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Temperature dependence of the electron specific heat and effective mass

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The temperature dependence of the electron specific heat is investigated. The Éliashberg method is used to obtain a general formula including the function $g(\omega)=\alpha^2(\omega)F(\omega)$ and the universal function $Z(T/\omega)$; an expression is found for $Z(x)$; this formula can be used to describe the nonmonotonic $C_e(T)$ dependence at all temperatures for an arbitrary electron-phonon interaction. Calculations are made for specific metals and the available experimental results are analyzed. The question of the temperature and carrier-density dependences of the mass is considered. An approximate method is suggested for obtaining information on the function $g(\omega)$.

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We shall consider the temperature dependence of the electron specific heat and effective mass. The electron–phonon interaction is known to affect considerably the characteristics of an electron system. The question of the renormalization of the electron mass associated with this interaction was investigated (for $T=0$) by Migdal.[1] The state of the phonon system varies with rising temperature. Excitation of thermal phonons results in temperature dependences of the electron characteristics. The temperature dependence of the effective mass has been observed in cyclotron resonance studies.[2-5] These studies have been carried out in the temperature range $T\lesssim 6°$ K, i.e., for $T\ll\omega_D$.

The electron–phonon interaction causes the temperature dependence of the electron specific heat to deviate from the simple linear law. This has been observed experimentally on several occasions.[6-9] Éliashberg[10] developed a method based on the exact expression for the thermodynamic potential. He obtained a general expression for the entropy and showed that allowance for the electron–phonon interaction results, in the $T\to 0$ limit, in an additional contribution $\Delta C_e(T)\propto T^3\ln(\omega_D/T)$. The question of the temperature dependence of the electron specific heat $C_e(T)$ and effective mass $m^*(T)$ has also been investigated by others[11,12] (see also Grimvall's review [13]). Fradin[14] calculated analytically the dependence $C_e(T)/T$ throughout the temperature range in the Einstein model of the phonon spectrum. The asymptotic behavior of $C_e(T)$ in the limits $T\to 0$ and $T\to\infty$ was investigated by Masharov.[15] We shall use the Éliashberg method to obtain relationships which describe the dependence $C_e(T)$ throughout the investigated temperature range for an arbitrary electron–phonon interaction and we shall apply these relationships to some specific substances.

The influence of the phonon system on the electron characteristics is known to be described by the function $g(\omega)=\alpha^2(\omega)F(\omega)$ [$F(\omega)$ is the density of the phonon states and $\alpha^2(\omega)$ describes the electron–phonon interaction]. The function $g(\omega)$ can be determined with high accuracy by investigating superconductors using the tunnel spectroscopy methods and it is known for many metals (see Figs. 2 and 3 below).[16-18]

The deviation of the dependence $C_e(T)$ from linearity can be expressed directly in terms of the function $g(\omega)$ (see below). We shall use the available tunnel data to plot curves describing the behavior of the specific heat and mass in several metals. We shall compare the predicted behavior with the available experimental data.

We shall present the results in three sections. In the first section we shall consider the temperature dependence of the electron specific heat. We shall discuss the available experimental data, the question of the separation of the phonon contribution, etc. In the second section we shall deal with the problem of the temperature and carrier-density dependences of the effective mass. In the third section we shall consider the possibility of obtaining information on the function $\alpha^2(\omega)$ describing the electron–phonon interaction.

1. ELECTRON SPECIFIC HEAT

The electron–phonon interaction results in deviation of the temperature dependence of the electron specific heat from the simple linear law. As mentioned above, the $C_e(T)$ dependence has been considered in several papers.[11-13] We shall solve this problem in its general form for an arbitrary electron–phonon interaction constant for all temperatures.

Éliashberg[10] showed that an analysis of the general expression for the thermodynamic potential gives the following expression for the entropy S_e:

$$S_e=\frac{\nu_0}{T^2}\int_0^\infty\frac{d\varepsilon\,\varepsilon}{\mathrm{ch}^2(\varepsilon/2T)}[\varepsilon-f(\varepsilon)],\tag{1}$$

where $\nu_0=m^0p_0/2\pi^2$ is the renormalized value of the density of states on the Fermi surface; $f(\varepsilon)$ is the odd part of the function $\Sigma_R(\varepsilon,p)$; $\Sigma_R(\varepsilon,p)$ is the self-energy part describing the electron-phonon interaction. According to Éliashberg,[19] $\Sigma_R(\varepsilon,p)$ satisfies the equation

