particular in the appearance of an accelerated-particle flux, was observed at $N_m \approx N_{cr}$. In this case the critical-concentration layer was a plane perpendicular to the propagation direction of the incident wave, and the interaction conditions were similar to those realized in the experiments described in[12]. Our results, however, differ from those of[12], and we still do not understand the cause of the discrepancy.

In a transparent plasma the compression of the Langmuir wave should become weaker as the particle concentration at the maximum of the layer decreases, so that the acceleration effectiveness should become lower, as in fact observed in experiment. The delay in the appearance of the hot electrons can be ascribed, within the framework of the considered model, to the increased time of establishment of the spatial harmonics of the principal scale of the modulation instability.

The experimentally observed nonstationary behavior of the bleaching is due to rapid changes in the macroscopic state of the plasma, and calls for further study.

The authors thank A. G. Litvak and A. M. Feĭgin for useful discussions and M. A. Miller for constant interest in the work.

[1)]The details of the measurement of ion-sound oscillations with a resonant probe are considered in [10].

[2)]The experiments did not yield a convincing $T_e(v_\sim/v_{Te})$ dependence at small excesses above threshold. The minimum temperature of the accelerated electrons registered by the analyzer was $T_e \approx 45$ eV.

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Ionizing shock waves in a transverse magnetic field

A. L. Velikovich and M. A. Liberman

Institute of Physics Problems, USSR Academy of Sciences
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Criteria are obtained for the existence of stationary and nonstationary magnetic structures in the front of a transverse ionizing shock wave. An additional relation is obtained between the magnetic field and the velocity on either side of the front; this relation eliminates the indeterminacy in the formulation of the Rankine-Hugoniot conditions. It is shown that the discontinuity H_2/H_1 of the magnetic field in the shock wave depends substantially on whether there is enough time for a stationary front structure to be established. This time is determined by the electric field ahead of the front, which is proportional to v_1H_1. The results of the theory are compared with experiment.

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1. INTRODUCTION

When an attempt is made to develop a theory of stationary ionizing shock waves (the magnetic field lies in the plane of the wave front) a difficulty is encountered immediately when it comes to write down the Rankine-Hugoniot conditions. In fact, on the discontinuity we have only three equations for the four quantities that characterize the state of the gas—the temperature T, the density ρ, the velocity v, and the magnetic field H. These are the momentum and energy conservation equations and the continuity equation. In the analogous problem of a transverse shock wave in a plasma the additional equation is the condition that the induction electric field vanish (on both sides of the discontinuity) in the coordinate system connected with the gas stream; this condition takes the form $v_1H_1 = v_2H_2$. In the case of an ionizing shock wave in a neutral gas there can apparently exist, generally speaking, any electric field ahead of the wave front.

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In fact, it follows from Maxwell's equations in a coordinate frame where the discontinuity is at rest (planar one-dimensional problem, flow along the x axis, magnetic field parallel to the z axis) that

$$E_y = \text{const}, \quad \frac{dH}{dx} = \frac{4\pi\sigma}{c}\left(\frac{vH}{c} - E_y\right). \tag{1.1}$$

Recognizing that all the derivatives ahead and behind the shock-wave front vanish, and writing down the second equation of (1.1) for the state behind the front (subscript 2), we have

$$E_{y1} = E_{y2} = v_2H_2/c.$$

Since the conductivity σ is zero ahead of the front, there is no analogous relation between E_{y1} and v_1H_1 (the subscript 1 labels the equilibrium values of the quantities ahead of the shock-wave front). Thus, the state of the gas behind the front (at $x=+\infty$) is not determined by the state of the gas ahead of the front ($x=-\infty$). The indeterminacy of the state 2, implied in the boundary conditions, can be restricted somewhat by showing from simple physical considerations (for rigorous arguments see [1-3]) that the magnetic field H_2 behind the shock-wave front lies in the interval between $H_2=H_1$ and $H_2=H_1v_1/v_2$. The lower limit $H_2=H_1$ corresponds to $E_{y1}=v_2H_1/c$ and to a maximum of the electric field ahead of the front in the immobile (laboratory coordinate frame, $E_{y1}{}^*=v_1H_1(1-v_2/v_1)/c$. The upper limit $H_2=H_1v_1/v_2$ corresponds to a zero electric field ahead of the front in the l.s. and to a value $E_{y1}=v_1H_1/c$ in the wave-front frame.

