Experimental investigation of quasistationary waves excited by periodic injection of electron bunches into plasma

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The interaction between a density-modulated electron beam and plasma has been investigated experimentally. It has been established that, in contrast to the usual beam-plasma systems with continuous injection, the injection of preformed electron bunches satisfying the equilibrium condition into plasma results in the excitation of nonlinear quasistationary waves of constant amplitude and profile (BGK waves). The instability of the stationary waves has been observed experimentally.

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It is well known that continuous injection of a monoenergetic electron beam into plasma will readily produce a longitudinal monochromatic wave that grows exponentially with distance. As the wave grows, the beam electrons are trapped by it and are grouped into dense bunches ("macroparticles"). The dynamics of these bunches determines the evolution of the wave during the nonlinear stage. Theoretical[1-4] and experimental[5] investigations show that the bunches into which the initially density-homogeneous beam divides are nonequilibrium formations which spread out in time, and this eventually leads to the attenuation of the wave amplitude.

On the other hand, there are many publications[6-8] in which the trapped-particle bunches are assumed to be in equilibrium, and this is used as the basis for developing a theory that predicts the existence in plasmas of longitudinal stationary nonlinear waves propagating without change in profile or amplitude (Bernstein-Green-Kruskal waves, often referred to as BGK waves). This theory predicts the possibility of waves with arbitrary potential profile (all that is necessary is to choose the necessary equilibrium distribution of trapped particles). However, the question of how such waves can be excited was left on one side[6,9] and, until now, stationary waves have not been observed.[1)]

In a previous paper,[12] we established the conditions for the existence of stationary waves in the case of a δ-function velocity distribution of trapped electrons, and reported the excitation of quasistationary waves by injection of preformed electron bunches into plasma. In this paper, we report the results of further experimental studies of this particular variety of BGK waves.

1. EXPERIMENTAL SETUP

The electron beam with energy $U_0 \sim 300$ eV and current $I_{b0} = 1-10$ mA was produced by a three-electrode electron gun (Fig. 1) consisting of a flat hot cathode 1, modulating grid 2, and anode grid 3. The separation between the gun electrodes was roughly 0.5 mm. The beam was allowed to enter the metal plasma chamber, 10 cm in diameter and filled with argon at 10^{-3} Torr, through a channel, 0.8 cm in diameter and 1.5 cm long. This channel was used to produce the pressure drop. The beam diameter at entry to the plasma chamber was about 0.5 cm. There were no external fields.

The beam was modulated by applying a constant bias voltage of between −100 and +10 V a microwave signal of between 0 and 100 V to the grid 2. The anode and cathode of the electron gun were high-frequency shorted. By varying the magnitude and sign of the bias voltage and the amplitude of the microwave signal, it was possible to produce both small sinusoidal modulation of the beam ($I_b/I_{b0} \ll 1$, where I_b is the alternating component of the beam current) or a deep modulation of the beam ($I_b/I_{b0} \sim 1$). In particular, it was possible to produce a beam in the form of a periodic sequence of electron bunches of adjustable length. Whenever necessary, the mean beam current I_{b0} could be made independent of the modulation amplitude by varying the cathode emission current.

The oscillations generated by the beam were detected and analyzed by the high-frequency probe 7 introduced into the plasma. The signal received by the plasma could be applied to a stroboscopic oscillograph with a pass band up to 3.5 GHz, or to the S4-5 and S4-8 spectrum analyzers.

A mobile collimated planar three-electrode probe (not shown in Fig. 1) was used to measure the radial distributions of beam current density at different distances from the point of injection into the plasma. The same probe, but with the entrance plane parallel to the beam axis, could be used to analyze the radial currents of plasma electrons and ions leaving the region of interaction. In all these measurements, the first grid was grounded, in order to reduce the perturbing effect of the probe, and the second grid was held at the potential

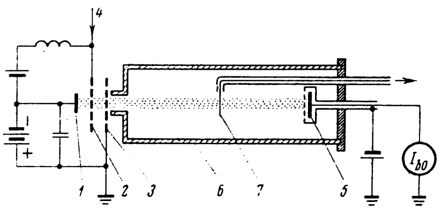

FIG. 1. Schematic diagram of experimental setup: 1—cathode; 2—grid; 3—anode; 4—microwave signal; 5—electron collector; 6—plasma chamber; 7—probe.

