[10]H. E. Dewitt, Phys. Rev. A14, 1290 (1976).
[11]L. D. Landau and E. M. Lifshitz, Kvantovaya mekhanika (Quantum Mechanics), Nauka, 1963 (English Transl., Addition-Wesley, Reading, Mass., 1965).
[12]I. N. Makarenko, V. A. Ivanov, and S. M. Stishov, Prib. Tekh. Eksp. No. 3, 202 (1974).
[13]I. N. Makarenko, A. M. Nikolaenko, and S. M. Stishov, in: Liquid Metals, ed. R. Evans and D. A. Greenwood, No. 30, Institute of Physics, Bristol, 1977, p. 79.
[14]V. A. Ivanov, I. N. Makarenko, A. M. Nikolaenko, and S. M. Stishov, Phys. Lett. 47A, 75 (1974).
[15]I. N. Makarenko, V. A. Ivanov, and S. M. Stishov, Pis'ma Zh. Eksp. Teor. Fiz. 18, 320 (1973) [JETP Lett. 18, 187 (1973)].
[16]D. L. Price, K. S. Singwi, and M. P. Tosi, Phys. Rev. B2, 2983 (1970).

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Influence of a microwave field on the critical current of superconducting contacts

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An investigation is reported of the influence of nonequilibrium electrons on the critical current of a superconducting contact subjected to an external hf field. It is shown that the effective cooling of electrons trapped in the region of the contact which has a lower value of the gap may result in a considerable increase of the critical current. The maximum critical current is found and a study is made of the dependence of this current on the power at various temperatures. On approach of the critical temperature there should be a transition from stimulation to suppression of superconductivity in the contact, which agrees with the experimental results. The frequency limits of the existence of the effect are estimated.

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1. INTRODUCTION

Experiments carried out on superconducting contacts (bridges, point contacts, etc.) demonstrate the possibility of increasing the critical current by irradiation with a microwave field (Dayem–Watt effect).[1-8] The influence of a microwave field on the critical current of a spatially homogeneous superconductor was considered by Éliashberg,[9] who showed that stimulation of superconductivity occurs because of a change in the electron energy distribution function under the influence of the electric field. We shall find the critical current of a contact between superconductors in a microwave field.

When a superconducting current flows through a contact, the order parameter in the region of the contact Δ becomes smaller than its value Δ_0 outside the contact. Electrons of energy $\epsilon < \Delta_0$ cannot escape outside the contact and execute finite motion in a potential well, being reflected by the contact edges. In an hf field the superconducting contact and the order parameter become alternating quantities. This "jitter" of the potential well results in energy diffusion of electrons, and their distribution function becomes of nonequilibrium type. This mechanism at the contacts is stronger than the direct influence of the electric field.

A considerable change in the electron distribution function occurs near the bottom of the potential well where the energy diffusion process results in an electron deficiency compared with the equilibrium distribution. This corresponds to effective cooling of the contact. On the other hand, diffusion may result in the accumulation of electrons at higher energies. The resultant effect on the critical current depends on the power and frequency of the incident radiation.

When this power is sufficiently high, the energy diffusion is a strong effect and all the electrons of energy $\epsilon < \Delta_0$ are cooled, whereas electrons of energy $\epsilon > \Delta_0$ do not accumulate in the contact region because of spatial diffusion and their distribution function remains of the equilibrium type. This stimulates superconductivity and the critical current of the contact rises.

At low radiation powers only the electrons of energies in a narrow region near the bottom of the well are cooled. These electrons are localized near the middle of the contact and the order parameter increases there. Elsewhere in the contact the electrons are heated and the order parameter decreases. It follows that the critical current of the contact decreases for a sufficiently low power.

When the frequency of the radiation field is increased, the range of energies where there are significant changes in the electron distribution begins to depend on the frequency and becomes wider. Electron cooling occurs in a wider region and, beginning from a certain frequency, the rise of the critical current occurs even at low radiation powers.

2. ENERGY DIFFUSION OF ELECTRONS IN A CONTACT

The distribution function of electrons in a contact subjected to an external alternating electromagnetic field

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is of the nonequilibrium type. If the energy relaxation time τ_ϵ is long compared with the field alternation period and with the electron spatial diffusion time along the contact, the electron distribution function $n(\epsilon)=[1-f(\epsilon)]/2$ depends weakly on time and coordinates and its dependence on the energy is described by the kinetic equation[10]

$$\frac{1}{\tau_\varepsilon}\left[f(\varepsilon)-\operatorname{th}\frac{\varepsilon}{2T}\right]\overline{\langle\varepsilon(\varepsilon^2-\Delta^2)^{-1/2}\rangle}=\frac{\partial}{\partial\varepsilon}\left(D_\varepsilon\frac{\partial f}{\partial\varepsilon}\right). \tag{1}$$

The angular brackets $\langle\dots\rangle$ denote averaging over the contact region, where $\Delta<\epsilon$, and the bar represents time averaging. In the case of smooth variations of the modulus of the order parameter Δ in the contact and sufficiently low frequencies ω, the energy diffusion coefficient D_ϵ is given by

$$D_\varepsilon=-\overline{\left\langle\frac{\partial\theta}{\partial t}\left(D\frac{\partial^2}{\partial\mathbf{r}^2}\right)^{-1}\frac{\partial\theta}{\partial t}\right\rangle},\qquad \theta=(\varepsilon^2-\Delta^2)^{1/2}-\langle(\varepsilon^2-\Delta^2)^{1/2}\rangle. \tag{2}$$

The validity of this formula is limited by the condition $\omega\ll D/a^2\ll\Delta$, which determines the permissible size a of the contact (here, $D=v_Fl_{tr}/3$ is the spatial diffusion coefficient). In this case the energy diffusion occurs mainly because of the reflection of electrons from the edges of the oscillating potential well and is described by the formula (2), whereas the direct influence of the electric field is weak.