It is obvious that at any arbitrarily small but finite value of the conductivity ahead of the shock-wave front (at $x=-\infty$) it follows from (1.1) that the equality $v_1H_1=v_2H_2$ should be satisfied. The boundary conditions and the structures of the shock-wave front are governed in this case by magnetohydrodynamics (MHD).[4]

The question arises of how small the conductivity ahead of the front must be to be able to regard it as equal to zero.

In[1-3] and in a number os subsequent papers (see the review[5]) there were considered stationary structures of transverse ionizing shock waves other than MHD, i.e., with finite values of the electric field at $x=-\infty$ in the l.s. and under the assumption that the conductivity of the gas ahead of a viscous discontinuity is zero.

Besides the impossibility of comparing quantitatively the theory of[1-3,5] with experiment, there are a number of discrepancies between this theory and the experimental data.[6,7] For example, it is seem from the oscillograms given in[6] that compression of the magnetic field precedes a viscous shock discontinuity, while nonequilibrium ionization of the gas was observed in[7] ahead of the viscous discontinuity.

It is shown in the present paper that a stationary structure of a transverse ionizing shock wave is possible only if the electric field E_{y1}^* ahead of the front (in the l.s.) is smaller than a certain critical value. A boundary condition is obtained for an ionizing shock wave propagating in an initially neutral gas, to replace the MHD condition $v_1H_1=v_2H_2$. In most cases of particular interest the threshold field is weak and the boundary conditions and the front structures are close to the magnetohydrodynamic ones investigated by us earlier.[4] In the general case, when account must be taken of the finite dimensions of the real experimental setup, the presence of an electric field ahead of the shock-wave front makes the existence of a stationary front structure impossible. We obtain in this paper a criterion for the existence of a stationary structure. The results of the theory are compared with the experimental data.

2. NONEQUILIBRIUM IONIZATION OF THE GAS AHEAD OF A SHOCK-WAVE FRONT AND THE CONDITION FOR THE EXISTENCE OF STATIONARY STRUCTURES

We consider a sufficiently strong plane ionizing shock wave propagating in an unbounded cold gas along the x axis and perpendicular to the magnetic field. Let the gas ahead of the viscous discontinuity have a small but finite conductivity σ_1 due, for example to photoionization by radiation coming from the hot region behind the shock-wave front. We assume that the photoelectron density and the ensuing gas conductivity are small enough to neglect the contribution of the Joule heating to the gas energy balance. The magnetic structure of the shock-wave front ahead of the viscous density discontinuity is[4]

$$x - x_0 = \frac{c^2}{4\pi\sigma_1}\int_{H_0}^{H}\frac{dH'}{H'v(H') - H_1v_1}, \tag{2.1}$$

where x_0 is the coordinate of the viscous density discontinuity, H_0 is the value of the magnetic field ahead of this discontinuity, and the function $v(H)$ is determined from the momentum and energy conservation laws (see[4]).

It follows from (2.1) that the width of the Joule region—the magnetic-field compression zone—is of the order of $\Delta_{j1}=c^2/4\pi\sigma_1 v_1$ and that $\Delta_{j1}\to\infty$ as $\sigma_1\to 0$. In this limit, an unperturbed value of the magnetic field H_1 is reached as $x\to-\infty$. Thus, at an infinitesimally small gas conductivity ahead of the viscous discontinuity, the compression of the magnetic field ahead of the discontinuity, at any finite distance, is a small quantity of the order of L/Δ_{j1}. In the co-moving coordinate system the electric field is then $E_y=v_2H_2/c$. In the immobile coordinate system the electric field far ahead of the viscous discontinuity is[1)]

$$E_{y1}{}^* = v_1H_1(1 - v_2H_2/v_1H_1)/c, \tag{2.2}$$

where v_2/v_1 and H_2/H_1 are the changes of the velocity and of the magnetic field ahead of the viscous discontinuity.

