Skin effects in metals in a perpendicular magnetic field

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The behavior of the surface impedance of anisotropic metals in a perpendicular magnetic field in the radio-frequency range is investigated theoretically and experimentally. It is found that below the helicon threshold there are two regions in which the skin effect has a different character. In weak fields, the skin effect is anomalous. However, the surface resistance R_+ and reactance X_+ vary rapidly with the magnetic field. In stronger fields, there is a skin effect of a novel type. Here the change in the impedance of the bulk metal with changing field is non-monotonic. The results of the calculations are in qualitative agreement with the experimental data for silver, copper, and aluminum.

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INTRODUCTION

The effect of a constant field H on the surface impedance of a metal under conditions of the anomalous skin effect was considered in Ref. 1. It follows from the results of Ref. 1 that the value of the impedance in weak fields differs little from its value at $H = 0$, i.e., that the presence of a magnetic field has practically no effect on the anomalous skin effect. This conclusion was confirmed by Alig,[2] who made a numerical calculation of the dependence of the impedance of an alkali metal on the value of a magnetic field perpendicular to the surface. According to Ref. 2, in the range of fields much weaker than the threshold field of the helicon, the surface resistance R_- and reactance X_- change slowly with H both for specular and for diffuse reflection of the electrons from the surface. This result also follows from the analytic expression obtained in Ref. 3 for the impedance.

It is shown in the present paper that the situation can be different in uncompensated metals with anisotropic Fermi surfaces. In the case of certain orientations of the vector H relative to the crystallographic axes, a number of such metals reveal a rapid change in the surface impedance in fields much smaller than the threshold field of the helicon. This pertains to the impedance of the bulk metal for both circular polarizations. In order that the impedance be strongly dependent on the value of H in the region of weak fields, the Fermi surface should possess the following feature: The principal group of carriers should have such a dispersion law that their displacements along the magnetic field in one cyclotron period are greater than a certain minimum value. This occurs if the function $\partial S/\partial p_z$ has a broad minimum ($S(p_z)$ is the cross sectional area of the Fermi surface cut by the plane $p_z = $ const, z axis parallel to H). We note that in the total absence of electrons with small displacements there would be no collisionless wave absorption to cause the anomalous skin effect. In real metals, there is always a group of carriers with small displacements. However, it frequently happens that the number of such carriers is very small. These carriers determine the dissipative part of the nonlocal conductivity, the value of which in weak magnetic fields is practically independent of H. The principal group of carriers with large displacements makes a contribution

proportional to H to the nondissipative part of the non-local conductivity. Therefore, the region of weak fields, in which the dissipative conductivity of the small group is predominant and the anomalous skin effect takes place, turns out to be significantly narrower than in alkali metals, and the impedance changes sharply with magnetic field. In stronger fields, the nondissipative component becomes dominant and the character of the skin effect changes. The damping of the radiofrequency field in the skin layer in this region is principally exponential, while the thickness of the skin layer is proportional to $(\omega H)^{-1/4}$, where ω is the frequency of the exciting field. We emphasize that such a skin effect is absent in alkali metals, in which the dissipative component of the conductivity is important up to the helicon threshold.

We shall carry out the calculation below for a comparatively simple model that accounts for the significant properties of real metals, demonstrate the features of the skin effect mentioned above, and make a comparison with the data of our experiments for silver, copper, and aluminum.

THEORY

1. The nonlocal conductivity of a metal with a spherical Fermi surface in a perpendicular magnetic field $(\mathbf{k} \| \mathbf{H} \| z)$ is described by the expression[4,1]

$$\sigma_\pm = \sigma_{xx} \pm i\sigma_{yx} = \frac{ne^2}{m[\nu - i(\omega \pm \Omega)]} F(q), \tag{1}$$

$$F(q) = \frac{3}{2q^2}\left(1 - \frac{1-q^2}{2q}\ln\frac{1+q}{1-q}\right), \tag{2}$$

$$q = \frac{kv_F}{\Omega \pm (\omega + i\nu)}, \tag{3}$$

where e is the absolute value of the charge, m is the effective mass of the conduction electrons, n is the concentration, v_F is the Fermi velocity, $\Omega = eH/mc$ is the cyclotron frequency, ν is the frequency of collisions with the lattice, k is the wave vector of the electromagnetic field in the metal, the upper sign refers to "plus" circular polarization, and the lower to "minus" polarization.

