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Phase transitions in complex magnetic structures

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High-temperature phase transitions are considered in alloys of rare-earth metals, of the type Er-Tb and Er-Dy, whose magnetic structure is a tilted spiral wave traveling along a preferred axis z of the crystal. For these compounds a phase diagram is constructed in the variables T and $|a_\parallel|$ ($a_\parallel$ is the anisotropy constant along the z axis); it describes phase transitions from the paramagnetic region to all possible states of the tilted spiral. The discontinuities of the order parameter and the critical exponents are calculated. There is a "tetracritical" point on the phase diagram. The behavior of the system in the vicinity of the "tetracritical" point is considered; and it is shown that in this system, along with first-order phase transitions to a planar spiral or to a state with a longitudinal sinusoidal wave, there can occur a second-order phase transition of the second kind directly to a tilted spiral.

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1. INTRODUCTION

The investigation of phase transitions in magnetic structures such as the helicoidal or sinusoidal (Dy, Er, Ho Tb, Cr, Eu, DyC_2, MnO_2, and $REAu_2$, where RE represents ions of rare-earth metals) has been the object of a large amount of research.[1-5] It has been shown that, depending on the symmetry of the system, phase transitions in materials with a complex magnetic structure may be either of first or of second order; the instabilities that lead to phase transitions of the first order are due to fluctuations of the short-range order.

In the present work, we have considered phase transitions in alloys of the type Er-Dy or Er-Tb, in which the magnetic structure is a spiral wave whose plane of polarization is oriented at an arbitrary angle Ψ to the direction of its wave vector $\mathbf{q}$ (see Fig. 1), a so-called tilted spiral. A phase diagram for these compounds has been constructed; it describes phase transitions from the paramagnetic region (P) to all possible states of the tilted-spiral structure. On the phase diagram there is a "tetracritical" point. The renormalization-group (RG) equations describing the behavior of the system near a tetracritical point are derived; and on the basis of these equations, transit-

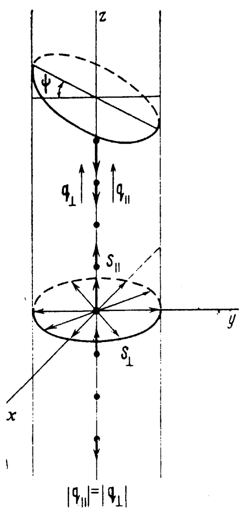

FIG. 1.

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ions are considered from the paramagnetic phase to a plane spiral (NS) or to a longitudinal sinusoidal wave (c-sin), and then to a tilted spiral (TS). It is shown that in the first case a first-order phase transition occurs, and then, after a lowering of the dimensionality of the projective phase space, a second-order phase transition occurs. The corresponding values of the discontinuity of the order parameter, and the critical exponents are calculated.

2. STATEMENT OF THE PROBLEM

We consider a magnetic structure described by two complex spin vectors. To them correspond two spin-density waves, traveling along a distinguished (tetragonal, orthorhombic, or hexagonal) axis $z(c)$ of the crystal (see Fig. 1):

$$\mathbf{S}_{\parallel}=\mathbf{s}_{\parallel}^{+}+i\mathbf{s}_{\parallel}^{-},$$
$$\mathbf{s}_{\parallel}^{+}=\mathbf{s}_{\parallel 0}^{+}\,\mathrm{Re}\,e^{i(qz+\varphi_0)},\qquad \mathbf{s}_{\parallel}^{-}=\mathbf{s}_{\parallel 0}^{-}\,\mathrm{Im}\,e^{i(qz+\varphi_0)}; \tag{1}$$

$$\mathbf{S}_{\perp}=\mathbf{s}_{\perp}^{+}+i\mathbf{s}_{\perp}^{-},$$
$$\mathbf{s}_{\perp}^{+}=\mathbf{s}_{\perp 0}^{+}\,\mathrm{Re}\,e^{iqz},\qquad \mathbf{s}_{\perp}^{-}=\mathbf{s}_{\perp 0}^{-}\,\mathrm{Im}\,e^{iqz}. \tag{2}$$

The following invariants of exchange and relativistic nature can be formed from them:

1. $\mathbf{S}_{\parallel}\mathbf{S}_{\parallel}^{*},\quad \mathbf{S}_{\perp}\mathbf{S}_{\perp}^{*},\quad S_{\parallel}^{z}S_{\parallel}^{z*},\quad S_{\perp}^{z}S_{\perp}^{z*};$
2. $(\mathbf{S}_{\parallel}\mathbf{S}_{\parallel}^{*})^2,\quad (\mathbf{S}_{\perp}\mathbf{S}_{\perp}^{*})^2,\quad S_{\parallel}^{2}S_{\parallel}^{2*},\quad S_{\perp}^{2}S_{\perp}^{2*},$

$(\mathbf{S}_{\parallel}\mathbf{S}_{\parallel}^{*})(\mathbf{S}_{\perp}\mathbf{S}_{\perp}^{*}),\quad (\mathbf{S}_{\parallel}\mathbf{S}_{\perp})(\mathbf{S}_{\parallel}^{*}\mathbf{S}_{\perp}^{*})$
$+(\mathbf{S}_{\parallel}\mathbf{S}_{\perp}^{*})(\mathbf{S}_{\parallel}^{*}\mathbf{S}_{\perp}),$

$S_{\parallel}^{z2}S_{\parallel}^{z2*},\quad S_{\perp}^{z2}S_{\perp}^{z2*},\quad S_{\parallel}^{z}S_{\parallel}^{z*}S_{\perp}^{z}S_{\perp}^{z*}.$

We shall hereafter neglect the relativistic invariants of the fourth order.