Although the concentration of the electrons due to photoionization may by itself be small, so that the corresponding conductivity ahead of the viscous discontinuity produces a negligibly small compression of the magnetic field at finite distances from the viscous discontinuity, the presence of a finite electric field (2.2) makes possible the development of ionization instability[8]—multiplication of the electrons ahead of the viscous density jump (a process analogous to non-autonomous discharge). The gas breakdown resulting from this process[2] leads to an increase of the conductivity, and to compression of the magnetic field ahead of the discontinuity, i.e., to a decrease of E_{y1}^* as $x\to-\infty$. In the stationary regime E_{y1}^* should be low enough to prevent electron multiplication in this field. This last condition leads to an equation for the threshold value of the electric field E_∞^* as $x\to-\infty$:

$$\nu_i(T_e)=\nu_a(T_e);\tag{2.3}$$

here ν_a is the electron-loss frequency and corresponds to effects linear in the electron concentration n_e (adhesion, dissociative recombination, etc), and ν_i is the frequency of ionization by electron impact. The electron temperature T_e is determined from the electron energy-balance equation in the quiescent gas as $x\to-\infty$

$$\sigma_1 E_\infty^{*2}=n_e\nu_i(T_e)\left(I+\frac{3}{2}T_e\right)+n_e\frac{2m_e}{m_a}\nu_{ea}(T_e-T_a),\tag{2.4}$$

where I is the ionization potential of the atom (molecule) and ν_{ea} is the frequency of the electron-atom collisions.

We shall neglect the dependence of the effective loss cross sections σ_a and of the elastic electron-atom collision cross sections on the electron temperature, since this dependence is slow compared with the exponential T_e-dependence of the impact-ionization cross section $\bar{\sigma}_i\sim\exp(-I/T_e)$ averaged over the Maxwellian distribution of the electrons. We assume

$$\sigma_1=\frac{e^2n_e}{m_e\nu_{ea}},\quad \nu_{ea}=N_1v_{Te}\sigma_{ea},\quad \nu_a=N_1v_{Te}\sigma_a,$$

$$\nu_i=N_1v_{Te}\left(\frac{d\sigma_i(\varepsilon)}{d\varepsilon}\right)_{\varepsilon=I}(I+2T_e)\exp\left(-\frac{I}{T_e}\right),$$

and obtain from (2.3) and (2.4), accurate to small terms of the order of $T_e/I\ll1$,

$$E_\infty^*=N_1\frac{I}{e}\left(\frac{8}{\pi\gamma_a}\sigma_a\sigma_{ea}\right)^{1/2},\tag{2.5}$$

where N_1 is the density of the atoms (molecules) as $x\to-\infty$, and

$$\gamma_a=\ln[(d\sigma_i/d\varepsilon)_{\varepsilon=I}I/\sigma_a].$$

For practical purposes it is convenient to rewrite (2.5) in terms of the temperature T_1 and the pressure p_1 of the cold gas ahead of the front:

$$\frac{E_\infty^*}{p_1}=\frac{I}{T_1}\left(\frac{8}{e^2\pi\gamma_a}\sigma_a\sigma_{ea}\right)^{1/2}.\tag{2.5a}$$

As seen from (2.5), E_∞^* depends little on the cross sections of all the processes. For example, at $N_1=10^{16}$ cm^{-3} calculations for hydrogen and oxygen, in which the losses are assumed to be due to adhesion, yield $E_\infty^*=0.4$ V/cm (hydrogen) and 3 V/cm (oxygen). For inert gases, the corresponding values of E_∞^* are substantially smaller.

