in the calculation. First, in Eq. (4), we used only a single relaxation time τ, which is clearly insufficient for the description, for example, of depolarized light scattering in a liquid.[7] However, this is valid in the APRL problem. Actually, in the model of continuous rotational diffusion[14] for APRL, solutions exist corresponding only to a single quantum number $l=2$ and, because of this, to only a single relaxation time. Moreover, in the important practical case $\omega\tau \ll 1$, the orientation of the molecules follows that of the spins; therefore, the matrix element of the transition (but not the width of the APRL line) does not generally depend on τ and on any details of the relaxation process. Finally, at $\omega\tau \gtrsim 1$, the latter can also affect the matrix element of the APRL.

Second, the mechanisms of APRL considered above in liquids consisting of rigid, undeformed molecules, in which the form of the spin Hamiltonian (2) is determined by the orientation of the molecules. These mechanisms naturally do not take into account the excited electron states in the intra- and intermolecular electric fields, which is valid at least for many free radicals by virtue of the large excitation energy ΔE of such states.[11] On the other hand, there are liquids in which the surrounding of the spin firms a sufficiently symmetric complex.[12] The quantity ΔE is then smaller and in the APRL problem it is necessary to take into account the spin-orbit interaction, the deformability and the rotational motion of the complex. These features, and also other possible mechanisms of APRL (anisotropic hyperfine, dipole-dipole, and other interactions) are easily taken into account by the method outlined above. As in APR in solids, this can lead to a significant increase in the effect.

Finally, relaxation processes not considered above can exist in liquids, with such small τ that their contribution to T_2 would be negligibly small, but significant in the matrix element of the APRL. Such processes also lead to an increase in $\Delta\alpha/\alpha$.

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Slow thermal wave in a helium flow

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The motion of the He I–He II interphase boundary is considered in a narrow capillary tube for a specified flow rate. A temperature diference between the capillary and the thermostat (the temperature T_0 of which is below T_c) is maintained by the heat produced by the friction between the flow and the capillary walls. The flow regime with a constant velocity of the boundary and of the thermal wave due to the motion is investigated. The wave velocity, the temperature distribution $T(x)$ along the capillary tube, and the distribution of the normal and superfluid velocities in the region $T(x) < T_c$ are found. In some respects the solution is similar to that for a combustion regime, that is, to ignition and extinction waves.

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INTRODUCTION

Steady flow of He I will take place along a capillary placed in a medium with a temperature below T_c if the heat produced by friction against the walls is sufficient to maintain the temperature of the liquid in the capillary above T_c. The temperature difference between the capillary and the thermostat, $T_1 - T_0$, is proportional to the

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square of the flow velocity. For a plane channel of width h, for example,

$$v^2 = \frac{\alpha h}{6\eta}(T_1 - T_0), \tag{1.1}$$

where α is the heat transfer coefficient from the channel to the medium and η is the dynamic viscosity of He I.

At this same temperature of the thermostat $T_0 < T_c$, flow of He II along the capillary is of course possible. Thus two stationary flow regimes correspond to each pair of values of v and T_0 (in a definite region of variation of these quantities). Both these regimes are stable relative to small perturbations. At the same time, there should exist conditions under which the heating of a finite mass of He II (or cooling of He I) produces a phase transition wave and, in the final analysis, a change in the flow regime.

For the elucidation of these conditions, we must consider the interphase boundary in a moving liquid. At large distances from the boundary, the temperature in He I approaches a constant value T_1. Lowering of the temperature on the portion adjoining the boundary, from T_1 to T_c, indicates the presence of a flow of heat across the boundary, toward the He II. At large distances from the boundary, the normal component in the He II can be assumed to be immobile. The temperature of the liquid here tends to T_0 and the pressure also reaches some constant value. There is a flow of the normal component on the portion adjoining the boundary, and is due to the temperature difference $T_c - T_0$ on this portion. The transfer of heat across the interphase boundary generally leads to boundary motion relative to the liquid. The entire structure of the temperature and velocity fields moves along with the boundary, forming a temperature wave of this type.

