Effect of electric field on exciton absorption

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The influence of an electric field on the absorption of light in semiconductors is investigated with account taken of the Coulomb interaction of the electron and hole. The electric field is assumed to be quasiclassical. It is shown that the Coulomb interaction leads to the appearance of a Sommerfeld factor in the continuous spectrum, just as in the absence of an electric field, and alters the character of the oscillations of the light-absorption coefficient in an electric field in such a way that the oscillations assume a sawtooth form near the edge of the continuous spectrum. Expressions for the light absorption coefficient in the forbidden band are derived and analyzed both far from the resonances and in their immediate vicinity.

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1. INTRODUCTION

The Keldysh-Franz effect has been the subject of many both theoretical and experimental studies. From the very outset it was clear that the Coulomb interaction between the produced electron and hole should alter the effect substantially. The undertaken numerical calculations provided practically no information on the influence of the interaction on the absorption in an electric field.

Substantial progress in this direction was made by Merkulov and Perel',[1,2] who obtained the asymptotic light absorption coefficient at large proton-energy deficits, and investigated the change of the absorption in transitions of the exciton to its ground state in an electric field. The principal results of these studies can be summarized briefly as follows: at large energy deficits the Coulomb interaction alters the pre-exponential factor in the asymptotic expression for the Keldysh-Franz effect, both as a result of the change of the wave function over short distances and as a result of the lowering of the potential barrier. At photon energies corresponding to exciton production in the ground state, the absorption has a resonant character. The width of the maximum is determined by the probability of ionization of the exciton by the electric field. However, the question of how the absorption varies in the continuous spectrum remains open. Notice should be taken here of the numerical calculations made by Blossey,[3] who investigated these questions.

In the present paper we construct a theory of exciton absorption in an electric field for both the continuous and discrete spectra; we also recalculate the results of Merkulov and Perel' for large energy deficits and refine the criteria for their applicability. Our method of solution differs from that of Merkulov and Perel' and is close to the method used to calculate the probability of ionization of the hydrogen atom (e.g.,[4]).

2. ABSORPTION OF LIGHT BY AN EXCITON IN THE CONTINUOUS SPECTRUM

A) Absorption spectrum far from the threshold. As usual, the light-absorption coefficient is proportional to the square, averaged over all possible states, of the modulus of the wave function of the electron-hole pair with coincident coordinates of the hole and electron:

$$K=K_0|\Psi(0)|^2. \tag{1}$$

The wave function of the pair is described by the Schrödinger equation

$$(\tfrac{1}{2}\Delta+1/r-Fz)\Psi(\mathbf{r})=-E\Psi(\mathbf{r}). \tag{2}$$

Here $E=\hbar\omega-E_g/2R_0$, $F=e\mathcal{E}a_0/2R_0$, a_0 is the exciton radius, r is the relative coordinate of the electron and hole expressed in units of a_0, and R_0 is the binding energy of the exciton.

It is well known that the variables in Eq. (2) separate in parabolic coordinates.[4] The light-absorption coefficient receives contributions only from states with azimuthal quantum number $m=0$, inasmuch as $\Psi(0)=0$ for other m. Equation (2) reduces to the two equations[4]

$$\chi_1''+\left(\frac{E}{2}+\frac{\beta_1}{\xi}+\frac{1}{4\xi^2}-\frac{F}{4}\xi\right)\chi_1=0,$$
$$\chi_2''+\left(\frac{E}{2}+\frac{\beta_2}{\eta}+\frac{1}{4\eta^2}+\frac{F}{4}\eta\right)\chi_2=0. \tag{3}$$

Here $\beta_1+\beta_2=1$, and

$$\Psi(r)=f_1(\xi)f_2(\eta)=\frac{\chi_1(\xi)}{\sqrt{\xi}}\frac{\chi_2(\eta)}{\sqrt{\eta}}. \tag{4}$$

The effective potentials $U_1(\xi)$ and $U_2(\eta)$ are shown in Fig. 1.

