Dissipative motion of superfluid ^{3}He in the A phase

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The equations of orbital hydrodynamics of the A phase of superfluid ^{3}He are obtained on the basis of the microscopic theory in the weak-coupling approximation. These equations are used to describe nonsingular vortices and solitons.

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1. INTRODUCTION

The existence of order-parameter structures corresponding to the phases A and B of superfluid ^{3}He is possible under nonstationary conditions only in the case of slow variations of the order parameter in time and at sufficiently large dimensions of the spatial inhomogeneities. The order parameter in the A phase is of the form

$$\hat{\Delta}_{\mathbf{p}}=i\hat{\sigma}^{(\nu)}\hat{\sigma}^{(2)}d_\nu\Delta_0(T)\,\Psi\hat{\mathbf{p}}, \tag{1}$$

where $\hat{\sigma}^{(\nu)}$ are Pauli matrices, d is a unit vector in spin space, and p is a unit vector in the direction of the momentum of the pairing particles. In spatially homogeneous stationary conditions, the complex vector Ψ takes the form $\Psi_0=\Delta'+i\Delta''$, where Δ' and Δ'' are two mutually perpendicular unit vectors. Their product $\Delta\times\Delta=1$ defines the anisotropy vector. The position of the triad of unit vectors Δ', Δ'', and l is specified by three parameters, for example by the rotation angle $\delta\theta$: $\delta\Psi_0=\delta\theta\times\Psi_0$. In the presence of spatial and temporal inhomogeneities, the complex vector Ψ has, generally speaking, six components. It is clear that one can speak of the existence of the A phase only when the components of the vector $\delta\Psi$ differ little from $\delta\Psi_0$. For this purpose it is necessary that the dimensions of the spatial inhomogeneities by larger than the coherence radius, $k\xi\ll1$. As to the frequencies ω, there are two regions where one can speak with assurance of the existence of the A-phase structure, namely $\omega\ll\tau^{-1}$ (the hydrodynamic regime) and $\tau^{-1}\ll\omega\ll\Delta$ (the so-called collisionless regime). Here τ is the characteristic relaxation time which is governed in this case by the quasiparticle collisions and is of the order of $\tau\sim E_F/T^2\gg\Delta^{-1}$. The existence of the A phase at $\omega\tau\sim1$ calls for a special analysis.

In the present paper we confine ourselves to the case $k\xi\ll1$ and $\omega\tau\ll1$ (near T_c it is necessary to satisfy the more stringent condition $\tau\omega\ll\Delta/T$), which corresponds, as is well known, to the hydrodynamic approximation. For the B phase the hydrodynamic equations were derived Combescot[1] with the aid of the Bogolyubov transformation. The collision integral for the B phase was written out by Geilikman and Chechetkin.[2] A phenomenological approach to the hydrodynamics of superfluid ^{3}He was developed in the papers of Graham[3] and of Khalatnikov and Lebedev.[4] Orbital hydrodynamics of the A phase is the subject of papers by Leggett and Takagi[5] and by Volovik and Mineev,[6] where this question was considered from the semiphenomenological point of view. The results of[3] and[4] disagree when it comes to the mechanisms of the relaxation of the orbital oscillations and the value of the orbital susceptibility. A kinetic equation for the quasiparticle distribution function was given in[6] in the τ-approximation. A collision integral for the A phase was derived only near T_c.[7] Combescot[7] obtained the equations of the orbital motion for the A phase with the aid of Bogolyubov transformations in the spatially homogeneous case.

It can thus be stated that despite the importance of the hydrodynamic approximation, the question of the microscopic derivation of the hydrodynamics equations for the A phase has not yet been solved. In the present paper we try to fill this gap deriving the hydrodynamics equations on the basis of the nonstationary theory of Gor'kov and Éliashberg,[9,10] which lends itself to s a simple generalization to the case of pairing with nonzero angular momentum. We assume that the condition $\tau\Delta\gg1$ is always satisfied, i.e., we exclude from consideration a very narrow temperature region near T_c: $1-T/T_c\sim10^{-4}$. It must be stipulated beforehand that we shall use the weak-coupling approximation without allowance for Fermi-liquid effects. We confine ourselves here for simplicity to the study of the motion of only the orbital part of the order parameter. The orbital motion is separated from the spin motion, with neglect of the dipole-dipole interaction that orients the vector d along l. This is possible at $kR_D\gg1$, where R_D is the characteristic distance over which the action of the dipole forces manifests itself, $R_D\approx120\xi_0$, or in the frequency region $g_D/v\Delta_0^2\ll\omega\tau\ll1$, when the energy of the nonequilibrium excitations exceeds the dipole energy (g_D is the dipole-interaction constant and $v=mp_F/2\pi^2$ is the density of states on the Fermi surface). Substituting the value of the constant $g_D=\rho^s/m^2R_D^2$, we obtain the inequality $\xi_0^2/R_D^2\ll\omega\tau\ll1$. The kinetic equations obtained in the present paper are valid also in the case when the spin vector d oscillates together with l, but remains parallel to it at all times. This occurs when the inhomogeneity dimensions exceed R_D and at frequencies $\omega\tau\ll\xi_0^2/R_D^2$. In this case the dipole-interaction energy $E_D=-(1/2)g_D(\mathbf{l}\cdot\mathbf{d})^2$ is constant and can be disregarded. The validity of these equations is governed in this case also by the fact that in the unitary state the ϵ_p spectrum (see below) does not depend on the spin.

The description of the orbital part of the A phase in the hydrodynamic approximation should contain equations that define the three components of the vector $\delta\theta$,

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the kinetic equation for the quasiparticle distribution function, and the conservation laws for the momentum, energy, and number of particles. One should add also the equation of state. We, however, are not interested here in the variation of the thermodynamic quantities, and confine ourselves to the study of the motion of the order parameter and of the corresponding dissipative processes.

We resolve the rotation $\delta\boldsymbol{\theta}$ into a component along $\mathbf{l}$ and a perpendicular component

$$\delta\theta = -\mathbf{l}\,\delta\Phi + \delta\theta_\perp, \quad \delta\theta_\perp = [\mathbf{l}\,\delta\mathbf{l}].$$

We have

$$\delta\Psi_0 = i\delta\Phi\,\Psi_0 - \mathbf{l}(\Psi_0\,\delta\mathbf{l}). \tag{2}$$

The function Φ introduced in this manner, as repeatedly noted (see, e.g.,[11]), is not single-valued in the coordinates, and it is therefore more convenient to formulate the hydrodynamics equations not for the phase Φ itself, but for its derivatives $\dot{\Phi}$ and $\nabla\Phi$, which are well-defined functions. The superfluid velocity is given by ($\hbar = 1$)

$$v^s = \frac{1}{4m}(\Delta_A'\nabla\Delta_A'' - \Delta_A''\nabla\Delta_A').$$

From this we obtain with the aid of (2) that the superfluid velocity $\mathbf{v}^s = \nabla\Phi/2m$ is connected with $\dot{\Phi}$ and with the derivatives of $\mathbf{l}$ in the following manner[12]:

$$\frac{\partial v_i^s}{\partial t} = \frac{1}{2m}\nabla_i\dot{\Phi} + \frac{1}{2m}\left[\frac{\partial\mathbf{l}}{\partial t}\times\nabla_i\mathbf{l}\right]\mathbf{l}. \tag{3}$$

From this follows, in particular,[12] the condition obtained by Mermin and Ho[11]

$$(\mathrm{rot}\,\mathbf{v}^s)_i = \frac{1}{4m}e_{ijk}\mathbf{l}[\nabla_j\mathbf{l}\times\nabla_k\mathbf{l}].$$

The hydrodynamics equations will be formulated for the quantities $\mathbf{v}^s$, $\dot{\Phi}$, and $\mathbf{l}$, with three independent variables ($\dot{\Phi}$ and the two components of the unit vector $\mathbf{l}$).

2. DERIVATION OF THE KINETIC EQUATIONS

We introduce the change of the total phase of the order parameter[13]

$$\delta\Phi_{\mathbf{p}} = \frac{(\Psi^*\hat{\mathbf{p}})(\delta\Psi\hat{\mathbf{p}}) - (\Psi\hat{\mathbf{p}})(\delta\Psi^*\hat{\mathbf{p}})}{2i|\Psi\mathbf{p}|^2}.$$

For the equilibrium A phase we have

$$\delta\Phi_{\mathbf{p}} = \delta\Phi - \frac{(\mathbf{l}\hat{\mathbf{p}})(\delta\boldsymbol{\theta}_\perp\hat{\mathbf{p}})}{|\mathbf{l}\times\mathbf{p}|^2}.$$

The final equations will contain only derivatives of $\delta\Phi_{\mathbf{p}}$, expressed in terms of $\dot{\Phi}$, $\mathbf{v}^s$, and derivatives of $\mathbf{l}$.