The so-called heterogeneous (taking place on a solid surface) exothermal reaction creates a picture with a similar distribution of thermal flows. A given temperature of a reacting gas mixture T_0 corresponds to two possible stationary reaction regimes. In one regime, the surface temperature is close to T_0, the reaction on a cold surface takes place with small intensity and almost does not heat the surface. A surface temperature, which greatly exceeds T_0, corresponds to the other (combustion) regime. The high surface temperature is maintained by the heat release of the reaction, which takes place very intensively at such a temperature. It was shown in Ref. 1 that, in the case of a nonuniformly heated surface, the boundary separating the regions of high-temperature and low-temperature regimes of the reaction moves as a function of T_0 and the other parameters either towards the cold region (ignition wave) or towards the hot region (extinction wave).

In contrast with temperature disturbances in a superfluid liquid, which propagate with a large velocity (second sound), the thermal wave under consideration, which is connected with the phase transition and the slow process of heat transfer in He I, has a small, essentially subsonic velocity (similar to slow-combustion waves). In He II, the heat is effectively transported by the normal component over large distance from the boundary. In this portion of the flow, which can be called the convective zone, the longitudinal temperature gradient turns out, in view of the smallness of the considered velocities, to be a quantity proportional to the pressure gradient. The latter in turn is proportional to the average (over the cross section) velocity of the normal motion, so that any two of these quantities can be expressed in terms of the third.

At the same time, the longitudinal profiles of the temperature and the normal velocity found from these considerations cannot be directly "joined" to the solution in the region $T > T_c$. It suffices to note that the velocity of the normal motion, which is responsible for the heat transfer, is directed opposite to the temperature gradient while along the other side of the interphase gradient, in the He I, the velocity of the liquid has the reverse direction if the region of He I is located downstream. Therefore, a narrow (in the temperature interval and, consequently, spatially) boundary layer should exist between the convective zone and the He I region (the heat release zone), in which the flow is rearranged. For calculation of the velocity of the thermal wave, it is not possible to investigate the structure of the boundary zone: in view of the narrowness of this zone, the momentum transfer and leakage of heat to the walls of the capillary can be neglected within it; the jumps in the temperature and velocity are then determined by application of the conservation laws to the quantities on both sides of the boundary zone.

The relations thus obtained yield a single value of the wave velocity for each choice of values of the specified parameters (flow rate of the liquid, temperature of the thermostat, etc). As will be shown in Sec. 4, the wave velocity and the direction of its propagation relative to the liquid depends on the flow direction. The phase located upstream grows.$^{1)}$ In this respect, the phenomenon under study differs strongly from the thermal wave of the "Superconductor–normal metal" phase transition in a linear ciruit with current.[2] The propagation of such a wave is connected with the transfer of the Joule heat released in the normal phase to the superconducting phase. Since this transfer is effected by ordinary thermal conduction, the velocity and other characteristics should depend only on the quantity and not on the direction of the flow.

It should also be noted that a thermal instability of the flow of a viscous liquid was observed in Refs. 3 and 4. This instability is due to the strong temperature dependence of the viscosity, and the corresponding critical phenomena have been investigated—jumpwise change in the flow regime under a smooth change in the pressure drop and in other parameters.

2. BOUNDARY CONDITIONS AND EQUATIONS

In a set of coordinates attached to the interphase boundary, the flow is stationary. For definiteness, we consider a plane channel of width h. The location of the He I and He II regions is shown in Fig. 1. The walls of the channel move with velocity $-u$, where u is the velocity of the thermal wave in the laboratory system of coordinates. The interphase boundary is determined by the condition

$$T(x, y) = T_c. \tag{2.1}$$

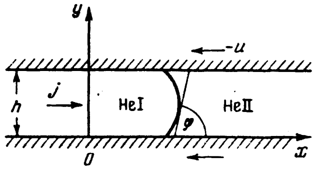

FIG. 1.