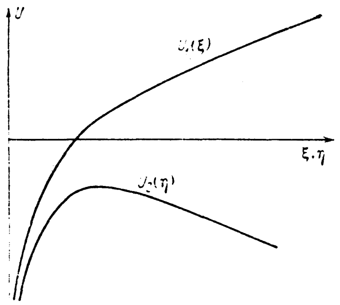

FIG. 1. Effective potentials for finite and infinite motions.

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To solve Eqs. (3) we use the fact that at short distances we can neglect the electric field and write down the exact wave functions in a Coulomb potential. If the influence of the electric field becomes substantial only in a region where the motion is quasiclassical, then it is possible, at short distances, to join together the quasiclassical Coulomb function with the quasiclassical function in which the electric field is taken into account. The condition for the applicability of the quasiclassical approximation is of the form

$$\frac{U_2'(\eta)}{k^3(\eta)}=\left(\frac{\beta_2}{2\eta^2}+\frac{1}{2\eta^3}-\frac{F}{4}\right)\times\left(\frac{k^2}{4}+\frac{\beta_2}{\eta}+\frac{1}{4\eta^2}+\frac{F}{4}\eta\right)^{-3/2}\ll 1, \tag{5}$$

where $E=k^2/2$.

It is seen from (3) that the influence of the field can be neglected if

$$F\eta/4\ll\max\{1/\eta^2,\ \beta^2/\eta,\ k^2/4\}. \tag{6}$$

It will be shown below that the significant values are $\beta_2\sim k$. Then (5) and (6) lead to the following condition for the applicability of the solution method described above:

$$k^3/F\gg 1. \tag{7}$$

This inequality is the only restriction of our theory. It requires that the external field be quasiclassical and imposes no limitations on the Coulomb energy.

The normalization conditions for the functions of the infinite and finite motion $\chi_2(\eta,\beta_2)$ and $\chi_1(\xi,\beta_1)$ are

$$\int_0^\infty \chi_{2,E'}(\eta,\beta_2')\chi_{2,E}(\eta,\beta_2)\,d\eta=\frac{2}{\pi}\delta_{\beta_2',\beta_2}\delta(E-E'), \tag{8}$$

$$\int_0^\infty \frac{d\xi}{\xi}\chi_1^2(\xi,\beta_1)=1. \tag{9}$$

Under the condition (6) we have

$$f_2(\eta,\beta_2)=f_2(0,\beta_2)e^{-ik\eta/2}F(1/2+i\beta_2/k,\ 1,\ ik\eta). \tag{10}$$

If furthermore $k\eta\gg 1$, then

$$f_2(\eta,\beta_2)=f_2(0,\beta_2)\left[\frac{2}{\pi k\eta}\left(1+\exp\left\{-\frac{2\pi\beta_2}{k}\right\}\right)\right]^{1/2}\times\cos\left(\frac{k\eta}{2}+\frac{\beta_2}{k}\ln k\eta-\frac{\pi}{4}-\delta\left(\frac{\beta_2}{k}\right)\right), \tag{11}$$

where $\delta(z)=\arg\Gamma(\frac{1}{2}+iz)$.

In the region $\eta>(k/F)^{1/2}$ it is necessary to take into account the field term, so that the function (11) goes over continuously into the quasiclassical function in an electric field. The normalization condition (8) yields directly

$$|f_2(0,\beta_2)|^2=\frac{1}{\pi(1+\exp\{-2\pi\beta_2/k\})}. \tag{12}$$

It is seen from (12) that $|f_2(0,\beta_2)|$ does not depend at all on the electric field, for owing to the smallness of the above-barrier reflection the function of the infinite motion can be normalized by calculating the probability flux at any point where the quasiclassical condition (5) is satisfied, including a point in which the electric field is no longer of significance.