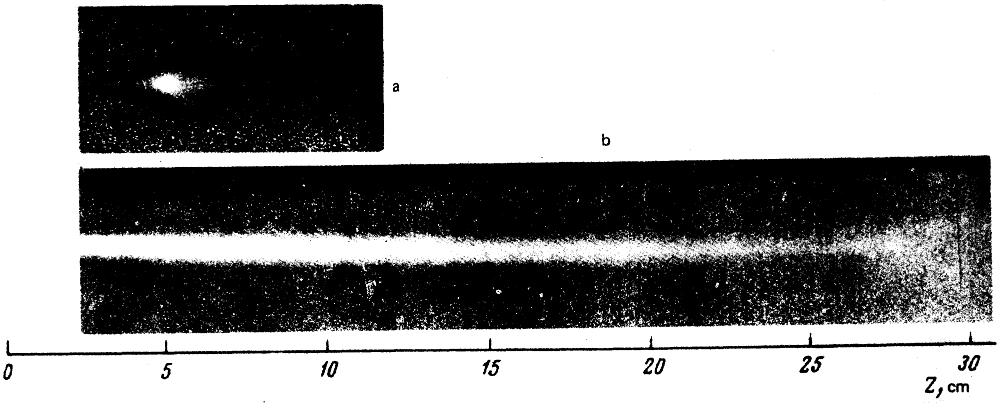

FIG. 2. Photographs of the beam with small (a) and deep (b) modulation of current density. The parameters of the beam-plasma system are as follows: $I_{b0}=3$ mA, $U_0=300$ eV, $P=5.8\times10^{-3}$ Torr. Modulation frequency $f=\Omega/2\pi=400$ MHz, a) $I_b/I_{b0}=3\times10^{-2}$, b) $I_b/I_{b0}\sim 1$.

necessary to extract a particular particle species (plasma electrons, beam electrons, ions).

2. EQUILIBRIUM OF ELECTRON BUNCHES

Our experiments have shown that the effects associated with beam plasma interactions are very dependent on the depth of preliminary modulation of the beam current injected into the plasma. When the modulation amplitude is small, a small region of beam-plasma discharge appears at a certain distance from the point of injection and is readily seen by eye (Fig. 2a). This is followed by a considerable angular spreading of the beam. The axial distribution of the oscillation amplitude (Fig. 3) has a sharp maximum, and the measured radial electron and ion currents leaving the discharge plasma are rapidly varying functions of distance along the beam.

All these effects are very similar to those that arise in the case of preliminary modulation of beam velocity,[13] and can be explained in terms of the electron bunching process. In particular, the experimental dependence of the position of the oscillation intensity maximum on the modulation amplitude is in good agreement with the following formula which we have derived for the coordinate S of the phase focus in the density-modulated beam:

$$S=\frac{1}{\gamma}\ln\frac{2j_0}{j_1}, \tag{1}$$

where γ is the spatial oscillation growth rate and j_0 and j_1 are respectively, the constant component and modulation amplitude of the beam current density.

Figure 4 shows a series of oscillograms illustrating the evolution of the oscillations in the potential, obtained at different distances from the point of injection of a beam with small density modulation. (We are interpreting the oscillations recorded by the probe as the potential oscillations on the basis of the data reported in a previous paper,[5] where the electron-beam method was used to explore the wave.) As in the experiments with the velocity-modulated beam,[5] we found that the wave profile became anharmonic at the point of strong beam bunching, and exhibited considerable variation during subsequent wave propagation, indicating that we were dealing with non-equilibrium bunches.

Waves excited by a beam with 100% density modulation are completely different in character. They start off with amplitude close to the maximum value and maintain this amplitude over distances over which the initially growing waves excited in the case of small modulation are found to decay (Fig. 3).

In an earlier paper,[12] we obtained a condition for

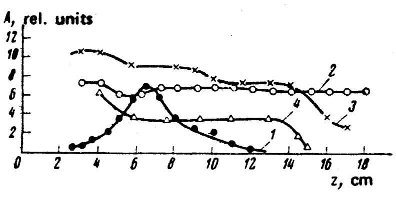

FIG. 3. Wave amplitude A as a function of distance from the point of injection of the beam for small (curve 1) and deep (curves 2, 3, 4) beam current-density modulation. For curves 1, 2, 3—$I_{b0}=10$ mA, $U_0=340$ eV; 4—$I_{b0}=5$ mA, $U_0=300$ eV; $f=420$ MHz, 1—$I_b/I_{b0}=6\times10^{-3}$; 2, 3, 4—$I_b/I_{b0}\sim 1$; the situations corresponding to curves 2 and 3 differ by bunch parameters.