In the region $T < T_c$, we shall describe the flow by means of two-fluid hydrodynamics.[5,6] On the surface of the waves, at $T > T_c$, we have the condition

$$v_x = -u, \quad v_y = 0, \quad \varkappa \frac{\partial T}{\partial y}\bigg|_{y=0} = -\varkappa \frac{\partial T}{\partial y}\bigg|_{y=h} = \alpha (T - T_0). \tag{2.2}$$

At $T < T_c$ we have

$$v_{nx} = -u, \quad \left(\varkappa \frac{\partial T}{\partial y} - \rho T s v_{ny}\right)_{y=0} = \left(-\varkappa \frac{\partial T}{\partial y} + \rho T s v_{ny}\right)_{y=h} = \alpha (T - T_0), \tag{2.3}$$

$$\rho_s v_{sy} + \rho_n v_{ny} = 0, \tag{2.4}$$

where s is the entropy of a unit mass of liquid, $\varkappa$ is the coefficient of thermal conductivity. For the considered flow, the equation (2.4) obviously holds not only on the walls, but throughout the entire volume of the He II. Moreover,

$$\rho_s v_{sx} + \rho_n v_{nx} = \mathrm{const} = j = \rho v_x|_{\mathrm{He\,I}}, \tag{2.5}$$

so that any two of the four components $\mathbf{v}_s, \mathbf{v}_n$ can be expressed in terms of the other two.

For convenience in subsequent references, we write down the relations

$$\frac{\partial \Pi_{xx}}{\partial x} + \frac{\partial \Pi_{xy}}{\partial y} = 0, \quad \frac{\partial \Pi_{xy}}{\partial x} + \frac{\partial \Pi_{yy}}{\partial y} = 0. \tag{2.6}$$

At $T > T_c$

$$\Pi_{xx} = P + \left(\frac{j^2}{\rho}\right), \quad \Pi_{xy} = \eta \frac{\partial v_x}{\partial y}, \quad \Pi_{yy} = P, \tag{2.7}$$

at $T < T_c$

$$\Pi_{xx} = P + \frac{1}{\rho_s} (j - \rho_n v_{nx})^2 + \rho_n v_{nx}^2 + \frac{2}{3} \eta \left(2 \frac{\partial v_{nx}}{\partial x} - \frac{\partial v_{ny}}{\partial y}\right). \tag{2.8}$$

$$\Pi_{xy} = \frac{\rho_n}{\rho_s} v_{ny} (\rho v_{nx} - j) + \eta \left(\frac{\partial v_{nx}}{\partial y} + \frac{\partial v_{ny}}{\partial x}\right), \tag{2.9}$$

$$\Pi_{yy} = P + \left(\frac{1}{\rho_s} + \frac{1}{\rho_n}\right) \rho_n^2 v_{ny}^2 + \frac{2}{3} \eta \left(2 \frac{\partial v_{ny}}{\partial y} - \frac{\partial v_{nx}}{\partial x}\right),$$

where P is the pressure in the liquid. In these (and subsequent) expressions the terms containing the second viscosity are omitted for reasons that we shall give below.

The equation

$$\mu(P, T) - \frac{\rho_n}{2\rho} (\mathbf{v}_n - \mathbf{v}_s)^2 + \frac{v_s^2}{2} = \mathrm{const}, \tag{2.10}$$

where μ is the chemical potential of the quiescent liquid, enables us to express the pressure in terms of the components of the velocity and the temperature. Taking it into account that as $x \to \infty$ the velocities take the form $v_{nx} = -u$, $v_{ny} = 0$, we find

$$P - P_0 = \rho \int_{T_0}^{T} s\,dT - \left(\frac{1}{\rho_s} - \frac{1}{\rho_{s0}}\right) j \left(\frac{1}{2} j - \rho u\right) + \frac{\rho \rho_n}{2\rho_s} (v_{nx}^2 + v_{ny}^2 - u^2), \tag{2.11}$$

where $\rho_{s0} = \rho_s (P_0, T_0)$. Finally, the equation for heat transfer

$$\rho T \left[\frac{\partial}{\partial x}(s v_{nx}) + \frac{\partial}{\partial y}(s v_{ny})\right] = \frac{\partial}{\partial x}\left(\varkappa \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(\varkappa \frac{\partial T}{\partial y}\right), \tag{2.12}$$

together with (2.6) and (2.11), form a set of four equations for the four quantities v_{nx}, v_{ny}, P, T. In (2.12), heating due to friction of the normal component of He II was not taken into account. As will be shown, the velocity $\mathbf{v}_n$ is small and the heat liberated, which is quadratic in the components of this velocity, is insignificant.