Under the same assumptions the Ginzburg–Landau equation for the modulus of the order parameter Δ with the nonequilibrium term $\Phi(\Delta)$ has the form[11]

$$\frac{\pi D}{8T}\frac{\partial^2\Delta}{\partial\mathbf{r}^2}-\frac{I_s^2}{4\pi e^2\rho^2DS^2\Delta^3f(\Delta)}+\frac{T_c-T}{T_c}\Delta-\frac{7\zeta(3)}{8\pi^2}\frac{\Delta^3}{T^2}+\Phi(\Delta)=0,$$
$$\Phi(\Delta)=\Delta\int\limits_\Delta^\infty\left[f(\varepsilon)-\operatorname{th}\frac{\varepsilon}{2T}\right]\frac{d\varepsilon}{(\varepsilon^2-\Delta^2)^{1/2}}. \tag{3}$$

The relationship of the gradient of the phase φ of the order parameter to the superconducting current I_s across a contact is used above:[12]

$$I_s=\pi e\rho DS\Delta f(\Delta)\nabla\varphi, \tag{4}$$

where ρ is the density of states and S is the cross-sectional area of the contact. In the case of small deviations of the distribution function $f(\epsilon)$ from the equilibrium function $\tanh(\epsilon/2T)$, Eqs. (3) and (4) reduce to the well-known expressions in the theory of superconductivity.

In the calculation of the energy diffusion we shall assume a fixed alternating voltage V across the investigated contact; this voltage determines the phase shift χ between the order parameter at the contact edges:

$$\dot\chi=\frac{2e}{\hbar}V. \tag{5}$$

The quantity $\dot\chi^2$ is proportional to the power absorbed by the contact. We can find D_ϵ from Eq. (2) if we deduce from Eq. (3) the relationship between the modulus of the order parameter Δ and the phase shift χ. This relationship depends on that between the length of the contact a and the size of a pair ξ, and also on the incident radiation power.

For a short contact $\xi\gg a$ and low radiation power, only the gradient terms are important in the Ginzburg–Landau equation (3). For a narrow contact, the solution of this equation satisfying the boundary condition $\Delta(a)=\Delta_0$, where Δ_0 is the value of the modulus of the order parameter of a bulk semiconductor, is

$$\Delta^2=\frac{\Delta_0^2}{2}\left[1+\frac{x^2}{a^2}+\left(1-\frac{x^2}{a^2}\right)\cos\chi\right]. \tag{6}$$

Using this expression, we find from Eq. (2) that

$$D_\varepsilon=\frac{a^2\Delta_0^2}{16D}\overline{\dot\chi^2(1+\cos\chi)}(c_1\alpha^2+c_2\alpha+c_3),\qquad \xi\gg a\gg\left(\frac{D}{\Delta_0}\right)^{1/2}, \tag{7}$$

where

$$\alpha=\frac{2(\varepsilon/\Delta_0)^2-1-\cos\chi}{1-\cos\chi},\quad c_1=\frac{1}{2}\int\limits_0^1(y(1-y^2)^{1/2}-\arcsin y)^2dy=0,1,$$
$$c_2=2\int\limits_0^1(y(1-y^2)^{1/2}-\arcsin y)\left(\arcsin y-\frac{\pi y}{4}\right)dy=-0,2,$$
$$c_3=2\int\limits_0^1\left(\arcsin y-\frac{\pi y}{4}\right)^2dy=4,1.$$

We shall now consider a long contact $a\gg\xi$ and low radiation power. For a narrow contact, the order parameter $\Delta=\Delta(x)$ is almost everywhere inside the contact close to the value Δ_∞ for an infinitely long contact (large deviations appear only in the region $a-x\lesssim\xi$ near the edges). If the superconducting current I_s is then close to the critical value for an infinite contact $I_{s\infty}^0=9.1\rho eSD^{1/2}(T_c-T)^{3/2}$, the order parameter is close to $\Delta_\infty=(2/3)^{1/2}\Delta_0$. Introducing $\delta=\Delta_\infty^2-\Delta^2(0)$ and the dimensionless variable $y(x)$ in accordance with the formula

$$\Delta^2(x)=\Delta_\infty^2+\delta[y^2(x)-1], \tag{8}$$

we obtain from the Ginzburg–Landau equation (3)

$$\frac{1}{(A\delta)^{1/2}}\int\limits_0^{y(x)}Z^{-1}(y)\,dy=x,\quad Z(y)=(y^4-3y^2+3+\alpha)^{1/2},$$
$$A=\frac{7\zeta(3)}{2\pi^3}\frac{1}{DT}. \tag{9}$$

The parameters α and δ are governed by the current I_s through the contact. Then,

$$\alpha=\frac{8\pi}{7\zeta(3)}\frac{I_s^2-I_{s\infty}^2}{\delta^2\Delta_\infty^2}\frac{T^3}{\rho^2e^2S^2D}, \tag{10}$$

and the value of δ is found from the condition $\Delta(a)=\Delta_0$. Since the integral in Eq. (9) is dominated by the region $y\sim1$, it follows that rewriting the boundary condition with the aid of Eq. (8), we can assume that the upper limit of this integral is equal to infinity. As a result, we obtain the following relationship between the parameters α and δ:

$$\frac{1}{(A\delta)^{1/2}}\int\limits_0^\infty Z^{-1}(y)\,dy=a. \tag{11}$$

A numerical solution of these equations shows that the maximum (critical) current corresponds to $\alpha_0=9.25$ and $\delta_0=1.25(a^2A)^{-1}\sim\Delta_0^2\xi^2/a^2$.

