}(E, E')$ from (2.3)

$$W_{LO}(E,E')=\begin{cases}\dfrac{\alpha\omega_{LO}^{3/2}\hbar^{1/2}}{2E^{1/2}(E'+\hbar\omega_{LO}-E)}\\ 0 \text{ outside the indicated interval}\end{cases}, \tag{2.7}$$

$$\varphi_-\left(\frac{E}{\hbar\omega_{LO}}\right)\leqslant\frac{E'}{\hbar\omega_{LO}}\leqslant\varphi_+\left(\frac{E}{\hbar\omega_{LO}}\right),$$

where

$$\varphi_\pm(x)=x-1+\frac{\bar\eta}{(1-\bar\eta)^2}[x^{1/2}\pm(x-1+\bar\eta)^{1/2}]^2. \tag{2.8}$$

If the energy E' is fixed, then $W_{LO}(E, E')$ differs from zero in the energy interval

$$\tilde\varphi_+\left(\frac{E'}{\hbar\omega_{LO}}\right)\leqslant\frac{E}{\hbar\omega_{LO}}\leqslant\tilde\varphi_-\left(\frac{E'}{\hbar\omega_{LO}}\right),$$
$$\tilde\varphi_\pm(x)=x+1-\frac{\bar\eta}{(1+\bar\eta)^2}[x^{1/2}\pm(x+1+\bar\eta)^{1/2}]^2. \tag{2.9}$$

The Hamiltonian of the interaction of an electron with light in a medium having a unity refractive index is given by

$$H_{el-l}=-\frac{e}{m_e c}\mathbf{Ap}+\frac{e^2A^2}{2m_e c^2},$$
$$\mathbf{A}=\left(\frac{2\pi\hbar c^2}{V_0}\right)^{1/2}\sum_{\varkappa,\lambda}\omega^{-1/2}\mathbf{e}_{\varkappa,\lambda}(a_{\varkappa,\lambda}e^{i\varkappa r}+a^+_{\varkappa,\lambda}e^{-i\varkappa r}), \tag{2.10}$$

where $\mathbf{p}$ is the electron-momentum operator, $\varkappa$ is the photon wave vector, $e_{\varkappa,\lambda}$ is its polarization vector, and $a_{\varkappa,\lambda}(a^+_{\varkappa,\lambda})$ is the operator of the annihilation (creation) of a photon with wave vector $\varkappa$ and polarization λ.

For the probability that an electron with wave vector K will emit in a unit time into a unit solid angle a secondary-radiation photon with wave vector $\varkappa$, summed over the polarizations of the emitted photon, we have

$$W_{i\mathbf{K},\varkappa_s}=\frac{2\pi}{\hbar}\sum_f|M|^2\delta(E_f-E_i), \tag{2.11}$$

where in lowest order of perturbation theory

$$M=\sum_n\frac{\langle f|H_{el-ph}|n\rangle\langle n|H_{el-l}|i\rangle}{E_i-E_n}+\sum_{n'}\frac{\langle f|H_{el-ph}|n'\rangle\langle n'|H_{el-l}|i\rangle}{E_i-E_{n'}}. \tag{2.12}$$

Here $|i\rangle$ is the initial system state with one electron with wave vector K; $|f\rangle$ is the final state and includes an electron, one LO phonon, and a quantum with $\varkappa_s$. In the intermediate states $|n\rangle$ are present an electron and one LO phonon, while in states $|n'\rangle$ are present an electron and a quantum with $\varkappa_s$. Calculations by formulas (2.11) and (2.12) yield at $\varkappa_s\approx 0$

$$W_{i\mathbf{K},\varkappa_s}=\frac{2\pi e^2\omega_{LO}^{1/2}\alpha}{m_e\omega_s^{3}\hbar^{1/2}V_0}E_{0e}^{1/2}f\left(\frac{E}{E_{0e}},\frac{E'}{E},\sin^2\theta,\bar\eta\right), \tag{2.13}$$

where

$$f(y,z,\sin^2\theta,\bar\eta)=y^{1/2}\Big\{[z+\bar\eta(1-z)]^{1/2}\frac{1+z+\bar\eta(1-z)}{(1-\bar\eta)^2}$$
$$-\frac{1}{2}(1-z)^2\ln\frac{1+[z+\bar\eta(1-z)]^{1/2}}{1-[z+\bar\eta(1-z)]^{1/2}}$$
$$+\sin^2\theta\Big[\frac{1}{2}[z+\bar\eta(1-z)]^{1/2}\frac{5-3(1-\bar\eta)z-3\bar\eta}{(1-\bar\eta)^2}$$
$$+\frac{3}{4}(1-z)^2\ln\frac{1+[z+\bar\eta(1-z)]^{1/2}}{1-[z+\bar\eta(1-z)]^{1/2}}\Big]\Big\}, \tag{2.14}$$
$$E=\hbar^2K^2/2m_e,\quad E'=E-\hbar\omega_{LO}-\hbar\omega_s; \tag{2.15}$$