On the basis of these invariants, one can construct the Landau free energy in the paramagnetic range:

$$\begin{aligned}F={}&{}^1/_2\tau_{\parallel}(s_{\parallel}^{+2}+s_{\parallel}^{-2})+{}^1/_2\tau_{\perp}(s_{\perp}^{+2}+s_{\perp}^{-2})+{}^1/_2a_{\parallel}(s_{\parallel z}^{+2}+s_{\parallel z}^{-2})\\
&+{}^1/_2a_{\perp}(s_{\perp z}^{+2}+s_{\perp z}^{-2})+{}^1/_8\Gamma_{1\parallel}(s_{\parallel}^{+4}+s_{\parallel}^{-4})+{}^1/_8\Gamma_{1\perp}(s_{\perp}^{+4}+s_{\perp}^{-4})+{}^1/_4\Gamma_{2\parallel}s_{\parallel}^{+2}s_{\parallel}^{-2}\\
&+{}^1/_4\Gamma_{2\perp}s_{\perp}^{+2}s_{\perp}^{-2}+{}^1/_2\Gamma_{3\parallel}(\mathbf{s}_{\parallel}^{+}\mathbf{s}_{\parallel}^{-})^2+{}^1/_2\Gamma_{3\perp}(\mathbf{s}_{\perp}^{+}\mathbf{s}_{\perp}^{-})^2\\
&+{}^1/_4\Gamma_4(s_{\parallel}^{+2}s_{\perp}^{+2}+s_{\parallel}^{-2}s_{\perp}^{-2}+s_{\parallel}^{+2}s_{\perp}^{-2}+s_{\parallel}^{-2}s_{\perp}^{+2})\\
&+{}^1/_2\Gamma_5[(\mathbf{s}_{\parallel}^{+}\mathbf{s}_{\perp}^{+})^2+(\mathbf{s}_{\parallel}^{-}\mathbf{s}_{\perp}^{-})^2+(\mathbf{s}_{\parallel}^{+}\mathbf{s}_{\perp}^{-})^2+(\mathbf{s}_{\parallel}^{-}\mathbf{s}_{\perp}^{+})^2].\end{aligned} \tag{3}$$

In the expression (3), $a_{\parallel}<0$ and $a_{\perp}>0$ are anisotropy constants, and the seed values of temperatures τ_{j0} and amplitudes Γ_{ij0} are

$$\tau_{\parallel 0}=\tau_{\perp 0}=\tau,\quad \Gamma_{1\parallel 0}=\Gamma_{1\perp 0}=\Gamma_{10},\quad \Gamma_{2\parallel 0}=\Gamma_{2\perp 0}=\Gamma_{10}-2\Gamma_{30},\quad \Gamma_{3\parallel 0}=\Gamma_{3\perp 0}=\Gamma_{30}.$$

In accordance with experimental data,[6-8] we shall consider the case of large anisotropy, either longitudinal or transverse ($|a_{\parallel}|\lesssim 1$, $|a_{\perp}|\lesssim 1$). In this case the invariant associated with the amplitude Γ_5 disappears. Then the RG equations in this case take the following form:

$$\begin{aligned}
-\Gamma_{1\parallel}'&=(n_{\parallel}+8)\Gamma_{1\parallel}^2+n_{\parallel}\Gamma_{2\parallel}^2+4\Gamma_{2\parallel}\Gamma_{3\parallel}+4\Gamma_{3\parallel}^2+2n_{\perp}\Gamma_4^2,\\
-\Gamma_{1\perp}'&=(n_{\perp}+8)\Gamma_{1\perp}^2+n_{\perp}\Gamma_{2\perp}^2+4\Gamma_{2\perp}\Gamma_{3\perp}+4\Gamma_{3\perp}^2+2n_{\parallel}\Gamma_4^2,\\
-\Gamma_{2\parallel}'&=2(n_{\parallel}+2)\Gamma_{1\parallel}\Gamma_{2\parallel}+4\Gamma_{1\parallel}\Gamma_{3\parallel}+4\Gamma_{2\parallel}^2+4\Gamma_{3\parallel}^2+2n_{\perp}\Gamma_4^2,\\
-\Gamma_{2\perp}'&=2(n_{\perp}+2)\Gamma_{1\perp}\Gamma_{2\perp}+4\Gamma_{1\perp}\Gamma_{3\perp}+4\Gamma_{2\perp}^2+4\Gamma_{3\perp}^2+2n_{\parallel}\Gamma_4^2,\\
-\Gamma_{3\parallel}'&=4\Gamma_{1\parallel}\Gamma_{3\parallel}+8\Gamma_{2\parallel}\Gamma_{3\parallel}+2(n_{\parallel}+2)\Gamma_{3\parallel}^2,\\
-\Gamma_{3\perp}'&=4\Gamma_{1\perp}\Gamma_{3\perp}+8\Gamma_{2\perp}\Gamma_{3\perp}+2(n_{\perp}+2)\Gamma_{3\perp}^2,\\
-\Gamma_4'&=[(n_{\parallel}+2)\Gamma_{1\parallel}+(n_{\perp}+2)\Gamma_{1\perp}]\Gamma_4+(n_{\parallel}\Gamma_{2\parallel}+n_{\perp}\Gamma_{2\perp})\Gamma_4+2(\Gamma_{3\parallel}+\Gamma_{3\perp})\Gamma_4+4\Gamma_4^2;\\
-\tau_{\parallel}'&=\tau_{\parallel}[(n_{\parallel}+2)\Gamma_{1\parallel}+n_{\parallel}\Gamma_{2\parallel}+2\Gamma_{3\parallel}]+2n_{\perp}\tau_{\perp}\Gamma_4,\\
-\tau_{\perp}'&=\tau_{\perp}[(n_{\perp}+2)\Gamma_{1\perp}+n_{\perp}\Gamma_{2\perp}+2\Gamma_{3\perp}]+2n_{\parallel}\tau_{\parallel}\Gamma_4,
\end{aligned} \tag{4}$$