The order parameter $\hat{\Delta}_{\mathbf{p}}(\omega, \mathbf{k})$ is expressed in term of the complete Green's function[10] $\hat{F}_{\epsilon_+,\epsilon_-}(\mathbf{p}_+, \mathbf{p}_-)$:

$$\frac{\hat{\Delta}_{\mathbf{p}}(\omega,\mathbf{k})}{3|g_0|} = \int\frac{d\epsilon}{4\pi i}\frac{d^3\mathbf{p}'}{(2\pi)^3}(\hat{\mathbf{p}}\hat{\mathbf{p}}')F_{\epsilon_+,\epsilon_-}(\mathbf{p}_+', \mathbf{p}_-'), \tag{4}$$

where $\epsilon_\pm = \epsilon \pm \omega/2$, $\mathbf{p}_\pm = \mathbf{p} \pm \mathbf{k}/2$, and g_0 is the Cooper-interaction constant. The function $\hat{F}$ is obtained from the system of the four Éliashberg equations[10] for the functions $\hat{G}$, $\hat{\bar{G}}$, $\hat{F}$, and $\hat{\bar{F}}^+$. It is convenient to eliminate immediately the dependence of $\hat{\Delta}_{\mathbf{p}}$ on the phase $\delta\Phi_{\mathbf{p}}$, by making the corresponding gauge transformation. We introduce the matrices

$$\mathfrak{G} = \begin{pmatrix} \hat{G} & \hat{F} \\ -\hat{\bar{F}}^+ & \hat{\bar{G}} \end{pmatrix}, \quad \Sigma = \begin{pmatrix} \hat{\Sigma}^{(1)} & \hat{\bar{\Sigma}}^{(2)} \\ -\hat{\Sigma}^{+(2)} & \hat{\bar{\Sigma}}^{(1)} \end{pmatrix},$$

$$H = \frac{1}{2m}\mathbf{p}\nabla\Phi_{\mathbf{p}}\tau_z + \frac{1}{2}\dot{\Phi}_{\mathbf{p}} + \begin{pmatrix} 0 & -\hat{\bar{\Delta}}_{\mathbf{p}} \\ \hat{\bar{\Delta}}^*_{\mathbf{p}} & 0 \end{pmatrix},$$

where $\hat{\bar{\Delta}}_{\mathbf{p}} = \hat{\Delta}_{\mathbf{p}}\exp(-i\delta\Phi_{\mathbf{p}})$, τ_z is a Pauli matrix that acts only on the components of the matrices $\mathfrak{G}$ (but not on the spin indices), and Σ are the self-energy parts due to the pair interaction. The Éliashberg equations in matrix form are

$$\mathbf{p}\mathbf{k}m^{-1}\mathfrak{G}_{\epsilon_+,\epsilon_-}(\mathbf{p}_+,\mathbf{p}_-) - \epsilon_+\tau_z\mathfrak{G}_{\epsilon_+,\epsilon_-}(\mathbf{p}_+,\mathbf{p}_-) + \mathfrak{G}_{\epsilon_+,\epsilon_-}(\mathbf{p}_+,\mathbf{p}_-)\tau_z\epsilon_- + \{H_\omega\mathfrak{G}_{\epsilon_+-\omega,\epsilon_-} - \mathfrak{G}_{\epsilon_+,\epsilon_-+\omega}H_\omega\}_{\omega,\mathbf{k}} = I_{\epsilon_+,\epsilon_-}(\mathbf{p}_+,\mathbf{p}_-), \tag{5}$$

where the "collision integral" is

$$I_{\epsilon_+,\epsilon_-}(\mathbf{p}_+,\mathbf{p}_-) = \{\Sigma^R\mathfrak{G} - \mathfrak{G}\Sigma^A + \Sigma\mathfrak{G}^A - \mathfrak{G}^R\Sigma\}_{\omega,\mathbf{k}}. \tag{6}$$

The curly brackets $\{\cdots\}_{\omega,\mathbf{k}}$ denote contraction with respect to ω and $\mathbf{k}$. The functions with the suffices R and A are respectively retarded and advanced (regular) Green's functions that satisfy Eq. (5), except that the right-hand side contains the "collision integral"

$$I^{R(A)}_{\epsilon_+,\epsilon_-}(\mathbf{p}_+,\mathbf{p}_-) = \{\Sigma^{R(A)}\mathfrak{G}^{R(A)} - \mathfrak{G}^{R(A)}\Sigma^{R(A)}\}_{\omega,\mathbf{k}}. \tag{7}$$

The retarded and advanced functions can also be calculated with the aid of the equations

$$\{\mathfrak{G}_0^{-1}\mathfrak{G}^{R(A)}\} - \{\Sigma^{R(A)}\mathfrak{G}^{R(A)}\} = 1,$$

where

$$\mathfrak{G}_0^{-1} = \xi_{\mathbf{p}} - \epsilon\tau_z + H, \quad \xi_{\mathbf{p}} = \mathbf{p}^2/2m - E_F,$$

i.e., with the aid of the ordinary diagram technique.

We change over in (4) and in (5)–(7) to Green's functions integrated over $d\xi_{\mathbf{p}}$[10], and furthermore take the Fourier transform with respect to $\mathbf{p}_+ - \mathbf{p}_- = \mathbf{k}$ in coordinate space

$$\mathcal{G}_{\epsilon_+,\epsilon_-}(\hat{\mathbf{p}}, \mathbf{r}) \equiv \begin{pmatrix} \hat{g} & \hat{f} \\ -\hat{f}^+ & \hat{\bar{g}} \end{pmatrix} = \int\frac{d\xi_{\mathbf{p}}}{\pi i}\mathfrak{G}_{\epsilon_+,\epsilon_-}(\mathbf{p}_+,\mathbf{p}_-)e^{i\mathbf{k}\mathbf{r}}\frac{d^3\mathbf{k}}{(2\pi)^3}.$$

We define here also some notation which will be encountered subsequently. The letters $\hat{g}$ and $\hat{f}$ with the carets stand for the functions $g_{\alpha\beta}$ and $f_{\alpha\beta}$ with allowance for the spin indices, while the letters without the carets denote only the orbital parts of the corresponding functions. Since we shall consider below only the unitary state, it follows that

$$\hat{g} = \delta_{\alpha\beta}g, \quad \hat{f} = i(\hat{\sigma}^{(\nu)}\hat{\sigma}^{(2)})_{\alpha\beta}d_\nu f,$$
$$\hat{\Delta}_{\mathbf{p}} = i(\hat{\sigma}^{(\nu)}\hat{\sigma}^{(2)})_{\alpha\beta}d_\nu\Delta_{\mathbf{p}}, \quad \Delta_{\mathbf{p}} = \Delta_0\Psi\hat{\mathbf{p}}$$

and analogously for $\hat{\bar{g}}$ and $\hat{f}^+$. The operation Sp will denote taking the trace over the matrix (not spin) indices, while the operation Tr stands for the trace over the spin indices.

We thus have

$$\frac{\Delta_0\Psi}{3|g_0|}=\nu\int\frac{d\varepsilon}{4}\frac{d\Omega_p}{4\pi}\hat{\mathbf{p}}f_{\varepsilon_+,\varepsilon_-}(\hat{\mathbf{p}},\mathbf{r}), \tag{8}$$

$$-iv_F\nabla\mathcal{G}_{\varepsilon_+,\varepsilon_-}(\hat{\mathbf{p}},\mathbf{r})-\varepsilon_+\tau_s\mathcal{G}_{\varepsilon_+,\varepsilon_-}(\hat{\mathbf{p}},\mathbf{r})+\mathcal{G}_{\varepsilon_+,\varepsilon_-}(\hat{\mathbf{p}},\mathbf{r})\tau_s\varepsilon_-$$
$$+\{H_{\omega_1}(\mathbf{r})\mathcal{G}_{\varepsilon_+,-\omega_1,\varepsilon_-}(\hat{\mathbf{p}},\mathbf{r})-\mathcal{G}_{\varepsilon_+,\varepsilon_-+\omega_1}(\hat{\mathbf{p}},\mathbf{r})H_{\omega_1}(\mathbf{r})\}_\omega=I_{\varepsilon_+,\varepsilon_-}(\hat{\mathbf{p}},\mathbf{r}). \tag{9}$$

The collision integral $\hat{I}$ is obtained from I by replacing $\mathcal{G}$ with $\mathcal{G}$. An analogous equation holds also for the regular functions. It leads to the relation[14]

$$\int\mathcal{G}^{R(A)}_{\varepsilon,\varepsilon'}(\hat{\mathbf{p}},\mathbf{r})\mathcal{G}^{R(A)}_{\varepsilon',\varepsilon_1}(\hat{\mathbf{p}},\mathbf{r})\frac{d\varepsilon'}{2\pi}=2\pi\delta(\varepsilon-\varepsilon_1)\delta_{\alpha\beta}. \tag{10}$$