Thus, the boundary condition that replaces the MHD relation $v_1H_1=v_2H_2$ for the stationary structure of a transverse ionizing shock wave takes the form

$$\frac{v_1H_1}{c}\left(1-\frac{v_2H_2}{v_1H_1}\right)=E_\infty^*\tag{2.6}$$

or

$$\frac{v_2H_2}{v_1H_1}=1-\frac{cE_\infty^*}{v_1H_1}\tag{2.7}$$

It is readily seen from (2.7) that this condition is close to the MHD condition already at moderately strong magnetic fields. For example, at $H_1=400$ Oe and $v_1=5\times10^6$ cm/sec we have $cE_\infty^*/v_1H_1=0.02$ for hydrogen with $N_1=10^{16}$ cm^{-5}, i.e., (2.7) agrees within 2% with the MHD relation.

In real experiments it is necessary to take into account the finite dimensions of the apparatus. For the ionization instability ahead of the viscous-discontinuity front to influence the magnetic structure of the front it is necessary that the electron density ahead of the viscous discontinuity, n_{e1}, have time to increase noticeably during the time of passage of the shock wave through the apparatus. If L is the path of the shock wave in the apparatus, ν_i is the frequency of ionization by electron impact, ν_a is the frequency of the losses to adhesion (recombination), ν_d is frequency of the diffusion departures of the electrons to the walls, and n_{ph} is the concentration of the priming electrons produced by photoionization, this condition reduces to the inequalities

$$\Delta_{j1}=\frac{c^2}{4\pi\sigma(n_{e1})v_1}\ll L,\tag{2.8}$$

$$\nu_i>\nu_a+\nu_d+\frac{v_1}{L}\ln\frac{n_{e1}}{n_{ph}}.\tag{2.9}$$

Comparing the different terms in the right-hand side of (2.9), choosing the largest one and equating to it the ionization frequency $\nu_i(T_e)$, we obtain the characteristic value of the field E_{cr}^*. If $E_{y1}^*<E_{cr}$, the multiplication of the electrons ahead of the viscous-discontinuity front is negligible and the structure of the shock-wave front agrees with the theory of[1-3,5,9] (i.e., the magnetic field is not compressed). It is only in this case that the structures considered in[1-3,5,9] with the electric field ahead of the front are quasistationary. In the opposite case, i.e., at $E_{y1}^*>E_{cr}^*$, the nonequilibrium ionization ahead of the viscous discontinuity influences strongly the shock-wave magnetic structure. This structure, generally speaking, is not stationary, but it can manage to approach the stationary structure considered above if the inequality $E_{y1}^*>E_{cr}^*$ is satisfied with a sufficiently large margin. The time of transition to the stationary regime is determined by the electron-multiplication stages that are nonlinear in n_e. This time can be calculated by means of an exact numerical solution of the nonstationary problem, but this is the subject of another study.

E_{cr}^* is determined from (2.4), (2.8), and (2.9). Its value corresponding to electron loss to adhesion agrees with E_∞^* in (2.5). If the loss frequency is determined by free diffusion of the electrons to the walls, then

$$E_d^*=\frac{I}{e\Lambda}\left(\frac{8}{\pi\gamma_d}\right)^{1/2},\tag{2.10}$$

where Λ is the characteristic diffusion length (of the order of the transverse dimensions of the apparatus), and

$$\gamma_d=\ln\left[\left(\frac{d\sigma_i}{d\varepsilon}\right)_{\varepsilon=I}I\sigma_{ea}N_1^2\Lambda^2\right].$$