the variable of the ϵ-expansion method is

$$x=\frac{2}{\varepsilon(\Lambda^2)^{\varepsilon/2}}\left\{\left(\frac{\Lambda^2}{\max[\lambda^2(\tau_{\parallel},\tau_{\perp}),s_{\perp 0}^2,s_{\parallel 0}^2]}\right)^{\varepsilon/2}-1\right\},\quad \varepsilon\to 0$$

and $\lambda^2(\tau_{\parallel},\tau_{\perp})$ is a scale parameter expressed as a function of $\tau_{\parallel}$ and $\tau_{\perp}$; $s_{\perp 0}^2=s_{\perp}^{+2}+s_{\perp}^{-2}$, $s_{\parallel 0}^2=s_{\parallel}^{+2}+s_{\parallel}^{-2}$. After the RG equations have been obtained, one can turn to the consideration of a specific magnetic structure.

3. TILTED SPIRAL

This magnetic structure is characteristic of the alloy Er–Tb and can be formed by longitudinal sinusoidal and helicoidal spin-density waves, traveling along a distinguished axis $z(c)$ of the crystal. Here, in the case of large anisotropy, the free energy of the system has the form

$$\begin{aligned}F={}&{}^1/_2\tau_{\parallel}(s_{\parallel}^{+2}+s_{\parallel}^{-2})+{}^1/_2\tau_{\perp}(s_{\perp}^{+2}+s_{\perp}^{-2})+{}^1/_8\Gamma_{1\parallel}(s_{\parallel}^{+2}+s_{\parallel}^{-2})^2\\
&+{}^1/_8\Gamma_{1\perp}(s_{\perp}^{+4}+s_{\perp}^{-4})+{}^1/_4\Gamma_{2\perp}s_{\perp}^{+2}s_{\perp}^{-2}+{}^1/_4\Gamma_4(s_{\parallel}^{+2}+s_{\parallel}^{-2})(s_{\perp}^{+2}+s_{\perp}^{-2}).\end{aligned} \tag{5}$$

Here

$$\tau_{\parallel 0}=\tau+a_{\parallel},\quad \tau_{\perp 0}=\tau+a_{\perp},\quad a_{\parallel}<0,\quad a_{\perp}<0,$$
$$\Gamma_{1\parallel 0}=\Gamma_{1\perp 0}=\Gamma_{10},\quad \Gamma_{2\perp 0}=\Gamma_{10}-2\Gamma_{30},\quad \Gamma_{10}>0,\quad \Gamma_{40}>0,$$
$$\mathbf{s}_{\parallel}^{+}=(0,0,s_{\parallel z}^{+}),\quad \mathbf{s}_{\perp}^{+}=(s_{\perp x}^{+},0,0),\quad \mathbf{s}_{\parallel}^{-}=(0,0,s_{\parallel z}^{-}),\quad \mathbf{s}_{\perp}^{-}=(0,s_{\perp y}^{-},0).$$

We shall consider the behavior of the system for various relations between the values of $|a_{\parallel}|$ and of $|a_{\perp}|$.

1. $|a_{\perp}|<|a_{\parallel}|$. The RG equations for $\Gamma_{ij}(x)$ and $\Gamma_j(x)$ can be easily obtained from equations (4):

$$\begin{aligned}&-\Gamma_{1\parallel}'=10\Gamma_{1\parallel}^2,\quad -\Gamma_{1\perp}'=2\Gamma_4^2,\quad -\Gamma_{2\perp}'=2\Gamma_4^2,\quad -\Gamma_4'=4\Gamma_{1\parallel}\Gamma_4,\\
&-\tau_{\parallel}'=4\tau_{\parallel}\Gamma_{1\parallel},\quad -\tau_{\perp}'=2\tau_{\parallel}\Gamma_4.\end{aligned} \tag{6}$$

The RG variable x in this case is found to be

$$x=\frac{2}{\varepsilon(\Lambda^2)^{\varepsilon/2}}\left\{\left(\frac{\Lambda^2}{\max[\lambda^2(\tau_{\parallel}),s_{\perp 0}^2]}\right)^{\varepsilon/2}-1\right\},\quad \varepsilon\to 0.$$

