Thermomagnetic coefficients in a size-quantized semiconducting film

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The linear diagonal and off-diagonal thermal currents are calculated for a size-quantized semiconducting or semimetallic film located in a perpendicular quantizing magnetic field. Special attention is paid to degenerate films with essentially discrete carrier-energy spectra. Both the diagonal and off-diagonal thermal currents in such films can be determined only by taking into acount the broadening of the magnetic-film levels. They are found to be strongly oscillating functions of the position of the chemical potential relative to the magnetic-film levels. The thermomagnetic coefficients that are obtained differ strongly from the bulk coefficients. Their amplitudes are comparable, but the phases are shifted and depend on the magnitude and nature of the broadening. Oscillations of the off-diagonal thermal current, of the de Haas–van Alphen type, are also considered.

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1. INTRODUCTION

Interest in thin films is connected both with their increasingly broad application in microelectronics and also with the appearance there of new physical phenomena that are absent in bulk samples. The application of a quantizing magnetic field perpendicular to the plane of the film makes the energy spectrum of the carriers (electrons) discrete, and leads to interesting new effects in the kinetic properties of such systems. In particular, in the calculation of the conductivity, it turns out to be necessary to take into account the broadening of the electron levels in self-consistent fashion.[1,2] In the present work, we have calculated the thermomagnetic kinetic coefficients of size-quantized films placed in a transverse quantizing magnetic field.

Thermomagnetic phenomena in thin films have also been studied in a work that appeared recently.[3] However, broadening of the electron states was not introduced in this research, so that answers could be obtained only in the range of parameters where the spectrum could be assumed to be quasicontinuous. We shall be interested primarily in the effects associated with the discreteness of the electron states, when a small number of the magnetic-film levels are occupied. Some $n_0=(\pi l^2 d_z)^{-1}$ electrons can be located on each of the levels in a unit volume, where $l=(c\hbar/e_0H)^{1/2}$ is the magnetic quantum length, d_z is the thickness of the film, and we do not take into account spin splitting. For $H\sim 10^4-10^5$ G and $d_z\sim 10^{-5}$ cm we have $n_0\sim 3\cdot 10^{15-17}\,\mathrm{cm}^{-3}$. Moreover, as a rule we shall consider degenerate films, where the temperature spread is less than the distance between levels $\Delta_{n,N}$ (n and N are the film and magnetic quantum numbers), which is equivalent to $T<\hbar\Omega_e\Delta_n$, where $\Omega_e=e_0H/\mathrm{cm}$ and $\Delta_n=\hbar^2\pi^2 n/md_z^2$.

2. OFF-DIAGONAL COEFFICIENTS

Since the size quantization affects only the z component of the motion of the electrons, the general formulas for the off-diagonal kinetic coefficients of the bulk samples in a quantizing magnetic field, obtained after a corresponding account of the diamagnetic fluxes,[4] remain valid also in the film:

$$j_i=\sigma_{ik}E_k-\beta_{ik}\nabla_k T, \tag{1}$$

$$\mathbf{j}_{nk}=-\frac{ce_0n_e}{H^2}[\mathbf{EH}]-\frac{cS}{H^2}[\nabla T\mathbf{H}], \tag{2}$$

$$\beta_{xy}=cS/H=\gamma_{xy}/T, \tag{3}$$

where n_e, $-e_0$ and S are the concentration, charge, and specific entropy of the electrons respectively. For nondegenerate films, β_{xy} was calculated in Ref. 3 for arbitrary thickness and magnetic fields.

