Absorption of Rayleigh sound in normal metals and superconductors

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An analysis is made of the absorption, in normal metals and superconductors, of the surface sound whose wavelength is much less than the mean free path of electrons. The relative role of the deformation and electromagnetic absorption mechanisms is considered. An analytic formula is obtained for the latter case and this formula shows that at low sound frequencies (when the wavelength of sound is greater than, or of the order of, the skin depth in the anomalous skin effect) both contributions are of the same order. The change in the absorption coefficient on transition to the superconducting state is discussed. An experimental study of this change makes it possible to separate the electromagnetic from the deformation absorption. Analytic formulas are obtained for electromagnetic fields which appear as a result of propagation of an acoustic wave.

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Many recent papers have dealt with the surface acoustic waves in conductors. Most of this work applies to semiconductors (see, for example, the review on this subject[1]). However, very careful experimental investigations have now been made of the Rayleigh waves in metals.[2] A method suitable for such investigations has now been developed. The absorption of the Rayleigh acoustic waves in metals has been investigated theoretically by Grishin, Kaner, and Tarasov.[3-5] They studied very thoroughly the deformation absorption of the surface sound in the absence and presence of an external magnetic field. In particular, they considered the possibility of determination of the reflection coefficient of electrons from the surface of a metal by comparing the theory with experiment.

The most interesting is the study of the surface sound in the case of strong spatial dispersion when the wavelength of sound $2\pi/k$ is much less than the mean free path l of electrons ($kl \gg 1$). Then, the interaction of a surface wave with electrons may differ considerably from the bulk case.

Under experimental conditions it is usual to have $k\delta \ll 1$, where δ is the skin depth (in the anomalous skin effect). It is also well known that when this condition is satisfied, the electromagnetic contribution to the absorption of bulk sound becomes important.[6] Therefore, it is natural to expect that the electromagnetic contribution is also important in the case of transverse sound with transverse and longitudinal displacement components.

The purpose of the present paper is to investigate this contribution and to compare it with the deformation contribution. We shall show that the electromagnetic and deformation contributions may be of the same order of magnitude. On the other hand, it is well known[7] that in the case of metals which can go over to the superconducting state the electromagnetic absorption falls strongly at the transition temperature T_c because of the Meissner screening of electromagnetic fields (rapid-fall effect). Thus, measurements of the absorption coefficient of the surface sound in normal and superconducting states may be used as an experimental method for separating the two absorption mechanisms. In view of this, the last part of the paper will be devoted to a study of the absorption of surface sound in a superconductor.

For simplicity, we shall consider a metal as an isotropic elastic medium. A Rayleigh wave in such a medium can be represented by a superposition of the potential $\mathbf{U}_l$ and vortex $\mathbf{U}_t$ displacement waves[8]:

$$\mathbf{U}=\sum_{\alpha=l,t}\mathbf{U}_\alpha(0)\exp[-\varkappa_\alpha z+i(kx-\omega t)]. \tag{1}$$

We shall assume that the metal occupies the half-space $z \geq 0$ and the wave travels along the x axis parallel to the surface; $\mathbf{U}_\alpha=(U_{x\alpha}, 0, U_{z\alpha})$ is the displacement vector of the component α. The components $\mathbf{U}_l$ and $\mathbf{U}_t$ are related by the boundary condition on the surface of the metal and this condition determines the dispersion law $\omega(k)$ of the Rayleigh wave. The exponential decay factors $\varkappa_\alpha$ are given by the expression

$$\varkappa_\alpha=(k^2-\omega^2/w_\alpha^2)^{1/2}, \tag{2}$$

where w_l and w_t are, respectively, the velocities of the longitudinal and transverse bulk waves. The phase velocity of a surface wave w can be represented in the form $w=w_t\gamma$, where γ depends only on the ratio w_l/w_t and is a characteristic of a given metal with a value close to unity.

The absorption coefficient of the surface sound is[3]

$$\Gamma=\frac{1}{4W}\mathrm{Re}\int\frac{2d^2p}{(2\pi\hbar)^3}\int_0^\infty dz\,\delta\varepsilon^* f_1(z), \tag{3}$$

where

$$W=|U_x^l(0)|^2A(\gamma)\rho\omega^2k^{-1},$$

ρ is the density of the crystal, f_1 is the nonequilibrium part of the distribution function expressed in a coordi-

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nate system linked to the deformed lattice, and $\delta\epsilon$ is the correction to the Hamiltonian of the electron, proportional to the displacement in the acoustic wave and containing a term describing the interaction with the electromagnetic field.