The Green's functions satisfy in turn[14]

$$\int[\mathcal{G}^{R}_{\varepsilon,\varepsilon'}(\hat{\mathbf{p}},\mathbf{r})\mathcal{G}_{\varepsilon',\varepsilon_1}(\hat{\mathbf{p}},\mathbf{r})+\mathcal{G}_{\varepsilon,\varepsilon'}(\hat{\mathbf{p}},\mathbf{r})\mathcal{G}^{A}_{\varepsilon',\varepsilon_1}(\hat{\mathbf{p}},\mathbf{r})]\frac{d\varepsilon'}{2\pi}=0. \tag{11}$$

We emphasize a circumstance of importance to what follows, namely that in the chosen gauge the condition $\mathrm{Sp}\mathcal{G}^{R(A)}$ that follows from (10) is accurate to terms of order ω/Δ inclusive. We write down here, for future reference, the equations[15] for the stationary functions $\mathcal{G}^{R(A)}_\omega$. Since $\tau\Delta\gg1$, we can neglect in (9) in the stationary case the collision integral. We have

$$(g^{R(A)}_\varepsilon)^2-f^{R(A)}_\varepsilon f^{+R(A)}_\varepsilon=1 \tag{12}$$

and

$$-iv_F\nabla f^{R(A)}_\varepsilon+v_F(\nabla\Phi_p)f^{R(A)}_\varepsilon-2\varepsilon f^{R(A)}_\varepsilon+2\Delta_p g^{R(A)}_\varepsilon=0,$$
$$-iv_F\nabla g^{R(A)}_\varepsilon+\Delta_p f^{+R(A)}_\varepsilon-f^{R(A)}_\varepsilon\Delta_p^*=0. \tag{13}$$

In the case of slow spatial variation of the order parameter, $k\xi\ll1$, we have in the zeroth approximation in $k\xi$

$$g^{R(A)}=\varepsilon/\xi^{R(A)},\quad f^{R(A)}=\Delta_p/\xi^{R(A)},\quad f^{+R(A)}=\Delta_p^*/\xi^{R(A)}, \tag{14}$$

where $\xi^{R(A)}=\pm(\epsilon^2-|\Delta_p|^2)^{1/2}+i\delta$ at $|\epsilon|>|\Delta_p|$ and $\xi^R=\xi^A=i(|\Delta_p|^2-\epsilon^2)^{1/2}$ at $|\epsilon|<|\Delta_p|$. Thus, at $|\epsilon|>|\Delta_p|$ the quantities $\xi^{R(A)}$ are the values of the function $(\epsilon^2-|\Delta_p|^2)^{1/2}$, which is analytic in the complex ϵ plane with cuts from $-\infty$ to $-|\Delta_p|$ and from $|\Delta_p|$ to $+\infty$, taken on the upper (lower) edge of the cut. We see that at $|\epsilon|>|\Delta_p|$ we have $\mathcal{G}^R=-\mathcal{G}^A$, and at $|\epsilon|<|\Delta_p|$ the functions are equal, $\mathcal{G}^R=\mathcal{G}^A$.

As a result of the condition (11) only two of the four functions $\hat{g},\hat{\bar{g}},\hat{f},\hat{f}^+$ are independent. To derive the kinetic equation we use a procedure proposed by Larkin and Ovchinnikov,[16] which makes it possible to recast the system (8) in the form of the usual kinetic equation. We put

$$\mathcal{G}_{\varepsilon_+,\varepsilon_-}=\int\{\mathcal{G}^{R}_{\varepsilon_+,\varepsilon'}(f^{(1)}_{\varepsilon',\varepsilon_-}+\tau_s f^{(2)}_{\varepsilon',\varepsilon_-})$$
$$-(f^{(1)}_{\varepsilon_+,\varepsilon'}+\tau_s f^{(2)}_{\varepsilon_+,\varepsilon'})\mathcal{G}^{A}_{\varepsilon',\varepsilon_-}\}\frac{d\varepsilon'}{2\pi}. \tag{15}$$

The spin suffices of $f^{(1)(2)}$ have been omitted for brevity. This expression satisfies the condition (11) identically. The calculations will consist of substituting (15) in (9) and taking into account the equations for the regular functions $\mathcal{G}^{R(A)}_{\varepsilon_+,\varepsilon_-}$ as well as of expanding the resultant equation up to second order in the temporal and spatial derivatives. In the collision integral, in view of the low collision frequencies $\tau^{-1}\ll\Delta$, we can confine ourselves in the principal order in ω/Δ and $k\xi$. In addition, the function $f^{(2)}$ is proportional to the derivation from equilibrium and is therefore small. For the orbital motion, under the conditions discussed in the Introduction, the distribution functions $f^{(1)}_{\alpha\beta}$ and $f^{(2)}_{\alpha\beta}$ are diagonal in the spin indices.

After performing the foregoing calculations we apply to the obtained matrix equation the Sp operation once with the unit matrix, and then again with τ_s, making use of the condition $\mathrm{Sp}\mathcal{G}^{R(A)}=0$. At $|\epsilon|>|\Delta_p|$ we have (cf.[16]):

$$g^R_\varepsilon\frac{\partial f^{(1)}_\varepsilon}{\partial t}+v_F\nabla(g^R_\varepsilon f^{(2)}_\varepsilon)+\frac{1}{2}\Big[g^R_\varepsilon v_F\frac{\partial}{\partial t}\nabla\Phi_p+\frac{\partial\hat{\Delta}_p}{\partial t}f^{+R}_\varepsilon+f^R_\varepsilon\frac{\partial\hat{\Delta}_p^*}{\partial t}\Big]\frac{\partial f^{(1)}}{\partial\varepsilon}$$
$$=\hat{J}_1\{f^{(1)}\}, \tag{16}$$

$$\frac{\partial}{\partial t}(g^R_\varepsilon f^{(2)}_\varepsilon)+g^R_\varepsilon v_F\nabla f^{(1)}_\varepsilon-\frac{1}{2}\Big(\hat{\Delta}_p\frac{\partial f^{+R}}{\partial\varepsilon}+\frac{\partial f^R}{\partial\varepsilon}\hat{\Delta}_p^*\Big)\frac{\partial f^{(2)}}{\partial t}$$
$$+\frac{1}{2}g^R_\varepsilon\frac{\partial^2\Phi_p}{\partial t^2}\frac{\partial f^{(1)}}{\partial\varepsilon}+\frac{1}{2}\Big[\frac{\partial f^R_\varepsilon}{\partial t}\hat{\Delta}_p^*+\hat{\Delta}_p\frac{\partial f^{+R}_\varepsilon}{\partial t}\Big]\frac{\partial f^{(2)}}{\partial\varepsilon}=J_2\{f^{(2)}\}. \tag{17}$$

We have changed over here to the temporal representation in the frequency difference $\epsilon_+-\epsilon_-=\omega$. The collision integrals $\hat{J}_1$ and $\hat{J}_2$ are equal to

$$\hat{J}_1=\frac{i}{4}\mathrm{Sp}[I-(I^R-I^A)f^{(1)}],$$
$$\hat{J}_2=\frac{i}{4}\mathrm{Sp}[\tau_s I-\tau_s(I^R-I^A)f^{(1)}]. \tag{18}$$

They will be calculated in the next section.

At $|\epsilon|<|\Delta_p|$ we get

$$\frac{i}{4}\Big(\frac{\partial f^R}{\partial\varepsilon}\frac{\partial^2\Delta_p^*}{\partial t^2}+\frac{\partial f^{+R}}{\partial\varepsilon}\frac{\partial^2\Delta_p}{\partial t^2}\Big)\frac{\partial f^{(1)}}{\partial\varepsilon}$$
$$-\frac{i}{4}\Big(\frac{\partial f^R}{\partial t}\frac{\partial\Delta_p^*}{\partial t}+\frac{\partial f^{+R}}{\partial t}\frac{\partial\Delta_p}{\partial t}\Big)\frac{\partial^2 f^{(1)}}{\partial\varepsilon^2}=J_1,$$
$$i(\Delta_p f^{+R}+f^R\Delta_p^*)f^{(2)}_\varepsilon=J_2.$$

The collision integral J_1 vanishes at $|\epsilon|<|\Delta_p|$ (see below). In the second equation, by virtue of $\tau\Delta\gg1$, we can neglect the term with J_2. Thus, the functions $f^{(1)}$ and $f^{(2)}$ at $|\epsilon|<|\Delta_p|$ correspond to the collisionless regime. Leaving out J_1 and J_2 from the right-hand sides of these equations we obtain $f^{(1)}=f^{(2)}=0$, i.e., $\mathcal{G}_{\epsilon_+,\epsilon_-}=0$, at $|\epsilon|<|\Delta_p|$ and $k\xi\ll1$. This can be verified also directly calculating $\mathcal{G}_{\epsilon_+,\epsilon_-}$ in this frequency region (and at $k\xi\ll1$) with the aid of the expression

$$\mathcal{G}_{\varepsilon_+,\varepsilon_-}=\mathcal{G}^R_{\varepsilon_+,\varepsilon_-}\,\mathrm{th}\frac{\varepsilon_-}{2T}-\mathrm{th}\frac{\varepsilon_+}{2T}\mathcal{G}^A_{\varepsilon_+,\varepsilon_-}+\mathcal{G}^{(a)}_{\varepsilon_+,\varepsilon_-},$$

where the anomalous function $\mathcal{G}^{(a)}$ takes in the collisionless case the form[9]

$$\mathcal{G}^{(a)}_{\varepsilon_+,\varepsilon_-}=-\Big\{\mathcal{G}^R_{\varepsilon+\omega/2,\varepsilon+\omega'/2}\Big(\mathrm{th}\frac{\varepsilon+\omega'/2}{2T}-\mathrm{th}\frac{\varepsilon-\omega'/2}{2T}\Big)H_{\omega'}\mathcal{G}^A_{\varepsilon-\omega'/2,\varepsilon-\omega/2}\Big\}.$$