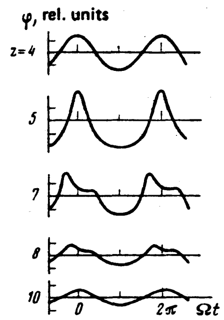

FIG. 4. Spatial evolution of the wave excited by a beam with small density modulation. The parameters of the beam-plasma system correspond to Fig. 2a and z is in centimeters.

stationary waves excited by a monoenergetic beam in plasmas. According to this condition, the electron beam must be transmitted by the plasma in the form of a periodic sequence of constant-density bunches. The bunch length L must satisfy the relation

$$L/\lambda = 1 - \Omega/\omega_p, \quad (2)$$

where λ is the wavelength, Ω is the modulation frequency, and ω_p is the plasma frequency. When this condition is satisfied, the electric field within the bunches is zero whereas, in the intervals between them, it is proportional to the electron density n_b in the bunches.

It was found that, by carefully choosing the bunch and plasma parameters, it was, in fact, possible to achieve a situation where the wave amplitudes underwent very small variation over large distances (curve 2, Fig. 3), i.e., the quasistationary wave was excited. The oscillograms of Fig. 5a show that, under these conditions, the observed wave profile is in good agreement with theoretical calculations (upper curve) and remain practically constant, as should be the case for equilibrium bunches in the wave. The small variation in the amplitude and profile suggest the presence of small bunch deformations, which is not unexpected because the equilibrium conditions are only approximately satisfied in a real-life experiment.

The excited quasistationary wave maintains the beam-plasma discharge in a much greater volume as compared with the case of variable-amplitude waves (Fig. 2b). In particular, the measured radial currents of ions and electrons leaving the plasma remain practically constant along the beam.

We note that the equilibrium condition for cold bunches, given by (2), was obtained previously by analyzing a one-dimensional system. The experimental radial beam current distribution[12] (see also Fig. 2b) suggests that the bunches are also in radial equilibrium. More detailed analysis shows that (2) is valid even for radially bounded systems. The self-focusing effect has been predicted for bunches under similar conditions.[14]

We have already noted that the quasistationary wave is excited only when certain definite experimental conditions are satisfied. When these equilibrium conditions, including, in particular, those corresponding to Fig. 5a, are violated by, for example, reducing n_b and hence ω_p [n_b is not present in (2)], the bunches experience considerable deformation along their path, and this leads to substantial amplitude oscillations with distance and to enhanced wave attenuation (Fig. 5b). Without changing the reduced value of n_b, it is possible to return to the equilibrium conditions by increasing ω_p through an increase in the gas pressure. Theory then shows that the amplitude of the new stationary wave should be smaller because of the smaller value of n_b (Fig. 5c).

3. INSTABILITY OF QUASISTATIONARY WAVE

The facts considered in the foregoing section suggest that the equilibrium of electron bunches was achieved in our experiments. Nevertheless, the wave amplitude was found to fall after a certain distance was reached. When this happens, there is an appreciable deterioration in the synchronism between the oscillations recorded by the probe and the modulating voltage (this is indicated by the more diffuse appearance of the oscillograms in Fig. 5a). Spectrum analyzers were used to show that, under these conditions, the probe receives both oscillations at the main frequency ($f \sim 400$ MHz) and "satellite" waves with frequencies near 30 and 400 ± 30 MHz (Fig. 6), which are spontaneously excited and grow along the beam.