$$\Sigma_R(\varepsilon,p)=-\frac{1}{(2\pi)^4}\int_{-\infty}^{\infty}d\varepsilon'\int d\mathbf{p}'\,\mathrm{Im}[G_R(\varepsilon',p')]$$
$$\times\alpha^2_{\mathbf{p}-\mathbf{p}'}\left[\frac{\mathrm{th}(\varepsilon'/2T)-\mathrm{cth}(\omega_{\mathbf{p}-\mathbf{p}'}/2T)}{\varepsilon'-\varepsilon-\omega_{\mathbf{p}-\mathbf{p}'}-i\delta}-\frac{\mathrm{th}(\varepsilon'/2T)+\mathrm{cth}(\omega_{\mathbf{p}-\mathbf{p}'}/2T)}{\varepsilon'-\varepsilon+\omega_{\mathbf{p}-\mathbf{p}'}-i\delta}\right],\tag{2}$$

where $G_R(\varepsilon,p)$ is the retarded Green function which is known to be of the form

$$G_R(\varepsilon,p)=[\varepsilon-\xi(p)-\Sigma_R(\varepsilon,p)]^{-1}.\tag{2'}$$

The expression for the function $f(\varepsilon)$ can be reduced to

$$f(\varepsilon)=\frac{\delta}{2}\int_0^\infty\int_0^\infty d\varepsilon'\,d\omega\,g(\omega)K(\varepsilon',\varepsilon,\omega),$$

where

$$\delta=(1+\lambda_0)^{-1},\quad \lambda_0=2\int_0^\infty\frac{d\omega}{\omega}g(\omega),\tag{3}$$

$$K(\varepsilon',\varepsilon,\omega)=\mathrm{th}\frac{\varepsilon'}{2T}\left[\frac{1}{\varepsilon'+\varepsilon+\omega}-\frac{1}{\varepsilon'-\varepsilon+\omega}+\frac{1}{\varepsilon'-\varepsilon-\omega}-\frac{1}{\varepsilon'+\varepsilon-\omega}\right].$$

We have introduced above the function $g(\omega)=\alpha^2(\omega)F(\omega)$. After integration by parts and several simple transformations, we obtain the following expression for the electron specific heat

$$C_e(T)=\gamma(T)T,\tag{4}$$

where the function $\gamma(T)$ describing the deviation of the $C_e(T)$ dependence from linearity due to the electron-phonon interaction has the form

$$\gamma(T)=\gamma^0\left[1+2\int_0^\infty\frac{d\omega}{\omega}g(\omega)Z(T/\omega)\right].\tag{5}$$

Equation (5) can conveniently be represented in the form

$$\frac{\gamma(T)}{\gamma(0)}=1+\rho\left[\frac{\varkappa(T)}{\varkappa(0)}-1\right],$$
$$\varkappa(T)=2\int_0^\infty\frac{d\omega}{\omega}g(\omega)Z(T/\omega),\tag{6}$$

where $\gamma(0)=m^*(0)p_0/3$, $m^*(0)$ is the renormalized value of the electron mass, $m^*(0)=m^{*0}(1+\lambda_0)$, m^{*0} is the unrenormalized electron mass,

$$\lambda_0=-\left(\frac{\partial\Sigma_R}{\partial\varepsilon}\right)\Big|_{T=0,\,\varepsilon=0,\,p=p_0}=2\int_0^\infty\frac{d\omega}{\omega}g(\omega)$$

is the renormalization parameter, and ρ is the renormalization coupling constant given by

$$\rho=\lambda_0/(1+\lambda_0).$$

The kernel $Z(T/\omega)$ is described by

$$Z(x)=\frac{6}{\pi^2}\int_0^\infty\frac{dt\,t}{\mathrm{ch}^2t}\int_0^\infty\frac{dt'}{\mathrm{ch}^2t'}\left\{\frac{t+t'}{1-[2x(t+t')]^2}+\frac{t-t'}{1-[2x(t-t')]^2}\right\},$$

or (7)

$$Z(x)\approx\Phi(9x^2),\quad \Phi(z)=\frac{1+2.6z+16z^2}{1+65z^3}-1.6\frac{z^3+4.1z^2}{z^4+9(z^3+1)}\,.\tag{7'}$$

The expression for the kernel $Z(x)$ can be simply transformed to

$$Z(x)=-\frac{3}{(\pi x)^2}\int_0^\infty\frac{dt\,t}{\mathrm{ch}^2t}\left\{\frac{G[(2t-x^{-1})^{-1}]}{2t-x^{-1}}+\frac{G[(2t+x^{-1})^{-1}]}{2t+x^{-1}}\right\},\tag{8}$$

where $G(x)$ is the function introduced by Grimvall[11] [see Eqs. (12) and (12′) below].