1. NORMAL METAL

In determining the absorption coefficient which is a linear function of the displacement vector, it is necessary to solve simultaneously the transport equation for f and the Maxwell equation for the vector potential $\mathbf{A}$ of the electromagnetic field which occurs in the expression for $\delta\epsilon$. The transport equation will be written down and solved in a coordinate system linked to the deformed lattice because the interaction of an electron with an acoustic wave is simplest in this system. Moreover, the same coordinate system is the most natural means for describing all the boundary conditions on the surface. On the other hand, it is more natural to write the Maxwell equations in the laboratory coordinate system. However, since, on the one hand, the difference between the fields in these coordinate systems $\sim c^{-1}[\mathbf{U}\times\mathbf{H}]$ is less than the precision of our theory and, on the other hand, the total current density in the laboratory coordinate system is equal to the current density of electrons in the deformed coordinate system, we shall solve the Maxwell equations without distinguishing between the two coordinate systems.

In the deformed coordinate system, we have[9]

$$\delta\varepsilon(\mathbf{p},\mathbf{r},t)=\Lambda_{ik}(\mathbf{p})U^{ik}(\mathbf{r},t)-\frac{e}{c}\mathbf{A}\mathbf{v}-m_0\mathbf{v}\mathbf{U}(\mathbf{r},t). \tag{4}$$

Here, e, $\mathbf{p}$, and $\mathbf{v}$ are, respectively, the charge, momentum, and velocity of an electron, m_0 is the mass of a free electron, c is the velocity of light, $\Lambda_{ik}(p)$ is the deformation potential tensor, and

$$U^{ik}=\frac{1}{2}\left(\frac{\partial U^i}{\partial x_k}+\frac{\partial U^k}{\partial x_i}\right)$$

is the strain tensor.

We shall begin by solving the transport equation. We shall consider a metal with an isotropic spectrum[1)] and bear in mind that the strain tensor of a Rayleigh wave (because of the boundary conditions at the metal–vacuum interface) is diagonal. Then, the term $\Lambda_{ik}U^{ik}$ can be represented in the form

$$\Lambda_{ik}U^{ik}=C[(\mathbf{v}\nabla)(\mathbf{p}\mathbf{U})-{}^1/_3 p_F v_F \operatorname{div}\mathbf{U}], \tag{5}$$

where $C\sim 1$ and $\mathbf{v}=\mathbf{p}/m$.

The solution of the transport equation can conveniently be sought in the form

$$f_p=f_0(\varepsilon_p)-\frac{\partial f_0}{\partial\varepsilon_p}\chi_p(z)\exp[i(kx-\omega t)]. \tag{6}$$

Here, f_0 is the Fermi function of the total energy and $\epsilon_p=\mathcal{E}_0(\mathbf{p})+\delta\epsilon(\mathbf{p},\mathbf{r},t)$. In the linear (in respect of the displacement vector) approximation, the transport equation is

$$\hat{B}\chi_p=\dot{\varepsilon}_p, \tag{7}$$

where $\hat{B}=\partial/\partial t+\mathbf{v}\nabla+\hat{I}$ is the transport equation operator and $\hat{I}$ is the collision operator (we are assuming that the dominant scattering mechanism is that involving impurities).

The solution of Eq. (7) subject to Eqs. (4) and (5), can be represented in the form

$$\chi_p=C(\mathbf{p},\dot{\mathbf{U}}(z))+\tilde{\chi}_p. \tag{8}$$

The function $\tilde{\chi}_p$ is found from

$$\hat{B}\tilde{\chi}_p=g(z,\mathbf{v}). \tag{9}$$

Here,

$$g(z,\mathbf{v})=e\mathbf{E}^{eff}\mathbf{v}-C\frac{p_F v_F}{3}\operatorname{div}\dot{\mathbf{U}}_l(z)-CI(\mathbf{p},\dot{\mathbf{U}}),$$
$$\mathbf{E}^{eff}=\mathbf{E}-\left(C+\frac{m_0}{m}\right)\frac{m\ddot{\mathbf{U}}}{e}. \tag{10}$$