The "distribution functions" $f^{(1)}$ and $f^{(2)}$ have unequal parity in ϵ and $\mathbf{p}$, namely $f^{(1)}_\epsilon(\hat{\mathbf{p}})=-f^{(1)}_{-\epsilon}(-\hat{\mathbf{p}})$, and $f^{(2)}_\epsilon(\hat{\mathbf{p}})=f^{(2)}_{-\epsilon}(-\hat{\mathbf{p}})$. As noted in[16], the kinetic equations (16) and (17) can be reduced to standard form, by putting

$$n(\hat{\mathbf{p}},\varepsilon_p,\mathbf{r},t)=\tfrac{1}{2}[1-(f^{(1)}+g^Rf^{(2)})],$$

where

$$\varepsilon_p=[(\xi_p+{}^1/_2\Phi_p)^2+|\Delta_p|^2]^{1/2}+{}^1/_2 v_F\nabla\Phi_p,$$

which agrees with the spectrum proposed in[6].

3. THE COLLISION INTEGRAL AND THE CONSERVATION LAWS

In the calculation of the self-energy parts Σ we shall consider only those quasiparticle collisions that lead to relaxation. Interactions that cause renormalization of the chemical potential and of the order parameter can be regarded as already accounted for in their proper places. Disregarding Fermi-liquid effects, we can assume that the vertices of the quasiparticle interaction remain unchanged on going into the superfluid state. We shall use below one more simplifying assumption, which is not of fundamental character, and is needed only to abbreviate the rather cumbersome expressions. Namely, we assume that the quasiparticle interaction that leads to relaxation is pointlike

$$\Gamma_{\alpha\beta;\gamma\delta}(p_1,p_2;p_3,p_4)=(2\pi)^3T^{-1}\delta(\mathbf{p}_1+\mathbf{p}_2-\mathbf{p}_3-\mathbf{p}_4)\times\delta_{\epsilon_1+\epsilon_2-\epsilon_3-\epsilon_4}\lambda(\delta_{\alpha\gamma}\delta_{\beta\delta}-\delta_{\alpha\delta}\delta_{\beta\gamma}),$$

where $\epsilon_k=2\pi iT(n_k+1/2)$.

The self-energy parts take in the imaginary-frequency representation the form

$$\Sigma_{\alpha\beta}^{(1)}(p,q)=-\delta_{\alpha\beta}\frac{\lambda^2}{2}\left[\frac{T}{(2\pi)^3}\right]^4\int dp_1\,dp_2\,dp_3\,dq_1\,dq_2\,dq_3\times\delta(p-p_1-p_2-p_3)\delta(q-q_1-q_2-q_3)\{2G(p_1,q_1)G(p_2,q_2)\bar G(p_3,q_3)+G(p_1,q_1)\mathbf{F}(p_2,q_2)\mathbf{F}^+(p_3,q_3)\},\quad(19)$$

$$\Sigma_{\alpha\beta}^{(2)}(p,q)=-\frac{\lambda^2}{2}\left[\frac{T}{(2\pi)^3}\right]^4\int dp_1\,dp_2\,dp_3\,dq_1\,dq_2\,dq_3\,\delta(p-p_1-p_2-p_3)\times\delta(q-q_1-q_2-q_3)\{2\mathbf{F}(p_1,q_1)F_{\alpha\beta}(p_2,q_2)\mathbf{F}^+(p_3,q_3)+G(p_1,q_1)F_{\alpha\beta}(p_2,q_2)\bar G(p_3,q_3)\},\quad(20)$$

and analogously for $\Sigma^{(1)}$ and $\Sigma^{+(2)}$. Here $p=\{\mathbf{p},\epsilon\}$, and the integration with respect to dp contains the sum over ϵ. In the derivation of these expressions we took it into account that in our case of the unitary state we have $G_{\alpha\beta}=\delta_{\alpha\beta}G$, and that $F_{\alpha\beta}=i\sigma^{(\nu)}\sigma^{(2)}F_\nu$, where $\mathbf{F}\times\mathbf{F}^+=0$. Continuing analytically the expressions for Σ in frequency from the upper half-plane to the real axis and changing over to functions integrated with respect to $d\xi_p$, we obtain in accord with Éliashberg[10]

$$\Sigma_{\epsilon,\epsilon-\omega}^{(1)R(A)}(\mathbf{p},\mathbf{p}-\mathbf{k})=-i\frac{\lambda^2\nu^2\pi}{32p_Fv_F}\delta_{\alpha\beta}\hat L\{2[g_{\epsilon_1}g_{\epsilon_2}\bar g_{\epsilon_3}]^{R(A)}+[g_{\epsilon_1}f_{\epsilon_2}f_{\epsilon_3}^+]^{R(A)}\},\quad(21)$$

$$\Sigma_{\epsilon,\epsilon-\omega}^{(1)}(\mathbf{p},\mathbf{p}-\mathbf{k})=-i\frac{\lambda^2\nu^2\pi}{32p_Fv_F}\delta_{\alpha\beta}\hat L\{2[g_{\epsilon_1}g_{\epsilon_2}\bar g_{\epsilon_3}]^{t}+[g_{\epsilon_1}f_{\epsilon_2}f_{\epsilon_3}^+]^{t}\},\quad(22)$$

where

$$[g_{\epsilon_1}g_{\epsilon_2}\bar g_{\epsilon_3}]^{R(A)}=[g_{\epsilon_1}^{R(A)}g_{\epsilon_2}\bar g_{\epsilon_3}+g_{\epsilon_1}g_{\epsilon_2}^{R(A)}\bar g_{\epsilon_3}+g_{\epsilon_1}g_{\epsilon_2}\bar g_{\epsilon_3}^{R(A)}+(g_{\epsilon_1}^R-g_{\epsilon_1}^A)(g_{\epsilon_2}^R-g_{\epsilon_2}^A)\bar g_{\epsilon_3}^{R(A)}],\quad(23)$$

$$[g_{\epsilon_1}g_{\epsilon_2}\bar g_{\epsilon_3}]^{t}=[g_{\epsilon_1}g_{\epsilon_2}\bar g_{\epsilon_3}+(g_{\epsilon_1}^R-g_{\epsilon_1}^A)(g_{\epsilon_2}^R-g_{\epsilon_2}^A)\bar g_{\epsilon_3}+(g_{\epsilon_1}^R-g_{\epsilon_1}^A)g_{\epsilon_2}(\bar g_{\epsilon_3}^R-\bar g_{\epsilon_3}^A)+g_{\epsilon_1}(g_{\epsilon_2}^R-g_{\epsilon_2}^A)(\bar g_{\epsilon_3}^R-\bar g_{\epsilon_3}^A)].\quad(24)$$

We have introduced here the operator

$$\hat L\{\dots\}=\int\frac{d\Omega_1\,d\Omega_2}{(4\pi)^2}\left[\delta\left(\frac{|\mathbf{p}_3|}{p_F}-1\right)\int d\epsilon_1\,d\epsilon_2\{\dots\}\right]_{\substack{\epsilon_3=\epsilon-\epsilon_1-\epsilon_2\\ \mathbf{p}_3=\mathbf{p}-\mathbf{p}_1-\mathbf{p}_2}},$$

and $g_\epsilon\equiv g_{\epsilon,\epsilon-\omega}(\mathbf{p},\mathbf{p}-\mathbf{k})$. The formulas for $\Sigma^{(2)R(A)}$ and $\Sigma^{(2)}$ (and analogously for $\Sigma^{(1)R(A)}$, $\Sigma^{(1)}$ and $\Sigma^{+(2)R(A)}$, $\Sigma^{+(2)}$) are obtained in (20) by replacing the expressions in the curly brackets of (20) by expressions of the type (21)-(24).