At low temperatures, the formula[5]

$$S=-\frac{1}{V}\frac{\partial\Omega}{\partial T}=\frac{1}{V}\frac{\partial}{\partial T}\left\{\sum_{n=0}^{\infty}\frac{T^{2n}}{(2n)!}\frac{\partial^{2n}\Omega}{\partial\mu^{2n}}\Big|_{T=0}B_{2n}\right\},\quad B_{2n}=2^{2n}\int_0^{\infty}\frac{dx\,x^{2n}}{\mathrm{ch}^2x} \tag{4}$$

is customarily employed. However, the thermodynamic potential $\Omega(\mu,H,T=0)$ as a function of the chemical potential μ and H, has discontinuities everytime the magnetic film level crosses the Fermi level. This is connected with the singularities in the density of states of the noninteracting electron gas and leads to nonphysical jumps in such quantities as S and the magnetic moment M. In order to surmount the difficulties that arise in the calculation of Ω, it is necessary to take into account the broadening of the discrete, strongly degenerate magnetic-film levels through one sort of mechanism or another of the electron scattering. By the definition of the thermodynamic potential, we have

$$\exp\left(-\frac{\Omega}{T}\right)=\mathrm{Sp}\left\{\exp\left(-\frac{\hat{H}-\mu\hat{N}}{T}\right)\right\}, \tag{5}$$

where $\hat{H}$ is the many-particle Hamiltonian of the electrons which interact with the impurities, phonons and (or) surface roughnesses, $\hat{N}$ is the operator of the number of particles, and the trace is taken over the entire set of many-particle states. We differentiate both sides of the relation (5) with respect to μ:

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$$-\frac{\partial\Omega}{\partial\mu}=\int dr\,\mathrm{Sp}\left\{\exp\left(\frac{\Omega-\hat{H}+\mu\hat{N}}{T}\right)\psi^{+}(r,\tau)\psi(r,\tau)\right\}$$
$$=\int dr\,G(r,\tau;r,\tau+0), \qquad (6)$$

where G is the exact single-electron Green's function. Transforming to the energy representation for G in (6), and again integrating with respect to μ, we obtain for the thermodynamic potential an expression that takes into account the broadening and renormalization of the single-electron spectrum:[1)]

$$\Omega=-T\sum_{\alpha}\frac{1}{\pi}\int dE\ln\left(1+\exp\frac{\mu-E}{T}\right)\mathrm{Im}\,G_{\alpha}(E), \qquad (7)$$

where α is the set of quantum numbers of the free electron in a thin film placed in a transverse magnetic field. Further calculations depend on the relations between the parameters T and Γ_{N_0,n_0} (Γ_{N_0,n_0} is the effective width of the magnetic-film level nearest the Fermi level) and on whether the energy regions in which the quantities

$$\sum_{\alpha}\mathrm{Im}\,G_{\alpha}(E)\neq 0,\quad \frac{\partial}{\partial T}\left\{T\ln\left[1+\exp\left(\frac{\mu-E}{T}\right)\right]\right\}\neq 0$$

differ appreciably from zero overlap.

If $T\ll\Gamma_{N_0,n_0}$ and the low-temperature expansion (4) converges well at any location of the chemical potential relative to the centers of the broadened levels,[2)] then it follows from (3), (4), and (7) that

$$\beta_{xy}=\frac{e_0T}{3\hbar d_z}\sum_{N,n}\mathrm{Im}\,G_{Nn}(\mu)=\frac{c\pi^2T}{3VH}\rho(\mu). \qquad (8)$$

At $\Gamma\ll\Delta$, the density of the electron states $\rho(E)$ oscillates strongly. Thus, for Lorentz broadening we have $\rho_{max}/\rho_{min}\approx\Delta^3/8\Gamma^2$. If the temperature scatter exceeds the collision broadening, $\Delta_{N_0,n_0}\gg\Gamma_{N_0,n_0}$, then the expression for β_{xy} becomes somewhat complicated; however, it is easy to show that β_{xy} will oscillate as before.