The conditions on the interphase boundary are of the form

$$[\Pi_{xx} \sin \varphi + \Pi_{xy} \cos \varphi] = 0, \quad [\Pi_{xy} \sin \varphi + \Pi_{yy} \cos \varphi] = 0, \tag{2.13}$$

$$[Q_x \sin \varphi + Q_y \cos \varphi] = 0, \tag{2.14}$$

where the square brackets indicate the differences between the corresponding quantities on the two sides of the boundary, Q_x and Q_y are the components of the energy flux, and φ is the angle between the plane touching the boundary at the given point and the wall (Fig. 1).

We take the density of the liquid ρ to be constant, neglecting the thermal expansion and the effect of the change in the pressure along the channel (the pressure deviation from P_0 is large only in the He I at large distances from the interphase boundary; this region has no significant effect on the characteristics of the thermal wave). Correspondingly, we shall assume ρ_s and ρ_n to be functions of the temperature only. If

$$\alpha \ll \varkappa / h, \tag{2.15}$$

the dependence of T on y can be neglected, a fact also taken into account in (2.7). Then the distribution of the temperature in the He I is determined by the equation

$$\varkappa \frac{d^2 T}{dx^2} - \rho (v - u) c_{\mathrm{I}} \frac{dT}{\partial x} + \frac{12 \eta v^2}{h^2} - \frac{2\alpha}{h} (T - T_0) = 0 \tag{2.16}$$

and the conditions

$$x \to -\infty, \ dT/dx = 0, \ x = 0, \ T = T_c, \tag{2.17}$$

where v is the mean velocity of the helium in the laboratory system of coordinates, c_1 is the specific heat of He I. As $x \to -\infty$, the temperature reaches a maximum value T_1, determined from (1.1), while the heat flux at the interphase boundary is

$$q_c = -\varkappa \frac{dT}{dx}\bigg|_{x=0} = \frac{\varkappa}{l_{\mathrm{I}}} (T_c - T_0) \left(\frac{v^2}{v^{\,2}} - 1\right), \tag{2.18}$$

where

$$v_m^2 = \frac{\alpha h}{6\eta} (T_c - T_0), \quad l_{\mathrm{I}}^{-1} = \frac{\rho c_{\mathrm{I}}}{2\varkappa} \left(\left[(v - u)^2 + \frac{8 \alpha \varkappa}{h \rho^2 c_{\mathrm{I}}^2}\right]^{1/2} + v - u\right). \tag{2.19}$$

3. CONVECTIVE ZONE

We now consider the flow of He II at a large distance from the interphase boundary.

At $\alpha=0$, the velocity v_{ny} vanishes and the transverse profile v_{nx} follows a Poiseuille pattern. Keeping in mind the smallness of α, we assume that $v_{nx}=-u+6w(x)(y/h)[1-(y/h)]$, where w is a slowly changing function of x (at $\alpha=0$ we would have $w=\text{const}$). Averaging (2.6) and (2.12) over y with account of (2.3) and (2.11), discarding the heat conductivity $\varkappa$ and taking into consideration the smallness of the velocity, we obtain approximately

$$\rho s(dT/dx)=-(12\eta w/h^2), \tag{3.1}$$

$$\rho T\frac{d}{dx}[s(w-u)]+\frac{2\alpha}{h}(T-T_0)=0; \tag{3.2}$$

as $x\to\infty$, we have $T=T_0$ and $w=0$.