For the finite motion, the conditions (5) and (6) also lead to the inequality (7). Therefore, acting in analogy with the preceding case, we can represent $f_1(\xi,\beta_1)$ in the region $1\ll k\xi\ll(k^3/F)^{1/2}$ in the form

$$f_1(\xi,\beta_1)=f_1(0,\beta_1)\left[\frac{1+\exp\{-2\pi\beta_1/k\}}{\pi\xi k(\xi)}\right]^{1/2}\cos S(\xi), \tag{13}$$

where

$$S(\xi)=\frac{k\bar\xi}{2}+\frac{\beta_1}{k}\ln k\bar\xi-\frac{\pi}{4}-\delta\left(\frac{\beta_1}{k}\right)+\int_{\bar\xi}^{\xi}d\xi'\left(\frac{k^2}{4}+\frac{\beta_1}{\xi'}-\frac{F\xi'}{4}\right)^{1/2}, \tag{14}$$

and $k(\xi)=(k^2/4+\beta_1/\xi-F\xi/4)^{1/2}$, with $\bar\xi$ an arbitrary value of ξ from the interval $1/k\ll\xi\ll(k/F)^{1/2}$.

The condition for quantization in the potential $U_1(\xi)$ is of the form

$$S(\xi_2)=(l+1/4)\pi, \tag{15}$$

where $\xi_2=k^2/F$ is the right-hand classical turning point.

Far from the edge, as we shall show, an important role is also played in the action $S(\xi_2)$ by values $\beta_1\sim k$. Then (14) and (15) yield

$$\pi l=\frac{k^3}{3F}+\frac{\beta_1}{k}\ln\frac{4k^3}{F}-\delta\left(\frac{\beta_1}{k}\right)-\frac{\pi}{2}. \tag{16}$$

We shall need henceforth the quasiclassical state density

$$\nu(\beta_1)=\frac{\partial l}{\partial\beta_1}=\frac{1}{\pi k}\left\{\ln\frac{4k^3}{F}-\frac{1}{2}\left[\psi\left(\frac{1}{2}+i\frac{\beta_1}{k}\right)+\psi\left(\frac{1}{2}-i\frac{\beta_1}{k}\right)\right]\right\}, \tag{17}$$

where

$$\psi(x)=\frac{d}{dx}\ln\Gamma(x).$$

To calculate $f_1^2(0,\beta_1)$ we use the normalization condition (9), and break up the entire region $0\leqslant\xi\leqslant\xi_2$, of integration with respect to ξ into two regions: 1) $0\leqslant\xi\leqslant\bar\xi$ and 2) $\bar\xi\leqslant\xi\leqslant\xi_2$. In the integration over the first region it is necessary, by virtue of the condition (6), to use the exact Coulomb function, while in the second region it is necessary to use the quasiclassical function in the electric and Coulomb fields. As a result of simple calculations, using expression (17) for $\nu(\beta_1)$, we obtain

$$f_1^2(0,\beta_1)=\frac{1}{\nu(\beta_1)}\frac{1}{1+\exp(-2\pi\beta_1/k)}. \tag{18}$$

We note that expression (18), or more accurately its connection with the state density, would be trivial if only the quasiclassical regions were to contribute to the normalization integral and to the action. In our case, however, the quasiclassical region also makes a substantial contribution [the second term in (17)]

$$|\Psi(0)|^2=\sum_{\beta_1+\beta_2=1}|f_1(0,\beta_1)f_2(0,\beta_2)|^2. \qquad (19)$$

To calculate the sum we use the Poisson formula

$$|\Psi(0)|^2=\int_{-\infty}^{+\infty}|f_1(0,\beta_1)f_2(0,1-\beta_1)|^2\nu(\beta_1)d\beta_1$$
$$+2\,\mathrm{Re}\sum_{m=1}^{\infty}\int_{-\infty}^{+\infty}d\beta_1\nu(\beta_1)\exp\{2\pi iml(\beta_1)\}|f_1(0,\beta_1)f_2(0,1-\beta_1)|^2. \qquad (20)$$

The integration in (20) is carried out not from the end point of the spectrum $\beta_1^{(0)}=-k^4/16f$, but from $-\infty$, since allowance for the fact that $\beta_1^{(0)}$ is finite leads to a correction on the order of $\exp(-k^3/F)$, which is smaller than the error in the quasiclassical approach.