We substitute Σ in the collision integrals $\hat J_1$ and $\hat J_2$ (18) and use formulas (14). With the aid of (15), after rather cumbersome calculations, we find that the integral $J_1=0$ at $|\epsilon|<|\Delta_p|$ and at $|\epsilon|>|\Delta_p|$ we have

$$\hat J=-\frac{\lambda^2\nu^2\pi}{2p_Fv_F}\hat L\left\{\frac{\varepsilon\varepsilon_1\varepsilon_2\varepsilon_3}{\xi^R\xi_1^R\xi_2^R\xi_3^R}M_1[f^{(1)}(f_1^{(1)}f_2^{(1)}+f_1^{(1)}f_3^{(1)}+f_2^{(1)}f_3^{(1)}+1)-f_1^{(1)}f_2^{(1)}f_3^{(1)}-f_1^{(1)}-f_2^{(1)}-f_3^{(1)}]\right\}\delta_{\alpha\beta},$$

$$\hat J=-\frac{\lambda^2\nu^2\pi}{2p_Fv_F}\hat L\left\{\frac{\varepsilon\varepsilon_1\varepsilon_2\varepsilon_3}{\xi^R\xi_1^R\xi_2^R\xi_3^R}M_2[f^{(2)}(f_1^{(1)}f_2^{(1)}+f_1^{(1)}f_3^{(1)}+f_2^{(1)}f_3^{(1)}+1)+f^{(1)}(f_1^{(1)}+f_2^{(1)})f_3^{(2)}-(f_1^{(1)}f_2^{(1)}+1)f_3^{(2)}]\right\}\delta_{\alpha\beta}.$$

The integration with respect to $d\epsilon_1$ and $d\epsilon_2$ is carried out here over the regions $|\epsilon_i|>|\Delta_{p_i}|\,(i=1,2,3)$, and

$$M_1=1+\frac{2\Delta\Delta_1^*\Delta_2^*}{4\varepsilon\varepsilon_1\varepsilon_2\varepsilon_3}\frac{\Delta_3+\Delta^*\Delta_1^*\Delta_2\Delta_3+\Delta^*\Delta_1\Delta_2^*\Delta_3}{}-\frac{\Delta_1^*\Delta_2+\Delta_1\Delta_2^*}{4\varepsilon_1\varepsilon_2}-\frac{\Delta\Delta_3^*+\Delta^*\Delta_3}{8\varepsilon\varepsilon_3}-\frac{\Delta\Delta_1^*+\Delta^*\Delta_1}{8\varepsilon\varepsilon_1},$$

$$M_2=1-\frac{\Delta_1^*\Delta_2+\Delta_1\Delta_2^*}{4\varepsilon_1\varepsilon_2}.$$

The collision integral J_1 vanishes when

$$f^{(1)}=\mathrm{th}\left(\frac{\varepsilon-\mathbf{pv}^n}{2T}\right),$$

a fact corresponding to an equilibrium Fermi distribution function of excitations that move with velocity $\mathbf{v}^n$. The following remark is in order here. In the derivation of the left-hand sides of the kinetic equations (16) and (17) it was implicitly assumed that the derivatives of the functions $f^{(1)}$ and $f^{(2)}$ with respect to the momenta, $\partial f^{(1)}/\partial\mathbf{p}$ and $\partial f^{(2)}/\partial\mathbf{p}$, are small. This is correct so long as the derivative of the equilibrium distribution function vanishes, $\partial f^{(0)}/\partial\mathbf{p}=0$. It follows therefore that the obtained equations are valid in a reference frame locally co-moving with the normal component, i.e., in a reference frame moving with a velocity $\mathbf{V}$ equal to $\mathbf{v}^n$ in the given point of space and at the given instant of time. In other words, for the substitution in (16) and (17) we must choose the equilibrium function in the form

$$f^{(0)}=\mathrm{th}\left(\frac{\varepsilon-\mathbf{pv}^n+\mathbf{pV}}{2T}\right).$$

Of course, the derivatives of $f^{(0)}$ with respect to the coordinates and the time do not contain $\mathbf{V}$:

$$\frac{\partial f^{(0)}}{\partial t}=-\frac{\partial f^{(0)}}{\partial\varepsilon}\mathbf{p}_F\frac{\partial\mathbf{v}^n}{\partial t},\qquad\nabla_i f^{(0)}=-\frac{\partial f^{(0)}}{\partial\varepsilon}\mathbf{p}_F\nabla_i\mathbf{v}^n.$$

The change to the laboratory frame is carried out in the final expression by the standard Galilean transformation procedure.

In the case of low frequencies $\omega\tau\ll1$ the deviations of $f^{(1)}$ from equilibrium are small, and we can linearize J_1 and J_2 near $f^{(0)}$. We put

$$f^{(1)}=f^{(0)}+\frac{\partial f^{(0)}}{\partial\varepsilon}\chi^{(1)},\qquad f^{(2)}=\frac{\partial f^{(0)}}{\partial\varepsilon}\chi^{(2)}.$$

Then

$$\hat{J}_1=-\delta_{\alpha\beta}\frac{\lambda^2\nu^2\pi}{2p_Fv_F}\frac{\varepsilon}{\xi^R}\frac{\partial f^{(0)}}{\partial\varepsilon}\hat{L}\{F(1,2,3)M_1[\chi^{(1)}-\chi_1^{(1)}-\chi_2^{(1)}-\chi_3^{(1)}]\}, \quad (25)$$

$$\hat{J}_2=-\delta_{\alpha\beta}\frac{\lambda^2\nu^2\pi}{2p_Fv_F}\frac{\varepsilon}{\xi^R}\frac{\partial f^{(0)}}{\partial\varepsilon}\hat{L}\{F(1,2,3)M_2[\chi^{(2)}-\chi_3^{(2)}]\}, \quad (26)$$

where

$$F(1,2,3)=\frac{\varepsilon_1\varepsilon_2\varepsilon_3}{\xi_1^R\xi_2^R\xi_3^R}(1+f_1^{(0)}f_2^{(0)}+f_1^{(0)}f_3^{(0)}+f_2^{(0)}f_3^{(0)}).$$

The integral $\hat{J}_1$ is made to vanish by a function of the type $\chi^{(1)}=A\epsilon+Bp$, and $\hat{J}_2$ by the function $\chi^{(2)}=C$, where A, B, and C do not depend on ϵ and p. This is a consequence of the arbitrary temperature of the excitation gas (the temperature is determined by the interaction with the thermostat), of the Galilean invariance of the collision integral, and of the arbitrary origin used for the excitation energy.

The conservation laws for the number of particles, for the momentum, and for the energy follow from the general equations of motion (5). It is easy to verify that the collision integrals satisfy the necessary requirements. It is most convenient to start directly with expression (6), from which we get

$$\int \mathrm{Sp}\{\tau_z \mathrm{Tr}\, I_{\varepsilon_+\varepsilon_-}(\mathbf{p}_+,\mathbf{p}_-)\}\frac{d\varepsilon}{4\pi i}\frac{d^3\mathbf{p}}{(2\pi)^3}=0$$

–the particle number conservation,

$$\int \mathbf{p}\,\mathrm{Sp}\{\mathrm{Tr}\, I_{\varepsilon_+\varepsilon_-}(\mathbf{p}_+,\mathbf{p}_-)\}\frac{d\varepsilon}{4\pi i}\frac{d^3\mathbf{p}}{(2\pi)^3}=0$$

–the momentum conservation, and

$$\int \varepsilon\,\mathrm{Sp}\{\mathrm{Tr}\, I_{\varepsilon_+\varepsilon_-}(\mathbf{p}_+,\mathbf{p}_-)\}\frac{d\varepsilon}{4\pi i}\frac{d^3\mathbf{p}}{(2\pi)^3}=0$$

–the energy conservation. Taking the corresponding moments of (5), we obtain the required conservation laws.

I. The continuity equation:

$$\frac{\partial(Nm)}{\partial t}+\mathrm{div}\,\mathbf{j}=0, \quad (27)$$

where the particle-number density (μ is the chemical potential) is

$$N=N_0(\mu)-\nu\dot{\Phi}-\nu\int\frac{d\mathrm{O}_p}{4\pi}\frac{d\varepsilon}{2}(g_\varepsilon^R-g_\varepsilon^A)f^{(2)}, \quad (28)$$

and the density of the total momentum is

$$\mathbf{j}=-\nu\int\frac{d\mathrm{O}_p}{4\pi}\frac{d\varepsilon}{2}\mathbf{p}_F(g_\varepsilon^R-g_\varepsilon^A)f^{(1)}. \quad (29)$$

(Unless indicated, the limits of integration with respect to $d\epsilon$ are $-\infty$ and $+\infty$.)

