conduction electrons that are compressed. At the same time, in metallic antiferromagnets the average polarization of the conduction electrons is zero, and the results of [5], where the field at the ^{57}Fe nuclei in the antiferromagnetic alloy Pt_3Fe was practically independent of the pressure, were explained naturally from this point of view. It appears that $FeSn_2$ is not a "good" metal and the strong dependence of the hyperfine field at the ^{57}Fe nuclei is connected with partial localization of the external magnetic electrons of the Fe atoms in directed chemical bonds.

In conclusion, the authors thank V. P. Mar'in for help with a number of experiments.

[1]N. N. Delyagin and É. N. Kornienko, Zh. Eksp. Teor. Fiz. **61**, 1946 (1971) [Sov. Phys. JETP **34**, 1036 (1972)].
[2]I. N. Nikolaev, V. P. Potapov, and V. P. Mar'in, Zh. Eksp. Teor. Fiz. **67**, 1190 (1974) [Sov. Phys. JETP **40**, 591 (1975)].
[3]I. N. Nikolaev, L. S. Pavlyukov, and V. P. Mar'in, Zh. Eksp. Teor. Fiz. **69**, 1844 (1975) [Sov. Phys. JETP **42**, 936 (1975)].
[4]I. N. Nikolaev, V. P. Potapov, and N. N. Delyagin, Zh. Eksp. Teor. Fiz. **70**, 241 (1976) [Sov. Phys. JETP **43**, 125 (1976)].
[5]I. N. Nikolaev, L. S. Pavlyukov, V. P. Potapov, and V. P. Mar'in, Fiz. Tverd. Tela (Leningrad) **18**, 1198 (1976) [Sov. Phys. Solid State **18**, 692 (1976)].
[6]I. N. Nikolaev and V. P. Potapov, Zh. Eksp. Teor. Fiz. **72**, 1600 (1977) [Sov. Phys. JETP **45**, 840 (1977)].
[7]V. I. Nikolaev, Yu. I. Shcherbina, and S. S. Yakimov, Zh. Eksp. Teor. Fiz. **45**, 1277 (1963) [Sov. Phys. JETP **18**, 878 (1964)];V. I. Nikolaev, Yu. I. Scherbina, and A. I. Karchevskiĭ, Zh. Eksp. Teor. Fiz. **44**, 775 (1963) [Sov. Phys. JETP **17**, 524 (1963)].
[8]G. Fabri, E. Germagnoli, M. Musei, and G. C. Locati, Nuovo Cimento B **40**, 178 (1965).
[9]G. Trumpy, E. Both, C. Djega-Mariadassou, and P. Lecocq, Phys. Rev. B **2**, 3477 (1970).
[10]W. Nicholson and E. Friedman, Bull. Am. Phys. Soc. **8**, 43 (1963).
[11]P. K. Iyengar, B. A. Dasannacharya, P. R. Vijayaraghavan, and A. P. Roy, J. Phys. Soc. Jpn. **17**, 247 (1962).
[12]M. B. Stearns, Phys. Rev. B **4**, 4081 (1971); V. Niculescu, K. Raj, J. I. Budnick, T. J. Burch, W. A. Hines, and A. H. Menotti, Phys. Rev. B **14**, 4160 (1976).
[13]V. N. Panyushkin, Fiz. Tverd. Tela (Leningrad) **10**, 1915 (1968) [Sov. Phys. Solid State **10**, 1515 (1968)].

Translated by J. G. Adashko

Rearrangement of the energy spectrum of $Pb_{1-x}Sn_xSe$ as a result of band inversion under pressure

N. B. Brandt, O. N. Belousova, V. P. Zlomanov, Ya. G. Ponomarev, E. P. Skipetrov, and V. I. Shtanov

Moscow State University
(Submitted 16 August 1977)
Zh. Eksp. Teor. Fiz. **74**, 646–657 (February 1978)

An investigation was made of the oscillatory and galvanomagnetic effects in single-crystal samples of *n*-type $Pb_{1-x}Sn_xSe$ ($x = 0.125$) at pressures up to 16 kbar. The dependences of the cyclotron masses at the Fermi level on the band gap ϵ_g were asymmetric relative to $\epsilon_g = 0$, as predicted in the theories of Dimmock and Martinez. The electron Fermi surface became spherical for negative values of ϵ_g. The experimental results obtained were combined with the data of other authors in a calculation of the parameters of the Dimmock dispersion law of carriers at the point L in the Brillouin zone.