The formula (6) is convenient because it contains only the renormalized quantities. It can be compared directly with the experimental data (as is done below). The kernel $Z(x)$ is given by Eqs. (7) and (8). Figure 1 shows graphically this universal function.

The formulas (6)–(8) describe completely the solution of the problem of the temperature dependence of the specific heat. In each concrete case it is governed only by the form of the function $g(\omega)$; this function is known for many metals from the results of the tunnel spectroscopy methods.[16-18] Figures 2 and 3 give the functions $g(\omega)$ for a number of metals and the dependence $\gamma(T)/\gamma(0)$ deduced from Eq. (6).

It is clear from Eqs. (6) and (7) that in the limit $T\to0$, we have $[\gamma(T)/\gamma(0)-1]\propto T^2\ln(\omega_D/T)$, in agreement with Éliashberg's treatment.[10] In the limit $T\to\infty$, we find that $Z(T/\tilde{\omega})\to0$. Then, $[\gamma(T)/\gamma(0)-1]\propto(-T^{-2})$. In this case $\gamma(T)$ tends to the unrenormalized value $\gamma^0=m^{*0}p_0/3$.

If the electron-phonon interaction is weak ($\rho\ll1$), Eq. (6) becomes

$$\frac{\gamma(T)}{\gamma(0)}=1+2\int_0^\infty\frac{d\omega}{\omega}g(\omega)[Z(T/\omega)-1].\tag{6'}$$

The temperature dependence $m^*(T)$ will be considered

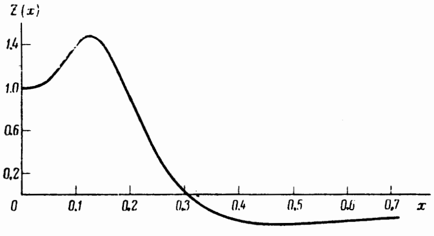

FIG. 1. Universal function $Z(x)$.

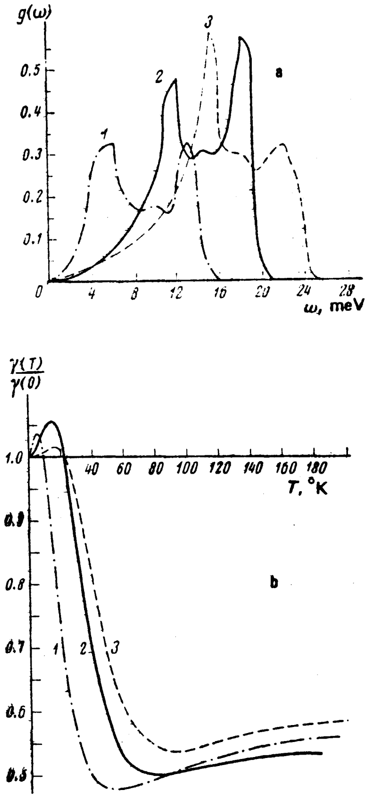

FIG. 2. Deviation of the electron specific heat from linearity: a) function $g(\omega)$; b) function $\gamma(T)=C_e(T)/T$. Curves 1, 2, and 3 represent In, Ta, and Nb, respectively.

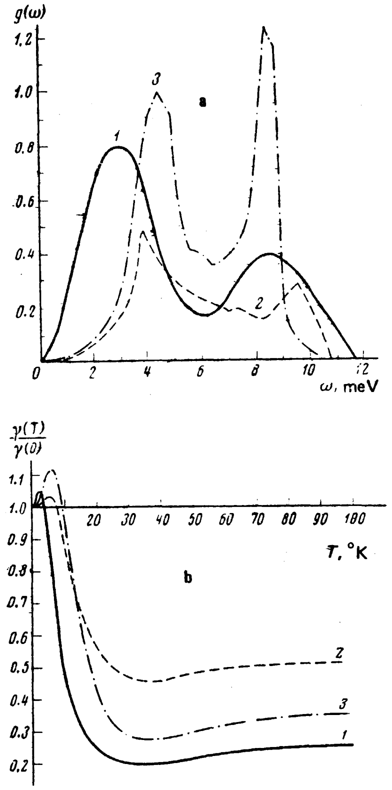

FIG. 3. Deviation of the electron specific heat from linearity: a) function $g(\omega)$; b) function $\gamma(T)=C_e(T)/T$. Curves 1, 2, and 3 represent Bi, Tl, and Pb, respectively.