The energy diffusion coefficient can be found from Eq. (2) if we know the time dependence of the order parameter. We then find that

$$\frac{\partial}{\partial t}\Delta^2(x)=\dot{\chi}\frac{d\Delta^2}{d\delta}\frac{d\delta}{d\chi}. \tag{12}$$

The derivative $d\Delta^2/d\delta$ is found from Eqs. (8) and (9):

$$\frac{d\Delta^2}{d\delta}=-(\alpha_0+3)\,yZ(y)\int_y^\infty y^{-2}Z^{-3}(y)\,dy. \tag{13}$$

The relationship between the phase shift χ of the order parameter and the quantity δ is found from Eq. (4) for the superconducting current:

$$\chi=\frac{4TI_s}{\pi e\rho SD}\int_0 \Delta^{-2}(x)\,dx. \tag{14}$$

In the calculation of the derivative $d\chi/d\Delta$ the current I_s can be regarded as constant (I_s is close to its maximum value) and $d\Delta^2/d\delta$ can be described by Eq. (13). Then, replacing the variable x in the integral of $y(x)$ in Eq. (9), we obtain

$$\frac{d\chi}{d\delta}=\frac{2T}{\pi e\rho SD}\frac{\alpha_0+3}{(A\delta_0)^{1/2}}\frac{I_s}{\Delta_\infty^4}\int_0^\infty Z^{-3}(y)\,dy=0.42\frac{aTI_s}{e\rho SD\Delta_\infty^4}. \tag{15}$$

The coefficient D_ϵ will be first calculated in the most important range of energies, which is not too close to the order parameter at the center of the contact (bottom of the potential well): $\epsilon^2-\Delta_\infty\gg\delta$. Using Eq. (2), we find that

$$D_\epsilon=\frac{1}{4D}\frac{1}{\varepsilon^2-\Delta_\infty^2}\overline{\left\langle\left[\int_0\left(\left\langle\frac{\partial\Delta^2}{\partial t}\right\rangle-\frac{\partial\Delta^2}{\partial t}\right)dx\right]^2\right\rangle}. \tag{16}$$

Substituting the expression for $\partial\Delta^2/\partial t$ from Eqs. (12), (13), and (15) we obtain

$$D_\varepsilon=\frac{1}{16D}\frac{(\alpha_0+3)^2}{aA\delta_0}\overline{\dot{\chi}^2}\frac{1}{\varepsilon^2-\Delta_\infty^2}\left(\frac{d\chi}{d\delta}\right)^{-2}C, \tag{17}$$

where

$$C=\int_0^a\left[\frac{x}{a}\int_0^\infty Z^{-3}(y)\,dy-\int_0^y Z^{-3}(y)\,dy\right]dx.$$

Substituting here the values of α_0 and δ_0, as well as Eq. (15) for $d\chi/d\delta$, and calculating the integral C by substituting the variable $x=x(y)$ in accordance with Eq. (9), we obtain

$$D_\varepsilon=0.11\rho^2e^2S^2D\frac{\overline{\dot{\chi}^2}\Delta_\infty^8}{T^2I_s^2}\frac{1}{\varepsilon^2-\Delta_\infty^2}=0.23\overline{\dot{\chi}^2}T\frac{\Delta_0^2}{\varepsilon^2-\Delta_\infty^2},\quad \varepsilon^2-\Delta_\infty^2\gg\delta. \tag{18}$$

In the other limiting case of $\epsilon^2-\Delta_\infty^2\ll\delta$, the important region is near the center of the contact, where the change in the order parameter obeys the quadratic law

$$\Delta^2(x)=\Delta_\infty^2-\delta+3A\delta^2x^2. \tag{19}$$

In the calculation of D_ϵ by means of Eq. (2) and Eq. (19) for $\Delta^2(x)$ as well as Eq. (15) for $d\chi/d\delta$, we obtain

$$D_\varepsilon=0.78\rho^2e^2S^2D\frac{\overline{\dot{\chi}^2}\Delta_\infty^8}{T^2I_s^2}\frac{1}{\delta},\ \varepsilon^2-\Delta_\infty^2\ll\delta, \tag{20}$$

which, apart from a numerical coefficient, is identical with Eq. (18) in the $\epsilon^2-\Delta_\infty^2\sim\delta$ case.

3. STIMULATION OF THE SUPERCONDUCTIVITY IN A CONTACT

If the radiation power is high, the left-hand side of the kinetic equation (1) can be regarded as equal to zero. Therefore, the flux of particles is $D_\epsilon\partial f/\partial\epsilon=\text{const}$. On the other hand, the boundary condition applicable to this kinetic equation requires that the particle flux vanishes at the bottom of the potential well. Consequently, $\partial f/\partial\epsilon=0$ or $f(\epsilon)=\text{const}$. Therefore, at a high radiation power a strong energy diffusion equalizes the level populations and the electron distribution function ceases to depend on the energy. Bearing in mind the second boundary condition $f(\Delta_0)=\tanh(\Delta_0/2T)\approx\Delta_0/2T$, we find that the distribution function at high radiation powers is

$$f(\varepsilon)=\Delta_0/2T,\quad \varepsilon<\Delta_0. \tag{21}$$

The nonequilibrium term of this distribution function is calculated from Eq. (3):

$$\Phi(\Delta)=\frac{\Delta\Delta_0}{2T}\left[\ln\frac{1+(1-\Delta^2/\Delta_0^2)^{1/2}}{\Delta/\Delta_0}-\left(1-\frac{\Delta^2}{\Delta_0^2}\right)^{1/2}\right]. \tag{22}$$