If the principal term in the right-hand side of (2.9) is the last one, i.e., E_{cr}^* is determined by the finiteness of the time of the experiment, we get

$$E_L^* = \frac{N_1 I}{e}\left(\frac{\sigma_{ea} v_1}{N_1 L}\ln\frac{n_{e1}}{n_{ph}}\right)^{1/2}\left(\frac{8}{\pi}\frac{m_e}{I\gamma_L}\right)^{1/4}, \qquad (2.11)$$

where $\gamma_L = \ln A - \frac{1}{2}\ln\ln A$,

$$A = \left(\frac{8}{\pi m_e}\right)^{1/2}\left(\frac{d\sigma_i}{d\varepsilon}\right)_{\varepsilon=I} I^{3/2}\frac{LN_1}{v_1}\Big/\ln\left(\frac{n_{e1}}{n_{ph}}\right).$$

The quantity n_{e1} in (2.11) must be estimated from (2.8). In practice the value of E_L^* calculated from (2.11) is not very sensitive to the actual value of the ratio n_{e1}/n_{ph}, so that we can put, for example, $\ln(n_{e1}/n_{ph}) = 20$.

For hydrogen at $N_1 = 10^{16}$ cm^{-3}, $v_1 = 5\cdot 10^6$ cm/sec, $L = 20$ cm, $\Lambda = 5$ cm we get from (2.5), (2.10), and (2.11) the respective values $E_a^* = 0.4$ V/cm, $E_d^* = 2$ V/cm, $E_L^* = 8$ V/cm. Thus, under these conditions the finite time of the experiment is more significant than the electron losses.

As a rule, questions concerning the existence of stationary structures are investigated with the aid of the equations of the stationary shock layer.[1-3] We emphasize that in our case it is impossible to solve the problem in this manner. The reason is that the initial state of the gas with $n_e = 0$ is unstable in a shock wave with an electric field ahead of the front. It can be verified with a simple model that the formal condition for the existence of the stationary structure of the transverse ionizing shock wave does not determine the compression of the magnetic field in the wave. On the other hand, if we choose as an additional condition the requirement that the conductivity σ and the degree of ionization α ahead of the viscous discontinuity be exactly equal to zero (this is justified at $E_{y1}^* < E_{cr}^*$, see above), then we arrive at the correct (albeit trivial) conclusion that the magnetic field is not compressed in the flow region Rm $\ll 1$.[9] Forgoing this requirement, we obtain an entire family of "stationary structures" corresponding to different compressions of the magnetic field.

We introduce the dimensionless variables

$$s = \frac{cE_{y1}}{v_1 H_1}, \quad h = \frac{H}{H_1}, \quad \alpha = \frac{n_e}{N_a + n_e}, \quad \omega = \frac{v}{v_1}, \quad \Theta_e = \frac{T_e}{T_1}.$$

Let M_1 and M_{a1} be respectively the acoustic and Alfven Mach numbers of the incident stream; assuming the shock wave to be strong enough, $M_1 \gg 1$, we neglect small quantities of order M_1^{-2}. The equations of the stationary shock layer can be written in the form[4,8]

$$\frac{d\alpha}{dx} = \frac{\alpha}{\Delta_{ion}}\left[1 - \alpha - \alpha^2\frac{1-\alpha_{eq}(\Theta_e,\omega)}{\alpha_{eq}^2(\Theta_e,\omega)}\right], \qquad (2.12)$$

$$\frac{dh}{dx} = \frac{h\omega - s}{\Delta_j}, \qquad (2.13)$$

$$\omega_\pm(h;s) = \frac{1}{8}\left\{5 - \frac{5}{2}\frac{h^2-1}{M_{a1}^2} \pm \left[\left(5 - \frac{5}{2}\frac{h^2-1}{M_{a1}^2}\right)^2 - 16 + \frac{32s(h-1)}{M_{a1}^2}\right]^{1/2}\right\}. \qquad (2.14)$$