in the next section. However, it should be pointed out that $\gamma(T)\neq m^*(T)p_0/3$ (see Figs. 5 and 7). This equality is valid only in the limits $T\to 0$ and $T\to\infty$. Thus, allowance for the electron–phonon interaction makes the formula $C_e=m^*p_0T/3$ invalid even if we take into account the temperature dependence of the effective mass. For this reason this temperature dependence $m^*(T)$ cannot be determined directly from the above formula (as is done erroneously in some cases[8,20]). The point is that allowance for the function $\Sigma_R(\varepsilon,p)$ [see Eq. (2′)] alters the electron dispersion law [it is found first as the root of the equation $\varepsilon=\xi(p)+\Sigma_R(\varepsilon,p)$]. Thus, the effective mass approximation itself becomes invalid. The excitation energy begins to depend in a complex manner on T. Therefore, it seems to us that the most consistent approach to the problem of the behavior of $C_e(T)$ is to use the Éliashberg method based on an investigation of the expression for the thermodynamic potential.

The deviation of the dependence $C_e(T)$ from linearity was investigated in greatest detail by Chernoplekov *et al.*,[6] who determined the specific heat of vanadium, and by Miller and Brockhouse,[7] who studied the specific heat of palladium and copper.

The main difficulty in these experiments is the need for accurate separation of the lattice component of the specific heat. As pointed out by Éliashberg,[10] in the limit $T\to 0$ the correction $\Delta C_e\propto T^3\ln(T/\omega_D)$ becomes considerable because of the logarithmic factor. At higher temperatures the problem of determination of C_{latt} becomes more complex.

In the cited investigations[6,7] the lattice component was separated as follows. The neutron method was used to find the density of the phonon states $F(\omega)$ (Fig. 4) and then to calculate C_{latt}. The electron contribution was then found by subtracting C_{latt} from the total specific heat C.

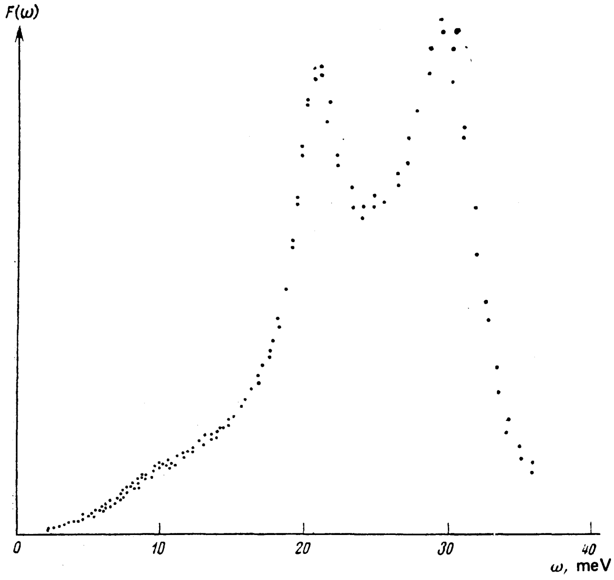

FIG. 4. Phonon density of states for V, obtained by the neutron method.

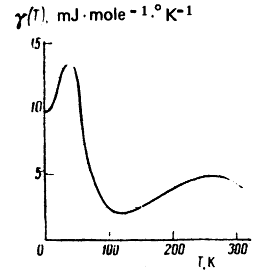

FIG. 5. Function $\gamma(T)$ for V found experimentally.[6]

It was shown by Chernoplekov *et al.*[6] that the error in the determination of $C_{1\mathrm{att}}$ was considerably less than the value of C_e. The selection of vanadium was very fortunate because neutron investigations of this metal could be carried out with a fairly high degree of precision and, moreover, V was a transition metal with a high density of states, so that the value of C_e was fairly large.