If $|\mu-E_{N_0,n_0}|\leq\Gamma_{N_0,n_0}$, then the region of energies in which Ω depends significantly on T overlaps with the narrow region of the spread of the level $\alpha_0=\{N_0,n_0\}$:

$$\beta_{xy}\approx\frac{e_0}{\pi\hbar d_z}\ln\left[1+\exp\left(\frac{\mu-E_{\alpha_0}}{T}\right)\right]\sim\frac{e_0}{\pi\hbar d_z}. \qquad (9)$$

If $E_{\alpha_0}+T\ll\mu\ll E_{\alpha_0}+\Delta_{\alpha_0}-T$, then these regions do not coincide and the integration in (5) must be carried out also over the regions of broadening of the levels near E_{α_0} and $E_{\alpha_0}+\Delta_{\alpha_0}$, and over the region $|E-\mu|\leq T$, where $\partial\Omega/\partial T$ is not small:

$$\beta_{xy}=\frac{\pi^2cT}{3HV}\rho(\mu)+\frac{\pi e_0T}{3\hbar d_z}[-E_1\exp(E_1)+E_2\exp(-E_2)],$$
$$E_1=\frac{E_{\alpha_0}-\mu}{T},\quad E_2=\frac{E_{\alpha_0}+\Delta_{\alpha_0}-\mu}{T}. \qquad (10)$$

Strictly speaking, the low-temperature expansion proves to be inapplicable also in the case $T<\Gamma$, if μ lies between the maxima of the density of states. In this case the contribution to (8) from integration over the broadening region can prove to be important near the minimum of β_{xy}:

$$\beta_{xy}=\frac{c\pi^2T}{3HV}\rho(\mu)+\frac{e_0}{\pi\hbar d_z}\frac{\partial}{\partial T}\left\{\frac{T^2}{\Gamma}\exp(\Gamma/T)\left[\exp(-E_2)+\exp(E_1)\right]\right\}, \qquad (11)$$
$$E_{\alpha_0}+\Gamma_{\alpha_0}\ll\mu\ll E_{\alpha_0}+\Delta_{\alpha_0}-\Gamma_{\alpha_0}.$$

Thus β_{xy} at low temperatures and at $\Delta>\Gamma$ undergoes strong oscillations with change in the magnetic field, concentration or film thickness. The amplitude of these oscillations can be much greater than unity. We note that the formula (8) can be obtained with the help of the generalized Wiedemann-Franz relation,[6] if the concentration of electrons in σ_{xy} is expressed in terms of the density of states.

For quasi-classical magnetic fields ($\mu\gg\hbar\Omega_e$) and "thick" ($\mu\gg\Delta_n$, Δ_n is the value of the film quantum) films, the quantity β_{xy} can be found with the help of the thermodynamic potential Ω, which has been obtained by a number of authors.[7,8] We give here only the expressions for $\Omega(T)$ and β_{xy} of a degenerate "thin" film in a quasiclassical magnetic field ($\mu,\Delta_n\gg\hbar\Omega_e,T$) obtained with the help of the Poisson formula[3)]; here the broadening can be disregarded because of the quasi-continuity of the Landau levels:

$$\Omega=-\frac{V\hbar\Omega_e}{\pi l^2d}\sum_{n=1}^{n_0}\left\{\frac{\tilde{\varepsilon}_n^2}{2}+\frac{\pi^2\tilde{T}^2}{6}+\sum_{s=1}^{\infty}\frac{(-1)^s}{2\pi^2s^2}\right.$$
$$\left.\times\left[f_F\left(\frac{\tilde{\varepsilon}_n}{\tilde{T}}\right)-\frac{a_s\cos(2\pi s\tilde{\varepsilon}_n)}{\mathrm{sh}\,a_s}\right]\right\}, \qquad (12)$$

$$\beta_{xy}=\frac{e_0}{\pi\hbar d_z}\sum_{n=1}^{n_0}\left\{\frac{\pi^2}{3}\tilde{T}+\sum_{s=1}^{\infty}\left[\frac{(-1)^{s+1}}{2\pi^2s^2}\frac{\tilde{\varepsilon}_n}{\tilde{T}}\frac{\partial f_F}{\partial\tilde{\varepsilon}_n}\right.\right.$$
$$\left.\left.+\frac{(-1)^s}{s}\cos(2\pi s\tilde{\varepsilon}_n)\frac{a_s}{\mathrm{sh}\,a_s}(\mathrm{cth}\,a_s-a_s^{-1})\right]\right\}. \qquad E_{n_0}\leq\mu<E_{n_0+1} \qquad (13)$$

where in formulas (12) and (13),

$$\tilde{\varepsilon}_n=\frac{E_n-\mu}{\hbar\Omega_e},\quad \tilde{T}=\frac{T}{\hbar\Omega_e},\quad a_s=2\pi^2s\tilde{T}.$$