E_{0e} is determined by formula (1.5), θ is the angle between the vectors K and $\varkappa_s$. The value of (2.13) averaged over the directions K is of the form

$$W_i(E,\omega_s)=\frac{2\pi e^2\omega_{LO}^{1/2}\alpha}{m_e\omega_s^{3}\hbar^{1/2}V_0}E_{0e}^{1/2}f\left(\frac{E}{E_{0e}},\frac{E'}{E},\frac{2}{3},\bar\eta\right). \tag{2.16}$$

3. BALANCE EQUATION FOR THE ELECTRON DISTRIBUTION FUNCTION

The balance equation (1.8) at $E>\hbar\omega_{LO}$ will be written in the form

$$\frac{\partial P(E)}{\partial t}=\int P(E')W_{LO}(E',E)dE'-P(E)\int W_{LO}(E,E')dE'+W_i\delta(E-E_{0e}). \tag{3.1}$$

For a stationary distribution, allowance for (2.6) changes (3.1) into

$$P(E)W_{LO}(E)-\int P(E')W_{LO}(E',E)dE'=W_i\delta(E-E_{0e}). \tag{3.2}$$

It is more convenient to solve (3.2) for the dimensionless quantity

$$\bar P=P\hbar\omega_{LO}\frac{W_{LO}(E_{0e})}{W_i}. \tag{3.3}$$

Introducing the dimensionless variables

$$x=E/\hbar\omega_{LO},\quad x_{0e}=E_{0e}/\hbar\omega_{LO}, \tag{3.4}$$

we have from (3.2), by taking (2.4), (2.7), and (2.9) into account.

$$\bar P(x)-\frac{x^{1/2}}{4\operatorname{arch}[x(1-\bar\eta)^{-1}]^{1/2}}\int_{\bar\varphi_+(x)}^{\bar\varphi_-(x)}\bar P(x')\frac{dx'}{(x')^{1/2}(x+1-x')}=\delta(x-x_{0e}). \tag{3.5}$$

From (3.5) we readily obtain

$$\bar P(x\approx x_{0e})=\delta(x-x_{0e}). \tag{3.6}$$

It is clear from the conservation laws that at not too large a phonon dispersion the quantity $\bar P(x)$ differs from zero in discrete energy intervals. With increasing number of the phonons that the electron must emit to reach an energy $E<E_{0e}$, these intervals increase, and if E_{0e} is large enough, then at energies below a certain value these intervals begin to overlap. For that energy region in which $\bar P(x)$ differs from zero only in discrete intervals, it is convenient to break it up in the following manner:

$$\bar P_0(x)=\delta(x-x_{0e}),$$
$$\bar P_1(x)=\bar P(x),\quad x_{0e}-1\leqslant x<x_{0e};\ \ldots \tag{3.7}$$
$$\ldots\bar P_k(x)=\bar P(x),\quad x_{0e}-k\leqslant x<x_{0e}-k+1;\ \ldots$$

Equation (3.5) interrelates the function $\bar P_k(x)$ with different k. Substituting in the second term of the left-hand side of (3.5) the known function $\bar P_0(x)$ from (3.7), we get for $\bar P_1(x)$

$$\bar P_1(x)=\frac{x^{1/2}}{4x_{0e}^{1/2}(x+1-x_{0e})\operatorname{arch}[x(1-\bar\eta)^{-1}]^{1/2}}$$
$$\text{at}\quad \varphi_-(x_{0e})\leqslant x\leqslant\varphi_+(x_{0e}), \tag{3.8}$$

$\bar P_1(x)=0$ at x outside the indicated interval.