We are interested in the case of frequencies ω that

are small in comparison with the frequency of collisions ν and such magnetic fields in which $\Omega \gg \nu$. Upon satisfaction of these conditions, the nonlocal conductivity $\sigma_{\pm}$ does not depend on ω, and the value of q reduces to the ratio of the maximum displacement of the electrons per cyclotron period to the electromagnetic wavelength, $q \approx k v_F / \Omega$.

As is well known, at values of the magnetic field corresponding to the inequality $q^2 < 1$, a helicon wave exists in the "minus" circular polarization. In the region $q^2 > 1$, the logarithm in (2) has a finite imaginary part due to the collisionless absorption that makes propagation of the helicon impossible and leads to the anomalous skin effect.

In the region of weak magnetic fields, where $q^2 \gg 1$, the approximate expression for the function F is of the form

$$F(q) = \mp \frac{3\pi i}{4q} + \frac{3}{q^2}. \tag{4}$$

The first term on the right side corresponds to collisionless absorption of the wave by electrons with small longitudinal velocities (Landau damping), while the second describes the dispersion of the nondissipative part of $\sigma_{\pm}$ due to electrons with large velocities.

Substituting (4) in (1), we have

$$\sigma_{\pm} = \frac{3ne^2}{m\Omega} \left(\frac{\pi}{4|q|} \pm \frac{i}{q^2} \right). \tag{5}$$

The first term in (5) does not depend on H, while the numerical coefficient in the second term is of the same order as in the first. Therefore, in the region $q \gg 1$ the second component is much smaller than the first, whence it follows that the magnetic field has a weak effect on the anomalous skin effect.

The situation is different for anisotropic metals if the magnetic field H is oriented relative to the Fermi surface in such a way that the plot of $\partial S / \partial p_z$ for the basic group of carriers has a minimum, i.e., there are no electrons among them with small displacements along the field. In particular, this takes place in noble metals at $k \| H$ [110][5] and in aluminum[6] at $k \| H$ [100]. In the study of dopplerons in copper in Ref. 7, a model of the Fermi surface was considered with minimum $\partial S / \partial p_z$, for which the function F, which describes the electron contribution to the conductivity, is of the form[1]

$$F(q) = 1 - \frac{q^2}{(q^2-1)^{1/2}} \text{arctg} \frac{1}{(q^2-1)^{1/2}}, \tag{6}$$

where

$$q = \frac{kc}{2\pi eH} \cdot \left(\frac{\partial S}{\partial p_z} \right)_{min}. \tag{7}$$

At large q, this function is real, $F \propto -q^{-2}$, i.e., the electrons make no contribution to the collisionless absorption. Moreover, it must be noted that the sign of F is the opposite of the sign of the second term in (4). This is also a consequence of the minimum of $\partial S / \partial p_z$.

Besides the electrons, copper has hole orbits of the "dog bone" type, located near the central cross section of the Fermi surface. Among these carriers there are holes with small displacements, causing collisionless absorption. However, the hole concentration in copper is small, therefore the dissipative conductivity term associated with them is numerically small. Thus, the asymptotic expression for the conductivity at $q \gg 1$ is of the form

$$\sigma_{\pm} = \frac{nec}{H} \left(\frac{\alpha}{|q|} \pm \frac{i}{q^2} \right), \tag{8}$$

where α is a small numerical coefficient.

We emphasize that the asymptotic form (8) is not the consequence of the model and occurs in all cases in which the basic group of carriers has a minimum of $\partial S / \partial p_z$.

2. We want to describe the behavior of the impedance not only in fields that are small in comparison with the helicon threshold field, but also in the region of the helicon and the doppleron that exists in the opposite polarization.[7-9] Therefore, we consider a simple Fermi-liquid model for which the nonlocal conductivity has the correct asymptotic forms at both large and small q:

$$\sigma_{\pm} = \frac{nec}{H} \left[\frac{\alpha q^2}{(1+q^2)^{3/2}} \mp \frac{1 \pm i\gamma}{(1 \pm i\gamma)^2 - q^2} \right], \quad \gamma = \frac{\nu}{\Omega}. \tag{9}$$

The second term represents the contribution of the electron Fermi surface, which has the shape of a paraboloid of revolution.[10] We have written unity in the denominator of the first term so that the collisionless absorption vanishes as $q \to 0$, as is the case in real metals.