Here Δ_{ion} and Δ_j are the characteristic lengths of the ionization and Joule-heating processes, and $\alpha_{eq}(\Theta_e, \omega)$ is the equilibrium (after Saha) degree of ionization for a given electron temperature Θ_e and for an atom concentration expressed with the aid of the continuity equation in terms of the velocity ω. The energy lost to ionization is disregarded. We assume, furthermore, that Rm $\ll 1$ in the viscous discontinuity and that its width is $l \ll \Delta_{ion}$, i.e., the viscous discontinuity can be regarded as a velocity and temperature discontinuity at constant α and h. The two branches of (2.14) (the plus and minus signs) correspond respectively to the gas-flow regions that are supersonic and subsonic relative to ordinary sound. The values of ω on the supersonic and subsonic branches are connected by the momentum and energy conservation laws, so that a transition from one branch to the other via a viscous discontinuity[8] is possible in the structures under consideration. Without writing down the concrete form of the equation of electronic thermal conductivity, we note that the presence of the electric field $E_{y1}^* = v_1 H_1(1-s)/c$ at $s < 1$ leads to heating of the electrons, which have a high temperature throughout the stationary shock layer. Therefore the quantity $\Delta_{ion} \sim \exp(I/T_e)$ also varies within a single order of magnitude throughout the stationary shock layer. Assuming that Θ_e is everywhere so large that $\alpha_{eq}(\Theta_e, \omega) = 1$, we introduce the stretched coordinate $d\xi = dx/\Delta_{ion}$ and replace $\alpha\Delta_j/\Delta_{ion}$ in the right-hand side of (2.13) by unity (the qualitative form of the structure remains unchanged, since the last quantity is positive and bounded at $\alpha \to 0$[8]). The equations then take the simple form

$$d\alpha/d\zeta = \alpha(1-\alpha), \qquad (2.12a)$$

$$dh/d\zeta = \alpha\psi^\pm(h;s), \qquad (2.13a)$$

where $\psi^\pm(h;s) = h\omega_\pm(h;s) - s$. The boundary condition at $x = -\infty$ is $\alpha = 0, h = 1$ (state 1), and at $x = +\infty$ (state 2) it is $\alpha = 1, h = h_2(s)$, the value that causes the right-hand side of (2.13) to vanish.

It is obvious that the incident stream is supersonic. It is easy to show that if the stream in state 2 is also supersonic then $\psi^+(h_2(s);s) = 0$ $(\psi^-(h_2(s);s) < 0)$. In this case the front structure is described by the solution

$$\alpha(h) = 1 - \exp\left[-\int_1^h\frac{dy}{\psi^+(y;s)}\right],$$
$$\zeta - \zeta_0 = \int_{h_0}^h\frac{dh'}{\alpha(h')\psi^+(h';s)}. \qquad (2.15)$$

The corresponding phase curves on the (h, α) plane are shown in Fig. 1. The singular point 2 is a node.

On the other hand if the stream at $x = +\infty$ is subsonic, then $\psi^-(h_2(s);s) = 0$ $(\psi^+(h_2(s);s) > 0)$, and an integral curve

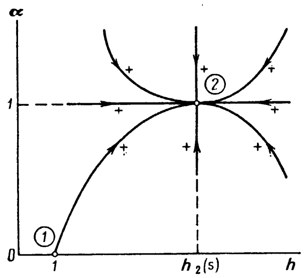

FIG. 1.

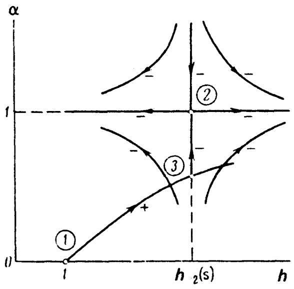

FIG. 2.

corresponding to the minus sign in (2.13a) enters in the point 2. From Fig. 2, which shows the field of the integral curves for this case (the plus and minus signs on Figs. 1 and 2 mark supersonic and subsonic integral curves, respectively), it is seen that the point 2 is a saddle and that the only integral curve that reaches it from the region $\alpha<1$ is the segment of the vertical line $h=h_2(s)=$const. Since a supersonic curve goes out of the point 1, a viscous discontinuity (point 3 of Fig. 2) is produced at the point where this curve crosses the integral curve going out of 2; in this discontinuity

$$h=h_2(s),\quad \alpha=\alpha_3(s)=1-\exp\left[-\int\limits_1^{h_2(s)}\frac{dy}{\psi^+(y;s)}\right].$$