Since the average of the function g over the constant-energy surface differs from zero, the collision operator in the transport equation should be represented in the form

$$If_p=\frac{1}{\tau}f_p-\sum_{p,p_1}W_{p,p_1}f_{p_1},$$

where $W_{p_1p_1}$ is the probability of a transition of an electron from a state p to a state p_1, and $\tau=1/\nu$ the departure time. Then, Eq. (9) can be solved by iteration over the arrival term. In this case we can show that, in the lowest order in respect to $1/kl$ and w/v_F, the arrival term can be ignored in the calculation of the absorption coefficient and of the current density given by

$$\mathbf{j}=Cen_0\dot{\mathbf{U}}+\frac{2e}{(2\pi\hbar)^3}\int d^3p\,\mathbf{v}\tilde{\chi}_p=\mathbf{j}_1+\mathbf{j}_2 \tag{11}$$

(n_0 is the total electron density).

It follows that we can calculate the quantities of interest to us by solving Eq. (9). The actual form of the solution depends on the nature of the boundary conditions for the reflection of electrons from the surface of a metal. We shall give the results for specular scattering and the corresponding generalization to nonspecular scattering will be discussed later. In our approximation,

$$\tilde{\chi}_p=\frac{1}{v_z}e^{-Lz}\Big\{\theta(-v_z)\int_{-\infty}^{z}e^{Lz'}g(v_z,z')\,dz'$$
$$+\theta(v_z)\Big[-2Cp_zv_z\dot{U}_z(0)+\int_0^z e^{Lz'}g(v_z,z')\,dz'$$
$$-\int_{-\infty}^{0}e^{-Lz'}g(-v_z,z')\,dz'\Big]\Big\}, \tag{12}$$

where $L=(ikv_x+\nu)/v_x$, and θ is the Heaviside function. The presence of the term $-2Cp_zv_z\dot{U}_z(0)$ in Eq. (12) is associated with the boundary conditions imposed on the complete function χ_p. The field $\mathbf{E}^{eff}$ can be determined by solving the Maxwell equations using the expression for the current (11). It is clear from Eqs. (11) and (12) that the relationship between the current and field is

nonlocal. Therefore, to solve the Maxwell equations, we shall continue formally $\mathbf{E}^{eff}(z)$ and $\mathbf{U}(z)$ over the whole space. In this case the x components of $\mathbf{E}^{eff}$ and $\mathbf{U}$ are continued in an even manner and the z components in an odd manner:

$$K_z(z)=K_z(|z|)\,\mathrm{sign}\,z, \quad K_x(z)=K_x(|z|), \tag{13}$$

where K is any vector.

We can show that such continuation ensures the vortical nature of $\mathbf{E}^{eff}$ and $\mathbf{U}_t$ and the potential nature of $\mathbf{U}_l$ throughout all space. We note that the z components of the continued fields lose their continuity over the whole space because they may have a discontinuity at the point $z=0$ (in the same way as the derivatives of the x components of the same fields). Therefore, all the differential equations should be redefined in order to remove nonphysical sources on the interface associated with the nature of our continuation. For example, the divergence has to be defined so that

$$\mathrm{Div}\,\mathbf{K}=\mathrm{div}\,\mathbf{K}-2\delta(z)K_z(z)\,\mathrm{sign}\,z. \tag{14}$$

Then, the relationship between the current and the field $\mathbf{E}^{eff}$ can be described by the difference kernel:

$$\begin{aligned} j_{zx}&=eN(\varepsilon_F)\int dS_F^+\Big\{-2Cp_zv_xe^{-L|z|}U_x(+0)\\ &+\int_{-\infty}^{\infty}dz'e^{-L|z-z'|}\frac{v_x}{v_z}[g_+(z',v_z)+g_-(z',v_z)\,\mathrm{sign}(z-z')]\Big\},\\ j_{zz}&=eN(\varepsilon_F)\int dS_F^+\Big\{-2Cp_zv_ze^{-L|z|}U_z(+0)\,\mathrm{sign}\,z\\ &+\int_{-\infty}^{\infty}dz'e^{-L|z-z'|}[g_-(z',v_z)\,\mathrm{sign}(z-z')+g_-(z',v_z)]\Big\}. \end{aligned} \tag{15}$$

