mon Press, Oxford (1975)).
[20]R. S. Hanni and R. Ruffini, Lett. Nuovo Cimento 15, 189 (1976).
[21]I. B. Khriplovich and É. V. Shuryak, Zh. Eksp. Teor. Fiz. 65, 2137 (1973) [Sov. Phys. JETP 38, 1067 (1974)].
[22]A. A. Sokolov and I. M. Ternov, Relyativistskiĭ Élektron (The Relativistic Electron), Nauka (1974).

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Quantum interferometer as detecting element of a gravitational antenna

A. V. Gusev and V. N. Rudenko

Moscow State University
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The properties of a quantum interferometer as a probe of small acoustic perturbations of a gravitational detector are studied. The low-frequency fluctuations of a Josephson contact are calculated, and on this basis formulas are obtained for the limiting sensitivity of the gravitational antenna. It is shown that when quantum restrictions are taken into account it is possible in principle to achieve the resolution necessary for second-generation antennas.

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1. INTRODUCTION

Braginsky *et al.*[1] have formulated a very general prediction for the parameters of bursts of gravitational radiation which can be reasonably expected as the result of cosmic catastrophes occurring with participation of superdense stars ($r \sim r_g$). For a frequency of events no less than ten events per year and a duration of bursts $\hat{\tau} \sim 10^{-3}$–10^{-4} sec the upper limit of their energy density at the Earth lies in the range $W \sim 1$–10^4 erg/cm² (Braginsky's estimate takes into account components with $M \sim 3$–$30M_\odot$ and a range $\epsilon \sim 0.1$–10^{-3} for the fraction of the total energy converted to gravitational radiation). For a quadrupole gravitational detector (GD) with masses m, linear dimension $l_g \sim 10^2$ cm, and a mean frequency $\omega_\mu \sim 3\times 10^4$ rad/sec this is equivalent to an external perturbation with a relative acceleration of the masses $F/m \sim 10^{-9}$ cm/sec² in the optimistic case ($W = 10^4$ erg/cm²) and $F/m \sim 3\times 10^{-11}$ cm/sec² in the pessimistic case ($W = 1$ erg/cm², $\tau \sim 2\times 10^{-4}$ sec). It is known that the potential sensitivity of GD permits detection of such excitations if the Brownian motion is reduced by cooling to $T_\mu \sim 3\times 10^{-3}$ K or as the result of a high mechanical quality factor $Q_\mu \sim 10^{10}$.[2] The reason for the delay in construction of second-generation antennas is the lack of efficient detecting elements which measure small vibrations of the GD. Actually, in the best converters of the parametric type with a pumping frequency ω_e under matched conditions, their intrinsic fluctuations at temperature T_e limit the sensitivity of the antenna at the level[3]

$$\left(\frac{F}{m}\right)_{min} \geqslant \frac{3\pi^{1/2}}{\hat{\tau}}\left(\frac{kT_e\omega_\mu}{m\omega_e}\right)^{1/2}. \tag{1}$$

Substitution into Eq. (1) of the values $T_e = 2$ K, $m \approx 3\times 10^4$ g, $\omega_e = 3\times 10^{10}$, and $\hat{\tau} \approx 2\times 10^{-4}$ sec gives $F/m \gtrsim 10^{-9}$ cm/sec², i.e., the sensitivity barely reaches the optimistic limit of the prediction. Increase of the mass of the GD to 5×10^6 g (Ref. 4) provides only $F/m \sim 10^{-10}$ cm/sec².

In addition there is an important limitation due to the possiblity of amplifying the probe signal. The amplifier noise temperature must satisfy the condition

$$T_n \leqslant \frac{2\pi G}{c^3 k}\omega_\mu\omega_e m l_g^2 \hat{\tau} W \sim 10^{-3} W. \tag{2}$$

For the best low-noise amplifiers in the frequency range considered, $T_n \sim 1$ K.[5] It is therefore clear that only $W \approx 10^3$ erg/cm² is accessible to detection; with increase of the mass $W \approx 10$ erg/cm².