II. The law of conservation of the total momentum:

$$\frac{\partial j_i}{\partial t}+\nabla_k\Pi_{ik}=0, \quad (30)$$

where the flux density of the total momentum is

$$\Pi_{ik}=-\frac{1}{2m}\int \mathrm{Sp}\,\mathrm{Tr}\left\{\left(p_i+\frac{1}{2}\nabla_i\Phi_p\tau_z\right)\left(p_k+\frac{1}{2}\nabla_k\Phi_p\tau_z\right)\times\mathcal{G}_{\varepsilon_+\varepsilon_-}(\mathbf{p}_+,\mathbf{p}_-)\right\}\frac{d\varepsilon}{4\pi i}\frac{d^3\mathbf{p}}{(2\pi)^3}-\frac{\Delta_0^2|\Psi|^2}{3|g_0|}\delta_{ik}.$$

Caution must be exercised when changing in this expression to Green's functions integrated with respect to $d\xi_p$, since the integral contains parts that diverge if the integration is initially with respect to $d\xi_p$, and then with respect to $d\epsilon$. In the tensor Π_{ik} we can separate the term corresponding to the dissipative processes. Since the correction that must be introduced in the distribution function to account for the disequilibrium decreases rapidly as $\epsilon > T$, it is possible to change over in the non-equilibrium part of the tensor Π_{ik} to the functions integrated with respect to $d\xi_p$. We obtain

$$\Pi_{ik}=\Pi_{ik}^{(0)}-\nu m\int\frac{d\mathrm{O}_p}{4\pi}\frac{d\varepsilon}{2}v_{Fi}v_{Fk}(g_\varepsilon^R-g_\varepsilon^A)f^{(2)}, \quad (31)$$

where

$$\Pi_{ik}^{(0)}=-\frac{1}{2m}\int \mathrm{Sp}\,\mathrm{Tr}\left\{\left(p_i+\frac{1}{2}\nabla_i\Phi_p\tau_z\right)\left(p_k+\frac{1}{2}\nabla_k\Phi_p\tau_z\right)\times[\mathcal{G}^R_{\varepsilon_+\varepsilon_-}(\mathbf{p}_+,\mathbf{p}_-)-\mathcal{G}^A_{\varepsilon_+\varepsilon_-}(\mathbf{p}_+,\mathbf{p}_-)]f^{(0)}\right\}\frac{d\varepsilon}{4\pi i}\frac{d^3\mathbf{p}}{(2\pi)^3}-\frac{\Delta_0^2|\Psi|^2}{3|g_0|}\delta_{ik}$$

is determined by the equilibrium properties of the liquid. We defer the calculation of $\Pi_{ik}^{(0)}$ to another paper.

III. The energy conservation law:

$$\frac{\partial\mathscr{E}}{\partial t}+\mathrm{div}\,\mathbf{j}_\mathscr{E}=0, \quad (32)$$

where

$$\mathscr{E}=-\frac{1}{2}\int \mathrm{Sp}\,\mathrm{Tr}\left\{\left(\varepsilon-\frac{1}{2}\dot{\Phi}_p\tau_z\right)\mathcal{G}_{\varepsilon_+\varepsilon_-}(\mathbf{p}_+,\mathbf{p}_-)\right\}\frac{d\varepsilon}{4\pi i}\frac{d^3\mathbf{p}}{(2\pi)^3}+\frac{\Delta_0^2|\Psi|^2}{3|g_0|} \quad (33)$$

and the energy flux density is

$$\mathbf{j}_\mathscr{E}=\mathbf{j}_\mathscr{E}^{(0)}-\nu\int\frac{d\mathrm{O}_p}{4\pi}\frac{d\varepsilon}{2}\varepsilon\mathbf{v}_F(g_\varepsilon^R-g_\varepsilon^A)f^{(2)}, \quad (34)$$

where

$$\mathbf{j}_\mathscr{E}^{(0)}=-\frac{1}{2m}\int \mathrm{Sp}\,\mathrm{Tr}\left\{\left(\varepsilon-\frac{1}{2}\dot{\Phi}_p\tau_z\right)\left(\mathbf{p}+\frac{1}{2}\nabla\Phi_p\tau_z\right)\times[\mathcal{G}^R_{\varepsilon_+\varepsilon_-}-\mathcal{G}^A_{\varepsilon_+\varepsilon_-}]f^{(0)}\right\}\frac{d\varepsilon}{4\pi i}\frac{d^3\mathbf{p}}{(2\pi)^3}.$$

4. EQUATIONS OF MOTION OF THE ORDER PARAMETER IN THE HYDRODYNAMIC APPROXIMATION

Taking the results of Sec. 2 into account, we get from (15)

$$f_{\varepsilon_+\varepsilon_-}=\{(f^R_{\varepsilon_+\varepsilon_-}-f^A_{\varepsilon_+\varepsilon_-})f_\varepsilon^{(1)}\}+\left\{\left[\frac{\omega^2}{8}(f^R_{\varepsilon_+\varepsilon_-}-f^A_{\varepsilon_+\varepsilon_-})\right]\frac{\partial^2 f_\varepsilon^{(1)}}{\partial\varepsilon^2}\right\}+\left\{\left[\frac{\omega}{2}(f^R_{\varepsilon_+\varepsilon_-}-f^A_{\varepsilon_+\varepsilon_-})\right]\frac{\partial f_\varepsilon^{(2)}}{\partial\varepsilon}\right\}-\left\{\left(\frac{\partial f^R}{\partial\varepsilon}-\frac{\partial f^A}{\partial\varepsilon}\right)\frac{\omega}{2}f_\varepsilon^{(2)}\right\}. \quad (35)$$

An expression for $f^+_{\varepsilon_+\varepsilon_-}$ is obtained from (35) by replacing $f^{R(A)}$ with $f^{+R(A)}$ and $f_\epsilon^{(2)}$ with $-f_\epsilon^{(2)}$. Substituting (35) in (8), we obtain an equation that determines the six components of the complex vector Ψ. Our task, however, is not a complete calculation of Ψ, but to separate, from the available six equations, those which describe the changes of the anisotropy vector l and of the phase Φ. As already mentioned, the changes of l and Φ reduce to rotation of the vector $\Psi_0=\Delta'+i\Delta''$. The sought equation obviously correspond to the components

of (8) that are orthogonal to Ψ_0, and the equation that describes the motion of l is obtained from the component orthogonal to the (Δ', Δ'') plane, while the equation that gives the phase Φ lies in the (Δ', Δ'') plane.

We consider first the reactive terms (which do not contain τ and are proportional to the frequency squared). The stem from the first and second terms of (35). We shall show that in the equation of motion of l their role is always small. For simplicity we confine ourselves to the case $T_c - T \ll T_c$, for its is precisely in this region, according to the results of[6], that one could expect a manifestation of the effects connected with the presence of the reactive terms. The calculations show that the corresponding contribution to the right-hand side of (8) takes near T_c the form

$$\frac{7\zeta(3)\Delta_0}{48\pi^2T^2}\frac{\partial^2\Psi}{\partial t^2}.$$

This leads to an "orbital susceptibility"

$$\chi_{orb}=\frac{7\zeta(3)\Delta_0^2\nu}{24\pi^2T^2}. \tag{36}$$

This expression differs from the formula given in[5] for χ_{orb}. This is quite natural, since the latter was obtained in the collisionless limit $\tau^{-1} \ll \omega \ll \Delta$. Formula (36) disagrees also with the result of Volovik and Mineev,[6] according to whom the susceptibility χ_{orb} is proportional at $\omega \ll \tau^{-1}$ to Δ/T raised to the zero power. According to (36), the contribution of the reactive terms if of the order of $\Delta\omega^2/T^2$, and the contribution due to the dissipative processes, as follows from (16) and (25), is of the order of $\Delta^2\tau\omega/T$. Their ratio is $\sim\Delta\tau T/\omega \gg 1$, and offers evidence in favor of the dissipative terms in the entire temperature region. Thus, the reactive terms should be discarded from the equation for l.

We return to the case of arbitrary temperatures. To separate the equation that determines the motion of l, we rewrite (8) in the form[1)]

$$\frac{\Delta_0\Psi}{3|g_0|}-\nu\int\frac{dO_p}{4\pi}\frac{d\varepsilon}{4}\hat{\mathbf{p}}\,(f_\varepsilon^R(\hat{\mathbf{p}},\mathbf{r})-f_\varepsilon^A(\hat{\mathbf{p}},\mathbf{r}))\,f_\varepsilon^{(0)}$$
$$=\nu\int\frac{dO_p}{4\pi}\int\limits_{\varepsilon>|\Delta_p|}d\varepsilon\,\hat{\mathbf{p}}f^R(\hat{\mathbf{p}},\mathbf{r})\frac{\partial f^{(0)}}{\partial\varepsilon}\chi^{(1)}. \tag{37}$$

The left-hand side of (37), as is well known, can be represented in the form of a variational derivative of the equilibrium functional of the free energy $\mathcal{F}$, and is equal to $\Delta_0^{-1}\delta\mathcal{F}/\delta\Psi^*$. We separate the component perpendicular to the (Δ', Δ'') plane, multiplying for this purpose (37) by $-1e_{ikm}l_k\Psi_m^*$ and the expression for Ψ^* by $-1e_{ikm}l_k\Psi_m$. Adding these two equations, we get

$$\Delta_0^{-1}\left[1\frac{\delta\mathcal{F}}{\delta 1}\right]\equiv\nu\int\frac{dO_p}{4\pi}\frac{d\varepsilon}{4}(1\hat{\mathbf{p}})\{[1\Psi^*](f_\varepsilon^R(\hat{\mathbf{p}},\mathbf{r})-f_\varepsilon^A(\hat{\mathbf{p}},\mathbf{r}))$$
$$+[1\Psi](f_\varepsilon^{+R}(\hat{\mathbf{p}},\mathbf{r})-f_\varepsilon^{+A}(\hat{\mathbf{p}},\mathbf{r}))\}f_\varepsilon^{(0)}=-\nu\int\frac{dO_p}{4\pi}\int\limits_{\varepsilon>|\Delta_p|}d\varepsilon(1\hat{\mathbf{p}})$$
$$\times\{[1\Psi^*]f_\varepsilon^R(\hat{\mathbf{p}},\mathbf{r})+[1\Psi]f_\varepsilon^{+R}(\hat{\mathbf{p}},\mathbf{r})\}\frac{\partial f^{(0)}}{\partial\varepsilon}\chi^{(1)}. \tag{38}$$

This equation is of standard form (cf. e.g.,[12]), its left-hand side contains the moment of the forces exerted on l by the superfluid part of the liquid, and the right-hand side contains the moment of the friction forces.