Equations (6) can be easily integrated, and their solutions have the form

$$\begin{aligned}&\Gamma_{1\parallel}(x)=\frac{\Gamma_{10}}{1+10\Gamma_{10}x},\quad \Gamma_{1\perp}(x)=\Gamma_{10}-\frac{\Gamma_{40}^2}{\Gamma_{10}}[(1+10\Gamma_{10}x)^{1/5}-1],\\
&\Gamma_{2\perp}(x)=\Gamma_{1\perp}(x)-2\Gamma_{30},\quad \Gamma_4(x)=\frac{\Gamma_{40}}{(1+10\Gamma_{10}x)^{2/5}},\\
&\tau_{\parallel}(x)=\frac{\tau_{\parallel 0}}{(1+10\Gamma_{10}x)^{2/5}},\quad \tau_{\perp}(x)=\tau_{\perp 0}-\frac{\tau_{\parallel 0}\Gamma_{40}}{\Gamma_{10}}[(1+10\Gamma_{10}x)^{1/5}-1].\end{aligned} \tag{7}$$

It follows from these expressions that the system can undergo a first-order transition to the helicoidal state. For this, it is necessary that $\Gamma_{30}<\Gamma_{10}$.[3] The stability limit x_h^* of the paramagnetic phase is determined by the condition $\Gamma_{1\perp}-\Gamma_{30}=0$ and is found to be

$$x_h^*=\frac{1}{10\Gamma_{10}}\left\{\left[1+\frac{\Gamma_{10}(\Gamma_{10}-\Gamma_{30})}{\Gamma_{40}^2}\right]^5-1\right\}. \tag{8}$$

Then the value of the discontinuity of the order parameter $s_{\perp 0}^2$ and the temperature $\tau_{\perp h}$ of transition to the helicoidal state (plane spiral) are, respectively,

$$s_{\perp 0}^2=\tau_{\perp h}/2b(x_h^*), \tag{9}$$

$$\tau_{\perp h}=2b(x_h^*)\left(\frac{1-\varepsilon/2}{1+{}^1/_2\varepsilon x_h^*}\right)^{2/\varepsilon},\quad \varepsilon\to 0; \tag{10}$$

here

$$b(x_h^*)={}^1/_4\Gamma_4^2(x_h^*).$$

After a transition to the normal spiral has occurred, the free energy of the longitudinal subsystem will have the form

$$F_\parallel={}^1/_2\tau_\parallel(s_\parallel^{+2}+s_\parallel^{-2})+{}^1/_8\Gamma_{1\parallel}(s_\parallel^{+2}+s_\parallel^{-2})^2. \tag{11}$$

Thus it follows from this expression that the system will transform smoothly to a state corresponding to a tilted spiral.

The problem of the second-order phase transition to a tilted spiral must be solved with the initial condition $x=x_{0h}$, where x_{0h} is determined by the equation $\tau_\perp(x_{0h})=\tau_{\perp h}$ and is

$$x_{0h}=\frac{1}{10\Gamma_{10}}\left\{\left[1+\frac{\Gamma_{10}}{\tau_{\parallel 0}\Gamma_{40}}(\tau_{\perp 0}-\tau_{\perp h})\right]^5-1\right\}. \tag{12}$$

Thus by solving the equations for $\tau_\parallel$ and $\tau_{1\parallel}$, one can determine the critical index for the second-order phase transition, which is equal to $\Upsilon=\frac{1}{2}+\epsilon/10$.[9]

2. $|a_\parallel|<|a_\perp|$. The RG equations have the following form:

$$\begin{gathered}-\Gamma_{1\parallel}'=2\Gamma_4^2,\quad -\Gamma_{1\perp}'=9\Gamma_{1\perp}^2+\Gamma_{2\perp}^2,\quad -\Gamma_{2\perp}'=6\Gamma_{1\perp}\Gamma_{2\perp}+4\Gamma_{2\perp}^2,\\ -\Gamma_4'=(3\Gamma_{1\perp}+\Gamma_{2\perp})\Gamma_4,\\ -\tau_\parallel'=2\tau_\perp\Gamma_4,\quad -\tau_\perp'=\tau_\perp(3\Gamma_{1\perp}+\Gamma_{2\perp}),\end{gathered} \tag{13}$$

and correspondingly

$$x=\frac{2}{\varepsilon(\Lambda^2)^{\varepsilon/2}}\left\{\left(\frac{\Lambda^2}{\max[\lambda^2(\tau_\perp),s_{\parallel 0}^2]}\right)^{\varepsilon/2}-1\right\}.$$

By introducing the new variables

$$y_{1\parallel}=\Gamma_{1\parallel}/\Gamma_{1\perp},\quad y_{2\perp}=\Gamma_{2\perp}/\Gamma_{1\perp},\quad y_4=\Gamma_4/\Gamma_{1\perp},\quad z=-\ln\Gamma_{1\perp},$$

one can put the system of equations (13) into the following form:

$$\begin{gathered}\frac{dy_{1\parallel}}{dz}=\frac{y_{1\parallel}(9+y_{2\perp}^2)-2y_4^2}{9+y_{2\perp}^2},\quad \frac{dy_{2\perp}}{dz}=y_{2\perp}\frac{(1-y_{2\perp})(3-y_{2\perp})}{9+y_{2\perp}^2},\\ \frac{dy_4}{dz}=y_4\frac{6-y_{2\perp}(1-y_{2\perp})}{9+y_{2\perp}^2}.\end{gathered} \tag{14}$$

We shall consider in detail the equation

$$\frac{dy_{2\perp}}{dz}=y_{2\perp}\frac{(1-y_{2\perp})(3-y_{2\perp})}{9+y_{2\perp}^2}. \tag{14a}$$