It follows from Eqs. (1) and (2) that an increase of the pumping frequency ω_e would be a radical measure. However, this is hindered by the following considerations. The first, which is technical in nature, is the unavailability to experimenters of pumping generators with sufficient stability in the range $\omega_e > 3\times 10^{10}$. A second, which is fundamental in nature, is the intrusion into the region of quantum limitations, according to which Eq. (1) is valid as long as $kT_e \gtrsim \hbar\omega_e$, and the maximum sensitivity of the antenna will not exceed the quantum limit[6]

$$\left(\frac{F}{m}\right)_{min} \geqslant \frac{1}{\hat{\tau}}\left(\frac{\hbar\omega_\mu}{m}\right)^{1/2}. \tag{3}$$

For $T_e \sim 2$–4 K the limiting value of the pumping frequency lies *a priori* somewhere near $\omega_e \sim 10^{11}$.

In recent years the hopes of a number of experimental groups have been based on the use of quantum magnetometers employing the Josephson effect—so-called SQUIDS.[1)] However, our analysis[7] has shown that single-contact SQUIDS with external pumping have the same

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properties as parametric converters, which lead to Eqs. (1)–(3). The purpose of the present article is to investigate the quantum interferometer (a magnetometer with two Josephson contacts) as the detecting element of a gravitational antenna. We show that in principle this element can solve the problem of detection of the oscillation of GDs in second-generation antennas.

A quantum interferometer[8,9] consists of a superconducting ring which contains two weak links—Josephson junctions supplied by a direct current bias J. The bias current is such that the currents in the weak links exceed the critical value J_c which destroys the superconductivity. As a consequence, there arises in the ring a potential difference which is an oscillating function of the magnetic flux penetrating the ring (the details can be found in books[8,9]).

As in Ref. 7, we shall discuss the connection of an interferometer to a GD with a coupling loop carrying a supercurrent J_0 and firmly mounted on the GD. The basic scheme of the entire antenna is shown in the figure. The interferometer is successively connected into the oscillatory circuit and turned to the GD resonance frequency ω_μ. The output signal is a current flowing in the inductance L_{ct}. We omit here the technical details of matching with the subsequent circuits, which have been discussed, for example, in Ref. 10; the latter can always be taken into account by an equivalent recalculation of the circuit parameters L_{ct} and R_{ct}.

In what follows the thermal noise of the GD will be assumed negligible ($H_\mu \to 0$, $T_\mu \to 0$). The sensitivity of the antenna is limited only by the fluctuations of the detecting element, and to evaluate these it is necessary to know the spectrum of voltage fluctuations of the quantum interferometer. For this reason at the beginning of the next section we present a calculation of the fluctuation spectrum of a Josephson contact under the condition of small fluctuations. The results are then used in Sec. III for calculation of the minimum observable force (acceleration) equivalent to the action of a gravitational wave. The parametric mechanism of operation of the interferometer as a probe with self-pumping and self-detection is discussed; the quantum sensitivity limit is evaluated in Sec. IV. Finally, in the conclusion we discuss the likelihood of use of the quantum interferometer for second-generation gravitational antennas.

2. LOW-FREQUENCY SPECTRUM OF VOLTAGE FLUCTUATIONS OF A JOSEPHSON JUNCTION

The behavior of the quantum-mechanical phase φ of a Josephson junction with negligible capacitance is described by the equation[8,9] (for $\alpha=1$)

$$\dot{\varphi}+\alpha\sin\varphi=I+\tilde{I}. \tag{4}$$

The dimensionless designations are as follows: $\tau=\omega_0 t$, where $\omega_0=2\pi RJ_c/\Phi_0$ is the characteristic frequency of the junction, R is the normal component of resistance, Φ_0 is the magnetic flux quantum, and $I=J/J_c$. The spectral intensity of the current fluctuations $\tilde{I}$ with inclusion of quantum noise is

$$S_I(\nu)=\Gamma(\beta\nu/2)\left[1+2(e^{\beta\nu}-1)^{-1}\right], \tag{5}$$

where $\Gamma=2\pi kT/J_c\Phi_0$, $\beta=\hbar\omega_0/kT$, $\nu=\omega/\omega_0$, and T is the temperature of the Josephson contact. Retention in Eq. (4) of the arbitrary constant $\alpha\neq 1$ is required for the subsequent calculation of the sensitivity of a two-contact magnetometer (Sec. 3).