To obtain an equation that describes the phase oscillations $\delta\Phi$, we must multiply (37) by Ψ^* and subtract the complex-conjugate expression, but retain in (35) the terms proportional to ω^2. There is no need, however, for an explicit expansion, since it can be verified with the aid of (17) that this equation is equivalent to the continuity equation (27) obtained above.

The explicit calculation of the friction-force moment is quite difficult, since the collision integral has a rather complicated structure. It simplifies considerably, however, near the critical temperature (this was pointed out by Pethick *et al.*[7,17]). In fact, the main contribution to the collision integral when the integration is with respect to $d\epsilon_1$ and $d\epsilon_2$ is made by frequencies $\epsilon_{1,2} \sim T$. Therefore the cators $M_{1,2}$ can be replaced near T_c by unity. Furthermore, the important role in the right-hand side of (38) is made by frequencies $\chi^{(1)} \propto \Delta/\epsilon$, in the case of large ϵ, the contribution of the terms $\chi^{(1)}_{\epsilon_1}$, $\chi^{(1)}_{\epsilon_2}$, and $\chi^{(1)}_{\epsilon-\epsilon_1-\epsilon_2}$ to J_1 is small compared with $c\chi^{(1)}_\epsilon$ (and the contribution of $\chi^{(2)}_{\epsilon-\epsilon_1-\epsilon_2}$ to J_2 is small compared with $c\chi^{(2)}_\epsilon$) to the extent that Δ/T is small. Thus, accurate to the principal terms in Δ/T, we have

$$\hat{J}_{1,2}=-\frac{1}{\tau_n(\varepsilon)}g^R\frac{\partial f^{(0)}}{\partial\varepsilon}\chi_\varepsilon^{(1),(2)}\delta_{\alpha\beta},$$

where

$$\tau_n^{-1}(\varepsilon)=\frac{\lambda^2\nu^2\pi}{2p_Fv_F}\hat{L}\{1+f_1^{(0)}f_2^{(0)}+f_1^{(0)}f_3^{(0)}+f_2^{(0)}f_3^{(0)}\}$$

is the free-path time of the quasiparticles in the normal state. The kinetic equation for $\chi^{(1)}$, accurate to the principal terms in $k\xi$, takes the form

$$\tau_n(\varepsilon)v_{Fi}v_{Fk}\nabla_i(g^R\nabla_k\chi^{(1)})-\frac{1}{\tau_n(\varepsilon)}g^R\chi^{(1)}=\frac{1}{2}\left(\frac{\partial\Delta_p}{\partial t}f^{+R}+f^R\frac{\partial\Delta_p^*}{\partial t}\right),$$

and the function $\chi^{(2)}$ at $\epsilon \sim \Delta \ll T$ is expressed in terms of $\chi^{(1)}$:

$$\chi^{(2)}=-\tau_n(\varepsilon)v_F\nabla\chi^{(1)}.$$

A local relation between $\chi^{(1)}$ and $\partial I/\partial t$[12] exists only in the case of sufficiently slow spatial variation of l, when the characteristic distance $R \gg 1$, where $l = v_F\tau_n(\epsilon)$. In this case we get

$$\chi_\varepsilon^{(1)}=-\frac{\tau_n(\varepsilon)}{2\varepsilon}\left(\Delta_p\frac{\partial\Delta_p^*}{\partial t}+\Delta_p^*\frac{\partial\Delta_p}{\partial t}\right). \tag{39}$$

Substituting (39) in (38) we obtain

$$\left[1\frac{\delta\mathcal{F}}{\delta 1}\right]=-\mu\left[1\frac{\partial 1}{\partial t}\right], \tag{40}$$

where the "cross-viscosity" coefficient[12] is

$$\mu=\nu\int\frac{dO_p}{4\pi}\int\limits_{\varepsilon>|\Delta_p|}d\varepsilon\,\tau_n(\varepsilon)\frac{\partial f^{(0)}}{\partial\varepsilon}\frac{\Delta_0^2(\hat{1\mathbf{p}})^2|\Delta_p|^2}{\varepsilon(\varepsilon^2-|\Delta_p|^2)^{1/2}}=\frac{\nu\tau_n(0)\Delta_0^3\pi^2}{64T_c}, \tag{41}$$

in agreement with the result of Pethick and Smith.[17] The expression for the free energy near T_c is well known and will not be presented here.

Thus, the system of hydrodynamic equations consists of the kinetic equations (16) and (17) with collision integrals (25) and (26), the equation (38), relation (3), and the conservation laws (27)–(34). We emphasize once

more that for the regular functions $g^{R(A)}, f^{R(A)}, f^{+R(A)}$, that enter in these equations it is not always sufficient to use formulas (14), which are valid in the zeroth approximation in the spatial gradients. Thus, for substitution in (38) we must calculate $f^{R(A)}$ and $f^{+R(A)}$ accurate to second order in $k\xi$, and for the momentum density–to first order, etc. The corresponding expression in each concrete case can be easily obtained from (12) and (13) or with the aid of the usual diagram technique.

With the aid of the obtained equations we can verify that the relaxation of the vector l is determined only by one characteristic time–the free path time of the quasiparticle. This is in disagreement with the assumption of Leggett and Takagi[5] that there is one other characteristic relaxation time (designated τ_K in[5]).

The equation of motion (38) for l does not contain the so-called "spontaneous angular momentum" L_{sp}. This indicates that L_{sp} is small and agrees with Ishikawa's calculations of the spontaneous angular momentum[18] (see also[6]).

5. MOTION OF NONSINGULAR VORTICES AND SOLITONS IN ^{3}He-*A*

We consider the motion of nonsingular vortices and solitons, which are macroscopic objects whose characteristic dimension is $R \gg 1$. Their velocity will be assumed small, $u \ll R/\tau$. We put for simplicity $\mathbf{v}^n = 0$.

The possible existence of nonsingular vortices in ^{3}He-A was discussed in[19-21]. The structure of such a vortex can be described by introducing the Euler angles α, β, γ, that define the position of the triad of unit vectors $\Delta', \Delta'', \mathbf{l}$. For example, $\mathbf{l} = (\sin\beta\cos\alpha, \sin\beta\sin\alpha, \cos\beta)$. The superfluid velocity, according to[20], is given by $\mathbf{v}^s = -(\cos\beta\nabla\alpha + \nabla\gamma)/2m$. Let $\alpha = -\gamma = \varphi$, where φ is the azimuthal angle in a cylindrical coordinate frame, and $\beta(\rho)$ ranges from zero at $\rho = 0$ to π at $\rho \gg R$. The superfluid velocity

$$\mathbf{v}^s = \frac{1-\cos\beta}{2m\rho}\hat{\mathbf{e}}_\varphi$$

vanishes at $\rho = 0$, and its value at $\rho \gg R$ is $v_\varphi^s = 1/m\rho$. In the presence of a superfluid flow with velocity $\mathbf{V}^s$ averaged over the volume, the vortex is acted upon by the Magnus force. The velocity of the steady motion of the vortex is determined by the balance of the Magnus force and the friction force due to the cross viscosity (this circumstance was pointed out in[21]). To derive the balance equation for the forces we write (37) in the form[2)]

$$\frac{\delta\mathcal{F}}{\Delta_0\delta\Psi^{*\alpha}} = \nu\int\frac{d\mathrm{O_p}}{4\pi}\int\limits_{\varepsilon>|\Delta_\mathbf{p}|} d\varepsilon\hat{\mathbf{p}} f_\varepsilon^R(\hat{\mathbf{p}}, \mathbf{r})\frac{\partial f^{(0)}}{\partial\varepsilon}\chi^{(1)}. \qquad (42)$$

The functional $\mathcal{F}$ is a quadratic form in the gradients of the components of the order parameter Ψ:

$$\mathcal{F} = \int d^3\mathbf{r} f_{ik}^{\alpha\beta}\{\Psi, \Psi^*\}(\nabla_i\Psi^\alpha)(\nabla_k\Psi^{*\beta}).$$