This equation has three fixed points: $y_{2\perp}^{(1)}=0$, $y_{2\perp}^{(2)}=1$, $y_{2\perp}^{(3)}=3$. The point $y_{2\perp}^{(2)}=1$ is stable, the points $y_{2\perp}^{(1,3)}=0,3$ unstable (see Fig. 2). If the seed values of Γ_{10} and $\Gamma_{2\perp 0}$ are such that $0<y_{2\perp 0}<3$, that is $-\Gamma_{10}<\Gamma_{30}<\frac{1}{2}\Gamma_{10}$, then $y_{2\perp}$ tends toward the stable point $y_{2\perp}^{(2)}=1$, or $\Gamma_{1\perp}\to\Gamma_{2\perp}\to 1/10x$; and this means that the system can undergo a first-order phase transition to a state with a longitudinal sinusoidal wave. In fact, on integrating equations (14) by quadratures, we get

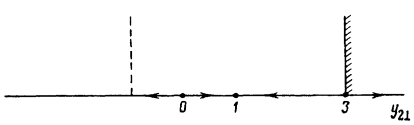

FIG. 2.

$$\begin{gathered}\frac{\Gamma_{1\parallel}(x)}{\Gamma_{10}}=1-\frac{2y_{40}^2}{y_{2\perp 0}(3-y_{2\perp 0})}\frac{y_{2\perp}-y_{2\perp 0}}{1-y_{2\perp}},\\ \frac{\Gamma_{1\perp}(x)}{\Gamma_{10}}=\left(\frac{3-y_{2\perp 0}}{3-y_{2\perp}}\right)^3\left(\frac{1-y_{2\perp}}{1-y_{2\perp 0}}\right)^5\left(\frac{y_{2\perp 0}}{y_{2\perp}}\right)^3,\\ \frac{\Gamma_4(x)}{\Gamma_{40}}=\left(\frac{3-y_{2\perp 0}}{3-y_{2\perp}}\right)\left(\frac{1-y_{2\perp}}{1-y_{2\perp 0}}\right)^2\left(\frac{y_{2\perp 0}}{y_{2\perp}}\right),\\ \frac{\tau_\parallel(x)}{\tau_{\parallel 0}}=1-\frac{2y_{40}\tau_{\perp 0}}{\tau_{\parallel 0}y_{2\perp 0}(3-y_{2\perp 0})}\frac{y_{2\perp}-y_{2\perp 0}}{1-y_{2\perp}},\\ \frac{\tau_\perp(x)}{\tau_{\perp 0}}=\left(\frac{3-y_{2\perp 0}}{3-y_{2\perp}}\right)\left(\frac{1-y_{2\perp}}{1-y_{2\perp 0}}\right)^2\left(\frac{y_{2\perp 0}}{y_{2\perp}}\right).\end{gathered} \tag{15}$$

The stability limit in this case is determined by the condition

$$\Gamma_{1\parallel}(x)/\Gamma_{10}=0,$$

whence

$$\Gamma_{10}x_s^*=\frac{(1-y_{2\perp 0})^5}{y_{2\perp 0}^3(3-y_{2\perp 0})^3}\varphi(y_{2\perp s}^*,y_{2\perp 0}), \tag{16}$$

$$\varphi(y_{2\perp s}^*,y_{2\perp 0})=\int_{y_{2\perp 0}}^{y_{2\perp s}^*}dy_{2\perp}\frac{y_{2\perp}^2(3-y_{2\perp})^2}{(1-y_{2\perp})^6}. \tag{17}$$

$$y_{2\perp s}^*=y_{2\perp 0}(2y_{40}^2+3-y_{2\perp 0})/[2y_{40}^2+y_{2\perp 0}(3-y_{2\perp 0})]. \tag{18}$$

On the basis of the expressions (16)–(18) one can easily obtain the value of the discontinuity of the order parameter and the critical temperature for the transition to the state with a longitudinal sinusoidal wave. In fact,

$$\tau_{\parallel s}=2b(x_s^*)\left(\frac{1-\varepsilon/2}{1+{}^1/_2\varepsilon x_s^*}\right)^{2/\varepsilon},\quad \varepsilon\to 0; \tag{19}$$

$$s_{\parallel 0}^2=\tau_{\parallel s}/2b(x_s^*),\quad b(x_s^*)={}^1/_4\Gamma_4^2(x_s^*). \tag{20}$$

After the transition to the c-sin state, the free energy of the helicoidal subsystem has the form

$$F_\perp={}^1/_2\tau_\perp(s_\perp^{+2}+s_\perp^{-2})+{}^1/_8\Gamma_{1\perp}(s_\perp^{+4}+s_\perp^{-4})+{}^1/_4\Gamma_{2\perp}s_\perp^{+2}s_\perp^{-2}. \tag{21}$$