The second term in the square brackets in (13) corresponds to the ordinary oscillations of the thermodynamic quantities in quasiclassical magnetic fields. However, whereas in bulk samples, the oscillating part of β_{xy} is small in comparison with the monotonic part (the rapid function in the small correction to Ω does not depend on T), in our case the first term in the square brackets in (13) at $\Delta_n\gg T$ is also a non-monotonic function of the location of the chemical potential relative to the film levels and can be significantly less than its amplitude value $\sim\hbar\Omega/2\pi^2T$. At $\Delta_n\gg\hbar\Omega_e$, the period of change of $\tilde{\varepsilon}_n\partial f_F/\partial\tilde{\varepsilon}_n$ is significantly smaller than the period of the $\cos 2\pi s\hat{\delta}_n$ oscillations. Therefore, at $|E_{n_0}-\mu|\gg T$, the contribution to (13) of the term that is oscillating rapidly with the magnetic field will be significant. We also note that at $a_s<1$, the amplitude of the oscillations depends weakly on the number of the harmonic and a decrease sets in only at $a_s>1$.

3. DIAGONAL COEFFICIENTS

As has already been noted, the calculation of the conductivity for a quasi-two-dimensional gas in a quantizing magnetic field requires the account of the broadening of the levels. This is equivalent to the separation, in the Kubo formula for σ_{xx} expanded in powers of the

interaction Hamiltonian, of terms corresponding to the broadening, and summation of this subsequence.[1,2] The application of a similar approach to the calculation of γ_{xx} encounters significant additional difficulties, associated with the necessity of taking into account the interaction energy fluxes in the expansion of the Kubo formula.[9] We can get around these difficulties by using the well-known results for σ_{xx}[1,2,10] and the general formal relations between the kinetic coefficients[11,6]:

$$\sigma_{ik}=e^2\int -\frac{\partial f}{\partial E}\Phi_{ik}(E)\,dE, \quad \beta_{ik}=\frac{e}{T}\int -\frac{\partial f}{\partial E}\Phi_{ik}(E)\,(E-\mu)\,dE. \tag{14}$$

The expressions for $\Phi_{xx}(E)$ have different forms for the different scattering mechanisms of the electrons, but also depend on the approximations used in obtaining σ_{xx}. We consider the case of elastic scattering from impurities and use $\Phi_{xx}(E)$ from the work of Gerhards[10]

$$\Phi(E)=\frac{1}{\pi^2\hbar d_z}\sum_{N,n}\left(N+\frac{1}{2}\right)\exp\left[-\frac{4(E-E_{N,n})^2}{\Gamma^2}\right] \tag{15}$$

($\Delta_n \gg \Gamma, T$). Then, in the case $T \ll \Gamma \ll \mu$, we have

$$\beta_{xx}\approx\frac{8\pi}{3}\frac{e_0 T}{\hbar d_z}\sum_{N,n}\left(N+\frac{1}{2}\right)\left(\frac{\mu-E_{N,n}}{\Gamma^2}\right)\exp\left[-4\left(\frac{\mu-E_{N,n}}{\Gamma}\right)^2\right]. \tag{16}$$