The low-frequency part of the voltage fluctuations in a Josephson junction has been calculated previously,[11] but the method of direct expansion in small fluctuations $\tilde{I}$ used in that work leads to an erroneous result. For this reason, rather than make a fundamental analysis, we prefer to calculate the low-frequency fluctuation spectrum $\dot{\varphi}$ by means of an asymptotic method in nonlinear oscillation theory.[12]

In Eq. (4) the series of substitutions

$$x=\tan\left(\varphi+\frac{\pi}{2}\right)\Big/2;\quad \dot{u}/u=-(I+\alpha+\tilde{I})x/2,$$

$$z=u\exp\left[-\frac{1}{2}\int_0 \frac{\tilde{I}}{I+\alpha}d\tau\right], \tag{6}$$

leads to the equation

$$\ddot{z}+(\nu_0/2)^2[1+\xi(\tau)]z=0,$$

$$\nu_0^2=I^2-\alpha^2,\quad \xi(\tau)\approx(2/\nu_0^2)\left[I\tilde{I}+\dot{\tilde{I}}/(I+\alpha)\right]. \tag{7}$$

Here the random variable $\xi(\tau)$ is written in the linear approximation in the fluctuations $\tilde{I}$, which we assume small. It is possible to discard terms of order $\tilde{I}^2$, $\tilde{I}\dot{\tilde{I}}$, and higher if we limit the bandwidth of the spectrum $\tilde{I}$. We shall assume that the spectrum (5) is cut off above somewhere in the region of frequencies $1<\nu<10$. A better defined condition of applicability of the subsequent calculations is $\xi\ll 1$. For this case Eq. (7) has been analyzed by Stratonovich.[13] The role of harmonics of the fundamental frequency ν_0 is small in this case, and the solution of Eq. (7) can be represented in quasiharmonic form:

$$z=A\cos(\tfrac{1}{2}\nu_0\tau+\theta)=A\cos\gamma,\quad \dot{z}=-\tfrac{1}{2}\nu_0 A\sin\gamma. \tag{8}$$

The variable u from Eq. (6) differs from z only in its amplitude:

$$u=A_u\cos\gamma,\quad A_u=\frac{A}{2}\int_0^\tau\frac{\tilde{I}}{I+\alpha}d\tau,$$

and for the phase θ and the amplitude logarithm $v=\ln A_u$ the following equations are valid (for details see Ref. 13):

$$\dot{v}=\frac{1}{2}\frac{\tilde{I}}{I+\alpha}+\frac{\nu_0}{4}\xi\sin 2\gamma,\quad \dot{\theta}=\frac{\nu_0}{4}\xi+\frac{\nu_0}{4}\xi\cos 2\gamma. \tag{9}$$

The smallness of $\tilde{I}$ assures slow variation of v and θ.

Using the relations which follow from Eqs. (6) and (8):

$$x\approx-\frac{2}{I+\alpha}\left(1-\frac{\tilde{I}}{I+\alpha}\right)(\dot{v}-\dot{\gamma}\tan\gamma),\quad \dot{\varphi}=I+\alpha+\tilde{I}-\frac{2\alpha}{x^2+1},$$

one can find for $\dot{\varphi}$ in first order in $\tilde{I}$:

$$\dot{\varphi}=\frac{\nu_0^2}{I+\alpha\cos 2\gamma}+\tilde{I}+\frac{\nu_0\alpha}{2(I+\alpha\cos 2\gamma)^2}\Big[\Big(2\theta-\frac{\nu_0\tilde{I}}{I+\alpha}\Big)(1-\cos 4\gamma)-(4\sin 2\gamma+2\sin 4\gamma)\dot{v}\Big]. \tag{10}$$

This formula relates $\dot{\varphi}$ with the random variables $\tilde{I}$, θ, and v, for which statistical characteristics are found from Eqs. (5), (7), and (9). In principle Eq. (10) solves the problem of the fluctuation spectrum of a Josephson contact. From this we can find both the low-frequency part of the fluctuations and the natural width of the generation line, together with the pedestal. In the present article we limit ourselves to calculation of the low-frequency part of the spectrum. For this purpose it is necessary to expand Eq. (10) in Fourier series in the harmonics of ν_0 and then average over the period ν_0^{-1}.