Now, knowing $\bar P_1(x)$, we can find $\bar P_2(x)$ by an analogous procedure:

$$\bar P_2(x)=\frac{x^{1/2}}{4\operatorname{arch}[x(1-\bar\eta)^{-1}]^{1/2}}\int_{\bar\varphi_+(x)}^{\bar\varphi_-(x)}\bar P_1(x')\frac{dx'}{(x')^{1/2}(x+1-x')}. \tag{3.9}$$

In principle, it is possible to find all $\bar P_k(x)$ for any k. The practical difficulty is that the obtained integrals (say from the right-hand side of (3.9)) can not be expressed in analytic form.

For the limits of the intervals where $\bar P_k(x)$ differ from zero (we designate these limit $x_{k,\min}$ and $x_{k,\max}$, with $x_{k,\min}<x_{k,\max}$), we can write the recurrence relations

$$x_{k+1,\min}=\varphi_-(x_{k,\min}),\quad x_{k+1,\max}=\varphi_+(x_{k,\max}). \tag{3.10}$$

In principle, these relations make it possible to determine the energy starting with which the intervals where $\bar P_k(x)\neq 0$ begin to intersect (if such an energy exists at the given dispersion parameter for the given value of E_{0e}), but even below this energy the procedure of successive substitutions makes it possible to determine the solution of (3.5).

For dispersionless LO phonons, i.e., at $\eta=0$, the probabilities $W_{LO}(E,E')$ are proportional to $\delta(E-E'$

$-\hbar\omega_{LO}$) and the electron is thrown off from the level E to the level $E-\hbar\omega_{LO}$, and not to an energy band as in the case of phonons with dispersion. Equation (3.2) reduces then to a system of algebraic equations whose solution is of the form

$$P(E)=\sum_{0\leqslant k<E_{0e}/\hbar\omega_{LO}-1}\frac{W_l\delta(E-E_{0e}+k\hbar\omega_{LO})}{W_{LO}(E_{0e}-k\hbar\omega_{LO})}. \tag{3.11}$$

4. SCATTERING CROSS SECTION

The scattering cross section is determined by formulas (1.6) and (1.7). Using (2.16) and (3.3), we have

$$\frac{d^2\sigma_e}{d\Omega\,d\omega_s}=\frac{V_0e^2\omega_{LO}^2\alpha n(\omega_s)}{4\pi^2c^4m_e\omega_s n(\omega_l)}\frac{W_l}{W_{LO}(E_{0e})}x_{0e}^{1/2}\Big\{f\Big(1,\frac{x_{0e}'}{x_{0e}},\frac{2}{3},\bar\eta\Big)+\sum_{k\geqslant1}\int_{x_{k,min}}^{x_{k,max}}\bar P_k(x)f\Big(\frac{x}{x_{0e}},\frac{x'}{x},\frac{2}{3},\bar\eta\Big)dx\Big\}, \tag{4.1}$$

where $\bar P_k(x)$ is determined from (3.7) and (3.5),

$$x_{0e}'=x_{0e}-1-\omega_s/\omega_{LO},\quad x'=x-1-\omega_s/\omega_{LO}.$$

For the numerical calculations it is convenient to express the probability of EHP production in the form

$$W_l=ck_0(\omega_l), \tag{4.2}$$

where $k_0(\omega_l)$ is the coefficient of light absorption via direct production of an EHP in the medium with unity refractive index. The expression for $k_0(\omega_l)$ is

$$k_0(\omega_l)=k(\omega_l)n(\omega_l), \tag{4.3}$$

where $k(\omega_l)$ is the experimentally determined coefficient of light absorption via direct EHP production in a medium with a refractive index $n(\omega_l)$. From (4.2), (4.3), and (4.1) we have

$$\frac{d^2\sigma_e}{d\Omega\,d\omega_s}=\frac{V_0e^2\omega_{LO}k(\omega_l)n(\omega_s)}{8\pi^2m_ec^3\omega_s}\frac{x_{0e}}{\operatorname{arch}[x_{0e}(1-\bar\eta)^{-1}]^{1/2}}\times\Big\{f\Big(1,\frac{x_{0e}'}{x_{0e}},\frac{2}{3},\bar\eta\Big)+\sum_{k\geqslant1}\int_{x_{k,min}}^{x_{k,max}}\bar P_k(x)f\Big(\frac{x}{x_{0e}},\frac{x'}{x},\frac{2}{3},\bar\eta\Big)dx\Big\}. \tag{4.4}$$