Transforming to the independent variable T, we write

$$\frac{d}{dT}[s(w-u)]=\frac{\alpha hs}{6\eta w}\left(1-\frac{T_0}{T}\right), \tag{3.3}$$

$$T=T_0,\quad w=0. \tag{3.4}$$

The separatrix, which emerges from the saddle point (3.4) in the region $T>T_0$, $w>0$, takes at small $T-T_0$ the form

$$w=v_m A\frac{T-T_0}{T_c-T_0},\quad A=\left[\left(\frac{u\tau}{2\sigma}\right)^2+\tau\right]^{1/2}+\frac{u\tau}{2\sigma}, \tag{3.5}$$

where $\tau=(T_c/T_0)-1$, $\sigma=s(T_0)/c(T_0)$, and c is the specific heat of He II.

We restrict ourselves to values of T_0 that are not far removed from T_c, so that τ would be less than σ. Then (3.5) can be extrapolated to the region adjoining the interphase boundary.

The width of the convective zone is, according to (3.1) and (3.5), a quantity of order

$$l=\rho s(T_0)(T_c-T_0)\frac{h^2}{12\eta v_m A}. \tag{3.6}$$

The thermal conductivity can be neglected if the first term on the left side of (3.2) is large in comparison with $\varkappa(d^2T/dx^2)$. With account of (3.1) and (3.6), we have the condition

$$h^2\gg 12\eta\varkappa/\rho^2 s(T_0)c(T_0)(T_c-T_0). \tag{3.7}$$

The right side of (3.7) is a quantity of order 10^{-12} cm^2 in the case of ^{4}He.

We have not considered the second viscosity along with the thermal conductivity in the region of He II. In an estimate of the corresponding terms in the equation, we must of course compare the terms in the equations, we must of course compare the derivatives $\partial v_{nx}/\partial x\sim w/l$, $\partial v_{ny}/\partial y\sim(\alpha/h\rho Ts)(T-T_0)$ and $\partial v_{ny}/\partial y\sim w/h$. Such a comparison shows that the second viscosity can be neglected if

$$\sqrt{\alpha}\ll(\rho s/A\zeta)[(\eta h/24)(T_c-T_0)]^{1/2}, \tag{3.8}$$

$$\sqrt{\alpha}\ll(\rho sAT_c/\zeta)[\eta h/6(T_c-T_0)]^{1/2}, \tag{3.9}$$

where $\zeta=\zeta_2-2\rho\zeta_1+\rho^2\zeta_3+(4/3)\eta$, and ζ_1, ζ_2, and ζ_3 are the second-viscosity coefficients introduced in Ref. 6. The assumption of small α, used above, corresponds to the conditions (2.15), (3.8), and (3.9).

4. VELOCITY OF THE WAVE

The relation (3.1) between w and dT/dx cannot hold at the interphase boundary itself. On this boundary, because of the equation of continuity, the velocity of the normal component must coincide with the constant (x-independent) velocity of the He I. The latter, being a given quantity (proportional to the flow rate), can take on both positive and negative values. But w, so long as (3.1) is valid, can only be positive, since $dT/dx<0$. Therefore, some boundary layer must exist between the region of He I and the convective zone in He II, and flow charges form in this layer.

In a mathematical sense, the unsuitability of Eqs. (3.1) and (3.2) in the boundary layer is connected with the fact that terms containing ρ_s in the denominator were discarded in their derivation. Near the interphase boundary, such terms obviously cannot be small. Along with this, since ρ_s is small only in the immediate vicinity of T_c, the boundary layer should be relatively narrow. Thanks to this narrowness, the momentum and heat losses at the walls should be small within the boundary layer, which enables us to assume the energy and momentum fluxes in it to be approximately constant, an assumption made without recourse to a study of the structure of this layer.

At a temperature T that is independent of y, the interphase boundary is planar and in the angle $\varphi=T/2$ the conditions (2.13), (2.14). In correspondence with what was said above, the values of Π_{xx} and Q_x averaged over y must be taken in these equations not on the interphase boundary at $T=T_c$, but on the boundary of the convective zone, i.e., at $\tau=T_c-\delta T$, where δT is the temperature jump in the boundary layer.