Using Eqs. (7) and (9), we obtain

$$\langle\dot{\varphi}\rangle=\nu_0+\frac{I}{\nu_0}\langle\tilde{I}\rangle+\langle\tilde{I}\cos 2\gamma\rangle\frac{I-\nu_0}{\nu_0}\Big[1+\frac{2}{\alpha^2}(I-\alpha)(I-\nu_0)\Big]+\ldots \tag{11}$$

The low-frequency voltage on the contact, which coincides with $\langle\dot{\varphi}\rangle$,[8,9] in accordance with Eq. (11) consists of a constant component, the contribution of the low-frequency part of the spectrum $\tilde{I}$, and also the contribution of the high-frequency fluctuations of $\tilde{I}$, whose spectrum is close to the generation frequency ν_0 (frequency conversion as the result of the nonlinear properties of the contact). Similarly there exist combination contributions of fluctuations resulting from higher harmonics, which in the approximation selected have been omitted in Eq. (11). Finally for the spectral intensity of the contact noise we have

$$S_{\dot{\varphi}}(0)=\frac{I^2}{\nu_0^2}\Big\{S_I(0)+\frac{(I-\nu_0)^2}{2I^2}\Big[1+\frac{2}{\alpha^2}(I-\alpha)(I-\nu_0)\Big]^2 S_I(\nu_0)\Big\}. \tag{12}$$

For $\alpha=1$ we can see a difference from the result obtained in Ref. 11, where the combination term has the form $(1/2I^2)S_I(\nu_0)$. The second term in the square brackets in Eq. (12) is small. The main difference is due to the factor $(I-\nu_0)^2$, which reduces the role of the combination effects, assuring that they fall off with increase of bias current as I^{-3} rather than as I^{-1} as in Ref. 11. Physically $I-\nu_0$ is the difference in the ordinates of the volt-ampere characteristics of an ordinary resistance $\dot{\varphi}=I$ and a Josephson junction $\dot{\varphi}=\nu_0$. With increase of I the characteristics approach each other and the nonlinear effects disappear.

The region of applicability of the quasiharmonic approximation (8) which we have used is limited by the conditions (see Ref. 13)

$$(\overline{\xi^2})^{1/2}\ll 1,\quad |\tilde{I}/(I+\alpha)|\ll 1. \tag{13}$$

For $\beta\gtrsim 1$, $\Delta\nu\gtrsim 1(\nu_0\sim 1)$ these relations lead to the requirement

$$(\hbar R)^{1/2}\ll\Phi_0(\nu_0/\Delta\nu)^3. \tag{14}$$

If we limit the bandwidth at the level of the second harmonic of the Josephson generation, i.e., $(\Delta\nu/\nu_0)\approx 2$, it is necessary to choose contacts whose resistance is $\leq 10^3$ ohms, which is technically achievable.

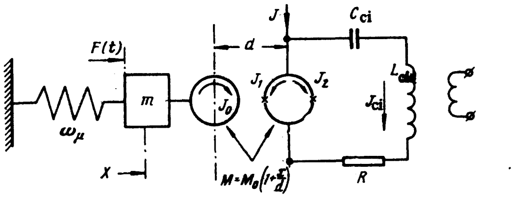

FIG. 1.

3. CALCULATION OF LIMITING OBSERVABLE FORCE

Let us turn to calculation of the sensitivity of a gravitational antenna with a quantum interferometer (see Fig. 1). The equations of the interferometer are as follows (for details see Ref. 14):

$$lj=\varphi_1-\varphi_2-\varphi_s,\quad 2v=\dot{\varphi}_1+\dot{\varphi}_2,$$
$$\dot{\varphi}_1+\sin\varphi_1=(I-I_{ci})/2-j+\tilde{I}_1,\quad \dot{\varphi}_2+\sin\varphi_2=(I-I_{ci})/2+j+\tilde{I}_2. \tag{15}$$

Here φ_1 and φ_2 are the phase jumps in the weak links, $\varphi_s=2\pi\Phi_s/\Phi_0$ is the magnetic flux signal, $v=V/V_0$ and $j=(J_1-J_2)/J_c$ are the voltage on the interferometer ring and the current circulating in it, $l=2\pi L J_c/\Phi_0$ is the inductance of the ring, and $\tilde{I}_1$ and $\tilde{I}_2$ are the independent fluctuation currents generated in the contact resistances (for simplicity we assume that the arms are symmetric, $V_0=J_c R$).