A solution exists for any s in the interval $1\geqslant s\geqslant\frac{1}{4}$, i.e., any compression of the magnetic field $1\leqslant H_2/H_1\leqslant v_1/v_2$ is admissible. This means that it is impossible to determine the compression of the magnetic field from the condition that a stationary structure exist, and this conclusion remains in force if we forego all the simplifying assumptions made. If we require that the points 1 and 3 coincide (i.e., $\alpha=0$ ahead of the viscous discontinuity), we obtain Hoffert's result[9] $h_2=1$, $s=v_2/v_1$.

Our results are perfectly natural, since it follows from rather general considerations[3,10] that the stability of the states on both sides of the shock-wave front and the existence of a single stationary shock transition between them are unambiguously related in this case. In other words, the possibility of having ahead of the viscous discontinuity an electron multiplication having the character of the development of ionization instability makes a formal consideration of stationary structures meaningless.

3. TRANSITION TO STATIONARY MHD STRUCTURE. COMPARISON OF THEORY WITH EXPERIMENT

We have derived above a criterion for the stationarity of a transverse ionizing shock wave in real experiment setups. The method of obtaining these shock waves (as a rule, a magnetic piston) is such that the shock wave starts out with a gasdynamic (viscous) discontinuity. The front of the as yet nonstationary shock wave does not have an MHD structure, but approaches the latter in the course of the evolution. The time required to reach the stationary regime is determined by the nonlinear stages of the ionization process ahead of the front: as the degree of ionization (the conductivity) decreases, the magnetic field is compressed, and the electric field ahead of the front decreases, resulting in a decrease in the electron temperature and in the rate of impact ionization. In addition, this time can depend significantly on the density n_{ph} of the priming electrons. It is not yet quite clear whether these electrons are due to radiation from the heated region behind the viscous discontinuity, or whether a substantial role is played by the radiation from the current sheath of the piston.[3)] A quantitative comparison with experiment can be obtained only on the basis of a numerical solution of the problem of the piston motion in the magnetic field, but on the whole the process can be understood, at least qualitatively, on the basis of the theory developed above.

Let us consider the experimental data of Stebbins and Vlases.[6] For all the shock waves investigated in their paper the initial value of the electric field ahead of the front exceeds the value E^*_{cr} (the experiments were performed with hydrogen at pressures 0.1 – 0.25 Torr, $v_1=4.2-13)\times10^6$ cm/sec, and H_1 from 420 to 2140 Oe). Thus, at least during the initial stage, it can be stated that stationary MHD structures are reached, other conditions being equal, faster the larger the value of v_1H_1 (and the larger v_1, as was customarily assumed). In addition the structure of the shock wave front will be closer to MHD the farther it moved away from the place where it was produced, i.e., more remote magnetic sensors should detect larger values of H_2/H_1, as was in fact observed in[6].

Figures 3 and 4 show plots, based on the experimental data of Stebbins and Vlases,[6] of the quantities

$$\lambda=(H_2/H_1)_{\rm exp}/(v_1/v_2)_{\rm MHD}$$

as functions of v_1H_1 (initial pressure $P_1=0.25$ Torr). The value $\lambda=1$ corresponds to a stationary MHD structure, and $\lambda=(v_1/v_1)_{\rm MHD}$ corresponds to a fixed magnetic field. The lower parts of the figures show the tempera-