From (2.13), with account of (2.11), we find

$$s\delta T=(\rho/\rho_s)(v-w)({}^1/{}_2v-{}^3/{}_2w+u). \tag{4.1}$$

In view of the smallness of δT we can set $s=s(T_c)$, $\rho_s=\rho R(\delta T/T_c)$, for ^{4}He we have $R\approx 6.6$.

In He I, on the interphase boundary, the average value over the cross section is

$$Q_x=\rho(v-u)[W_c+{}^1/{}_2(v-u)^2]+q_c, \tag{4.2}$$

where $W_c=W(T_c)$ is the specific enthalpy. On the boundary of the convective zone,

$$\begin{aligned}Q_x=&j[W-Ts-(\rho_n/2\rho)(\mathbf{v}_n-\mathbf{v}_s)^2+(v_s^2/2)]\\&+\rho Tsv_{nx}+v_{nx}(\mathbf{v}_n,\ \mathbf{j}-\rho\mathbf{v}_s)-\varkappa(dT/dx).\end{aligned} \tag{4.3}$$

Setting $W-W_c=-c\,\delta T$, eliminating $\mathbf{v}_s$ from (4.3) with the aid of (2.4), (2.5), then averaging over y and substituting the resultant expression in (4.2) we find

$$q_c=\rho s(w-v)T_c-\rho c(v-u)\delta T+\frac{\rho^2}{\rho_s}(w-u)^2\left(w-v+3\frac{v-u}{2}\right)+\frac{12\eta\varkappa}{h^2\rho s}w. \tag{4.4}$$

In (4.4), the derivative dT/dx on the boundary of the convective zone is expressed in terms of w in accord with (3.1).

Assuming $T=T_c-\delta T$ in (4.1) and (4.4), substituting in them q_c according to (2.18) and w according to (3.5) with $T=T_c-\delta T$ and eliminating δT from these two equations, we find the dependence of the velocity of the thermal wave u on the rate of flow v and on other parameters.

Figure 2 shows the approximate form of the dependence on v of the velocity $v_0=v-u$ of the wave relative to the liquid. The values $v^2<v_m^2$ are not considered, since at such small velocities T_1 in (1.1) is less than T_c and the stationary flow of He I is small. At $v=-v_m$, we have $v_0\sim(RsT_cv_m)^{1/2}\sigma^{1/2}$; as the absolute value of v increases (at $v<0$), v_0 falls off, reaching negative values if

$$h^3<108R(\varkappa T_0/\alpha s)[\eta c_1/\rho c(T_c-T_0)]^2, \tag{4.5}$$

and in the case of the opposite inequality, v_0 falls off, remaining a positive quantity, as shown in Fig. 2. For ^{4}He, the quantity on the right side of (4.5) is of the order of 10^{-10} cm^2.

In the region $v>v_m$, the velocity is $v_0<0$. At $v=v_m$ we have $v_0/v_m\approx(\sigma/\tau)-1$; then the absolute value of the velocity v_0 falls off somewhat and, passing through a minimum, increases again. The asymptotic expression at large v is

$$v_0=-{}^2/_3\sigma v. \tag{4.6}$$

The obtained solution shows that the velocity of the normal component w is always smaller in absolute value than the velocity of He I. This enables us, bearing in mind only a qualitative description of the thermal wave, to neglect the viscous heat release in He II, which is proportional to w^2.

Over almost the entire range of v (with the exception of negative values that are large in absolute value, under the condition (4.5)) the sign of v_0 is opposite that of v. The thermal wave is propagated in such a fashion that the downstream phase located is transformed into the upstream phase.

We now make some numerical estimates. At $\alpha=10^{-5}$ W/cm^2-deg, $T_0=2$ K, $h=10^{-4}$ cm, we obtain $v_m\approx3$ cm/sec; $v_0\approx1.5$ m/sec at $v=-v_m$ and $v_0\approx-4$ cm/sec at $v=v_m$. A layer of porous material with helium filtering through it can apparently serve as a system suitable for the excitation of the considered thermal waves.