Here, $\int dS_{F^+}$ denotes integration over half the Fermi surface for $v_z\geq 0$; $g_+(z,v_z)$ and $g_-(z,v_z)$ are, respectively, the even and odd (with respect to z and v_z) parts of the function $g(z,v_z)$; $N(\epsilon_F)=2p_F^2/(2\pi\hbar)^3v_F$ is the density of states on the Fermi surface.

It should be noted that this procedure can be carried out also in the case of nonspecular reflection of electrons from the surface described by the boundary condition

$$\rho\chi(0,v_z)=\chi(0,-v_z), \quad v_z>0.$$

Then, instead of Eq. (13), it is sufficient to define all the fields and displacements outside the metal in accordance with the rule:

$$K_x(z)=\rho^{-1}K_x(|z|), \quad K_z(z)=-\rho^{-1}K_z(|z|), \quad z\leq 0.$$

This transformation results in a slight change in the subsequent expressions. However, several experiments have indicated[10] that the reflection of electrons from the surface of a single crystal is specular over a wide range of angles. Therefore, we shall give the actual results for the specular reflection case.

The existence of the difference kernel makes it possible to solve the Maxwell equations by expanding them as Fourier series in terms of the variable z. The Fourier component of the current is

$$j_i(q)=Ce n_0U_i(q)+\sigma_{ik}(q)E_k^{eff}(q)+F_i(q)\,\mathrm{Div}\,U_t(q)+S_i(q)U_z(+0), \tag{16}$$

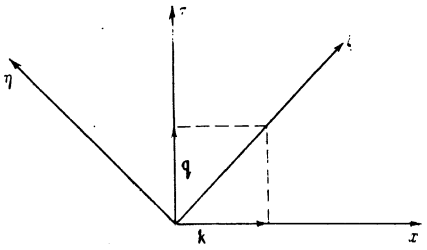

FIG. 1.

where

$$\sigma_{ik}(q)=e^2N(\varepsilon_F)\int dS_F^+\frac{v_iv_k}{v_z}\frac{2\,\mathrm{Re}\,L}{(q+\mathrm{Im}\,L)^2+\mathrm{Re}^2L}, \tag{17}$$

$$F_i(q)=\frac{2}{3}iCeN(\varepsilon_F)p_Fv_F\int dS_F^+\frac{v_i}{v_z}\frac{q+\mathrm{Im}\,L}{(q+\mathrm{Im}\,L)^2+\mathrm{Re}^2L}, \tag{18}$$

$$S_i(q)=4iCeN(\varepsilon_F)\int dS_F^+p_zv_i\frac{q+\mathrm{Im}\,L}{(q+\mathrm{Im}\,L)^2+\mathrm{Re}^2L}. \tag{19}$$

We can easily show that $\sigma_{ik}(q)$ is degenerate. This is due to the fact that the x and z components of the field and current are connected by the solenoidal conditions. However, we can see that each Fourier component has its own coordinate system in which this matrix has one nonzero matrix element. This coordinate system is shown in Fig. 1. If we apply these coordinates, we find that

$$\begin{aligned} F_i&=iCen_0\delta_{i\zeta}/(q^2+k^2)^{1/2},\\ S_i&=2iCen_0\frac{1}{(q^2+k^2)^{1/2}}\left(\delta_{i\zeta}+\frac{2kq}{q^2+k^2}\delta_{i\eta}\right), \end{aligned} \tag{20}$$

and the only nonzero component of the conductivity tensor is

$$\sigma_{\eta\eta}=\frac{3\pi e^2}{4p_F}\frac{n_0}{(q^2+k^2)^{1/2}}. \tag{21}$$

We can easily show that $j_\zeta=0$ and the Maxwell equations for E_ζ^{eff} are satisfied identically. The equation for E_η^{eff} is

$$\begin{aligned} &\left(1+\frac{q^2}{k^2}-3\pi i\frac{w}{4v_F}\frac{1}{(k\lambda_0)^2(1+q^2/k^2)^{1/2}}\right)E_\eta^{eff}(q)\\ &-\frac{2q/k^2}{(1+q^2/k^2)^{1/2}}E_x'(+0)=\frac{4\pi\omega^2}{c^2k^2}Cen_0\Big[U_\eta(q)\\ &+\frac{4ikq}{(k^2+q^2)^{1/2}}U_z(+0)\Big]+\left(1+\frac{q^2}{k^2}\right)\left(C+\frac{m_0}{m}\right)\frac{mU_\eta(q)}{e}. \end{aligned} \tag{22}$$