We now employ a precedure used earlier[22] to analyze the motion of vortices in superconductors. We put

$$\Psi(\mathbf{r}) = \Psi^{(0)}(\mathbf{r}-\mathbf{u}t) + \Psi_1,$$

where $\Psi^{(0)}$ corresponds to the immobile vortex and the correction Ψ_1 is proportional to the vortex velocity $|\mathbf{u}|$, after which we linearize (42) with respect to Ψ_1. The left-hand side of (42) becomes after linearization

$$\left(\frac{\delta\mathcal{F}}{\delta\Psi^{*\alpha}}\right)_1 = \left(\frac{\partial^2 f_{mn}^{\gamma\delta}}{\partial\Psi^{*\alpha}\partial\Psi^\beta}\Psi_1^\beta + \frac{\partial^2 f_{mn}^{\gamma\delta}}{\partial\Psi^{*\alpha}\partial\Psi^{*\beta}}\Psi_1^{*\beta}\right)(\nabla_m\Psi^\gamma)(\nabla_n\Psi^{*\delta})$$
$$+\frac{\partial f_{mn}^{\gamma\delta}}{\partial\Psi^{*\alpha}}[(\nabla_m\Psi_1^\gamma)(\nabla_n\Psi^{*\delta}) + (\nabla_m\Psi^\gamma)(\nabla_n\Psi_1^{*\delta})]$$
$$-\nabla_n\left[\left(\frac{\partial f_{mn}^{\gamma\alpha}}{\partial\Psi^\beta}\Psi_1^\beta + \frac{\partial f_{mn}^{\gamma\alpha}}{\partial\Psi^{*\beta}}\Psi_1^{*\beta}\right)\nabla_m\Psi^\gamma + f_{mn}^{\gamma\alpha}\nabla_m\Psi_1^\gamma\right].$$

It is furthermore obvious that if the function $\Psi^{(0)}(\mathbf{r})$ satisfies the equation $\delta\mathcal{F}/\delta\Psi^* = 0$, then the function $\Psi^{(0)}(\mathbf{r}+\mathbf{a}) = \Psi^{(0)} + (\mathbf{a}\nabla)\Psi^{(0)}$ satisfies the same equation, where a is an arbitrary constant vector. We have

$$\left(\frac{\delta\mathcal{F}}{\delta\Psi^{*\alpha}}\right)_\mathbf{a} = 0,$$

where $(\delta\mathcal{F}/\delta\Psi^{*\alpha})_\mathbf{a}$ is obtained from $(\delta\mathcal{F}/\delta\Psi^{*\alpha})_1$ by replacing Ψ_1 with $\Psi_\mathbf{a} = (\mathbf{a}\nabla)\Psi^{(0)}$. We make up the combination

$$\int d^3\mathbf{r}\left\{\Psi_\mathbf{a}^\alpha\left(\frac{\delta\mathcal{F}}{\delta\Psi^\alpha}\right)_1 - \Psi_1^\alpha\left(\frac{\delta\mathcal{F}}{\delta\Psi^\alpha}\right)_\mathbf{a} + \text{c.c.}\right\}, \qquad (43)$$

where the integration is over a cylinder surrounding the vortex and having a radius much larger than R. It is easy to verify that this combination is an integral of the divergence of some vector and reduces to an integral over a remote surface at large distances from the vortex we have $\mathbf{l} = \text{const}$. and $\Psi_1 = i\Phi_1\Psi^{(0)}$, $\Psi_\mathbf{a} = i\Phi_\mathbf{a}\Psi^{(0)}$, where $\Phi_1 = 2m\mathbf{V}^s\mathbf{r}$, which corresponds to the presence of a superfluid flow with velocity $\mathbf{V}^s$ at infinity, and $\Phi_\mathbf{a} = (\mathbf{a}\nabla)\Phi_0$, $\Phi_0 = 2\varphi$. Using the definition of the superfluid current $j_i^s = 2m\delta\mathcal{F}/\delta\nabla_i\Phi$, we can show that expression (43) is equal to

$$-\frac{1}{2m}\int dS_i(\Phi_\mathbf{a} j_{1i}' - \Phi_1 j_{\mathbf{a}i}') = b2\pi\rho_\perp'[\mathbf{a}\mathbf{V}']_z,$$

where $\rho_\perp^s$ is the component of the superfluid-density tensor in a direction perpendicular to l, and b is the length of the vortex. With the aid of (43) we get

$$\rho_\perp^s[\mathbf{V}^s\boldsymbol{\varkappa}]\mathbf{a} = \nu\int d^2\mathbf{r}\int\frac{d\mathrm{O_p}}{4\pi}\int\limits_{\varepsilon>|\Delta_\mathbf{p}|} d\varepsilon(\Delta_\mathbf{a} f^{+R} + \Delta_\mathbf{a}^* f^R)\frac{\partial f^{(0)}}{\partial\varepsilon}\chi^{(1)}, \qquad (44)$$

where $\varkappa = (2\pi/m)\mathbf{n}_z$ is the circulation of $\mathbf{v}^s$ over an infinitely remote contour.

In the general case it is difficult to solve the kinetic equation, and we confine ourselves therefore to temperatures close to T_c. With the aid of (39) we get

$$\rho_\perp'[\mathbf{V}'\boldsymbol{\varkappa}]\mathbf{a} = \mu\int d^3\mathbf{r}(\mathbf{a}\nabla)\mathbf{l}(\mathbf{u}\nabla)\mathbf{l},$$

where we have replaced the derivative with respect to time by $-(\mathbf{u}\cdot\nabla)$. Expressing l in terms of the Euler angles and integrating with respect to the azimuthal angle φ, we arrive at the balance equation for the forces

$$\rho_\perp'[\mathbf{V}'\boldsymbol{\varkappa}] - \mu\alpha\mathbf{u} = 0,$$

where

$$\alpha=\pi\int_0^\infty \rho\, d\rho\left[\left(\frac{\partial\beta}{\partial\rho}\right)^2+\frac{\sin^2\beta}{\rho^2}\right].$$

is a number of the order of unity.

The nonsingular vortex in ^{3}He-A moves perpendicular to the incident superfluid flow. This situation is analogous to the motion of vortices in dirty superconductors (see, e.g.,[22]), but differs from ordinary superfluid helium, where the vortex mainly with a mass-averaged velocity $\mathbf{u}=(\rho^s\mathbf{v}^s+\rho^n\mathbf{v}^n)/\rho$ (see[23]). The difference is due to the fact that the vortex in ordinary He-II is a quantum object whose motion is determined by scattering of excitations by such an object. In our situation, on the other hand, the probability of scattering of the excitations by the vortex is small compared with the probability of their collisions with one another. In this sense, the nonsingular vortex is a hydrodynamic object.

By way of a soliton example, we consider a particle-like soliton, the possible existence of which was noted by Volovik and Mineev[24] (soliton of the first type). It has an energy $\mathcal{F}=A\rho^s R/m^2$ and a velocity $|\mathbf{u}|=B/mR$ in the direction of the vector l at infinity (z axis), where A and B are constants on the order of unity. When the soliton moves, the friction with the normal excitation decreases its energy, and the soliton grows smaller. It is clear that one can speak of a soliton only in the case when the "collapse" rate is smaller than $|\mathbf{u}|$, and we assume therefore in first-order approximation that the soliton is displaced as a whole: $\Psi=\Psi^{(0)}(\mathbf{r}-\mathbf{u}t)$. Let us calculate the free energy lost by the soliton

$$\frac{\partial\mathcal{F}}{\partial t}=\int d^3r\left(\frac{\delta\mathcal{F}}{\delta\Psi^*}\frac{\partial\Psi^*}{\partial t}+\frac{\delta\mathcal{F}}{\partial\Psi}\frac{\partial\Psi}{\partial t}\right).$$

Expressing $\delta\mathcal{F}/\delta\Psi^*$ with the aid of (42) in terms of the correction to the distribution function (39), we obtain near T_c

$$\frac{\partial\mathcal{F}}{\partial t}=-\mu\int d^3r\left(\frac{\partial\mathbf{l}}{\partial t}\right)^2=-\eta\mathbf{u}^2,$$

where $\eta=\mu\zeta R$, and ζ is a constant of the order of unity:

$$\zeta=\frac{1}{R}\int d^3r\left(\frac{\partial}{\partial z}\mathbf{l}\right)^2.$$

The expression for η recalls the Stokes formula for viscous friction in flow around a solid.

It is easy to obtain the lifetime of the soliton

$$t_0=\frac{A}{2\zeta B^2}\frac{\rho' R^2}{\mu}.$$

The rate of soliton collapse is

$$-\dot{R}=\frac{\eta u^2 m^2}{A\rho^s}\sim\frac{\tau_n T^2}{E_F}\frac{\Delta}{T}n.$$

Near T_c we actually have $|\dot{R}|\ll|\mathbf{u}|$, but at $T\sim\Delta$ it may turn out that $|\dot{R}|\sim|\mathbf{u}|$. Under these conditions the soliton is either strongly distorted by the friction, or its existence becomes impossible.

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[1)]It follows from (37) that the corrections to Ψ are small compared with Ψ_0 when $\omega\tau\ll1$ (or $\omega\tau\ll\Delta/T$ near T_c) in accord with the statement in the Introduction.

[2)]In this section we use the usual gauge for the order parameter.

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