Substituting in (20) the explicit expressions (12), (18), and (16), we obtain after simple transformations

$$|\Psi(0)|^2=\frac{1}{\pi}\frac{1}{1-\exp(-2\pi/k)}-\frac{k}{\pi^2}\mathrm{Re}\exp\left(i\frac{2}{3}\frac{k^3}{F}\right)$$
$$\times\int_{-\infty}^{+\infty}dt\frac{\Gamma^2(1/2-it)}{1+\exp[2\pi(t-1/k)]}\exp\left(\pi+2i\ln\frac{4k^3}{F}\right)t. \qquad (21)$$

Going around the point 0 in the upper half of the complex t-plane along a semicircle of radius $1\ll R\ll k^3/F$ and neglecting, relative to the parameter $(k^3/F)^{-1}$, both the integral along this semicircle and the residues of all the poles of the integrand that lie inside the contour, except the one closest to the real axis, we obtain

$$K=K_0\frac{1}{\pi(1-e^{-2\pi/k})}\left\{1-\frac{F}{2k^3}\cos\left[\frac{2}{3}\frac{k^3}{F}+\frac{2}{k}\ln\frac{4k^3}{F}+2\arg\Gamma\left(1-\frac{i}{k}\right)\right]\right\}. \qquad (22)$$

We have neglected all $m>1$ relative to the same parameter $(k^3/F)^{-1}$. In dimensional variables we have

$$k=\left(\frac{\hbar\omega-E_g}{R_0}\right)^{1/2},\quad \frac{k^3}{F}=\left(\frac{\hbar\omega-E_g}{\Theta}\right)^{3/2},$$

where

$$\Theta=(e\mathscr{E}\hbar)^{2/3}/(2m^*)^{1/3}.$$

B) Absorption near the threshold. We now consider the light absorption spectrum at $k\ll1$. We assume, as before, that k^3/F is large and show that in this region the principal parameter is the quantity k^4/F, so that at $k^4/F\gg1$ the results obtained for $k\gtrsim1$ can be joined together with the results obtained for $k\ll1$.

In the case of infinite motion, expression (12) was obtained by us without restrictions on the value of k. For finite motion, the form of formula (18) is likewise preserved. All that changes is expression (17) for the state density $\nu(\beta_1)$. In fact, the quantization condition (16) was derived under the condition $\beta_1\sim k$, which was verified in the course of the integration in (21). On the other hand, in the case $k\ll1$, the important values in the integration are $\beta_1\sim1$, so that the quantization condition must be derived anew.

At $\xi\ll\beta_1/k^2$, $(\beta_1/F)^{1/2}$, the wave function in the Coulomb field is

$$f_1(\xi)=f_1(0)J_0(2\sqrt{\beta_1\xi}). \qquad (23)$$

If furthermore $\xi\gg1/\beta_1$, then we can write the asymptotic form of (23):

$$f_1(\xi)\approx f_1(0)\pi^{-1/2}(\beta_1\xi)^{-1/4}\cos[2(\beta_1\xi)^{1/2}-\pi/4]. \qquad (23a)$$

It is seen that (23a) is the quasiclassical wave function with

$$k(\xi)=\left(\frac{\beta_1}{\xi}\right)^{1/2},\quad S(\xi)=\int_0^{\xi}d\xi'k(\xi').$$

Proceeding further to values of ξ where the electric field comes into play, and matching (23a) with the quasiclassical function in this region, we find that

$$S(\xi)=\int_0^{\xi}(k^2/4+\beta_1/\xi'-F\xi'/4)^{1/2}d\xi'. \qquad (24)$$

The quantization condition takes the form

$$S(\xi_1)=\pi(l+1/2),$$

where

$$\xi_1=\frac{k^2}{2F}\left[1+\left(1+\frac{16\beta_1F}{k^4}\right)^{1/2}\right]$$

is the right-hand turning point. After simple calculations we obtain for $S(\xi_1)$

$$S(\xi_1)=\frac{k^3}{3F}\left(\frac{1+(1+x)^{1/2}}{2}\right)^{3/2}(1+y)^{1/2}$$
$$\times\left\{yK\left(\frac{1}{(1+y)^{1/2}}\right)+(1-y)E\left(\frac{1}{(1+y)^{1/2}}\right)\right\}; \qquad (25)$$

here

$$y=\frac{(1+x)^{1/2}-1}{(1+x)^{1/2}+1},\quad x=\frac{16F}{k^4}\beta_1=\frac{\beta_1}{\gamma k}=x_0\beta_1;$$