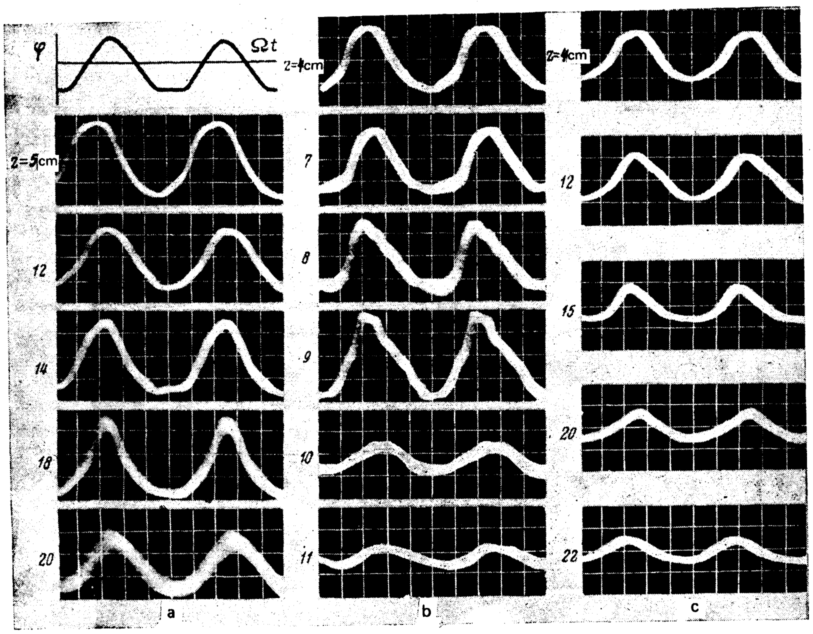

FIG. 5. Potential profiles in the case of deep modulation of the beam current density: $U_0 = 300$ eV, $f = 420$ MHz, $I_b/I_{b0} \sim 1$; a) $I_{b0} = 1.8$ mA, $P = 6.5 \times 10^{-3}$ Torr; b) $I_{b0} = 1.5$ mA, $P = 6.5 \times 10^{-3}$ Torr; c) $I_{b0} = 1.5$ mA, $P = 8.0 \times 10^{-3}$ Torr.

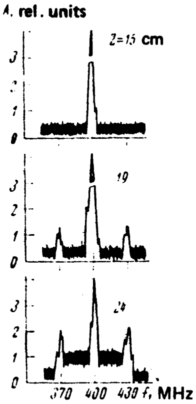

FIG. 6. Oscillation spectra excited by an electron beam with deep modulation in the neighborhood of the main frequency: $I_{b0}=2.3$ mA, $U_0=300$ eV, $P=5\times10^{-3}$ Torr, $f=400$ MHz.

The excitation of satellites in beam-plasma systems has been discussed theoretically.[15,16] In particular, the following dispersion relation has been established[15] for a model in which the electron beam is in the form of undeformed charged layers oscillating on the bottom of the potential wells produced by the wave:

$$\frac{\omega_b^2}{(\omega-kv_0)^2-\omega_0^2}\left[\frac{1}{\varepsilon(k,\omega)}+\frac{1}{\varepsilon(k-2k_0,\omega-2\Omega)}\right]=1 \qquad (3)$$

where ω_b is the plasma frequency of the beam corresponding to its mean density, ω_0 is the layer oscillation frequency, ϵ is the permittivity of plasma for longitudinal oscillations, v_0 is the beam velocity, $k_0=\Omega/v_0$, and k and ω are the perturbation wave parameters. It is assumed that $(\omega_p-\Omega)/\omega_p\ll 1$.

For the purposes of comparison with experiment, it is interesting to consider the spatial amplification of satellite waves. Neglecting thermal velocities of plasma electrons, and assuming that ω is real, we find from (3) that

$$k=\frac{\omega}{v_0}\pm\frac{\omega_b}{v_0}\left[\left(\frac{\omega_0}{\omega_b}\right)^2+\left(\frac{1}{\varepsilon(\omega)}+\frac{1}{\varepsilon(\omega-2\Omega)}\right)\right]^{1/2}. \qquad (4)$$

If we now substitute the amplitude of the potential in the stationary wave,[12] $\varphi_0\approx 4\pi e n_b/k_0^2$, into the expression for the oscillation frequency $\omega_0=k_0(e\varphi_0/m)^{1/2}$, and then use the equilibrium condition (2), we find that the ratio $(\omega_b/\omega_0)^2$ in (4) becomes equal to $1-\Omega/\omega_p$.

The frequency band of the growing waves and the spatial growth rates can readily be determined from (4) by introducing the new variables $\delta=\omega_p-\Omega$ and $\beta=\omega_p-\omega$, and using the fact that δ/ω_p and β/ω_p are small. Figure 7 shows the calculated dependence of Imk on β. It is clear that maximum amplification occurs for waves separated by approximately $\pm\delta$ from the frequency of the main wave. The fact that the growth rate tends to infinity is due to the fact that the thermal motion of plasma electrons and the dissipation of the oscillations were not taken into account in the calculation. These effects will restrict the growth rate at the resonance points, as in the problem of wave amplification during the usual beam-plasma interaction.

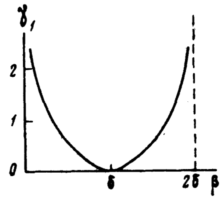

FIG. 7. Spatial growth rate Im $k=\gamma_1(\omega_b/v_0)(\sqrt{\omega_p/\delta})$ as a function of the perturbation wave frequency.