This term is large compared with the first term in the Ginzburg–Landau equation describing spatial variation of the modulus of the order parameter if $a\gg\eta$, where $\eta=(D/\Delta_0)^{1/2}$. Therefore, in the case of sufficiently high radiation power we find that for long ($a>\xi$) and short ($\xi>a>\eta$) contacts the order parameter depends weakly on the coordinate almost throughout the contact. The value of this parameter Δ is governed by the superducting current I_s found from the Ginzburg–Landau equation:

$$I_s^2=4\pi e^2\rho^2DS^2\Delta^2f(\Delta)\left[\tau\Delta-\frac{7\zeta(3)}{8\pi^2}\frac{\Delta^3}{T^2}+\Phi(\Delta)\right], \tag{23}$$

where $\tau=(T_c-T)/T_c$; when the radiation power is high, only the nonequilibrium term is important in Eq. (23). The critical current is found from the maximum of this expression which is always reached at some value of $\tilde{\Delta}$ if $\Phi(\Delta)>0$.

Using this expression and Eq. (22) for $\Phi(\Delta)$, we find that $\tilde{\Delta}=0.7\Delta_0$ and the critical current is

$$I_s^*=0.43e\rho SD^{1/2}\Delta_0^{5/2}/T. \tag{24}$$

This current is independent of the length of the contact and exceeds the critical value in the absence of irradiation by a factor of $\tau^{-1/4}$ for a long contact ($a>\xi$) and by a factor $a(\Delta_0/D)^{1/2}$ for a short contact ($\xi>a>\eta$). It thus follows that at high radiation powers the critical current increases considerably and ceases to depend on this power.

When the radiation power is reduced, the nonequilibrium term in the Ginzburg–Landau equation (3) is still quite large in a certain range of powers but we can no longer use the limiting expression (21) for the distribution function. We can find this function only if we know the energy diffusion coefficient D_ϵ and solve the kinetic equation (1).

We can obtain D_ϵ using Eq. (18) but subject to some modification since it is derived on the assumption that the nonequilibrium term is small. As pointed out earlier, the modulus of the order parameter depends weakly on the coordinates throughout the contact when the radiation power is high. Then, near the maximum of the superconducting current (23) the dependence $\Delta^2(x)$ is described by the same formulas (8) and (9) as for a long contact and low radiation power except that now Δ_∞ should be replaced with $\tilde{\Delta}$ and the parameter A now depends on the nonequilibrium term $\Phi(\Delta)$. However, Eq. (17) for D_ϵ includes only the product $A\delta$, which—as indicated by Eq. (11)—is a universal parameter. Therefore, at high radiation powers we can describe D_ϵ again by Eq. (18)—using the first part—but now we have $\Delta_\infty = \tilde{\Delta}$ and I_s has to be found from the condition for the maximum of Eq. (23).

Before solving the kinetic equation (1) with this coefficient D_ϵ, we must identify the important range of energies. We can find the correction to the critical current due to hf radiation by employing Eq. (23) but we must calculate the nonequilibrium term $\Phi(\tilde{\Delta})$. We can then write

$$\Phi(\tilde{\Delta}) = \int_{\tilde{\Delta}}^{\Delta_0} \frac{\tilde{\Delta}}{(\varepsilon^2-\tilde{\Delta}^2)^{1/2}} \left[f(\varepsilon) - \operatorname{th}\frac{\varepsilon}{2T} \right] d\varepsilon$$
$$= \int_{\tilde{\Delta}}^{\Delta_0} \left\langle \frac{\varepsilon}{(\varepsilon^2-\Delta^2)^{1/2}} \right\rangle \left[f(\varepsilon) - \operatorname{th}\frac{\varepsilon}{2T} \right] d\varepsilon + \int_{\tilde{\Delta}}^{\Delta_0} \left(\frac{\tilde{\Delta}}{(\varepsilon^2-\tilde{\Delta}^2)^{1/2}} - \left\langle \frac{\varepsilon}{(\varepsilon^2-\Delta^2)^{1/2}} \right\rangle \right) \left[f(\varepsilon) - \operatorname{th}\frac{\varepsilon}{2T} \right] d\varepsilon. \qquad (25)$$

It follows from the kinetic equation (1) that the first integral in Eq. (25) is the total derivative of $D_\epsilon \partial f/\partial\epsilon$. This integral is small so that for $\epsilon = \tilde{\Delta}$ the derivative is $\partial f/\partial\epsilon = 0$ (corresponding to the vanishing of the particle flux at the bottom of the well) and for $\epsilon = \Delta_0$ the value of D_ϵ is now small [it is clear from Eq. (18) that D_ϵ falls rapidly with rising ϵ].

In the calculation of the second integral the important values of ϵ are those far from the bottom of the potential well $\tilde{\Delta}$. In this range of energies we may assume that $\langle(\epsilon^2-\Delta^2)^{-1/2}\rangle = (\epsilon^2-\tilde{\Delta}^2)^{-1/2}$ and solve the kinetic equation (1) using the perturbation theory:

$$f(\varepsilon) - \operatorname{th}\frac{\varepsilon}{2T} = -0.1\rho^2 e^2 S^2 DP \frac{\tilde{\Delta}^8}{T^2 I_s^2} \frac{1}{(\varepsilon^2-\tilde{\Delta}^2)^{3/2}}, \quad \varepsilon - \tilde{\Delta} \gg \varepsilon_P, \qquad (26)$$

where the dimensionless parameter $P = \tau_\epsilon \overline{\mathring{\chi}^2}/T$ is proportional to the radiation power, $\epsilon_P = \tilde{\Delta}^3(\rho^2 e^2 S^2 DP / \times T^2 I_s^2)^{2/5}$, and the restrictions on the energy are found from the conditions of validity of the perturbation theory [the derivative of the distribution function $f'(\epsilon)$ should differ little from $\tanh'(\epsilon/2T) \sim 1/T$].