We now consider a portion of He I of finite but sufficiently great length, bounded on the left and right by He II (see Fig. 3). A sufficiently large [in comparison with the quantity l, from (2.19)] length of the segment of the channel filled with He I enables us to consider the motion of the left and right boundaries independently. Let the flow velocity $v>0$ (i.e., the flow is from left to right). Relative to the right boundary, the region of He I is upstream; therefore, transformation of He II into He I occurs at the right boundary. On the left boundary, He I is converted into He II. At small flow velocities v, as is seen from Fig. 2, the rate of transformation v_0 at the left boundary is greater (in absolute value) than at the right. The region of He I will decrease and, finally, disappear. At larger v, the rate of conversion at the right boundary is larger and the mass of He I increases. Consequently, there exists a critical velocity v_{cr}, beginning with which the flow of He II in the capillary is unstable relative to local heating of sufficient intensity. At small α, the velocity v_{cr} becomes smaller than the critical velocity, corresponding to destruction of the superconductivity as a consequence of the formation of vortex rings.

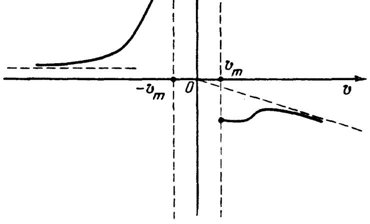

FIG. 2.

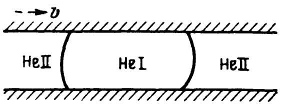

FIG. 3.

In the derivation of (4.1) and (4.4), we used the narrowness of the boundary layer in which, in contrast with the convective zone the inertial terms are important in the momentum flux. Their contribution is of the order of $(\rho^2/\rho_s)v_n^2$ and, since v_n is much smaller than the sound velocity, it should be taken into account only at small values of ρ_s/ρ, i.e., in a narrow temperature range near T_c which forms the boundary layer. Actually, according to (4.1) and (4.4), the width of this interval δT turns out to be a quantity of the order of $T_c[v/(RsT_c)^{1/2}]$ and does not exceed $\sim10^{-2}$K at reasonable values of the velocity $v\lesssim10$ cm/sec. So far as the spatial width of the boundary layer is concerned, it is a quantity of the order of

$$l_1\delta T/(T_1-T_c),$$

where l_1 is determined from (2.19): $l_1\lesssim\varkappa/\rho cv$.

The jump in the temperature at the temperature at the boundary He I – He II in a chamber of centimeter dimensions, where a region of He I developed around the heater, was observed by Peahkov.[7] Convective turbulence was generated in the He I and equalized the temperature. Undoubtedly, significant interest would attach to a study of the He I – He II boundary in a narrow capillary, where the turbulence and formation of vortices are impossible and the flow is quasi-one-dimensional. In the one-dimensional case with convective transfer of heat into the He II from the interphase boundary, the formation of a boundary layer, in which the finite interval of temperatures occurs over a length not exceeding $\varkappa/\rho cv_n\sim10^{-4}$ cm at $v_n\sim1$ cm/sec, is apparently inevitable, just as in the case considered here.

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1) One has in mind, of course, the growth as a result of the phase transition, and not of the motion of the interphase boundary along with the liquid in the laboratory system of the coordinates.

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Investigation of the nature of homotropic orientation of nematic liquid crystal molecules, and the possibility of applying it in modulation spectroscopy

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The electroreflection method has been used to investigate the spontaneous polarization of nematic liquid crystals (NLC) as a function of the orientation of the molecules. It is found that electroreflection in a NLC–semiconductor system is observed only with homotropic orientation of the molecules. The existence of the phenomenon of electroreflection is considered in its dependence on various conditions; the fixed bias, the frequency and amplitude of the modulating field, the temperature of the specimen, and the form of the modulating signal. An explanation of the observed phenomena is given from the point of view of the polar structure of the NLC. A theoretical analysis of electroreflection is made on the basis of a bipolar model of the NCL. Good verification is obtained of the experimental results and of the observed electrooptical effects of the first to sixth orders. A new technique of modulation spectroscopy, using nematic liquid crystals, is proposed.