The equation for the current in the circuit $I_{ci}=J_{ci}/J_c$ has the form

$$\ddot{I}_{ci}+2\Delta\dot{I}_{ci}+\nu_{ci}^2 I_{ci}=l_{ci}^{-1}(\dot{v}-\dot{e}),$$
$$\Delta=\delta_{ci}/\omega_0=R_{ci}/2L_{ci}\omega_0\ll\nu_{ci}=\omega_{ci}/\omega_0,\quad e=E_{fl}/V_0, \tag{16}$$

E_{fl} is the fluctuation emf in the resistance of the circuit. The calculation is carried out most simply with neglect of the ring inductance $l=0$ (in actual circuits a typical operating regime corresponds to the condition $l\lesssim 1$). Then from Eq. (15) it follows that

$$\dot{\varphi}+\cos(\varphi_s/2)\sin\varphi=(I-I_{ci})/2+(\tilde{I}_1+\tilde{I}_2)/2,$$
$$v=\dot{\varphi},\quad \varphi=(\varphi_1+\varphi_2)/2. \tag{17}$$

Equation (17) is identical to the equation for a single contact considered in Sec. II. The role of α is played by the quantity $\cos(\varphi_s/2)$, which is slow in comparison with the changes of φ at the Josephson generation frequency $(\omega_\mu\sim\omega_s\ll\omega_0)$. Solution of Eq. (17) gives the function v entering into Eq. (16), and in view of the selective properties of the circuit only the slow part of the function is needed:

$$v^{(sl)}\approx\nu_0+(1/4\nu_0)(\tilde{\varphi}_s\sin\varphi_b-I\dot{I}_{ci})+\chi. \tag{18}$$

We assumed above that φ_s consists of a constant bias and a weak harmonic signal: $\varphi_s=\varphi_b+\tilde{\varphi}_s$, $\varphi_b\gg|\tilde{\varphi}_s|$, $\tilde{\varphi}_s=\tilde{\varphi}_m\cos\nu_s\tau$, and χ is a random variable which has the spectrum (12) (with replacement of Γ by $\Gamma/2$).

Substitution of (18) into (16) permits evaluation of the intrinsic sensitivity of a two-contact SQUID as a magnetometer. For $\nu_0 \sim 1$, $\beta \lesssim 1$, and $\sin(\varphi_b/2) \sim 1$ we obtain for the minimum detectable flux

$$\delta\Phi_{min} \geqslant (kT/V_0 J_c)^{1/2} \Phi_0 (\Delta f)^{1/2}. \tag{19}$$

Typical parameter values $T \sim 4$ K, $V_0 \sim 10^{-3}-10^{-4}$ V, $J_c \sim 10^{-4}-10^{-6}$ A give a numerical estimate $\delta\Phi_{min} \sim (10^{-7}-10^{-8})\Phi_0\,\mathrm{Hz}^{-1/2}$, which is two to three orders of magnitude better than the limiting resolution of single-contact magnetometers in the hysteresis mode.[7]

The sensitivity of the gravitational antenna can be found from simultaneous solution of Eqs. (17), (18), and the equation for the mechanical degree of freedom. The energy of interaction of the electrical and mechanical degrees of freedom we shall assume proportional to the sum of the currents $J_1 + J_2$ flowing in the arms of the interferometer, which can be assured by choice of the geometry in the experimental arrangement. Altogether we have the system of equations

$$\begin{gathered}(-\lambda C_0/4l_{ci}\nu_0)\sin\varphi_b\,\dot y + \ddot I_{ci} + 2\tilde\Delta \dot I_{ci} + \nu_{ci}^2 I_{ci} \\ = l_{ci}^{-1}(\dot\chi - e^{(sl)}), \quad \ddot y + \nu_\mu^2 y + \lambda\nu_\mu^2 I_{ci} \approx \nu_\mu^2 f_s,\end{gathered} \tag{20}$$

where $y = x/d$ is the mechanical coordinate, $\tilde\Delta = \Delta(1 + I/2\nu_0)$ is the equivalent damping, and the parameters are the same as in Ref. 7:

$$\lambda = M_0 J_0 J_c / m\omega_\mu^2 d^2, \quad C_0 = 2\pi m(\omega_\mu d)^2 / J_c \Phi_0; \quad f_s = F(t)/m\omega_\mu^2 d$$

is the external force equivalent to action of a gravitational wave (here $\tilde\varphi_s = \lambda C_0 y$).