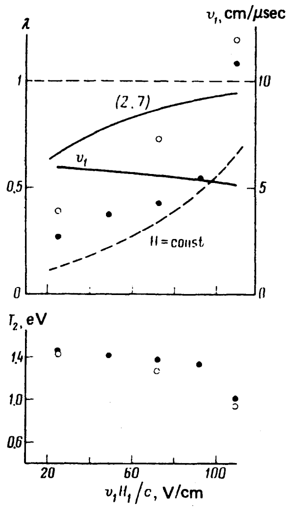

FIG. 3. Values of $\lambda=(H_2/H_1)_{\rm exp}/(\rho_2/\rho_1)_{\rm MHD}$ at different values of v_1H_1/c according to the data of[6] at $v_1\approx$ const (points) and according to formula (2.7). The lower part of the figure shows the temperature behind the shock front, calculated from the measured value of $(H_2/H_1)_{\rm exp}$. Position of magnetic sensor: ● – $R=10$ cm, ○ + $R=14$ cm.

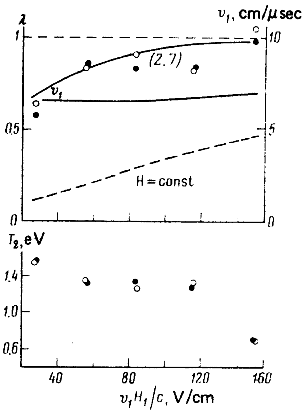

FIG. 4. Value of λ at different values of v_1H_1/c and different sensor positions, from the data of[6] (● – $R = 10$ cm, ○ – $R = 14$ cm) and from formula (2.7). The points closer to $\lambda = 1$ in the figure correspond to even smaller values of v_1 and T_2.

ture behind the shock-wave front, calculated from the measured value of H_2/H_1. The data in Figs. 3 and 4 pertain to different voltages on the discharge gap that produces the magnetic piston (10, 18, and 25 kV, respectively; obviously, a higher voltage ionizes the gas more strongly by the discharge radiation and decreases the time needed to obtain an equilibrium structure, as is in fact observed). As seen from Figs. 3 and 4, the proximity to the MHD structure is determined not by the shock wave intensity (by the front velocity, by the temperature behind the front), but by the value of v_1H_1. At practically constant front velocity (Fig. 4) or at a decreasing front velocity (Fig. 3) and a decreasing temperature behind the front (Figs. 3 and 4) λ is closer to unity, at a given sensor position, the larger v_1H_1.

CONCLUSION

We note in conclusion that the value of E_∞^* given by (2.5), as well as the values of E_{cr}^* calculated in accord with (2.10) and (2.11), was obtained under the assumption that the hydrodynamics equations are valid at small n_e. In addition, no account was taken in (2.4) of the loss to excitation of the electronic levels of the atoms (and also the vibrational and rotational levels in molecules), which in many cases exceed the ionization energy loss. The figure presented for E_∞^* and E_{cr}^* are therefore tentative. Incidentally, as demonstrated, these are only order-of-magnitude values.

A detailed discussion of the questions connected with the critical breakdown fields in gases can be obtained, for example, in[12]. We note also that in very strong ionizing shock waves, in which the assumptions that $T_e \ll 1$ and $\mathrm{Rm} \ll 1$ are not satisfied in the viscous discontinuity, the times of ionization ahead of the front are quite short and the observed structures are magnetohydrodynamic.[13]

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1) Calculations show that in the case of practical interest (e. g., under the conditions of the experiment of Stebbins and Vlases[6]), the magnetic Reynolds number in the viscous discontinuity is $\mathrm{Rm} = 4\pi\sigma v_1 l_a/c^2 \ll 1$. Consequently, the magnetic field does not change in the viscous discontinuity, and E_{y1}^* has a maximum: $E_{y1}^* = v_1H_1(1 - v_2/v_1)/c$.

2) Gas breakdown ahead of the front of a transverse ionizing shock wave was observed in [14].

3) An attempt to explain the results of[6] by taking into account the photoionization of the gas ahead of the shockwave front by the radiation from behind the front was undertaken in [4,11].

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