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It is known that under certain boundary conditions it is possible, in thin layers of nematic liquid crystals (NLC), to attain homotropic orientation of the molecules.[1] Homotropic layers have been obtained and investigated[2] that originated spontaneously in specimens of a thickness less than a certain critical value during the phase transition from the isotropic phase to the nematic; their nature was associated with the polar structure of the NLC. But so far there is no direct experimental confirmation of such a structure. In the present paper, therefore, a direct test is made of the presence of polarity, and a comprehensive investigation of it is made in a homotropically orientated NLC by the method of electroreflection from a semiconductor–nematic liquid crystal boundary; also, the possibility of application of NLC for investigation of electroreflection (the Franz-Keldysh effect) is clarified. The essence of this method in the present case was as follows.

It is known that the change of the reflection coefficient of a semiconductor placed in an electric field is determined by the change of the imaginary part ϵ_i and real part ϵ_r of the permittivity. In a field of intensity E, the changes of these quantities are respectively[3]

$$\Delta\varepsilon_i(\omega,E)=\frac{B\theta^{1/2}}{\omega^2}F\left(\frac{\omega_g-\omega}{\theta}\right),\quad \Delta\varepsilon_r(\omega,E)=\frac{B\theta^{1/2}}{\omega^2}G\left(\frac{\omega_g-\omega}{\theta}\right),\tag{1}$$

and the change of the reflection coefficient is

$$\Delta R/R=\alpha(\omega)\Delta\varepsilon_i+\beta(\omega)\Delta\varepsilon_r,\tag{2}$$

where ω is the frequency of the light, $\theta=E^2/2\mu^*$; μ^* is the effective mass, $B=2|\mathbf{P}_{cv}\mathbf{n}|^2(2\mu^*)^{3/2}$, $\mathbf{P}_{cv}(\mathbf{k})$ is the matrix element of the momentum $\mathbf{P}$ between the Bloch functions of the conduction band and of the valence band for the vector $\mathbf{k}$, $\mathbf{n}$ is the unit vector along the field, $\alpha(\omega)=\partial\ln R/\partial\epsilon_i$, and $\beta(\omega)=\partial\ln R/\partial\epsilon_r$. The functions $F[(\omega_g-\omega)/\theta]$ and $G[(\omega_g-\omega)/\theta]$ are expressed in terms of the Airy functions $Ai(x)$ and $Bi(x)$ and the unit step function $H(x)$:

$$F\left(\frac{\omega_g-\omega}{\theta}\right)=\pi\left[Ai'^2\left(\frac{\omega_g-\omega}{\theta}\right)-\left(\frac{\omega_g-\omega}{\theta}\right)Ai^2\left(\frac{\omega_g-\omega}{\theta}\right)-\left(\frac{\omega-\omega_g}{\theta}\right)^{2}H\left(\frac{\omega-\omega_g}{\theta}\right)\right],$$

$$G\left(\frac{\omega_g-\omega}{\theta}\right)=\pi\left[Ai'\left(\frac{\omega_g-\omega}{\theta}\right)Bi'\left(\frac{\omega_g-\omega}{\theta}\right)-\left(\frac{\omega_g-\omega}{\theta}\right)\cdot Ai\left(\frac{\omega_g-\omega}{\theta}\right)Bi\left(\frac{\omega_g-\omega}{\theta}\right)-\left(\frac{\omega_g-\omega}{\theta}\right)^{1/2}H\left(\frac{\omega_g-\omega}{\theta}\right)\right].\tag{3}$$

The value of the field $E=\partial\phi/\partial z$ at the surface of the semiconductor in the semiconductor—NLC system is determined by the volume distribution of dipole moment $\mu(z)$; see Fig. 1 (following Ref. 2, we shall suppose that $\mu(\pm\sigma/2)=\mu_0$ and $\mu(0)=0$). The value of the potential in

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