Expression (4.4) yields the scattering cross section in the case when the photons are emitted by electrons only. It should be noted that this cross section does not depend on α. When account is taken of the hole contribution, the total cross section is

$$\frac{d^2\sigma}{d\Omega\,d\omega_s}=\frac{d^2\sigma_e}{d\Omega\,d\omega_s}+\frac{d^2\sigma_h}{d\Omega\,d\omega_s}, \tag{4.5}$$

where the expression for $d^2\sigma_h/d\Omega d\omega_s$ differs from the first term of (4.5) only in that m_e is replaced everywhere by m_h. Putting $\eta=0$ and using (3.11), we readily obtain an expression for the scattering cross section given in Ref. 10 for the case of dispersionless LO phonons:

$$\frac{d^2\sigma_e}{d\Omega\,d\omega_s}=\frac{V_0e^2\omega_{LO}k(\omega_l)n(\omega_s)}{8\pi^2m_ec^3\omega_s}x_{0e}^{1/2}\times\sum_{0\leqslant k<x_{0e}-1}\frac{x_{ke}^{1/2}}{\operatorname{arch}x_{ke}^{1/2}}f\Big(\frac{x_{ke}}{x},\frac{x_{ke}'}{x},\frac{2}{3},0\Big), \tag{4.6}$$

where $x_{ke}=x_{0e}-k$.

Formula (4.5) was obtained with the dispersion law (2.1). In order for the analysis to remain valid within the framework of the model it is necessary that the LO phonon with the maximum vector $q=q_{max}$ be describable by the dispersion law (2.1). If the approximation (2.1) is valid only up to $q=q_c$, then our results are valid for

$$\omega_l\leqslant\omega_{lc}=(E_g+E')/\hbar, \tag{4.7}$$

where

$$E'=\frac{\hbar^2}{2m_e}\Big[q_c\frac{1-\bar\eta}{2}+\frac{m_e\omega_{LO}}{\hbar q_c}\Big]^2\Big(1+\frac{m_e}{m_h}\Big),$$

as follows from (1.5) and from the expression for q_{max}

$$q_{max}=\{K+[K^2-2m_e\omega_{LO}\hbar^{-1}\times(1-\bar\eta)]^{1/2}\}(1-\bar\eta)^{-1}, \tag{4.8}$$

where K is the absolute value of the wave vector of the electron that emits the LO phonon.

5. DISCUSSION OF NUMERICAL RESULTS

On the basis of (3.8) and (3.9) we calculated the functions $\bar P_1(x)$ and $\bar P_2(x)$. The results of the calculations at $\bar\eta=0.02$ and $x_{0e}=3.4$ are shown in Fig. 2. It is seen from this figure that the electron energy spread increases with each emitted phonon. The maximum value of $\bar P_k(x)$ decreases at the same time. The energy spread of the electrons depends not only on the dispersion parameter but also on the level E_{0c}. At $x_{0c}=10$ and $\bar\eta=0.02$ the electron energy distribution becomes continuous already after emission of two LO phonons. At $x_{0e}=5$ a continuous distribution sets in after four successively emitted LO phonons.

At the numerical values

$$m_e/m_h=0.2,\quad x_{0e}=3.4,\quad \bar\eta=0.02$$

we used (4.4) to calculate the quantity

$$\Lambda=\Big(\frac{e^2\omega_{LO}V_0k(\omega_l)n(\omega_s)}{8\pi^2m_ec^3\omega_s}\Big)^{-1}\frac{d^2\sigma}{d\Omega\,d\omega_s} \tag{5.1}$$

as a function of ω_s/ω_{LO}. At the chosen values of the parameters, the holes make no contribution whatever. The results of the calculation are shown in Fig. 3 by

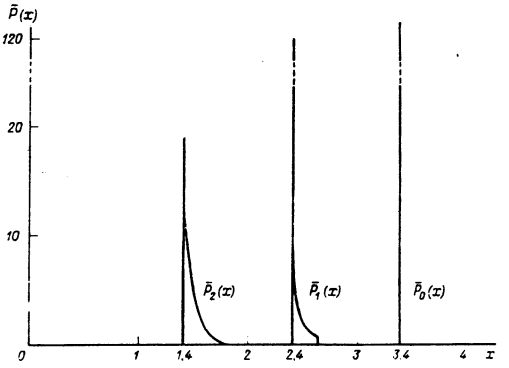

FIG. 2. Dimensionless density $\bar P$ of the distribution of the electron energy, as determined by (3.3), as a function of $x=E/\hbar\omega_{LO}$ at

$$x_{0c}=\frac{\omega_l-E_g/\hbar}{\omega_{LO}(1+m_e/m_h)}=3.4,\quad \bar\eta=0.02$$

(result of numerical calucations). The parts $\bar P_1(x)$ and $\bar P_2$ of the curve correspond to electrons after emission of one and two LO phonons, $\bar P_0(x)=\delta(x-x_{0e})$. The range of x from 0 to 1 was not considered, since in this region electrons can not emit LO phonons.