The dispersion equation for circular polarizations of the electromagnetic field $k^2 c^2 = 4\pi i \omega \sigma_{\pm}$ can be written in this model in the form

$$D_{\pm}(q) = 0, \tag{10}$$

$$D_{\pm}(q) = q^2 - \xi \left[\frac{i \alpha q^2}{(1+q^2)^{3/2}} \mp \frac{1}{1-q^2} \right], \tag{11}$$

where

$$\xi = 4\pi \omega n p_0^2 c / e H^3 \tag{12}$$

for the paraboloid $\partial S / \partial p_z = 2\pi p_0 = \text{const}$. In the second term in square brackets, we have neglected the small imaginary terms of order γ.

We consider first the region of fields satisfying the inequality

$$\alpha \xi^{1/4} \ll 1. \tag{13}$$

Under the condition (13), the first component in the square brackets in (11) is small and can be neglected in comparison with the second. Here the dispersion equation has two solutions: q_H and q_D, which describe the distribution of the radiofrequency field in the metal and the value of the impedance. In the minus polarization, both roots are real at $\xi < \frac{1}{4}$. One of them refers to the

helicon, and the other to the doppleron. At $\xi>\tfrac{1}{4}$, both roots are essentially complex. In the plus polarization, the root q_H is imaginary; it is connected with the "damped helicon." The root q_D, which is real, characterizes the doppleron.

We assume the reflection of the electrons from the surface to be diffuse. Then the surface impedance $Z_\pm$ of the semi-infinite metal is determined by the expression of Ref. 11, which can be written in the form

$$Z_\pm = a\left[\frac{i}{2\pi}\int_{-\infty}^{\infty} dq \ln\frac{D_\pm(q)}{q^2}\right]^{-1},\qquad(14)$$

$$a=4\pi\omega p_0/ceH.\qquad(15)$$

In the considered region of fields, the integral in (14) is simply expressed in terms of the solution of the dispersion equation:

$$Z_\pm = a(q_H+q_D\mp1)^{-1}\qquad(16)$$

If we substitute the explicit expressions for q_H and q_D in (16) (the roots of the dispersion equation, which have positive imaginary parts), we then obtain

$$Z_+=a\{[\tfrac{1}{2}+(\tfrac{1}{4}+\xi)^{1/2}]^{1/2}-1+i[(\tfrac{1}{4}+\xi)^{1/2}-\tfrac{1}{2}]^{1/2}\}^{-1};$$
$$Z_-=a\{[\tfrac{1}{2}-(\tfrac{1}{4}-\xi)^{1/2}]^{1/2}+1-[\tfrac{1}{2}+(\tfrac{1}{4}-\xi)^{1/2}]^{1/2}\}^{-1},\quad \xi<\tfrac{1}{4},\qquad(17)$$

$$Z_-=\frac{a}{2\sqrt{\xi}}[1-i(2\xi^{1/2}-1)^{1/2}],\quad \xi>\tfrac{1}{4}.\qquad(18)$$

These are in fact interpolation formulas. They describe correctly the behavior of the impedance both in the region of existence of the helicon in strong fields ($\xi\ll1$) and in fields that are much smaller than the threshold value ($\xi\gg1$). The dependence of the impedance Z_- on the field in the vicinity of the helicon threshold is sensitive to the model of the Fermi surface and one should not attribute a special significance to it.

We consider in more detail the skin effect in the region of comparatively weak fields, where

$$\xi\gg1\gg\alpha\xi^{1/2}.\qquad(19)$$

Here the expressions (17)–(18) for the impedance are simplified and take the form

$$Z_+=a\frac{1-i}{2\xi^{1/2}},\quad Z_-=a\left(\frac{1}{2\xi^{1/2}}-\frac{i}{2^{1/2}\xi^{1/4}}\right).\qquad(20)$$

It follows from (20) with account of (12) and (15) that R_+ and $X_\pm$ are proportional to $(\omega^3/H)^{1/4}$, while $R_-\propto(\omega H)^{1/2}$. In the minus polarization, the electromagnetic field decays exponentially over a distance $\delta=1/k''$, where k'' is the imaginary part of k and is a solution of the dispersion equation. This distance is

$$\delta=\frac{\sqrt{2}}{\xi^{1/4}}\frac{cp_0}{eH}\sim(\omega H)^{-1/4}.\qquad(21)$$

In the plus polarization, the field is the sum of the doppleron and skin terms. The doppleron term has small damping, while the skin term decays over a dis-

tance of the order of δ (21).