Figure 5 shows the experimental data obtained by Chernoplekov *et al.*[6] We can see that the temperature dependence of C_e is in agreement with the above relationships (compare Figs. 2 and 3). The dependence $\gamma(T)$ has a low-temperature maximum $[T \approx 0.1\tilde{\omega}_1$, where $\tilde{\omega}$ corresponds to the frequency of the lower peak of $g(\omega)]$ and a minimum $(T \approx \omega_2)$, where $\gamma(0) \neq \gamma(\infty)$. Only the value of $\gamma(T_{\max})$ disagrees with Eq. (6). This may be due to insufficient precision of the neutron measurements at low values of ω (this was pointed out also by Chernoplekov *et al.*[6] and elsewhere [21]). Moreover, the asymptotic values $\gamma(\infty)/\gamma(0)$ may be too high [the asymptotic behavior of $\gamma(T)$ may be established independently by means of a formula which gives the critical temperature of the transition to the superconducting state: $\gamma(0)/\gamma(\infty) = 1 + \lambda_0$]; clearly, more accurate data on the thermal expansion of V are required because these can be used to estimate the contribution of the anharmonic effects which are important in the $T \gg \omega_D$ range. The value of $T_{\min}$ and the dependence $\gamma(T)$ are established more accurately in the range $100°\mathrm{K} > T > 50°\mathrm{K}$. A detailed comparison with Eq. (6) cannot yet be made because the function $g(\omega)$ is not yet known for V. Therefore, after suitable tunnel experiments and reconstruction of the function $g(\omega)$ for vanadium we may calculate the specific heat of V using Eq. (6) and compare it with the experimental data.[6] The results reported by Miller and Brockhouse[7] are also in agreement with Eq. (6). It is also possible to use the experimental data to obtain information on the function $g(\omega)$ (see third section below).

2. DEPENDENCES OF THE EFFECTIVE MASS ON T AND n

1. We shall begin from Eq. (2) for the retarded Green function. We shall then assume that the usual dispersion law unperturbed by the electron–phonon interaction is quadratic, i.e., $\xi(p) = (p^2 - p_0^2)/2m^{*0}$ (m^{*0} is the renormalized effective mass corresponding to a negligible electron–phonon interaction).

We shall write $G_R^{-1}(\varepsilon, p)$ in the form

$$G_R^{-1}(\varepsilon, p) = \varepsilon(1+\beta) - \xi(p) - \Sigma_R'(\varepsilon, p),$$

where

$$\beta = \left(\frac{\partial \Sigma_R}{\partial \varepsilon}\right)\Big|_{\varepsilon=0,\, p=p_0}, \quad \Sigma_R' = \Sigma_R + \beta\varepsilon,$$

so that $(\Sigma_R'/\varepsilon)\big|_{\varepsilon\to 0} \to 0$. It is important to note that the quantity β depends on temperature (see below).

The renormalized effective mass is given by

$$m^*(T) = m^{*0}[1 - (\partial\Sigma_R/\partial\varepsilon)\big|_{\varepsilon=0\ p=p_0}]. \tag{9}$$

We shall ignore the quantity

$$\left(\frac{\partial \Sigma}{\partial p}\frac{m}{p}\right)\Big|_{p=p_0,\, \varepsilon=0} \sim \left(\frac{\omega_D}{\varepsilon_F}\right)^2$$

The dependence $m^*(T)$ is considered by Grimvall.[11, 13] We shall discuss this dependence is a somewhat different way. The results will be obtained in a form which makes it possible to carry out a direct comparison with the experimental data.

Allowance for the electron–phonon interaction generally alters the electron dispersion law, as indicated by Eq. (2). However, in measurements carried out by the cyclotron resonance method one has to allow only for the function β,[22] so that we can determine directly the effective mass $m^*(T)$ from Eq. (9). Allowance for the function Σ_R' and, therefore, for the corrections associated with the change in the dispersion law, is essential in the calculation of the electron specific heat (see above, Sec. 1).

It is clear from Eq. (9) that the calculation of the required function reduces to the calculation of the self-energy part $\Sigma_R(\varepsilon, p)$. This function satisfies Eq. (2). Next, we substitute Eq. (2) for $\Sigma_R(\varepsilon, p)$ into Eq. (9) and separate Σ_R into the odd f_0 and even $\mu + if$ parts; we thus allow also for the contribution of the imaginary part (see also the treatments of Migdal[1] and Éliashberg[19]). After simple calculations, subject to Eq. (3), we obtain the formula

$$\frac{m^*(T)}{m^{*0}} = 1 - \frac{1}{4\pi^2}\int_0^\infty d\varepsilon' \int_0^{k_1} dq\, q\alpha^2 \frac{m^{*0}}{2Tp_0}\,\mathrm{ch}^{-2}\frac{\varepsilon'}{2T}\left[\frac{1}{\varepsilon' - \omega_q} - \frac{1}{\varepsilon' + \omega_q}\right]. \tag{10}$$

We shall now integrate over the phonon momentum q and assume that $k_1 = \min\{q_m, 2p_0\}$.