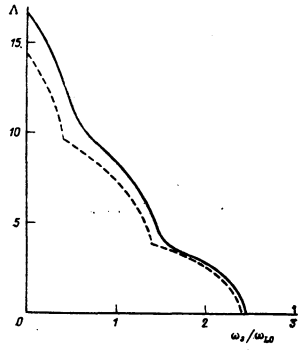

FIG. 3. Dependence of the dimensionless quantity Λ (5.1) on ω_s/ω_{LO} at $m_e/m_h = 0.2$ $x_{0e} = 3.4$ for LO phonons without dispersion (dashed line) and with dispersion (solid line) at $\bar{\eta} = 0.02$ (result of numerical calculations).

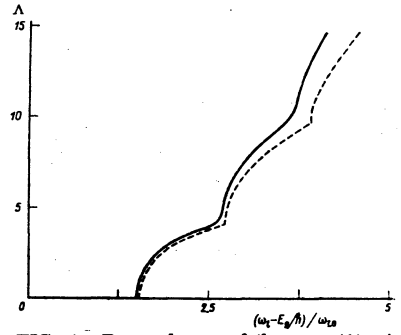

FIG. 4. Dependence of the quantity Λ (5.1) on $(\omega_l - E_g/\hbar)\omega_{LO}$ at $m_e/m_h = 0.2$ $\omega_s/\omega_{LO} = 0.3$ for LO phonons without dispersion (dashed line) and with dispersion at $\bar{\eta} = 0.02$ (solid line). Result of numerical calculations.

the solid line. The dashed line shows the same dependence, but for the case of dispersionless LO phonons calculated by formula (4.6) (see also Ref. 10). For the case of LO phonons with dispersion, the dependence continues to have a steplike form. Each higher step corresponds to turning-on of a process in which the number of emitted LO phonons increases by unity. The presence of dispersion leads to a smoothing of the threshold singularities characteristic of the case of dispersionless LO phonons and to a shift of the entire curve towards larger ω_s. The first threshold appears at

$$\frac{\omega_s}{\omega_{LO}} = \frac{x_{0s}}{1-\bar{\eta}} - 1 = \frac{\omega_l - E_g/\hbar}{\omega_{LO}(1+m_e/m_h)(1-\bar{\eta})} - 1. \qquad (5.2)$$

Figure 4 shows a plot (solid curve) of Λ against ω_l/ω_{LO} at a fixed value of the ratio $\omega_s/\omega_{LO} = 0.3$ and at $\bar{\eta} = 0.02$. For comparison, the dashed curve shows the same dependence but for dispersionless LO phonons. The first threshold appears at

$$\frac{\omega_l}{\omega_{LO}} = \frac{E_g}{\hbar\omega_{LO}} + \left(1+\frac{\omega_s}{\omega_{LO}}\right)(1-\bar{\eta})\left(1+\frac{m_e}{m_h}\right). \qquad (5.3)$$

It must be noted that the chosen value of $\bar{\eta} = 0.02$ is much larger than those really existing ($\approx 10^{-4}$). Increasing the dispersion parameter leads to greater smoothing of the thresholds but, as seen from Figs. 3 and 4, even at a larger dispersion they manifest themselves well enough. We emphasize that the distances between the thresholds on Figs. 3 and 4 are different: they are exactly equal to unity at $\eta = 0$ on Fig. 3, while in Fig. 4 they are equal to $1 + m_e/m_h$.

[1)]Attention must be called to the difference between the parameter $\bar{\eta}$ and the customarily encountered disperssion parameter. If the dispersion law is written in the form

$$\omega_{LO}(q) = \omega_{LO}\left[1 - \frac{\Delta}{2}\left(\frac{qa}{2\pi}\right)^2\right],$$

then the usual values of Δ range from 1/3 to 1/10; relating, however, the quantities $\bar{\eta}$ and Δ, we get

$$\bar{\eta} = \Delta\left(\frac{a}{2\pi}\right)^2 \frac{m_e\omega_{LO}}{\hbar} \approx 10^{-4} \ll \Delta,$$

where a is the lattice constant.

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