Comparison of the experimentally established evolution of the spectra (Fig. 6) with the calculated dependence (Fig. 7) shows that the two are in qualitative agreement. The oscillograms of wave profiles shown in Fig. 5a can be used for quantitative comparison of calculated and measured frequencies of growing oscillations. If we use these oscillograms to determine L/λ as the ratio of the duration of the flat segments on the potential wells to their period, we can calculate δ from (2). This turns out to be approximately 100 MHz, which is not in agreement with the measured spectrum. However, one of the probable reasons for this discrepancy is that the theory which was constructed for small δ is not accurate enough for the values of δ realized in our experiments.

The simplifying assumptions used in the theory also result in the fact that the theory does not predict the amplification of oscillations in the low-frequency part of the spectrum at frequencies near 30 MHz. On the other hand, these oscillations are very important for the dynamics of the system, and this is confirmed by the following experiment. In addition to the microwave voltage exciting the quasistationary wave, we applied a small 30-MHz voltage to the modulating grid in the gun. This resulted in an appreciable narrowing of the spectra of all the satellite waves, and the region in which the main wave was attenuated moved toward the point of injection of the beam (Fig. 8). Figure 9 shows the spatial dependence of the amplitudes of the main and low-frequency waves. The amplitudes of waves with the combination frequencies (400 ± 30 MHz) are found to vary along the beam by analogy with the curve shown by the broken line. The data of Figs. 8 and 9 show that the reason for the attenuation of the main wave is the growth of the satellite wave. These waves take the bunches away from the equilibrium state, and they break up. An analogous effect of satellites on the main wave (but not a stationary wave) has been investigated numerically.[16]

It is important to note that, in the experiment with

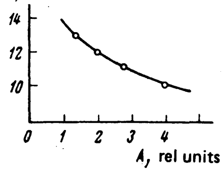

FIG. 8. Distance over which the quasistationary wave propagates without appreciable change in amplitude as a function of the initial amplitude of the low-frequency wave. The experimental conditions correspond to those in Fig. 9.

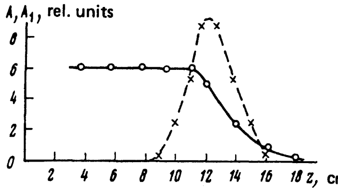

FIG. 9. Spatial distribution of the amplitudes of the main (solid line) and low-frequency (broken line) waves. The scale along the vertical axis is different for the two curves. $I_{b0}=4.5$ mA, $U_0=300$ eV, $f=400$ MHz, $P=5.8\times10^{-3}$ Torr. The frequency of the perturbation wave is $f_1=\omega_1/2\pi=30$ MHz, $I_b/I_{b0}\sim 1$.

nonequilibrum bunches, we found two further effects which additionally enhance the attenuation of the main wave. The first of these is the angular spread of the electron beam, which can be seen by eye (Fig. 2a) for $z=25-30$ cm. The second effect is a strong change in the beam-plasma discharge conditions, so that the plasma parameters also change substantially in the region in which the amplitude decays. The measured axial dependence of the radial electron and ion currents in this region has a decaying character. This is why the direct effect of the satellites on the main wave is clearly seen only during the initial stage of attenuation of this wave, and is subsequently masked by the effects mentioned above.

The breakup of the electron bunches naturally leads to the attenuation of the satellite waves as well (Fig. 9).

CONCLUSIONS

Our experiments have shown that the injection of a density-modulated electron beam into plasma may result in the excitation of both initially growing waves (for $I_b/I_{b0}\ll 1$), which are usually observed in other experiments as well, and nonlinear quasistationary BGK waves. The condition for the excitation of such waves by monoenergetic beams is given by (2). Because of the equilibrium distribution of trapped particles, the BGK wave propagates with constant profile and amplitude over distances for which waves in ordinary plasma systems are almost completely attenuated. The quasistationary wave maintains the beam-plasma discharge over its entire path.

We have established that the attenuation of waves excited by the equilibrium electron bunches is due to the growth of satellite waves as a result of instabilities. This is in qualitative agreement with numerical analyses of the nonlinear parametric instability of the nonstationary main wave, although the spectra of excited waves are only partially in agreement with previously developed theory.[15]

1) Fedorchenko *et al.* [10] have reported experiments with BGK waves, but the waves investigated by them were not stationary. This is indicated, in particular, by the reported axial dependence of their amplitudes. [11] In fact, Fedorchenko *et al.* [10] arbitrarily define BGK waves as waves "containing an appreciable number of trapped particles," but this is not the essence of stationary waves as given in generally accepted definitions. [6,7]

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