Here, $\lambda_0^2=mc^2/4\pi e^2n_0=c^2/\omega_p^2$ is the London depth of penetration at $T=0$ and

$$E_x'(0)=\lim_{z\to+0}\frac{dE_x(z)}{dz}.$$

A comparison of the last term on the right-hand side of Eq. (22) with the first one readily shows that their ratio is $\sim(k\lambda_0)^2$. Therefore, we can ignore the last term, representing the Steward-Tolman effect, up to frequencies $w/\lambda\sim 10^{11}$ sec^{-1}. In subsequent analysis we shall assume that the condition $k\lambda_0\ll 1$ is satisfied. An analysis of the boundary conditions on the metal—vacuum interface shows that, in this range of frequencies, $E_x'(+0)=0$ and $E_z(+0)=0$.

In the calculation of the absorption coefficient it is also convenient to continue the fields and displacements over the whole space. The absorbed power consists

of the deformation and electromagnetic contributions. The application of Eqs. (3) and (22) gives the results

$$\Gamma_{def}=\frac{N(\varepsilon_F)}{8\pi W}\mathrm{Re}\int_{-\infty}^{\infty}dq\,U_{ik}U_{jm}^{*}\int dS_F^{+}\Lambda_{ik}\Lambda_{jm}\frac{\nu}{\nu^2+(kv_x+qv_x)^2}, \tag{23}$$

$$\Gamma_{em}=\frac{1}{16\pi W}\mathrm{Re}\int_{-\infty}^{\infty}dq\,E_i(q)E_k^{*}(q)\sigma_{ik}(q). \tag{24}$$

The most interesting case is when $k\delta\ll 1$, because it is in this situation that the electromagnetic absorption is important. Then, on the left-hand side of Eq. (22) we can leave only the third term in parentheses. Then,

$$\Gamma_{em}=\frac{C^2mn_0kv_F}{3\pi^2\rho A(\gamma)}\left[\frac{|U_x^t(0)|^2}{|U_x^l(0)|^2}\left(1+\frac{\varkappa_t^4}{k^4}\right)I_1(1)+4I_2(1)\right], \tag{25}$$

where

$$I_1(f)=\int_{-\infty}^{\infty}dy\,f(y)\frac{y^2}{(1+y^2)^{1/2}(\varkappa_t^2/k^2+y^2)^2},$$

$$I_2(f)=\int_{-\infty}^{\infty}dy\,f(y)\frac{y^2}{(1+y^2)^{3/2}},\quad I_2(1)=\frac{4}{3}.$$

The asymptotic form of this expression for $\alpha=1-\gamma\ll 1$ is

$$\Gamma_{em}=\frac{C^2mn_0kv_F}{36\pi^2\rho\alpha}\frac{1}{1-(w_t/w_l)^2}. \tag{26}$$

The formula for Γ_{def} was first obtained by Grishin and Kaner[3] so that we shall simply give the asymptote of α obtained from Eq. (23)

$$\Gamma_{def}\approx 0.07\frac{C^2n_0mkv_F}{\rho}\frac{1}{1-(w_t/w_l)^2}. \tag{27}$$

Hence, we can see that

$$\Gamma_{em}/\Gamma_{def}\approx 0.04\alpha^{-1}. \tag{28}$$

Since α varies from 0.14 to 0.05, this ratio can range from 0.3 to 1.

It follows that the electromagnetic absorption in the $k\delta\ll 1$ case is generally of the same order as the deformation absorption. This means that in analyzing the experimental results at frequencies lower than $f\sim w/\delta\sim 300$ MHz one has to allow for the electromagnetic absorption.