Introducing in the usual way the function $g(\omega)$, we find that simple transformations yield

$$\frac{m^*(T)}{m^{*0}} = 1 + 2\int_0^\infty \frac{d\omega}{\omega} g(\omega) G(T/\omega), \tag{11}$$

where

$$G(x) = \int_0^\infty \frac{dz}{\mathrm{ch}^2 z}[1 - (2zx)^2]^{-1} \tag{12}$$

is the function introduced by Grimvall.[11]

We shall represent the function $G(x)$ in a form convenient for calculations:

$$G(x) = (2\pi x)^2 \sum_{n=0}^{\infty} (2n+1)/\{1 + [(2n+1)\pi x]^2\}^2. \tag{12'}$$

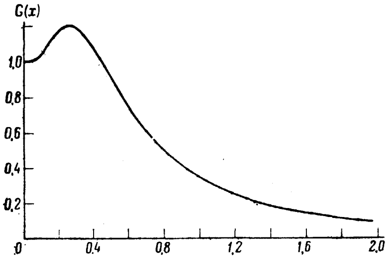

FIG. 6. Universal function $G(x)$.

Figure 6 shows a graph of the function $G(x)$. At $T=0$, Eq. (11) reduces to the well-known expression

$$m^*(0)/m^{*0}=1+\lambda_0,\quad \lambda_0=2\int_0^\infty\frac{d\omega}{\omega}g(\omega).$$

If we use Eq. (11), we obtain the final expression

$$\frac{m^*(T)}{m^*(0)}=1+\rho\left[\frac{\varphi(T)}{\varphi(0)}-1\right],\quad \varphi(T)=2\int_0\frac{d\omega}{\omega}g(\omega)G(T/\omega),\qquad(13)$$

$$\rho=\lambda_0/(1+\lambda_0).$$

Equation (13) includes the experimentally measurable quantities. In the case of a weak electron–phonon interaction ($\rho \ll 1$), Eq. (13) becomes

$$\frac{m^*(T)}{m^*(0)}=1+2\int_0^\infty\frac{d\omega}{\omega}g(\omega)[G(T/\omega)-1].\qquad(13')$$

Clearly, this influence of the electron–phonon interaction appears most explicitly in metals exhibiting tight binding (for example, Pb, Hg, Bi, etc.). Therefore, in a detailed comparison of the theory with experiment one should use Eq. (13).

The formulas (13), (12), and (12′) allow us to investigate the question of the temperature dependences $m^*(T)$. In the limit $T\to 0$, we have $G(T/\tilde{\omega})\approx T^2$ and $m^*(T)$ increases quadratically. The quadratic dependence of $[m^*(T)/m^*(0)-1]$ has been observed experimentally.[2-5]

The function $g(\omega)$ is usually characterized by maxima (normally there are two peaks corresponding to the longitudinal and transverse branches: see, for example, the work reported in [17]). The dependence $[m^*(T)/m^*(0)-1]\propto T^2\ln(\tilde{\omega}/T)$ is valid in the range $T\ll\tilde{\omega}_1$ (a rougher estimate is given by the inequality $T\ll\omega_D$). In the other limiting case of $T\gg\omega_D$ we have $G(T/\tilde{\omega})\propto T^{-2}$ and $m^*(T)$ decreases quadratically. In the intermediate temperature range there should be a maximum of $m^*(T)$.

Figure 7 gives the dependences $m^*(T)$ for lead and tantalum calculated by means of Eq. (13) using the tunnel spectroscopy data for Pb and Ta. We can see that $T_{\max}\approx 12$°K for Pb and $T_{\max}\approx 30$°K for Ta.

The cyclotron resonance measurements were carried out at somewhat lower temperatures. For example, Krasnopolin and Khaĭkin[2] studied lead in the range $T\lesssim 4$°K. A comparison of the theory with experiment can naturally be made also in this temperature range. It is clear from Fig. 8 that Eq. (13) describes the experimental data quite satisfactorily.

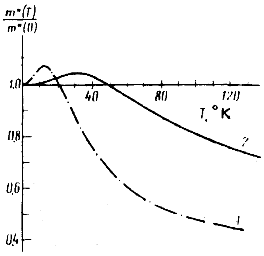

FIG. 7. Temperature dependence of the effective mass. Curves 1 and 2 represent Pb and Ta, respectively.

An increase in temperature results in broadening of the cyclotron resonance line so that studies of this kind cannot be carried out at high temperatures. However, the attainment of $T\sim T_{\max}\sim 10$°K, i.e., the observation of a maximum of $m^*(T)$, is possible and this makes it interesting to carry out the relevant experiments.