The maximum value of the output signal, which is the current I_{ci}, corresponds to the relation $\lambda^2 = \lambda_{opt}^2 = 16 l_{ci} \nu_0 (\omega_\mu \hat\tau)/C_0 \nu_\mu \sin\varphi_b$; here

$$L_{ci} J_{ci}^2 \approx [(F_0\hat\tau)^2/2m](\omega_0/\omega_\mu)\sin\varphi_b\,(\delta_{ci}\hat\tau)^{-1}.$$

As in Ref. 7, we shall consider a short external action $F(t)$ of duration $\hat\tau$ with a spectrum concentrated in the region $\omega_\mu \pm \hat\tau^{-1}$. From solution of the system (20) we find the spectrum of the response signal $I_{ci}^s(\nu)$ and the power spectrum of fluctuations $S_{ci}(\nu)$. For the ratio of the signal to noise after optimal processing we obtain

$$\mu = \pi^{-1} \int\limits_{\nu_\mu - (\omega_\mu\hat\tau)^{-1}}^{\nu_\mu + (\omega_\mu\hat\tau)^{-1}} \frac{|I_{ci}^s(\nu)|^2}{S_{ci}(\nu)} d\nu \approx (f_0 \omega_\mu \tau)^2 \left(\frac{\lambda}{\lambda_{opt}}\right)^2 \frac{C_0 \nu_\mu \sin\varphi_b}{4\pi\nu_0 (S_\chi + S_e)}. \tag{21}$$

In the limiting case of small circuit noise $S_e \to 0$ the observable amplitude of external force following from Eq. (21) for $\mu = 1$ and $\lambda = \lambda_{opt}$ is

$$\begin{gathered}F_0 \geqslant \frac{4}{\hat\tau}\left(mkT\frac{\omega_\mu}{\omega_0}\right)^{1/2} \frac{I}{\sin\varphi_b}\left\{1 + \frac{(I-\nu_0)^2}{4I^2}\left[1 + \frac{2}{\alpha^2}(I-\alpha)(I-\nu_0)\right]^2 \right. \\ \left. \times \beta\nu_0[1 + 2(e^{\beta\nu_0} - 1)^{-1}]\right\}^{1/2}.\end{gathered} \tag{22}$$

For not too large $\beta < 10$ in the regime $1 < I < 2$, $\sin\varphi_b \sim 1$, Eq. (22) gives

$$(F_0)_{min} \geqslant \frac{4}{\hat\tau}\left(mkT\frac{\omega_\mu}{\omega_0}\right)^{1/2}. \tag{23}$$

The same relation is valid when the circuit noise is important:

$$S_e = kT_{ci} R_{ci} \omega_0 V_0^{-2} = 2\pi k T_{ci} / J_c \Phi_0 \gg S_\chi;$$

In the above equation the contact temperature T must be replaced by the circuit noise temperature T_{ci}.

4. THE PARAMETRIC MECHANISM AND THE QUANTUM SENSITIVITY LIMIT

Equation (23) has the form characteristic of an antenna with a parametric converter, whose makeup includes a pumping generator working at frequency ω_0.[3,7] This fact compels us to consider the quantum interferometer as a probe of the parametric type. Analysis shows that such a representation is possible. In fact, in the presence of a magnetic bias flux in the ring of the interferometer a high-frequency circulating current appears as the result of Josephson generation (self-pumping). In the working regime at $\nu_0 \sim 1$ only the fundamental harmonic of the current is of great importance. It follows from Eq. (15) for $l = 0$ that $j \approx -\sin(\varphi_s/2)\cos\varphi$, from which, using the main equation (4), we can find

$$j_1 = 2\tan(\varphi_s/2)\,(I - \nu_0)\,[I + (I - \nu_0)\alpha]\sin\nu_0\tau. \tag{24}$$