The qualitative behavior of β_{xx}—alternating oscillations with $(\beta_{xx})_{max}\sim e_0 T/\hbar d_z\Gamma$—is maintained if $\Phi(E)$ different from (15) are used. If the condition $\Gamma \ll T$ is satisfied for the degenerate film, then

$$\beta_{xx}\approx\frac{e_0}{\hbar d_z}\frac{\Gamma}{(4\pi)^{1/2}T}\sum_{N,n}\left(N+\frac{1}{2}\right)(\mu-E_{N,n})/T\,\mathrm{ch}^2\frac{\mu-E_{N,n}}{2T}, \tag{17}$$

and now β_{xx} oscillates with $(\beta_{xx})_{max}\sim e_0\Gamma/\hbar d_z T$. In contrast with Eqs. (10) and (11), integration over the region $|\mu-E|\leqslant T$ at small density of states on the Fermi surface gives an insignificant contribution to β_{xx} in most cases because of the odd parity of the function $E/\cosh^2 E$.

With decrease in the magnetic field or increase in the film thickness, the expression for $\Phi(E)$ will become complicated because of the necessity of taking into account the effect of the other levels on the broadening. However, numerical calculation shows[12] that the oscillations of $\rho(E)$ and σ_{xx} diminish slightly with decrease in the magnetic field. Since, in contrast with the bulk sample, the principal contribution to the diagonal terms in the degenerate film in a strong magnetic field under the condition $\Delta > \Gamma$ are made by the one or two magnetic-film levels close to ϵ_F, independently of the number of occupied levels, we can expect that β_{xx} will be described by formulas of the type (16) and (17) even in quasiclassical fields ($\mu, \Delta_n \gg \hbar\Omega_e$).

When the degeneracy is lifted the oscillations of β_{xx} vanish and the result, which can be obtained from the relaxation[4)]

$$\beta_{xx}=\frac{1}{e}\frac{\partial}{\partial T}(\sigma_{xx}T),$$

which follows from (9) at $-\mu+\hbar\Omega_e/2+\Delta_{n=1}\gg T$ and the expression for σ_{xx} obtained previously,[2] has qualitatively the following form:

$$\frac{\beta_{xx}}{\beta_{xy}}\sim\frac{\Gamma}{\max(T,\hbar\Omega_e)}\ll 1. \tag{18}$$

4. THE THERMOELECTRIC POWER AND THE NERNST-ETTINGSHAUSEN COEFFICIENT

From the viewpoint of comparison with experiment, it is of interest to know the theoretical formulas for the thermoelectric power α and the transverse Nernst-Ettingshausen coefficient Q:

$$\alpha=\frac{\sigma_{xy}\beta_{xy}+\sigma_{xx}\beta_{xx}}{\sigma_{xx}^2+\sigma_{xy}^2}, \quad Q=\frac{\sigma_{xy}\beta_{xx}-\sigma_{xx}\beta_{xy}}{H(\sigma_{xx}^2+\sigma_{xy}^2)}. \tag{19}$$

For bulk samples in a strong magnetic field ($\Omega_e\tau \gg 1$) the diagonal thermal coefficient β_{xx} can be important for the thermoelectric power in a strongly compensated semiconductor with $n_e \approx n_i$ and in the oscillating part of α.[9] The situation is different in degenerate, size-quantized semiconducting films. There, σ_{xx} undergoes strong oscillations and $(\sigma_{xx})_{max}\sim\sigma_{xy}$.[1,2] Moreover, as follows from the results of this work, the diagonal and off-diagonal thermal currents also undergo oscillations with comparable amplitudes and shifted phases. Therefore, in both α and Q, it is generally necessary to keep all terms both in the numerator and in the denominator. One should expect a complicated oscillating behavior of α and Q in the change of the film parameters and (or) magnetic field even for the simplest forms of symmetric broadening and without account of spin splitting. In particular, because of the reversal of the sign of β_{xx} when the chemical potential goes through a maximum $\Phi(E)$, the quantities α and Q can vanish and will be nonsymmetric relative to $\mu=E_{N,n}$. Because of the sensitivity of ρ_{xx}, α and Q to the electron spectrum and the scattering mechanism of the carriers in thin films, combined galvano- and thermomagnetic experiments can give valuable information on these quantities.