$K(z)$ and $E(z)$ are complete elliptic integrals of the first and second kind, respectively.

The state density is of the form

$$\nu(\beta_1)=\frac{\partial l}{\partial\beta_1}=\frac{1}{\pi}\frac{\partial S(\xi_1)}{\partial\beta_1}=\frac{2}{\pi k}(1+x)^{-1/4}K\left(\frac{1}{(1+y)^{1/2}}\right). \qquad (26)$$

The condition $\beta_1\gg k$ is equivalent to the condition $x\gamma\gg1$. By direct calculation we can again verify that the normalization integral, as before, is proportional to $\nu(\beta_1)$, and therefore, in accordance with (21),

$$|\Psi(0)|^2=\frac{1}{\pi}+\sum_{m=1}^{\infty}(-1)^m\frac{2}{\pi x_0}\mathrm{Re}\int_{-\infty}^{+\infty}dx\exp\left[i\frac{32}{3}\gamma m\varphi(x)\right]$$
$$\times[1+\exp(-2\pi\gamma x)]^{-1}[1+\exp\{-2\pi\gamma(x_0-x)\}]^{-1}, \qquad (27)$$

where $\varphi(x)=(3F/k^3)S(\xi_1)$. At $x\ll1$ we have

$$\varphi(x)=1+{}^3/_{16}x[\ln(64/x)+1], \qquad (28a)$$

and at $x\gg1$

$$\varphi(x)\approx{}^1/_2K(2^{-1/2})x^{3/4}+{}^3/_2x^{1/4}[E(2^{-1/2})-{}^1/_2K(2^{-1/2})]. \qquad (28b)$$

Therefore in the upper half-plane the integrand in (27) attenuates exponentially, and in the calculation of the integral we can close the contour in the upper half-plane. It turns out that there are two sets of poles inside the contour:

$$x_j=\frac{i}{\gamma}\left(j+\frac{1}{2}\right), \quad x_n=x_0+\frac{i}{\gamma}\left(n+\frac{1}{2}\right); \quad j,n=0,1,2\ldots$$

The residues of the poles of the first set are small in the parameter $k \ll 1$ in comparison with the corresponding residues of the second set, and can be neglected if $k \ll 1$, so that the expression under the summation sign with respect to m takes the form

$$A_m=(-1)^m\frac{2k}{\pi}\,\mathrm{Im}\sum_{n=0}^{\infty}\exp\left\{i\frac{32}{3}\gamma m\varphi\left[x_0+\frac{i}{\gamma}\left(n+\frac{1}{2}\right)\right]\right\}. \quad (29)$$

At small k, the expression in the exponential can be expanded in the form:

$$\varphi\left[x_0+\frac{i}{\gamma}\left(n+\frac{1}{2}\right)\right]=\varphi(x_0)+\frac{3i}{8\gamma}K\left(\frac{1}{(1+y)^{1/2}}\right)(1+x)^{-1/4}\left(n+\frac{1}{2}\right). \quad (30)$$

Substituting (30) and (29) in (27) and changing the order of the summation, we obtain

$$|\Psi(0)|^2=\frac{1}{\pi}\left\{1-2k\sum_{n=0}^{\infty}\mathrm{Im}\frac{1}{1+\exp[-i\theta+(2n+1)q]}\right\}, \quad (31)$$

where

$$\theta={}^{32}/_3\gamma\varphi(x_0), \quad q=2K((1+y)^{-1/2})(1+x_0)^{-1/4}.$$

We consider first the case $x_0=16F/k^4 \ll 1$. Using asymptotic expressions for the complete elliptic integrals, we obtain

$$\theta=\frac{2}{3}\frac{k^3}{F}+\frac{2}{k}\ln\frac{4k^3}{F}+\frac{2}{k}(1+\ln k), \quad q=\ln\frac{4k^4}{F}. \quad (32)$$