2. SUPERCONDUCTOR

As pointed out earlier, it is convenient to use the measured value of the change in the absorption coefficient on transition through the superconducting temperature in experimental separation of the electromagnetic and deformation contributions to the absorption.

To analyze the situation, we shall consider a fairly pure superconductor and sound of moderately high frequency:

$$\hbar\omega\ll\Delta(T),\quad l\gg\xi_0,\quad k\xi\ll 1, \tag{29}$$

where $\Delta(T)$ is the superconductor gap, $\xi=\hbar v_F/\pi\Delta(T)$ is the coherence length, and $\xi_0=\xi(T=0)$. When the conditions of Eq. (29) are obeyed, we can apply the transport equation for quasiparticle excitations[11] which is

$$\frac{\partial n_p}{\partial t}+\frac{\partial\tilde{\varepsilon}_p}{\partial\mathbf{p}}\frac{\partial n_p}{\partial\mathbf{r}}-\frac{\partial\tilde{\varepsilon}_p}{\partial\mathbf{r}}\frac{\partial n_p}{\partial\mathbf{p}}+In_p=0, \tag{30}$$

where[2)]

$$\tilde{\varepsilon}_p=\varepsilon+\mathbf{p}_s\mathbf{v}-m_0\mathbf{v}\dot{\mathbf{U}},\quad \varepsilon(\mathbf{p},\mathbf{r})=\{[\mathscr{E}_0(\mathbf{p})+\Lambda_{ik}U^{ik}-\mu]^2+\Delta^2\}^{1/2}, \tag{31}$$

$$\mathbf{p}_s=-\frac{e}{c}\mathbf{A}. \tag{32}$$

The collision operator of the transport equation for the scattering by impurities reduces, on the basis of the same considerations as for a normal metal, to the outgoing term equal to $\nu|\xi_p|/\epsilon_p$, where $\xi_p=E_0(p)-\mu$. Bearing this point in mind, we can represent the distribution function in the form

$$n_p=f_0(\tilde{\varepsilon}_p)-C(\mathbf{p},\dot{\mathbf{U}})\frac{\partial f_0}{\partial\varepsilon}-\tilde{\chi}\frac{\partial f_0}{\partial\varepsilon}, \tag{33}$$

where $\tilde{\chi}$ is found from the equation

$$\left(\frac{\partial}{\partial t}+\frac{\xi_p}{\varepsilon_p}(\mathbf{v},\nabla)+\nu\frac{|\xi_p|}{\varepsilon_p}\right)\tilde{\chi}=e\mathbf{E}^{el}\mathbf{v}-C\frac{\xi_p}{\varepsilon_p}\frac{p_Fv_F}{3}\mathrm{div}\,\dot{\mathbf{U}}^t. \tag{34}$$

The electric current density can be expressed in terms of $\tilde{\chi}$ and $\mathbf{v}_s$:

$$\mathbf{j}=e\sum_{p,\sigma}\mathbf{v}\tilde{\chi}\left(-\frac{\partial f_0}{\partial\varepsilon}\right)+eN_s\left(\mathbf{v}_s-\frac{m_0}{m}\dot{\mathbf{U}}\right)+CeN_n\dot{\mathbf{U}}. \tag{35}$$

Here, $\mathbf{v}_s=\mathbf{p}_s/m$, and N_n and N_s are the densities of the "normal" and "superconducting" electrons:

$$N_n=n_0\int_0^{\infty}\mathrm{ch}^{-2}\left[x^2+\left(\frac{\Delta}{2T}\right)^2\right]^{1/2}dx,\quad N_s=n_0-N_n.$$

The rest of the calculation is fully analogous to that given above for a normal metal. The expression for F_i and S_i are fully analogous to Eqs. (18) and (19) and differ only by the replacement of n_0 with N_n. In the calculation of the electrical conductivity tensor $\sigma_{ik}(q)$, ignoring the time dispersion, a divergence occurs at the lower limit of the energy integral:

$$\frac{1}{2T}\int_{\Delta}^{\infty}d\varepsilon\frac{\varepsilon^2}{\varepsilon^2-\Delta^2}\mathrm{ch}^{-2}\frac{\varepsilon}{2T}.$$