3. We now study the region of weak magnetic fields

$$(\alpha^2\xi)^{1/3}\gg1.\qquad(22)$$

In this case, the dispersion equation (10) can be written approximately in the form

$$q^2-\xi\left[\frac{i\alpha}{(1+q^2)^{1/2}}+\frac{1}{q^2}\right]=0.\qquad(23)$$

The expression in square brackets, which is connected with the nonlocal conductivity, is defined on the real q axis. We continue it analytically into the complex q plane, defining the first sheet by the cuts shown in Fig. 1. Then the dispersion equation for the plus polarization has in the upper halfplane of the first sheet the solution

$$q_1^+=i(\alpha\xi)^{1/3}+i/3\alpha,\qquad(24)$$

and two solutions q_0 and q_1^- for the minus polarization:

$$q_0=i/\alpha,\quad q_1^-=i(\alpha\xi)^{1/3}-i/3\alpha.\qquad(25)$$

The locations of these roots are shown in Fig. 1. The remaining roots of the dispersion equations lie in the lower halfplane and we shall not be interested in them in what follows.

The surface impedance is determined by the integral (14). We carry out integration by parts and deform the contour of integration to the cut in the upper halfplane, isolating the pole components. Then, using the smallness of α, we displace the cuts to the real semi-axis. Finally, replacing q^2 by x, we obtain

$$a(Z_\pm)^{-1}=\sum_i q_i^\pm+\frac{\alpha\xi}{2\pi}\int_0^\infty\frac{3x^2\pm\xi}{(x^2\mp\xi)^2+(\alpha\xi)^2x}dx,\qquad(26)$$

where q_i are the dispersion-equation roots lying in the upper halfplane on the first sheet.

Further calculation of the integrals for different polarization is carried out separately. As a result of uncomplicated but tedious calculations with the use of the inequality (22), we find

$$Z_\pm=Z_0(1+\Delta_\pm),\qquad(27)$$

$$Z_0=2\sqrt{3}e^{-\pi i/3}a(\alpha\xi)^{-1/3},\qquad(28)$$

$$\Delta_\pm=\pm\frac{\sqrt{3}e^{2\pi i/3}}{8\pi(\alpha^2\xi)^{1/3}}\ln[\pm(\alpha^2\xi)^{1/3}e^{1+\pi i/3}].\qquad(29)$$

The quantity Z_0 does not depend on H. It represents the impedance of a metal with carrier concentration αn
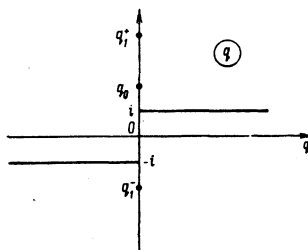

FIG. 1. Cuts and roots of the dispersion equation in the plane of q.

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under the conditions of the anomalous skin effect at $H = 0$. The functions $\Delta_{\pm}$ describe the change in the impedance with the magnetic field. Thanks to the fact that ξ enters into (29) only in combination with the small factor α^4, the functions $\Delta_{\pm}$ (in contrast with the case of the alkali metal,[3] change rapidly with H. It should be emphasized that the impedance for linear polarization, $Z_{xxx} = Z_{yy} = (Z_+ + Z_-)/2$, does not contain a logarithmic term and therefore is a more slowly varying function of the field. Moreover, we note that in specular reflection of the carriers from the surface, the impedance has likewise no logarithmic singularities.