Since $q \gg 1$, it suffices to retain in the sum (29) only the term with $n=0$, and the remaining terms will be small in terms of the parameter $F/4k^4 \ll 1$. Moreover, using the asymptotic expression for $\Gamma(1-i/k)$ in (29), we verify that the asymptotic forms of (31) at $x_0 \ll 1$ and of (22) at $k \ll 1$ coincide. Thus, expression (22) describes the absorption of light at arbitrary k under the condition $k^4/F \gg 1$.

If $x_0 \gg 1$, then $q \ll 1$, therefore the sum over n can be replaced by an integral. As a result we have

$$|\Psi(0)|^2=\frac{1}{\pi}\left\{1+\frac{F^{1/4}}{K(1/\sqrt{2})}\,\mathrm{arctg}\left[\frac{(1-q)\sin\theta}{1+(1-q)\cos\theta}\right]\right\} \quad (33)$$

If $\theta-\pi(2j+1) \gg 1$, then

$$\mathrm{arctg}\left\{\frac{(1-q)\sin\theta}{1+(1-q)\cos\theta}\right\}\approx\left[\frac{\theta}{2\pi}+\frac{1}{2}\right]\pi-\frac{\pi}{2},$$

but if $\theta-\pi(2j+1) \leq q \ll 1$, $j=0,1,2,\ldots$, then

$$\mathrm{arctg}\left\{\frac{(1-q)\sin\theta}{1+(1-q)\cos\theta}\right\}\approx\frac{\pi(2j+1)-\theta}{q}.$$

Here $[z]$ is the non-integer part of z. Thus, at small k the oscillations in the Keldysh-Franz effect, with allowance for the Coulomb interaction, are transformed into a sawtooth curve.

C) Absorption in the forbidden band. If the photon frequency is less than the width of the forbidden band, then if no account is taken of the Coulomb interaction the absorption edge becomes smeared out because of the possibility of electron-hole pair tunneling. The change of the absorption in an electric field, with account taken of the Coulomb interaction of the electron and the hole, was investigated by Merkulov and Perel'[1] at large photon-energy deficits $(E_g-\hbar\omega)/e\mathcal{E}a_0$ and by Merkulov[2] near resonance with the ground level of the exciton.

In the present section we obtain these results anew by our method, and also investigate the absorption near resonances with excited states of the exciton in an electric field.

When finite motion is considered at sufficiently large energy deficits $\varkappa^3/F \gg 1$ and far from resonance, we can neglect the electric field completely and use the wave function of the electron in an unperturbed Coulomb potential:

$$f_1(\xi, n_1)=f_1(0)e^{-\varkappa\xi/2}F(-n_1, 1, \varkappa\xi), \quad n_1=0, 1, 2\ldots \quad (34)$$

The quantum number is $\beta_1=\varkappa(n_1+\frac{1}{2})$, where $\varkappa^2/2=-E$. The normalization condition yields

$$f_2^2(0)=\varkappa. \quad (35)$$

In the region $\eta \ll (\beta_2/F)^{1/2}$ we can neglect the electric field also for infinite motion, and then the wave function takes the form

$$f_2(\eta)=f_2(0)e^{-\varkappa\eta/2}F(-{}^1/_2+\beta_2/\varkappa, 1, \varkappa\eta). \quad (36)$$

at $\varkappa\eta \gg 1$ we have

$$f_2(\eta)=\frac{f_2(0)}{(\varkappa\eta)^{1/2}}\exp\left[\frac{\varkappa\eta}{2}-\frac{\beta_2}{\varkappa}\ln\varkappa\eta\right]\left[\Gamma\left(\frac{1}{2}-\frac{\beta_2}{\varkappa}\right)\right]^{-1} \quad (37)$$