The equations for the amplitudes $\Gamma_{1\perp}$ and $\Gamma_{2\perp}$ remain unchanged; and since $0<y_{2\perp 0}<3$, the system will, as before, tend toward the stable point $y_{2\perp}^{(2)}=1$. All that changes is the initial condition, which is determined by the equation

$$\frac{\tau_{\parallel s}}{\tau_{\parallel 0}}=1-\frac{2y_{40}\tau_{\perp 0}}{\tau_{\parallel 0}y_{2\perp 0}(3-y_{2\perp 0})}\frac{\overset{0}{y}_{2\perp s}-y_{2\perp 0}}{1-\overset{0}{y}_{2\perp s}}, \tag{22}$$

and accordingly

$$\Gamma_{10}x_{s0}=\frac{(1-y_{2\perp 0})^5}{y_{2\perp 0}^3(3-y_{2\perp 0})^3}\varphi(\overset{0}{y}_{2\perp s},y_{2\perp 0}), \tag{23}$$

$$\overset{0}{y}_{2\perp s}=y_{2\perp 0}\frac{2y_{40}\tau_{\perp 0}+\tau_{\parallel 0}(3-y_{2\perp 0})(1-\tau_{\parallel s}/\tau_{\parallel 0})}{2y_{40}\tau_{\perp 0}+\tau_{\parallel 0}y_{2\perp 0}(3-y_{2\perp 0})(1-\tau_{\parallel s}/\tau_{\parallel 0})}. \tag{24}$$

Thus, the system undergoes a second-order phase transition to a tilted spiral, TS. The critical exponent γ remains the same as in the case considered above, $|a_\perp|<|a_\parallel|$: namely, $\gamma=\frac{1}{2}+\epsilon/10$.

We shall consider the case $-1<y_{2\perp 0}<0$. This means that the function $y_{2\perp}(x)$ falls into the region of unstable solutions of equation (14a) (Fig. 2). The helicoidal subsystem becomes unstable and will undergo a first-order transition. The limit of its stability is determined by the condition $y_{2\perp}=-1$. The value of $y_{2\perp}$ at the transition point, $y_{2\perp h}^0$, is accordingly

$$y_{2\perp h}^{0}=\frac{2+3a+(9a^2+8a)^{1/2}}{2(1+a)},\quad a=\frac{\tau_{\perp h}^{0}}{\tau_{\perp 0}}\frac{(1-y_{2\perp 0})^2}{y_{2\perp 0}(3-y_{2\perp 0})},\tag{25}$$

where $\tau_{\perp h}^0$ is the temperature of transition of the helicoidal subsystem to the ordered state.

From the expression (25) it follows that the values of $y_{2\perp h}^0$ and $\tau_{\perp h}^0/\tau_{\perp 0}$ vary over the following ranges:

$$-3<y_{2\perp h}^{0}<-1,$$
$$-\frac{8y_{2\perp 0}(3-y_{2\perp 0})}{9(1-y_{2\perp 0})^2}<\frac{\tau_{\perp h}^{0}}{\tau_{\perp 0}}<-\frac{y_{2\perp 0}(3-y_{2\perp 0})}{(1-y_{2\perp 0})^2}.\tag{26}$$

The discontinuity of the order parameter is determined by the expression

$$s_{\perp 0}^2=\tau_{\perp h}^0/2b(x_{\perp h}^{*0}),\tag{27}$$

$$b(x_{\perp h}^{*0})=-\frac{1}{16}\Gamma_{10}^2\frac{(3-y_{2\perp 0})^6y_{2\perp 0}^6}{(1-y_{2\perp 0})^{10}}\frac{(1-y_{2\perp})^{11}}{y_{2\perp}^5(3-y_{2\perp})^5},\quad x\to x_{\perp h}^{*0};$$

$x_{\perp h}^{*0}$ is a renormalized value of $x_{\perp h}^{*}$, given by

$$\Gamma_{10}x_{\perp h}^{*}=\frac{(1-y_{2\perp 0})^5}{y_{2\perp 0}^3(3-y_{2\perp 0})^3}\varphi(-1,y_{2\perp 0}),\tag{28}$$

and accordingly $x_{\perp h}^{*0}>x_{\perp h}^{*}$. The value of $x_{\perp h}^{*0}$ is determined by the equation

$$\tau_{\perp h}^{0}=2b(x_{\perp h}^{*0})\left\{\frac{1-\frac{1}{2}\varepsilon[1+f(x_{\perp h}^{*0})]}{1+\frac{1}{2}\varepsilon x_{\perp h}^{*0}}\right\}^{2/\varepsilon},\quad \varepsilon\to 0,\tag{29}$$

$$f(x_{\perp h}^{*0})=-\frac{(1-y_{2\perp 0})^5}{\Gamma_{10}y_{2\perp 0}^3(3-y_{2\perp 0})^3}\frac{y_{2\perp}^2(1+y_{2\perp})(3-y_{2\perp})^2}{(1-y_{2\perp})^6},\quad x\to x_{\perp h}^{*0}.$$

We shall now consider the behavior of the longitudinal sinusoidal subsystem. In order that the inequalities $y_{2\perp s}^{*}<0$ and $y_{2\perp s}^0<0$ may be satisfied, the following conditions must be satisfied:

$$\frac{3}{2}-\left(\frac{9}{4}+2y_{40}^2\right)^{1/2}<y_{2\perp 0}<0,$$
$$\frac{3}{2}-\left[\frac{9}{4}+\frac{2y_{40}\tau_{\perp 0}}{\tau_{\parallel 0}(1-\tau_{\parallel s}/\tau_{\parallel 0})}\right]^{1/2}<y_{2\perp 0}<0.\tag{30}$$

From these conditions it follows that the value of $1-\tau_{\parallel s}/\tau_{\parallel 0}$ must vary within the range

$$\tau_{\perp 0}/y_{40}\tau_{\parallel 0}<1-\tau_{\parallel s}/\tau_{\parallel 0}<1.$$