4. Plots of the surface resistance and the reactance for both circular polarizations are shown in Fig. 2. These are obtained by numerical integration of (14) with the conductivity (9). The calculation was performed for the following values of the parameters: $n = 5 \times 10^{22}$ cm^{-3}, $\alpha = 0.2$; $p_0 = 0.47\hbar$ Å^{-1}, the free path length of the electrons $l = p_0/m\nu = 0.5$ mm, $m = 10^{-27}$g, frequency $\omega/2\pi = 200$ kHz. Since the chosen value of α is not very small, it is impossible to expect numerical agreement of these curves with the asymptotic formulas calculated earlier. However, the character of the change of $R_{\pm}$ and $X_{\pm}$ with the field H is in complete agreement with the expressions (17)–(18) and (27)–(29). It is seen from the drawing that in weak fields the functions $Z_{\pm}(H)$ undergo a sharp change; hence the signs of the derivatives of $dR_{\pm}/dH$ and $dX_{\pm}/dH$ are opposite to the signs of these
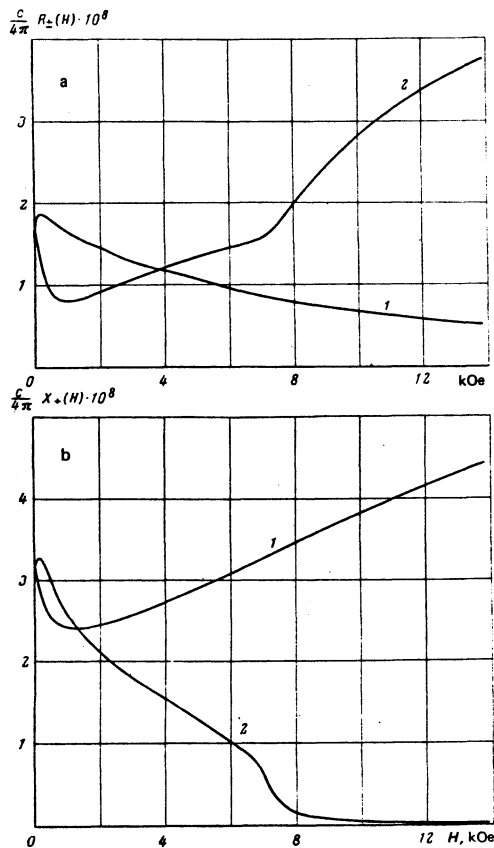

FIG. 2. Plots of the magnetic field dependence of the surface resistance (a) and reactance (b) of a semi-infinite metal. Curves 1 correspond to the plus polarization, curves 2 to the minus.

functions in the alkali metal.[2] The significant difference from the alkali metal lies in the fact that the functions $R_{\pm}(H)$ and $X_{\pm}(H)$ have extrema on going to the region of moderate fields. It should be noted in this case that, in correspondence with Eqs. (17)–(18) and (27)–(29), the extrema of R_+ and X_- are located at smaller fields than the extrema of R_- and X_+.

5. We noted above that the character of the reflection of the electrons affects the change in the impedance of the semi-infinite metal in weak fields. It is also known that the behavior of the impedance in the vicinity of the helicon and doppleron thresholds[2,12] is very sensitive to the character of the reflection. It was shown in Refs. 13, 14 that in diffuse reflection, the amplitude of the doppleron oscillations in compensated metals in strong fields is much greater than that in specular reflection. Is is of interest to study the effect of diffuse reflection on the doppleron oscillations of the uncompensated metal. We do this with the help of the model (9) described above. We determine the amplitude of the doppleron oscillations of the impedance of the plate for the plus polarization in the region of moderate fields (13). Since the holes play a small role in this region, we can use the results of Ref. 13 for the amplitude of the doppleron oscillations in the paraboloidal model. In order to obtain the formula for the impedance of the plate in the plus polarization we must replace the quantity 1 $- ij$ in Eq. (29) of Ref. 13 by $- (1 + ij)$ and q_s by q_H. Assuming that the thickness of the plate d is large in comparison with the damping length of the doppleron, we can represent the oscillatory part of the impedance ΔZ_+ in the form

$$\Delta Z_+ = 4a \frac{1 - q_D^2}{q_D(q_D^2 - q_H^2)} \exp(ik_D d) \tag{30}$$

in the case of specular reflection, and

$$\Delta Z_+ = 4a \frac{q_D(q_D - 1)(q_D + q_H)}{(q_H + q_D - 1)^2(q_D + 1)(q_H - q_D)} \exp(ik_D d) \tag{31}$$

in the case of diffuse reflection, where $k_D = q_D eH/p_0 c$ is the wave vector of the doppleron.

The solutions of the dispersion equations (10)–(11) for the plus polarization in the region (13) are of the form

$$q_H = i[(1/4 + \xi)^{1/2} - 1/2]^{1/2}, \tag{32}$$

$$q_D = [(1/4 + \xi)^{1/2} + 1/2]^{1/2} + 1/4 i\alpha\xi[(1/4 + \xi)^{1/2} + 1/2]^{-1}. \tag{33}$$

In (33) we have taken into account a small imaginary term (proportional to α) that is due to the dissipative hole conductivity. This term determines the damping of the doppleron and, consequently, the value of the exponential in (30)–(31), although it does not affect the pre-exponential factors. Here and below, we shall not write for the damping of the doppleron the obvious component ij due to collisions.