This divergence is removed by allowing for the time dispersion (the corresponding parameter which follows from the transport equation is $\xi\sim w\Delta/v_F$). Consequently, at temperatures close to T_c the tensor σ_{ik} acquires the following factor[12]

$$g(T)=1+\frac{\Delta}{2T}\ln\left(\beta\frac{Tv_F}{\Delta w}\right). \tag{36}$$

It should be noted that for $k\xi\gg 1$ the corresponding expression cannot be obtained from the transport equation and should be determined by a quantum-mechanical calculation. Then,[13]

$$g(T)=1+\frac{\Delta}{2T}\left(\ln\frac{8\Delta}{\hbar\omega}-1\right). \tag{37}$$

When $k\xi_0 \gg 1$, there is also a change in the expression for N_s. In this range, we have[14]

$$\frac{N_s(q)}{n_0} = \frac{3\pi^2}{4\xi_0} \frac{1}{(q^2+k^2)^{1/2}} \frac{\Delta(T)}{\Delta(0)} [1-2f_0(\Delta)]. \quad (38)$$

The electric field in a superconductor (for $k\delta \ll 1$) at a temperature near T_c is [compare with Eq. (22)]

$$E_n(q) = \left[\frac{N_s}{n_0} - \frac{3\pi i}{4} \frac{w}{v_F} g(T) \frac{1}{(1+q^2/k^2)^{1/2}} \right]^{-1} \times \frac{C\omega^2 m}{e} \left[U_n(q) + \frac{4iqU_s(+0)}{k^2(1+q^2/k^2)^{1/2}} \right]. \quad (39)$$

We can see that the electromagnetic absorption is important only in the immediate vicinity of T_c, where $N_s/n_0 \lesssim w/v_F$ (region of the rapid fall effect). In this region it is governed by the expressions (25) and (26) with

$$I_j \left\{ \left[1+ \left(\frac{4}{3\pi g} \frac{N_s}{n_0} \frac{v_F}{w} \right)^2 (1+y^2) \right]^{-1} \right\}, \quad j=1,2$$

and with the factor g^{-1} in front of the formula. Thus, in the limit $T \to T_c$, we have $\Gamma^s_{em}/\Gamma_{em} \to 1$ and when the temperature is lowered, the value of $\Gamma^s_{em}/\Gamma_{em}$ drops rapidly:

$$\Gamma_{em}^s/\Gamma_{em} \sim g \left(\frac{3\pi}{4} \frac{n_0}{N_s} \frac{w}{v_F} \right)^2. \quad (40)$$

Far from T_c we have only the deformation contribution Γ^s_{def}, which is equal to $2f_0(\Delta)\Gamma_{def}$.

It follows from the above analysis that an investigation of the absorption of the surface sound in a normal metal and in a superconductor allows us to deduce, from the sudden change in the absorption at the temperature T_c, the separate electromagnetic and deformation contributions to the absorption, which are of the same order for $k\delta \ll 1$.

We wish to draw attention to the fact that there is one other method for investigating the electromagnetic interaction of surface sound with conduction electrons. This method is based on the direct recording of electromagnetic fields near the surface of a sample. Such a method has been used in a study of the propagation of transverse bulk waves in superconducting tin.[15] Therefore, we shall give an estimate for the amplitude of the resultant magnetic fields. A typical estimate of the field (for $k\delta \ll 1$) gives

$$H \sim mcv_F k U^{ik}/e.$$

For typical metal parameters $U^{ik} \sim 10^{-5}$ (corresponding to sound intensity of the order of 1 W/cm^2) and $k \sim 10^3$ cm^{-1}, we have $H \sim 0.1$ Oe. In the superconducting range of temperatures this field decreases by a factor of $1+N_s v_F/n_0 w$. Fields of this kind can be recorded by modern detectors.

[1)] In the case of an anisotropic spectrum, the results obtained below can be regarded as order-of-magnitude estimates.

[2)] In fact, Eq. (31) includes an additional term $\hbar\dot{\varphi}$ in the parentheses (in the radicand) and Eq. (32) includes the term $\frac{1}{2}\hbar\nabla\varphi$, where φ is the phase of the order parameter. However, since $w/v_F \ll 1$, the first term can be ignored and the second can be removed by a gauge transformation.

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