When $y_{2\perp 0}\to-\epsilon$, the values of $\tau_{\parallel s}$ and $s_{\parallel 0}^2$ are determined by the expressions

$$\tau_{\parallel s}=2b(x_s^*)\left(\frac{1-\varepsilon/2}{1+\frac{1}{2}\varepsilon x_s^*}\right)^{2/\varepsilon},$$
$$s_{\parallel 0}^2=\frac{\tau_{\parallel s}}{2b(x_s^*)},\tag{31}$$

where $\epsilon\to 0$,

$$b(x_s^*)=\frac{1}{4}\Gamma_4^2(x_s^*),$$
$$b(x_s^*)=\Gamma_{40}^2\frac{y_{40}^4}{(2y_{40}^2+3)^2},$$
$$y_{40}<1,\quad y_{2\perp 0}\to-\varepsilon.$$

The parameters x_s^*, $y_{2\perp s}^*$, and $y_{2\perp s}^0$ are determined by the expressions

$$\Gamma_{10}x_s^*=\frac{1}{9}\left[\left(\frac{2y_{40}^2+3}{2y_{40}^2}\right)^3-1\right],$$
$$y_{2\perp s}^{*}=-\varepsilon\frac{2y_{40}^2+3}{2y_{40}^2},\quad y_{2\perp s}^{0}=-\varepsilon\frac{3\tau_{\parallel 0}}{2\tau_{\perp 0}}.\tag{32}$$

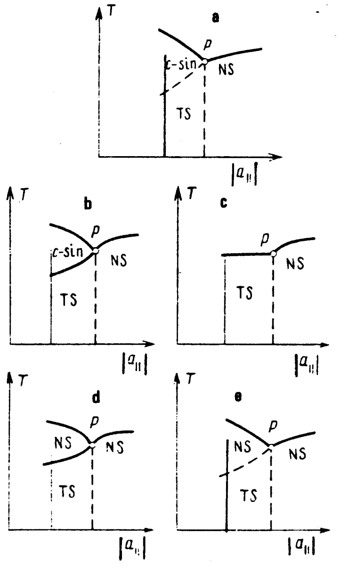

FIG. 3.

After the analysis presented above, the following conclusions can be drawn for the case $-1<y_{2\perp 0}<0$ (see Fig. 3b, c, d, e):

b) $y_{2\perp s}^0>-1$. In this case the system undergoes first a first-order phase transition to the c-sin state, and then a first-order phase transition to the tilted spiral.

c) $-3<y_{2\perp s}^0<-1$. Under these conditions there is a change of the phase diagram of the system, and a situation may arise in which $y_{2\perp s}^0\to y_{2\perp h}^0$. This means that a first-order phase transition from the paramagnetic phase to the tilted spiral is possible and that a tricritical point occurs on the phase diagram.

d) $y_{2\perp s}^0<-3$. The system undergoes a first-order phase transition from the paramagnetic region to a plane spiral, and then a first-order transition to the

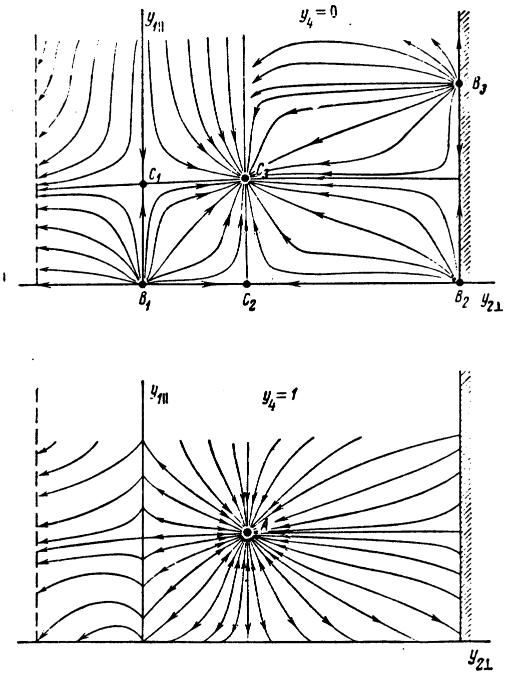

FIG. 4.

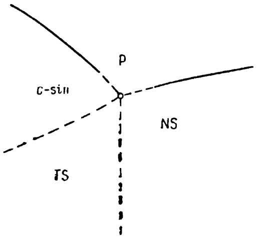

FIG. 5.

tilted spiral is possible.

e) $-1 < y_{2\perp 0} < \frac{3}{2} - (\frac{9}{4} + 4y_{40}^2)^{\frac{1}{2}}$. The system undergoes a first-order phase transition to a plane spiral, and then a second-order phase transition to the tilted spiral.