Substituting (32)–(33) in (30)–(31), we get for the amplitude of the doppleron oscillations

$$|\Delta Z_+| = 2a[1 - 1/2(1/4 + \xi)^{-1/2}][(1/4 + \xi)^{1/2} + 1/2]^{-1/2} \exp(-k_D'' d) \tag{34}$$

in the case of specular reflection, and

$$|\Delta Z_+| = 2a\{1 + [\tfrac{1}{2} + (\tfrac{1}{4} + \xi)^{\frac{1}{2}}]^{\frac{1}{2}}\}^{-1} \exp(-k_D''d) \qquad (35)$$

in the case of diffuse reflection; $k_D'' - \mathrm{Im}\, k_D$.

In the range of fields in which $\xi \gg 1$, the amplitudes of the oscillations in both cases are the same:

$$|\Delta Z_+| = \frac{4\pi}{c}\left(\frac{4\omega^3 p_0^2}{\pi n e^3 cH}\right)^{\frac{1}{2}} \exp\left[-\alpha\left(\frac{\pi \omega n e}{4cH}\right)^{\frac{1}{2}} d\right]. \qquad (36)$$

At small ξ, the amplitude of the oscillations in the diffuse case falls off as $1/H$, while in the specular case it contains the extra factor 4ξ and falls off as H^{-4}.

EXPERIMENT AND DISCUSSION

The $R(H)$ and $X(H)$ dependences of silver, copper and aluminum were studied experimentally in the case of circular polarizations of the exciting field. The samples were plane-parallel plates, cut by the electric spark method from single crystal blanks. After the cutting, the plates of silver and copper were treated by the method described in Refs. 15 and 16. The normals to their surfaces coincide with the direction of the [110] axis. The dimensions of the samples of silver and copper after treatment amounted to $14 \times 7 \times 0.811$ and $10 \times 8 \times 0.25$ mm, respectively, while the resistance ratios $\rho_{300}/\rho_{4.2}$ were 1.6×10^4 and 1.4×10^4, respectively. The normal to the aluminum plate coincided with the [100] axis. The sample was etched in concentrated nitric acid before the measurements. Its dimensions were $12 \times 6 \times 0.47$ mm and the resistance ratio $\rho_{300}/\rho_{4.2} = 7 \times 10^3$.

The measurements of the impedance were carried out in a magnetic field perpendicular to the surface of the plate, at a temperature of 4.2 K and in the radiofrequency range. The real and imaginary parts of the impedance were studied by means of an amplitude bridge and an autodyne, in the coil of the tank circuit of which the sample was placed. The measurement apparatus assured a reliable discrimination of the signals proportional to $R(H)$ and $X(H)$. The method of measurements and the method of calibration of the impedance changes are described in detail in Ref. 17. The coil of the tank circuit, and also the auxiliary coil which permits production of a circularly polarized field, were wound of brass wire of diameter 0.08 mm. The use of copper coils led to a significant distortion of the experimental plots as a consequence of the dependence of the impedance of the wire on the magnetic field.

Figures 3 and 4 show the $R_\pm(H)$ and $X_\pm(H)$ curves for silver and aluminum. We subtracted from the plotted values a correction connected with the weak dependence of the impedance of the brass coils on the magnetic field. Above all, note should be made of the presence of a strong dependence of the impedance of silver on the magnetic field intensity over the entire interval of its change.

The $R_\pm$ and $X_\pm$ curves in Fig. 3 are nonmonotonic; in fields $H < 3$ kOe they have extrema. In stronger fields, the resistance R_+ decreases with increase in H (if we disregard small oscillations) and R_- increases. The
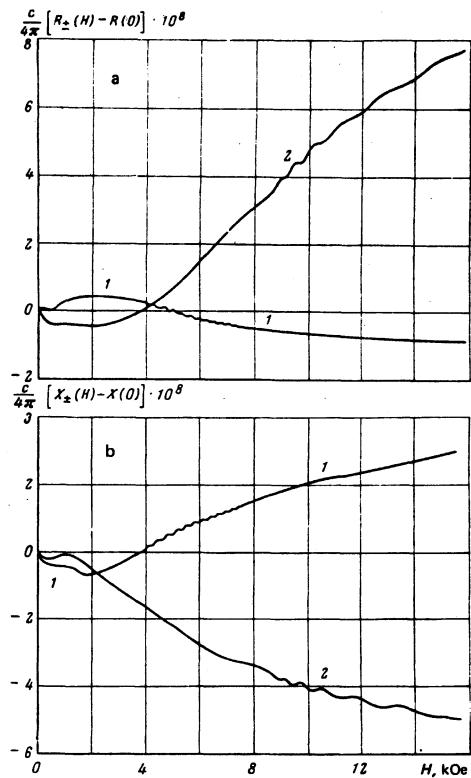