The variations of $\varphi_s = \varphi_b + \tilde\varphi_s$ in the presence of a signal $\tilde\varphi_s$ lead to amplitude-frequency modulation of the circulating current. The inverse demodulation and separation of the low-frequency voltage in the interferometer ring are accomplished as the result of the nonlinear properties of the Josephson junction,

$$\nu = \dot\varphi = \frac{\nu_0^2}{I + \alpha}\exp\left[\cot\!\left(\frac{\varphi_s}{2}\right)\int\limits_0^\tau j(\tau)\,d\tau\right]. \tag{25}$$

It is easy to show that taking into account of only the first harmonic already provides a low-frequency voltage close in value to the exact value of ν_0. For this purpose we substitute (24) into (25) and after straightforward transformations obtain

$$\begin{gathered}\nu \approx \frac{\nu_0^2}{I + \alpha}\exp\left[-\frac{2I(\nu_0 - I)}{\nu_0}\right]\left\{\mathcal{J}_0\left[\frac{2I(\nu_0 - I)}{\nu_0}\right]\right. \\ \left. + 2\mathcal{J}_1\left[\frac{2I(\nu_0 - I)}{\nu_0}\right]\cos\nu_0\tau + \text{higher harmonics}\right\}\end{gathered} \tag{26}$$

($\mathcal{J}_{0,1}$ are Bessel functions). From this we obtain for the low-frequency part of the voltage

$$\nu^{(sl)} = \nu_0\left(1 - \frac{\alpha(1-\alpha)}{I} + \left(\frac{\alpha}{2I}\right)^2 (\alpha^2 - 4\alpha + 2) + \ldots\right). \tag{27}$$

In the working regime: $1 < I < 2$, $\alpha = \cos(\varphi_s/2) \sim 0.7$, $\nu_0 \sim 1$, the contribution of higher harmonics to $\nu^{(sl)}$ does not exceed 20%.

Equations (24)–(27) demonstrate the parametric nature of the operation of a quantum interferometer as a probe with self-pumping and self-detection. This is exact for $l = 0$. If the ring inductance is different from zero, there also appears a mechanism due directly to the constant compensating current in the ring. However, for $l \lesssim 1$ (which is satisfied in experimental circuits) the parametric principle must be considered dominant.

As was noted in Sec. I, the rise in sensitivity of parametric probes with increase of the pumping frequency in reality is limited by quantum fluctuations. The latter are taken into account by Eq. (22), which for $\beta \to \infty$ ($\omega_0 \to \infty$, $T \to 0$) gives a relation similar to Eq. (3). It is important, however, to evaluate the critical pumping frequency for which Eq. (23) is still satisfied. This critical frequency depends on the efficiency of conversion of high-frequency fluctuations into the low-frequency region. It obviously corresponds to the value β_{cr} at which the two terms in the curly brackets of Eq. (22) become equal. In majorized form this condition is

$$\beta_{cr} > l^2/\nu_0(l-\nu_0)^2. \quad (28)$$

For $\nu_0 \sim 1$ it follows from this that $\beta_{cr} > 10$ or $\hbar(\omega_0)_{cr} > 10kT$. Thus, for a quantum interferometer the critical self-pumping frequency turns out to be an order of magnitude higher than the value expected *a priori* (see Sec. I).

5. DISCUSSION OF RESULTS

The principal result of the analysis carried out above is the conclusion that a quantum interferometer in principle can serve as the parametric converter for a gravitational antenna with an extremely high pumping frequency. In fact, the Josephson generation frequency, for example, for contact parameters $V_0 \sim 10^{-3}$ V and $J_c \sim 10^{-6}$ A, amounts to $\omega_0 = 2\pi V_0/J_c\Phi_0 \approx 3\cdot 10^{12}$, which is an order of magnitude higher than for inductive converters working in known antenna models.[2,4] At the same time it exceeds the critical frequency of the quantum sensitivity limit (28), for which $(\omega_0)_{cr} > 10kT/\hbar \approx 6\times 10^{12}$ ($T \sim 4$K). It is notable also that a quantum interferometer is a type of parametric probe which does not require an external pumping generator and demodulator (detector), which are sources of additional fluctuations limiting the sensitivity of the best experimental models of parametric converters.[15] This feature of a quantum interferometer is unique.

Returning to the estimates (1) and (2) and substituting in these equations instead of ω_e the frequency $\omega_0 = 3\times 10^{12}$ while retaining the remaining parameters, we can see that in principle it becomes possible to detect accelerations $(F/m)_{min} \gtrsim 10^{-10}$ cm/sec^2 for $m \approx 3\times 10^4$ g and $(F/m)_{min} \gtrsim 10^{-11}$ cm/sec^2 for $m \approx 5\times 10^6$ which completely satisfies the requirements imposed on second-generation antennas.