2. The electron–phonon interaction gives rise to a dependence of the effective mass on the electron density. This may be most important in semiconductors and semimetals. For example, studies of Si by the cyclotron resonance method[5] report the dependence $m^*(n)$. An investigation of the nature of this dependence will be made starting from Eq. (10) and assuming that $T=0$:

$$\frac{m^*(n)}{m^{*0}}=1+\frac{m^{*0}}{2\pi^2 n}\int_0 dq\,q\alpha_q^2\omega_q^{-1}.\qquad(14)$$

We shall next assume that

$$\alpha_q^2=I^2/2M\omega_q,\quad I\approx\varepsilon_F(q,\ e_{qj}),\quad k_1=\min\{2p_0,\ q_D\}\qquad(15)$$

(see, for example, Ziman[23] and Geĭlikman[24]). We shall ignore the umklapp processes; the necessary generalization presents no difficulties.

We can thus see that the function $m^*(n)$ is governed primarily by the direct dependence of m^* on $p_0\propto n^{1/3}$. Moreover, it is necessary to allow for the dependence of the phonon frequency on the electron density: $\omega_q\approx\Omega_p/\varepsilon(q,0)$, where Ω_p is the plasma frequency. In the random phase approximation, $\varepsilon=1+4\pi e^2\Pi/q^2$ [Π is the polarization operator equal to $\Pi=1+\Lambda^2/q^2$, where $\Lambda=4\pi e^2\nu_0$ (see, for example, Ziman [23])]. Substituting next Eq. (15) into the expression for ω_q in Eq. (14), we

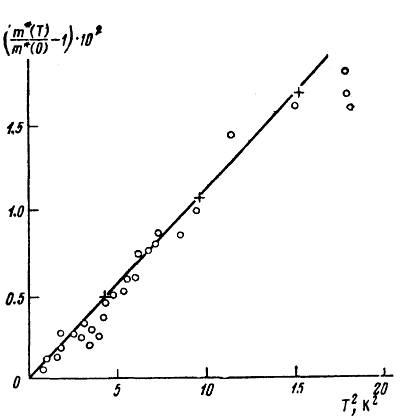

FIG. 8. Dependence $m^*(T)$ for Pb; the experimental data obtained in Ref. 2 are given; the continuous line is plotted according to Ref. 13.

can investigate the dependence $m^*(n)$. We shall first consider the range of carrier densities in which $2p_0 < q_D$, so that $k_1 = 2p_0$. We can easily see that then m^* rises with increasing n, so that $\delta m \propto p_0 \propto n^{1/3}$. At higher carrier densities (when $k_1 = q_D$), m^* decreases with increase in n in accordance with the law $m^* \propto p_0^{-2}$. Thus, the dependence $m^*(n)$ is nonmonotonic and has a maximum. At a carrier density corresponding to a Kohn singularity ($2p_0 = q_D$) there is a sudden change in the derivative of $m^*(n)$.

3. INVESTIGATION OF THE FUNCTION $g(\omega)$

The function $g(\omega) = \alpha^2(\omega)F(\omega)$ can be found by the tunnel spectroscopy methods. In the case of superconductors a method has been developed on inversion of the Éliashberg equation.[16-18] In this way the form of the function $g(\omega)$ has been found for certain superconductors; we have used the relevant data above (see Figs. 2 and 3). It should also be pointed out that a series of papers have appeared (see, for example, Geerk *et al.*[25]) in which the tunnel measurements for establishing $g(\omega)$ are compared with the neutron data which give directly the phonon density of states $F(\omega)$. This comparison makes it possible to determine the function $\alpha^2(\omega)$ describing the electron–phonon interaction and to estimate the relative intensity of the interaction of electrons with the various phonon branches.

A new method of microcontact spectroscopy,[26] which makes it possible to obtain information on the function $g(\omega)$ from the nonlinearities of the current-voltage characteristics, seems a very promising way of finding the function $g(\omega)$ for normal metals.

However, in the case of some substances (for example, V) the use of the tunnel method for reconstructing the function $g(\omega)$ is a very complex task. This is due to the difficulty of the preparation in these cases of tunnel junctions of sufficiently high quality. For this reason the function $g(\omega)$ has not yet been determined for V. Therefore, the search for methods not involving tunnel spectroscopy and enabling the determination of the behavior of the function $g(\omega)$ at least approximately is of considerable interest.