FIG. 3. Plots of the surface resistance (a) and reactance (b) of a plate of silver in the case of plus circular polarization (curves 1) and minus (curves 2). The magnetic field is $H \| n \|$ [100], the frequency $\omega/2\pi = 190$ kHz, $d = 0.811$ mm, $T = 4.2$ K.
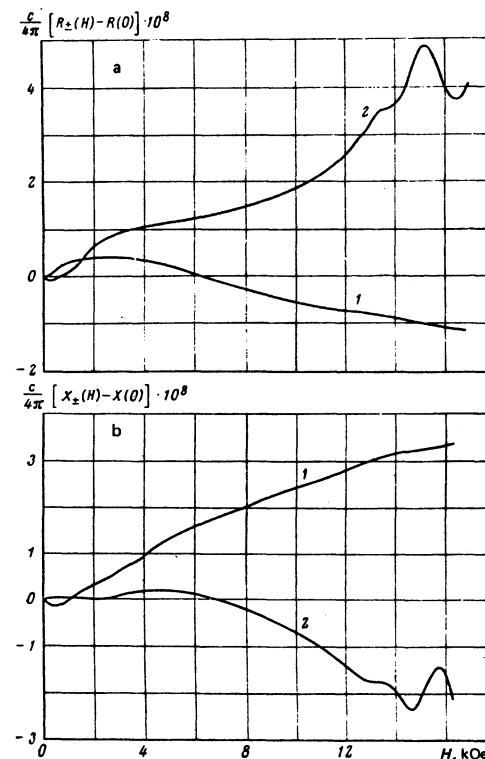

FIG. 4. Plots of the resistance (a) and reactance (b) of an aluminum plate for plus polarization (curves 1) and minus (curves 2). The magnetic field is $H \| n \|$ [100], the frequency $\omega/2\pi = 190$ kHz, $d = 0.47$ mm, $T = 4.2$ K.

signs of the change in the reactances $X_\pm$ in the region $H > 3$ kOe are proportional to the signs of the change of the corresponding resistances R. Since the signs of the change of R_+ and R_-, and also of X_+ and X_-, are different, the dependence of the impedance on the field in the linear polarization turns out to be much weaker (the impedance in the linear polarization is equal to the half-sum of Z_+ and Z_-). It should be noted that in most experimental studies of the impedance of metals in a magnetic field, the derivative dR/dH or dX/dH was measured for linear polarizations with copper coils. This apparently makes difficult the observation of the singularities of the impedance of uncompensated metals in a magnetic field.

The oscillations of the impedance on the curves of Fig. 3 are connected with the penetration of the electromagnetic field through the plate. The oscillations of R_- and X_- in strong fields are due to the excitation of the helicon. In the plus polarization, oscillations are seen that are connected with the doppleron wave. The properties of dopplerons in silver were studied in Ref. 8.

We do not present the plots of the impedance for copper, since the behavior of $Z_\pm(H)$ for it does not differ qualitatively from the corresponding curves for silver. The basic difference is that the singularities near zero field were more clearly pronounced than in the curves of Fig. 3. It is probable that this is connected with the small free path length of the carriers in copper.

Figure 4 shows the $R_\pm(H)$ and $X_\pm(H)$ curves for aluminum. In general outline, the behavior of the impedance of aluminum is the same as that of the impedance of silver. In a comparison of the plots in Figs. 4 and 3, one should compare the corresponding curves of opposite polarizations, since the majority carriers in aluminum are holes, while those in silver are electrons. The impedance of aluminum in weak fields changes more smoothly than that of silver. The doppleron oscillations in aluminum have a relatively small amplitude and are observed only in the derivatives of the impedance. In the curves of Fig. 4 for the plus polarization, in strong fields, large helicon oscillations are observed.

A comparison of the curves in Figs. 3 and 4 with the corresponding curves on Fig. 2 shows that the extremal and theoretical results are in qualitative agreement. An exception is the region of very weak fields ($\Omega \sim \nu$), where our theory is inapplicable.

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