and the motion in this region is quasiclassical. Just as in the preceding section, the wave function (37) can be joined together with the quasiclassical function in the region where the electric field comes into play. This quasiclassical function is, as usual, of the form

$$f_2(\eta)=f_2(0)(2\eta)^{-1/2}|k(\eta)|^{-1/2}e^{S(\eta)}.$$

The complete "below the barrier" action up to the right-hand turning point is equal to

$$S(\eta_2)=\frac{\varkappa\bar\eta}{2}-\frac{\beta_2}{\varkappa}\ln\varkappa\bar\eta+\int_{\bar\eta}^{\eta_2}\left(\frac{\varkappa^2}{4}-\frac{F\eta}{4}-\frac{\beta_2}{\eta}\right)^{1/2}d\eta\approx\frac{\varkappa^3}{3F}-\frac{\beta_2}{\varkappa}\ln\frac{4\varkappa^3}{F}, \quad (38)$$

where $\eta_2=\varkappa^2/F$ is the right-hand turning point and $\bar\eta$ is chosen such that $\varkappa\bar\eta \gg 1$ but $F\bar\eta/4 \ll \beta_2/\bar\eta$, $\varkappa^2/4$.

To the right of the turning point we obtain the following quasiclassical function:

$$f_2(\eta)=\frac{f_2(0)}{(2\eta)^{1/2}}\frac{\exp\{S(\eta_2)\}}{\Gamma({}^1/_2-\beta_2/\varkappa)}\frac{2\cos S(\eta)}{|k(\eta)|^{1/2}},$$

but according to the flux normalization condition the coefficient of $|k(\eta)|^{-1/2}\cos S(\eta)$ should be equal to $1/\pi$. Therefore

$$f_2^2(0)=\frac{1}{2\pi^2}\exp\{-2S(\eta_2)\}\Gamma^2\left(\frac{1}{2}-\frac{\beta_2}{\varkappa}\right), \quad (39)$$

$$\beta_2/\varkappa = 1/\varkappa - n_2 - {}^1/_2. \quad (40)$$

Thus, using (35), (38), and (39), we find that

$$|\Psi(0)|^2 = \sum_{n_1=0}^{\infty} f_1^2(0) f_2^2(0)$$
$$= \frac{\varkappa}{2\pi^2} \sum_{n_1=0}^{\infty} \Gamma^2\left(n_1 + 1 - \frac{1}{\varkappa}\right) \exp\left\{-\frac{2}{3}\frac{\varkappa^3}{F} + \frac{2}{\varkappa}\ln\frac{4\varkappa^3}{F} - (2n_1+1)\ln\frac{4\varkappa^3}{F}\right\}. \quad (41)$$

In our approximation $F/4\varkappa^3$, we retain in the sum in (41) only one term with $n_1 = 0$. The result is

$$|\Psi(0)|^2 = \frac{F}{8\pi^2\varkappa^2}\Gamma^2\left(1 - \frac{1}{\varkappa}\right)\exp\left\{-\frac{2}{3}\frac{\varkappa^3}{F} + \frac{2}{\varkappa}\ln\frac{4\varkappa^3}{F}\right\}. \quad (42)$$

The last expression coincides with the result of Merkulov and Perel', which they obtain by another method under the assumption $\varkappa^2/F \gg 1$.

Expression (42) is incorrect when the photon frequency is very close to the energy of a transition into any bound state of the exciton. Near resonance it is necessary, first, to take into account the Stark effect, and, second, one cannot neglect the damped exponential in the quasiclassical Coulomb wave function, since the coefficient of the growing exponential vanishes at resonance. Indeed, although the wave function of the finite motion remains the same as before and is described by expressions (34) and (35), while the wave function in the Coulomb region $f_2(\eta)$ is described as before by expression (36), the form of $f_2(\eta)$ changes near resonance at $\eta\varkappa \gg 1$:

$$f_2(\eta) = \frac{f_2(0)(-1)^{n_2}}{n_2!(\varkappa\eta)^{1/2}}\left\{\exp\left[-\frac{\varkappa\eta}{2} + \frac{\beta_2}{\varkappa}\ln\varkappa\eta\right] - (n_2!)^2\frac{(y-\Delta\beta_2)}{\varkappa}\exp\left[\frac{\varkappa\eta}{2} - \frac{\beta_2}{\varkappa}\ln\varkappa\eta\right]\right\}, \quad (43)$$

where $y = \beta_2 - \varkappa(n_2 + \frac{1}{2})$ and $\Delta\beta_2$ is the perturbation-theory correction on account of the Stark effect.[4]