The temperature dependence of the electron specific heat and its deviation from the linear law are governed by the form of the function $g(\omega)$ [see Eq. (6)]. Moreover, neutron measurements have given us the phonon density of states $F(\omega)$ (this is important in the separation of the phonon component of the specific heat; see above). We shall now consider the possibility of obtaining information on the function $\alpha^2(\omega)$ and, consequently, on $g(\omega)$ from the experimental measurements of the electron specific heat and from the neutron measurements of $F(\omega)$. The direct inversion problem (see, for example, Lifshitz [27]) is unstable and can be solved only if additional information is available on the required function. Therefore, it seems natural to solve this problem in a different way.

The behavior of the current-voltage characteristic of a tunnel junction in a superconductor makes it possible to deduce singularities of the function $g(\omega)$ from sudden changes in this characteristic. We know the function $F(\omega)$. It represents approximately two Lorentzians describing the density of states of the transverse and longitudinal phonon branches. The positions of the peaks $g(\omega)$ and $F(\omega)$ coincide because they are known to be associated with the Van Hove singularities of the phonon dispersion law.

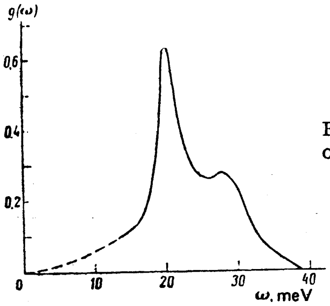

FIG. 9. Phonon spectrum of V.

We shall adopt an approximate method for the description of $g(\omega)$ first suggested by Scalapino *et al.*[28] The function $\alpha^2(\omega)$ is approximated by two constants $\alpha_t \approx \alpha(\bar{\omega}_1)$ and $\alpha_l \approx \alpha(\bar{\omega}_2)$, representing the interaction of electrons with the transverse and longitudinal branches, respectively. Then, we apply Eq. (6) and select the values of α_t and α_l in such a way as to obtain a curve close to the experimental dependence $\gamma(T)$. It is important to note that

$$\lambda_0=2\int\limits_0 \frac{d\omega}{\omega} g(\omega)$$

can be determined from an analysis of the asymptotic behavior $\gamma(T)$ because [see Eqs. (5) and (6)] $\gamma(0)/\gamma(\infty) = 1 + \lambda_0$. A different way of finding λ_0 applicable to superconductors is based on the use of the formula for T_c; we can then apply the McMillan expression[29] or similar formulas.[30,31] The temperature T_c is related directly to λ_0. The problem of finding $g(\omega)$ for V was solved by us by the second method because the results of high-temperature measurements of $\gamma(T)$ did not seem to be sufficiently reliable.

We carried out the relevant calculations for V using the experimental data given by Chernoplekov *et al.*[6]. This approach was of intrinsic interest because the function $g(\omega)$ for V was not yet known. It was found (the calculations were carried out on a computer) that $\xi = \alpha_t^2/\alpha_l^2 \approx 3.7$, i.e., that electrons in V interacted more intensively with the lower branch. The function $g(\omega)$ for V is shown in Fig. 9. It agrees satisfactorily with the neutron data and the values of $\gamma(T)$ and T_c.

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Cholesteric liquid crystal in a magnetic field near the phase transition into a smectic-*A*

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The dependence of the period of the cholesteric helix on the magnetic field perpendicular to the axis of the field is calculated above the point of the phase transition into the smectic-*A*. The dependence of the critical field H_c on the temperature is obtained.

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The behavior of a cholesteric liquid crystal in a magnetic field perpendicular to the axis of the choloesteric helix was investigated both theoretically[1] and experimentally.[3] With increasing magnetic field, period of the cholesteric structure increases and above the critical field H_c the crystal has nematic ordering. In weak magnetic fields, the period increases like the fourth power of the field and diverges logarithmically near the critical field. We examine this problem from the point of view of the phase transition of a cholesteric liquid crystal into a smectic one. In the phase transition into the smectic-*A* phase, the order parameter is the Fourier component ψ of the crystal density, with wave vector $\mathbf{q}_0 = 2\pi\mathbf{n}_0/d$, where d is the distance between planes and $\mathbf{n}_0$ is the average director and is perpendicular to the equidistantly disposed layers. To take into account the quadratic fluctuations of the order we use the de Gennes Hamiltonian.[4]

$$H=\int d^3r\left\{a\rho^2+\frac{1}{2m_\parallel}|(\mathbf{n}\nabla-iq_0)\rho|^2+\frac{1}{2m_\perp}[\mathbf{n}\nabla\rho]^2\right\},\qquad(1)$$

$$\mathbf{n}=(\cos\varphi(z),\,\sin\varphi(z),\,0),$$