3. $|a_\parallel| \to |a_\perp|$. From the phase diagrams presented in Fig. 3, it follows that they have a tetracritical point. The case to be considered corresponds to the behavior of the system in the vicinity of this point. The fields $S_\parallel$ and $S_\perp$ are subject to large fluctuations, and the RG equations take the following form:

$$-\Gamma_{1\parallel}' = 10\Gamma_{1\parallel}^2 + 2\Gamma_4^2, \quad -\Gamma_{1\perp}' = 9\Gamma_{1\perp}^2 + \Gamma_{2\perp}^2 + 2\Gamma_4^2,$$
$$-\Gamma_{2\perp}' = 6\Gamma_{1\perp}\Gamma_{2\perp} + 4\Gamma_{2\perp}^2 + 2\Gamma_4^2, \quad -\Gamma_4' = (4\Gamma_{1\parallel} + 3\Gamma_{1\perp} + \Gamma_{2\perp})\Gamma_4 + 4\Gamma_4^2, \tag{33}$$
$$-\tau_\parallel' = 4\tau_\parallel\Gamma_{1\parallel} + 2\tau_\perp\Gamma_4, \quad -\tau_\perp' = \tau_\perp(3\Gamma_{1\perp} + \Gamma_{2\perp}) + 2\tau_\parallel\Gamma_4.$$

The variable x is defined by formula (4). In the variables $y_{1\parallel}$, $y_{2\perp}$, y_4, and z, the equations for the amplitudes take the form

$$\frac{dy_{1\parallel}}{dz} = \frac{y_{1\parallel}(9 + y_{2\perp}^2 - 10y_{1\parallel}) - 2y_4^2(1 - y_{1\parallel})}{9 + y_{2\perp}^2 + 2y_4^2},$$
$$\frac{dy_{2\perp}}{dz} = (1 - y_{2\perp})\frac{y_{2\perp}(3 - y_{2\perp}) - 2y_4^2}{9 + y_{2\perp}^2 + 2y_4^2}, \tag{34}$$
$$\frac{dy_4}{dz} = y_4\frac{6 - 4y_{1\parallel} - (1 - y_{2\perp})y_{2\perp} - 4y_4 + 2y_4^2}{9 + y_{2\perp}^2 + 2y_4^2}.$$

These equations have seven fixed points:

$$A(1,1,1)$$
$$B_1(0,0,0), \quad B_2(0,3,0), \quad B_3(9/5,3,0),$$
$$C_1(9/10,0,0), \quad C_2(0,1,0), \quad C_3(1,1,0).$$

The phase trajectories of the system are shown in Fig. 4. The points $B_1 - B_3$ and $C_1 - C_3$ are unstable, the point A weakly stable. This means that if the seed values of $y_{1\parallel 0}$, $y_{2\perp 0}$, and y_{40} are such that $y_{1\parallel} > 0$, $0 < y_{2\perp 0} < 3$, and $y_{40} > 0$ (diagram a of Fig. 3), then the system can approach this point infinitely slowly in time x. Figure 4 shows the phase trajectories of the system in the planes $y_4 = 0$ and $y_4 = 1$. We shall consider the behavior of the system in the plane $y_4 = 1$. When $y_{1\parallel 0} > 0$, $0 < y_{2\perp 0} < 3$, and $y_{40} > 0$, the system will tend to fall into this plane. Being in the plane $y_4 = 1$, the system will arrive at the point A if it has fallen into an ϵ-neighborhood of it. In this case there occurs a second-order phase transition to the tilted spiral, with critical exponent $\gamma = \frac{1}{2} + \epsilon/8$. If the system is outside an ϵ-neighborhood of the point A in the plane $y_4 = 1$, it will wander for a long time and may reach the stability limit of the paramagnetic phase (a first straight line $y_{1\parallel} = 0$, $-1 < y_{2\perp} < 3$ and a second, $y_{1\parallel} > 0$, $y_{2\perp} = -1$). There then occurs a first-order phase transition to the c-sin state or to the plane spiral, respectively. If $y_{1\parallel 0} > 0$, $-1 < y_{2\perp 0}$, and $y_{40} > 0$, the system cannot fall into an ϵ-neighborhood of the point A but will move to the stability boundary of the paramagnetic region (the planes: 1) $y_{1\parallel} = 0$, $y_4 > 0$, $-1 < y_{2\perp} < 0$ and 2) $y_{1\parallel} > 0$, $y_4 > 0$, $y_{2\perp} = 1$). In this case there will occur a first-order phase transition to states plotted in diagrams b, c, and d of Fig. 3, and then the system will behave the same as in §2.

4. CONCLUSION

The analysis presented above shows that the presence of a large anisotropy makes it possible to isolate all the states of a complex magnetic structure and provides a possibility of constructing a phase diagram of a new type in the variables $(T, |a_\parallel|)$. This diagram permits prediction of phase transitions at high temperatures for a whole class of materials (for example, besides the phase transitions in the alloys of rare-earth elements mentioned above, this diagram describes phase transitions in Ho and Er.[10] A correct calculation of fluctuations of the short-range order, within the framework of the ϵ-expansion method, provides a possibility of correctly describing all possible transitions in the magnetic structure considered. It is interesting to note that all transitions from the paramagnetic region, with the exception of transitions in the vicinity of the "tetracritical" point, are fluctuational first-order transitions. This is explained by the fact that despite the strong anisotropy, the number of fluctuating fields is large enough so that instabilities, leading to a first-order phase transition, arise in the system. After a first-order transition has occurred and one of the subsystems—helicoidal or sinusoidal—has transformed to a condensate, the number of fluctuating fields decreases. Therefore transitions to the tilted spiral are, as a rule, second order. The critical exponents of these transitions are nearly equal.

On the phase diagrams presented, there is a tetracritical point. In order to describe correctly the behavior of the system in its vicinity, one must write the RG equations to order ϵ^2. As a result we find that lines of first-order transitions change, near the tetracritical point, to lines of second-order transitions (Fig. 5).

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