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[1)] Along with the calculation of the entropy, we can also use Eq. (7) in the calculation of such thermodynamic quantities of the degenerate quasi-two-dimensional electron gas in a magnetic field as the magnetic moment and the heat capacity.

[2)] We note that the criterion for the applicability of the low-temperature expansion

$$\left[\rho(E)\gg T^{2n}\frac{\partial^{2n}\rho}{\partial E^{2n}}\right]_{\varepsilon=\mu},$$

$n = 1, 2, \ldots$ depend strongly also on the shape of the line also.

[3)] Gogadze and Kulik[8] give only the oscillating part of Ω.

[4)] When using this relation, one must not differentiate with respect to the "phonon" temperature.

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Experimental results and analysis of structural fluctuations in photosensitive Josephson junctions

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Large and small structural fluctuations in Pb–CdS–In junctions were investigated. It was found that such photosensitivity junctions were particularly suitable for investigations of this kind. The dependence of the maximum Josephson current on the magnetic field was determined and a good agreement was obtained between the experimental and theoretical results.

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1. INTRODUCTION

Influence of inhomogeneities in the barrier layer on the properties of a Josephson tunnel junction has been investigated comprehensively by Yanson.[1,2]1) Of special interest are "structural fluctuations," which are randomly distributed inhomogeneities altering greatly the properties of a contact. In particular, Yanson studied the influence of such structural fluctuations on the dependence of the critical current on the applied magnetic field. Since these fluctuations disturb the homogeneity along a Josephson junction, the observed effect was particularly noticeable for low values of the barrier energy, namely near the minimum of the dependence of the superconducting current on the magnetic field. Although the experiments described above were in agreement with the theory, some of the effects predicted by the theory have not yet been confirmed experimentally.

We investigated photosensitive semiconductor junctions[3] which made it possible, as shown below, to study more effectively the problem mentioned above. Yanson's theory was considered from a somewhat different point of view and it was developed further. Experimental dependences of the critical current on the magnetic field in the presence of fluctuations were obtained and discussed. A comparison was made with the theory and a very good agreement was obtained.

2. THEORY

We shall consider the main results of Yanson[1,2] in a somewhat modified form. The relationship between the Josephson current and the applied magnetic field is known to be given by the modulus of the Fourier component of the maximum current density:

$$I_J(\varphi)=\left|W\int_{-L/2}^{L/2}J_1\exp\left(\frac{2\pi i}{L}\varphi x\right)dx\right|. \tag{1}$$

It is assumed that the barrier is in the (x,y) plane; L and W are the dimensions of the junction along the x and y axes; φ is the normalized magnetic flux $\varphi=\Phi/\Phi_0$, where Φ is the flux produced by an external magnetic field B directed along the y axis and $\Phi_0=h/2e$ is a quantum of this flux.

We shall allow for the presence of structural fluctuations by rewriting the current in the one-dimensional form:

$$J_1=J_1(x)+J_f(x)=J_1(x)+\sum_{n=1}^{\infty}\left(a_n\cos\frac{2\pi n}{L}x+b_n\sin\frac{2\pi n}{L}x\right); \tag{2}$$

here, $J_1(x)$ is the distribution on the current density in the absence of fluctuations and $J_f(x)$ is the random distribution of inhomogeneities, such that $\langle J_f(x)\rangle=0$, where the angular brackets represent averaging over the junction area. We shall assume that the correlation function of $J_f(x)$ is of the form[4]

$$\langle J_f(x_1)J_f(x_2)\rangle=\langle J_f^2\rangle\exp(-|x_1-x_2|/r); \tag{3}$$

$\langle J_f^2\rangle$ is the average value of the square of the amplitude of the fluctuations (it is assumed that $\langle J_f^2\rangle$ is constant over the whole junction) and r is the correlation radius characterizing the size of inhomogeneities.

Substituting Eq. (2) into Eq. (1), and allowing for Eq. (3), we obtain

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