Substituting Eq. (26) into Eq. (25), we find that with logarithmic precision

$$\Phi_1(\tilde{\Delta}) = -0.1\rho^2 e^2 S^2 DP \frac{\tilde{\Delta}^8}{T^2 I_s^2} \int_{\tilde{\Delta}+\varepsilon_P}^{\Delta_0} \frac{\tilde{\Delta}-\varepsilon}{(\varepsilon^2-\tilde{\Delta}^2)^{1/2}} \frac{1}{(\varepsilon^2-\tilde{\Delta}^2)^{3/2}} d\varepsilon$$
$$= 0.03\rho^2 e^2 S^2 DP \frac{\tilde{\Delta}^6}{T^2 I_s^2} \ln\frac{\Delta_0 - \tilde{\Delta}}{\varepsilon_P}. \qquad (27)$$

The contribution to the nonequilibrium term $\Phi_2(\tilde{\Delta})$ in the energy range $\epsilon - \tilde{\Delta} < \epsilon_P$ originates because of the difference between $\langle(\epsilon^2-\Delta^2)^{-1/2}$ and $(\epsilon^2-\tilde{\Delta}^2)^{-1/2}$, and it is small for sufficiently long contacts.

Substituting Eq. (27) into the formula for the current (23), we obtained a self-consistent equation for the calculation of the critical current. In the case of high pumping rates, only the nonequilibrium term is important in Eq. (23) and the maximum of the current is reached at $\tilde{\Delta}$ close to Δ_0. Solution of this equation gives

$$I_s = 0.53 I_{s\infty}^0 \tau^{-3/8} P^{1/4} \ln^{1/4}[\tau^{1/2}/P]. \qquad (28)$$

In the case of sufficiently long contacts the above formula is valid for radiation powers $\tau^{3/2} \ll P \ll \tau^{1/2}$, when the nonequilibrium term in the Ginzburg–Landau equation is large and the distribution function can still be found from the perturbation theory.

In the range $P \ll \tau^{3/2}$ the nonequilibrium term in the Ginzburg–Landau equation is small and the critical current found from the condition for the maximum of Eq. (23) is identical with $I_{s\infty}^0$ and we have $\tilde{\Delta} = \Delta_\infty$. Substituting the values of $I_{s\infty}^0$ and Δ_∞ in Eq. (27), we find that $\Phi_1(\Delta_\infty)$ is given by

$$\Phi_1(\Delta_\infty) = 0.1PT \ln\frac{\Delta_0}{\varepsilon_P^0}, \quad \varepsilon_P^0 = T\tau^{1/10} P^{2/5}.$$

At low powers the contribution $\Phi_2(\Delta_\infty)$ may also be important. We can find it by determining the distribution function $f(\epsilon)$ near the bottom of the well where $\epsilon - \Delta_\infty \ll \epsilon_P^0$ and the kinetic equation can no longer be solved using the perturbation theory. If $\epsilon_P^0 \gg \delta/\Delta_0$, the bottom of the potential well can be regarded as flat and the coefficient D_ϵ can be described by Eq. (18). The solution of the kinetic equation with this type of dependence of D_ϵ on the energy can be found in the paper by Ivlev and Éliashberg.[13] Using their results, we obtain

$$f(\varepsilon) - \operatorname{th}\frac{\varepsilon}{2T} = 0.1\tau^{1/10} P^{2/5}, \quad \varepsilon - \Delta_\infty \ll \varepsilon_P^0. \qquad (30)$$

Calculating $\langle(\epsilon^2-\Delta^2)^{-1/2}\rangle$ from the known dependence $\Delta(x)$ for a long contact [Eqs. (8) and (9)], we find that $\Phi_2(\Delta_\infty)$ is described by

$$\Phi_2(\Delta_\infty) = 0.1\tau^{1/10} P^{2/5} \Delta_\infty \int_{\Delta_\infty}^{\Delta_\infty + \varepsilon_P} [(\varepsilon^2-\Delta_\infty^2)^{-1/2} - \langle(\varepsilon^2-\Delta^2)^{-1/2}\rangle] d\varepsilon = 0.08 \frac{(D\Delta_0)^{1/2}}{a} \tau^{-3/20} P^{2/5} \ln\left(\frac{\varepsilon_P^0 \Delta_0}{\delta}\right), \qquad (31)$$

where the lower integration limit can be regarded with logarithmic precision to be equal to $\Delta_\infty + \delta/\Delta_0$. Thus, at moderately high radiation powers we have two contributions to the nonequilibrium term $\Phi(\Delta)$ and they differ in sign.

The correction to the critical current, deduced from Eq. (23), is

$$\frac{I_s - I_{s\infty}^0}{I_{s\infty}^0} = 0.06\tau^{-3/2} P \ln\frac{\Delta_0}{\varepsilon_P^0} - 0.14\frac{1}{a}\left(\frac{D}{\Delta_0}\right)^{1/2} \tau^{-23/20} P^{2/5} \ln\left(\frac{\varepsilon_P \Delta_0}{\delta}\right). \qquad (32)$$

The above expression has a minimum when the power is $P \sim (\xi/a)^{5/3}\tau$. Then, Eq. (32) is valid if $[(I_s - I_{s\infty})/I_{s\infty}]_{\min} \ll 1$, which limits the length of the contact: $a > \xi\tau^{-3/10}$. Thus, for a sufficiently long contact obeying $a > \xi\tau^{-3/10}$ we find that the critical current increases linearly on increase of the radiation power beginning from powers of the order of $P_1=(\xi/a)^{5/3}\tau$.