It is important to note that, in contrast to the case of a single-contact SQUID in a hysteresis-free regime (see Ref. 7), Eq. (23) determines not the limiting achievable sensitivity, but the force detectable with retention of the maximum absolute value of response signal. The condition (2) shows that an amplifier with noise temperature of 10 K will be sufficient for detection of pulses at a level 10 erg/cm^2 for a mass $m \approx 3\times 10^4$ g and of 10^{-1} erg/cm^2 for a mass $m \approx 5\times 10^6$ g.

In conclusion we note that although the main calculation (Sec. III) has been carried out for the case $l=0$, the results should not change greatly for $l<1$, which is typical for practical circuits. Realization of an antenna with a quantum interferometer in practice, of course, involves difficulties in preparation of contacts with the specified characteristics J_c and R and also difficulties in matching them with the subsequent amplifier.

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[1]SQUID is an acronym for superconducting quantum interference device.

[1]V. B. Braginsky, Proc. of the Conf., E. T. G., Pavia, Italy, 1976 (in press).

[2]V. B. Braginskiĭ, A. B. Manukin, E. I. Popov, V. N. Rudenko, and A. A. Khorev, Zh. Éksp. Teor. Fiz. 66, 801 (1974) [Sov. Phys. JETP 39, 387 (1974)].

[3]A. V. Gusev and V. N. Rudenko, Radiotekhnika i élektronika 21, 1865 (1976).

[4]S. P. Boughn, W. M. Fairbank, R. P. Giffard, J. N. Hollenhorst, M. S. McAshan, H. J. Paik, and R. C. Taber, Phys. Rev. Lett. 38, 454 (1977).

[5]V. M. Rudenko, D. B. Khalyapin, and V. R. Magnushevskiĭ, Maloshumyashchie vkhodnye tsepi SVCh priemnykh ustroistv (Low-Noise Input Circuits for Microwave Receivers), Svyaz' (Communications) Publishing House, 1971.

[6]V. B. Braginskiĭ, Yu. I. Vorontsov, and F. Ya. Khalili, Zh. Éksp. Teor. Fiz. 73, 1340 (1977) [Sov. Phys. JETP 46, 705 (1977)].

[7]A. V. Gusev and V. N. Rudenko, Zh. Éksp. Teor. Fiz. 72, 1217 (1977) [Sov. Phys. JETP 45, 637 (1977)].

[8]R. Feynman, R. B. Leighton, and M. Sands, The Feynman Lectures on Physics, Vol. 3, Addison Wesley, Reading, Massachusetts, 1963. Russ. transl., Vol. 9, Mir, 1967, p. 247.

[9]I. O. Kulik and I. K. Yanson, Effekt Dzhozefsona v sverkhprovdyashchikh tunnel'nykh strukturakh, Nauka, 1970. English translation, Josephson Effect in Superconducting Tunneling Structures, Halsted Press, 1972.

[10]V. V. Danilov, K. K. Likharev, O. V. Snegirev, and E. S. Soldatov, Radiotekhnika i élektronika 22, 2383 (1977).

[11]K. K. Likharev and V. K. Semenov, Pis'ma v Zh. Éksp. Teor. Fiz. 15, 625 (1972) [JETP Lett. 15, 442 (1972)].

[12]N. N. Bogolyubov and Yu. M. Mitropol'skiĭ, Asimptoticheskie metody v nelineĭnoĭ mekhanike (Asymptotic Methods in Nonlinear Mechanics), Nauka, 1966.

[13]R. L. Stratonovich, Izbrannye voprosy teorii fluktuatsiĭ v radiotekhnike, Sov. Radio, 1961. English translation, Topics in the Theory of Random Noise, 1963–7, Gordon.

[14]C. D. Tesche and J. Clark, IEEE Transactions, MAG-13, 859 (1977).

[15]V. B. Braginskiĭ, V. G. Petnikov, V. I. Panov, and V. D. Popel'nyuk, Prib. Tekh. Éksp., No. 1, 234 (1977) [Instrum. Exp. Tech.].

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