The quasiclassical asymptotic form of the Coulomb function (43) can be drawn together with the quasiclassical function in an electric field first under the barrier, and the latter—with the function to the right of the right-hand turning point. Once matched, the wave function in this region takes the form

$$f_2(\eta) = \frac{2f_2(0)(-1)^{n_2}}{\sqrt{2}[\eta k(\eta)]^{1/2}}\left\{\frac{\exp\{-S(\eta_2)\}}{2n_2!}\cos\left(S - \frac{\pi}{4}\right) - n_2!\frac{y-\Delta\beta_2}{\varkappa}\exp\{S(\eta_2)\}\cos\left(S + \frac{\pi}{4}\right)\right\}, \quad (44)$$

where $S(\eta_2)$ is determined from expression (38). Using the normalization condition, we obtain

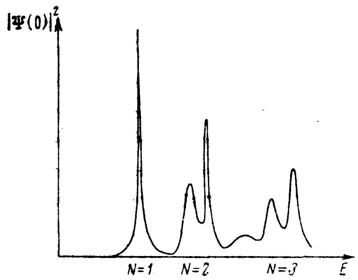

FIG. 2. The schematic form of the absorption spectrum in the forbidden band.

$$|\Psi(0)|^2 = \frac{2\varkappa}{\pi^2}\Gamma\left[\Gamma^2 + \left(\frac{2(y-\Delta\beta_2)}{\varkappa}\right)^2\right]^{-1};$$
$$\Gamma = \frac{1}{(n_2!)^2}\exp\{-2S(\eta_2)\}. \quad (45)$$

Thus, the absorption coefficient has a Lorentz shape with a width equal to the ionization probability of the given level. Near the resonances, expression (45) must also be transformed by substituting the explicit expression for $\Delta\beta_2$ and by using the connection between the parabolic quantum numbers and the radial quantum number N:

$$N = n_1 + n_2 + 1.$$

After simple transformations, we ultimately obtain

$$|\Psi(0)|^2 = \frac{2\varkappa}{\pi^2 N} \times \frac{\gamma_{n_1,n_2}}{\gamma^2_{n_1,n_2} + [(E - E_{n_1n_2})/E_{n_1,n_2}]^2}, \quad (46)$$

$$\gamma_{n_1n_2} = \frac{1}{(n_2!)^2}\frac{4}{FN^2}\left(\frac{4}{e^2N^3F}\right)^{2n_2} \times \exp\left[-\frac{2}{3FN^3} + 3(N-1)\right]. \quad (47)$$

Figure 2 shows the schematic form of the spectrum. It is seen that the linewidth increases with increasing number of the level in the given multiplet. The latter is natural, inasmuch as for large values of the parabolic quantum number n_2 the wave functions are shifted relative to the field, and their tunneling probability is therefore larger. At $N = 1$, expression (47) coincides with the corresponding expression for the light absorption coefficient in Merkulov's paper.[2]

[1] I. A. Merkulov and V. I. Perel', Phys. Lett. **45A**, 83 (1973).
[2] I. A. Merkulov, Zh. Eksp. Teor. Fiz. 66, 2314 (1974) [Sov. Phys. JETP **39**, 1140 (1974)].
[3] D. F. Blossey, Phys. Rev. B **2**, 3976 (1970).
[4] L. D. Landau and E. M. Lifshitz, Kvantovaya mekhanika, Fizmatgiz, (Quantum Mechanics) 1963, pp. 320–328.

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