This linear stimulation continues up to powers of the order of $P_2=\tau^{3/2}$, when the nonequilibrium term in the Ginzburg–Landau equation becomes large. For $P\gg P_2$, the critical current can be calculated allowing only for this term and the current rises in accordance with the law $P^{1/4}$ [in determining the dependence $I_s(P)$ in the intermediate range we have to allow for all the terms in Eq. (23)]. Finally, if $P>P_3=\tau^{1/2}$, the critical current reaches saturation. The dependence of the critical current on the radiation power is shown in Fig. 1.

In the case of a contact obeying $a<\xi\tau^{-3/10}$ there is no region with the linear dependence of I_s on P since the term Φ_2 may be important at high radiation powers when the nonequilibrium component is still large. In estimating Φ_2 we shall use Eq. (18) for D_ϵ and substitute there I_s from Eq. (28) and use the value $\Delta_\infty=\tilde{\Delta}$. Calculating then $\Phi(\tilde{\Delta})$ from Eq. (25), we find that, as in the derivation of Eq. (31), the value of $\Phi_2(\tilde{\Delta})$ is described by

$$\Phi_2(\tilde{\Delta})\approx-\tau^{41/40}Ta^{-1}(D/\Delta_0)^{1/2}P^{-1/20}. \tag{33}$$

Thus, Eq. (28) for a short contact applies in the power range $P>(D/\Delta_0a^2)^{10/11}\tau^{1/2}$. At lower powers the value of $\Phi_2(\tilde{\Delta})$ becomes large and this suppresses the superconductivity of the contact.

4. CRITICAL CURRENT AT LOW RADIATION POWERS

For a low radiation power we can solve the kinetic equation (1) for all important energies using the perturbation theory and substituting $f=\tanh(\epsilon/2T)$ on the right-hand side. Consequently, we find that $\Phi(\Delta)$ is given by

$$\Phi(\Delta)=\tau_\varepsilon\frac{\Delta}{2T}\int\limits_{\Delta}^{\Delta_0}\frac{1}{(\varepsilon^2-\Delta^2)^{1/2}}\left\langle\frac{1}{(\varepsilon^2-\Delta^2)^{1/2}}\right\rangle^{-1}\frac{1}{\varepsilon}\frac{\partial D_\varepsilon}{\partial\varepsilon}d\varepsilon. \tag{34}$$

For a short contact characterized by $a\ll\xi$ we can use Eq. (7) for D_ϵ. We then obtain

$$\Phi(\Delta)=-\frac{1}{8\pi}\frac{a^2\Delta}{D}P\int\limits_{\Delta}^{\Delta_0}\frac{0.66-0.62\varepsilon^2/\Delta_0^2}{(\varepsilon^2-\Delta^2)^{1/2}}\left(\varepsilon^2-\frac{\Delta_0^2}{2}\right)^{1/2}d\varepsilon. \tag{35}$$

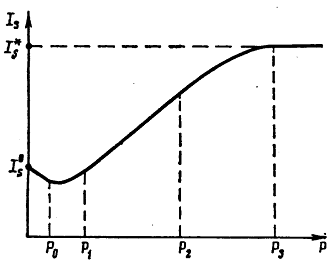

FIG. 1. Dependence of the critical current through a long contact on the radiation power.

The negative sign in the above expression appears because of the accumulation of excitations $[f(\epsilon)-\tanh(\epsilon/2T)<0]$ throughout the range of significant energies. The sign of $f(\epsilon)-\tanh(\epsilon/2T)$ becomes positive only in a narrow range of energies near the bottom of the potential well, where there are significant deviations of the distribution function from equilibrium and we can no longer use the perturbation theory. This range makes no significant contribution to the nonequilibrium term $\Phi(\Delta)$ when the radiation power is low.

In calculating the change in the critical current we shall use Eqs. (3) and (4). Then, in the case of a short contact only the first two terms and the nonequilibrium contribution are important in the Ginzburg–Landau equation (3). Integration of this equation gives the dependence $\Delta(x)$ for a given current. The boundary condition $\Delta(a)=\Delta_0$ allows us to relate the current to the order parameter $\Delta(0)$ at the center of the contact:

$$\left(\frac{\pi D}{T}\right)^{1/2}\frac{\Delta(0)}{8}\int\limits_{\Delta(0)}^{\Delta_0}\left[\frac{TI_s^2[\Delta^2-\Delta^2(0)]}{4\pi e^2\rho^2DS^2}-\Delta^2\Delta^2(0)\int\limits_{\Delta(0)}^{\Delta}\Phi(\Delta)d\Delta\right]^{-1/2}d\Delta^2=a. \tag{36}$$

Since the nonequilibrium term $\Phi(\Delta)$ is small at low radiation powers, we can simplify Eq. (36) to the linear term of the expansion of the integrand in respect of $\Phi(\Delta)$; assuming also that $\Delta(0)=\Delta_0/\sqrt{2}$, which is the value of the order parameter in the absence of radiation, we find that

$$I_s=I_s^0\left[1+\frac{32}{\pi}\frac{a^2}{D}\frac{T}{\Delta_0^3}\int\limits_{\Delta_0/\sqrt{2}}^{\Delta_0}\frac{\Delta_0^2-\Delta^2}{(2\Delta^2-\Delta_0^2)^{1/2}}\Phi(\Delta)d\Delta\right], \tag{37}$$

where $I_s^0=\pi e\rho DS\Delta_0^2/4aT$ is the critical current for a short contact in the absence of radiation. Substituting in this formula Eq. (35), we obtain the following expression for I_s:

$$I_s=I_s^0\left[1-k\left(\frac{a^2\Delta_0}{D}\right)^2P\tau^{-1/2}\right],$$
$$k=\frac{2\sqrt{2}}{\pi^2}\int\limits_{1/\sqrt{2}}d\Delta\left[\frac{1-\Delta^2}{(\Delta^2-1/2)^{1/2}}\Delta\int\limits_{\Delta}\frac{0.66-0.62\varepsilon^2}{(\varepsilon^2-\Delta^2)^{1/2}}\left(\varepsilon^2-\frac{1}{2}\right)^{1/2}d\varepsilon\right]=0.04. \tag{38}$$

To find the nonequilibrium term in a long contact at a low radiation power we shall again use Eq. (22), where the coefficient D_ϵ is given by Eq. (18). We can see that the value of D_ϵ falls rapidly on increase of the electron energy ϵ. Therefore, in the integral of Eq. (34) at values of Δ close to Δ_∞ the important range of energies is $\epsilon^2-\Delta_\infty^2\sim\delta$. However, in this range the coefficient D_ϵ is known only to an order of magnitude and, consequently, we can find the nonequilibrium term with the same precision:

$$\Phi(\Delta)\approx-\frac{\tau_\varepsilon}{T}D_\varepsilon|_{\varepsilon^2-\Delta_\infty^2\sim\delta}\approx-P\Delta_0\frac{a^2}{\xi^2}\tau^{-1/2}. \tag{39}$$

The correction to the critical current is found from Eq. (23). Substituting in this equation the expression for $\Phi(\Delta)$, we obtain

$$I_s \approx I^0_{s\infty}[1 - P\tau^{-3/2}a^2/\xi^2]. \tag{40}$$

Equation (40) is valid if the correction to the critical current due to the incident radiation is small, $(I_s - I_s^0)/I^0_{s\infty} \ll 1$, and if the distribution function in the important energy range differs little from the equilibrium form $(\epsilon_p^0 < \delta/\Delta_0)$. For a contact of length $a > \xi\tau^{-1/6}$ the second condition is violated earlier and Eq. (40) is valid up to powers $\sim P_0 = (\xi/a)^5\tau$. In the opposite case of $a < \xi\tau^{-1/6}$, Eq. (40) ceases to be valid at powers $P \sim (\xi^2/a^2)\tau^{3/2}$ lower than P_0, when the nonequilibrium term becomes large and considerable suppression of the superconductivity in the contact takes place.

5. CONDITIONS FOR STIMULATION OF THE SUPERCONDUCTIVITY IN A CONTACT

The results obtained above show that the energy diffusion of electrons in a contact subjected to a microwave field is due to oscillations of the potential well and this may increase considerably the critical current of the contact. When the radiation power is sufficiently high, the critical current reaches saturation and becomes equal to I_s^* given by Eq. (24). This saturation value is independent of the size of the contact and it decreases in accordance with the law $\tau^{5/4}$ as the temperature approaches T_c.

The critical current of a contact in the absence of radiation also decreases on approach to T_c but the law now depends on the contact size. For a short contact $(a < \xi)$ the critical current obeys the law $I_s^0 \propto \tau$ and a long contact $(a > \xi)$ obeys $I^0_{s\infty} \propto \tau^{3/2}$. Therefore, for $a > \xi$ the relative correction to the critical current far from T_c increases on increase of temperature in accordance with the law $\tau^{-1/4}$ and then reaches a maximum $\sim(a/\xi_0)^{1/2}$ at a temperature such that $a \sim \xi$ but in the range $\xi > a > \eta$ the correction decreases in accordance with the law $\tau^{1/4}$; for $a < \eta$, the relative correction to the critical current is small. Thus, the effect of stimulation of the superconductivity of a contact appears more strongly in a certain temperature range when $a \sim \xi$ (Fig. 2).

The behavior of the dependences $I_s(P)$ in the initial part also depends on the proximity of the temperature to the critical value. Close to T_c, when the contact size is $a < \xi\tau^{-3/10}$, a considerable suppression of the superconductivity in the contact takes place. At lower temperatures and for $a > \xi\tau^{-3/10}$ the formulas for a long contact are applicable; the suppression occurs only at low powers and the relative minimum of the critical current is now small and continues to decrease as a result of cooling. Stimulation of the superconductivity in this temperature range is first linear and the slope of the linear part depends weakly on temperature. The attainment of the saturation value by the critical current occurs in accordance with the law $P^{1/4}$ and is reached at radiation powers $\sim\tau^{1/2}T/e^2R\tau_\epsilon$ (Fig. 1).

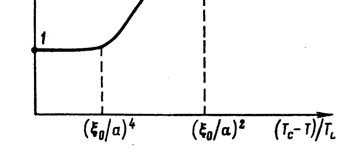

FIG. 2. Temperature dependence of the stimulation effect of the critical current.

This behavior of the critical current in a radiation field agrees with the experimental observations. The maximum critical current, measured in relative units, has a peak near T_c.[2-4] A transition from stimulation to suppression of the superconductivity in a contact occurs on approach of the temperature to T_c.[3,4] The initial stimulation obeys a linear law and then the critical current reaches saturation. However, a detailed comparison of the theory and experiment is made difficult by the direct heating of a contact in a microwave field.[14] This heating is clearly responsible for the fact that the critical current does not always reach saturation and begins to fall steeply when a certain power is reached. The critical current may also decrease because of the effective heating of electrons of energies $\epsilon > \Delta_0$, which is particularly important in films in which the spatial diffusion is difficult.

The results obtained are valid subject to certain restrictions on the field frequency. A reduction in the frequency ω increases the alternating part $\Delta_1(t)$ of the modulus of the order parameter and the condition $\Delta_1 < \delta/\Delta_0$ is valid only at frequencies $\omega^2\tau_\epsilon > \Delta_0 a^2/\xi^2$. If this condition is not obeyed, the value of the order parameter throughout the contact varies in the same way with time (the bottom of the potential well oscillates as one unit). Consequently, Eq. (16) simplifies by the cancellation of the terms $\langle\partial\Delta^2/\partial t\rangle$ and $\partial\Delta^2/\partial t$, so that the energy diffusion coefficient decreases. There is a corresponding reduction in the nonequilibrium term in the Ginzburg–Landau equation and the critical current reaches saturation at higher radiation powers. Then, the Éliashberg term[9] may be important and this results in linear stimulation at lower powers even when the temperature is very close to T_c. This is confirmed by the experimental results since at low frequencies there is no suppression of the superconductivity near T_c.[2,3] The same situation occurs also for very long contacts and at high frequencies.[5-7]

The maximum critical current also depends on the frequency because an increase in the amplitude of the alternating current $I_1 \propto I_s^*(D/\Delta a^2)^{1/2}\chi/\omega$ makes the average current through the contact smaller than the maximum value. This restricts the frequency at which stimulation can be observed:

$$\omega^2\tau_\epsilon > \frac{D}{a^2}\frac{T}{\Delta}P.$$

This effect may also account for the reduction in the critical current of a contact on increase of the radiation power even when there is no field heating of the contact.

The validity of the results is limited also on the high-frequency side. When the frequency becomes comparable with the reciprocal of the electron diffusion time in a contact, D/a^2, we have to allow for the frequency dependence of the energy diffusion coefficient D_ϵ. However, this does not affect the results qualitatively. Another effect appears when the radiation frequency becomes comparable with the width of the range of energies near the bottom of the potential well where considerable changes take place in the electron distribution function. Then, the energy diffusion is no longer described by a differential equation. In a short contact, in which the energies $\epsilon \sim \Delta_0$ are important, this effect becomes significant at frequencies $\omega \sim \Delta_0$. In the case of long contacts at lower temperatures there is a considerable change in the contribution of electrons of energies $\epsilon - \bar{\Delta} < \omega$ but the contribution of electrons with high energies is not greatly affected. Consequently, at a frequency $\omega > \delta/\Delta_0 \sim \Delta_0 \xi^2/a^2$ even at low radiation powers there is an increase in the range of energies near the bottom of the potential well where the distribution function differs greatly from the equilibrium value and stimulation of the superconductivity in the contact takes place.

It should also be noted that a change in the critical current in a microwave field and the appearance of static current regions in the current-voltage characteristics of the contacts are correlated: the two effects become significant only when the contact size is $a > (D/\Delta_0)^{1/2}$.[4,11,15]

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[1]A. F. G. Wyatt, V. M. Dmitriev, W. S. Moore, and F. W. Sheard, Phys. Rev. Lett. **16**, 1166 (1966).
[2]A. H. Dayem and J. J. Wiegand, Phys. Rev. **155**, 419 (1967).
[3]Yu. I. Latyshev and F. Ya. Nad', Zh. Eksp. Teor. Fiz. **71**, 2158 (1976) [Sov. Phys. JETP **44**, 1136 (1976)].
[4]V. N. Gubankov, V. P. Koshelets, and G. A. Ovsyannikov, Zh. Eksp. Teor. Fiz. **73**, 1435 (1977) [Sov. Phys. JETP **46**, (1977)].
[5]K. W. Shepard, Physica (Utrecht) **55**, 786 (1971).
[6]T. M. Klapwijk and J. E. Mooij, Physica (Utrecht) B+C **81**, 132 (1976).
[7]T. M. Klapwijk, J. N. van den Bergh, and J. E. Mooij, J. Low Temp. Phys. **26**, 385 (1977).
[8]B. R. Fjordboge, T. D. Clark, and P. E. Lindelof, Phys. Rev. Lett. **37**, 1302 (1976).
[9]G. M. Éliashberg, Pis'ma Zh. Eksp. Teor. Fiz. **11**, 186 (1970) [JETP Lett. **11**, 114 (1970)]; Zh. Eksp. Teor. Fiz. **61**, 1254 (1971) [Sov. Phys. JETP **34**, 668 (1972)].
[10]A. I. Larkin and Yu. N. Ovchinnikov, Zh. Eksp. Teor. Fiz. **68**, 1915 (1975) [Sov. Phys. JETP **41**, 960 (1975)].
[11]L. G. Aslamazov and A. I. Larkin, Zh. Eksp. Teor. Fiz. **70**, 1340 (1976) [Sov. Phys. JETP **43**, 698 (1976)].
[12]A. I. Larkin and Yu. N. Ovchinnikov, Zh. Eksp. Teor. Fiz. **73**, 299 (1977) [Sov. Phys. JETP **46**, 155 (1977)].
[13]B. I. Ivlev and G. M. Eliashberg, Pis'ma Zh. Eksp. Teor. Fiz. **13**, 464 (1971) [JETP Lett. **13**, 333 (1971)].
[14]M. Tinkham, M. Octavio, and W. J. Skocpol, J. Appl. Phys. **48**, 1311 (1977).
[15]M. Octavio, W. Skocpol, and M. Tinkham, Nonequilibrium-Enhanced Critical Currents in Short Superconducting Weak Links (Preprint), Harvard University (1977).

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