PACS numbers: 71.25.Pi, 72.15.Gd, 62.50.+p

INTRODUCTION

Lead selenide (PbSe) and its alloys $Pb_{1-x}Sn_xSe$ ($x < 0.43$) crystallize in the NaCl lattice and are semiconductors with a narrow direct gap ϵ_g at the point L in the Brillouin zone. In the case of PbSe and $Pb_{1-x}Sn_xSe$ with $x < x_0$ ($x_0 = 0.15$ at $T = 4.2$°K) the terms L_6^- and L_6^+, which correspond to the bottom of the conduction band and the top of the valence band at atmospheric pressure,[1] become inverted at higher pressures.[2-5] The transition to the zero-gap state [$\epsilon_g \equiv E(L_6^-) - E(L_6^+) = 0$] at a pressure $p = p_0$, which depends on the composition x and temperature T, is accompanied by vanishing of the effective carrier masses $m^*(0)$ at the bottom of the conduction band and at the top of the valence band. The effective masses at the Fermi level $m^*(\epsilon_F)$ are affected much less by transition to the zero-gap state and the relative reduction in these masses decreases with increasing carrier density in the bands.[5]

The nature of the rearrangement of the carrier spectrum as a result of inversion of the terms at L depends on the strength of the interaction of the inverted terms with the more distant bands at the point L.[1-4,6] If this interaction can be ignored, the carrier dispersion law is of the Kane type[7] and the sign of the band gap ϵ_g is unimportant. Then, all the ϵ_g-dependent quantities vary symmetrically relative to the value $\epsilon_g = 0$ [the effective masses $m^*(\epsilon_F)$ pass through a minimum and the Fermi energy ϵ_F passes through a maximum if the carrier

 0038-5646/78/4702-0340$02.40

density remains constant]. It should be added that in the case of the two-band Kane model the electron and hole dispersion laws are mirror images of one another, the constant-energy surfaces are strictly ellipsoidal, and the anisotropy of these surfaces is independent of the Fermi energy ϵ_F and of the magnitude and sign of ϵ_g.

The influence exerted on the electron and hole dispersion laws of the four more distant bands at the point L, arranged in pairs above and below the Fermi level,[1] complicates greatly the nature of the changes in the spectrum as a result of band inversion at L.[2-4,6,8,9] Theoretical calculations carried out in the six-band approximation[4,6] predict a reduction in the anisotropy of the constant-energy surfaces at L on transition to negative values of ϵ_g and transformation of the Fermi surface to the perfectly spherical shape at some value of $\epsilon_g^* < 0$. This effect has been observed in studies of the anisotropy of the Fermi surface of $Pb_{1-x}Sn_xSe$ alloys of variable composition ($0 \leqslant x \leqslant 0.2$) at atmospheric pressure[8,9] and is clearly the most convincing proof of the validity of the models of the energy spectrum based on the six-band approximation.[4,6] A further increase in the absolute value of ϵ_g should split the extremum at L into three equivalent extrema located in the hexagonal plane of the Brillouin zone[4] (this represents a transition to a spectrum of the SnTe type).

It follows from the theory[2,4] that the interaction with the more distant bands results in an asymmetric, relative to $\epsilon_g = 0$, dependence of the effective masses at the Fermi level $m^*(\epsilon_F)$ (for $N = \text{const}$) on the energy gap: the average rate of rise of $m^*(\epsilon_F)$ with increasing $|\epsilon_g|$ beyond the point of inversion should be considerably greater than the average rate of change of $m^*(\epsilon_F)$ before inversion. Moreover, the minima in the band-gap dependences of $m^*(\epsilon_F)$ should shift toward the positive values of ϵ_g on increase in the carrier density N.[2,4]

To detect experimentally the theoretically predicted[2,4,6] asymmetry of the band-gap dependences of the effective masses and thus confirm anew the validity of the six-band Dimmock[6] and Martinez[4] models of the carrier spectrum, it is necessary to carry out detailed investigations of the band-gap dependences of the cyclotron masses in $Pb_{1-x}Sn_xSe$ in a wide range of values of $|\epsilon_g|$ before and after passing through the zero-gap state. Such investigations have not yet been carried out. In an earlier paper[5] we reported the results of a study of the Shubnikov-de Haas effect in n-type PbSe on transition to the zero-gap state under pressure. The dependences of the cyclotron masses m_{c100} on ϵ_g obtained[5] in the range $145\ \text{meV} \geqslant \epsilon_g \geqslant 0$ were insufficient to make an assured selection between the Kane model and the six-band Dimmock model.

It should be pointed out that at present there are far too few experimental data to determine reliably the validity of the six-band models.[4,6] Some of these data are only in qualitative agreement with the models or are even in conflict with them.[9,10] For example, the theory predicts an increase in the anisotropy of the Fermi surface of $Pb_{1-x}Sn_xSe$ on increase in the carrier density,[6] whereas it is reported[9] that the anisotropy of the hole Fermi surface decreases on increase in the hole density P in the alloy with $x = 0.06$. Deviations of the shape of the Fermi surface from an ellipsoid, due to the presence of the terms proportional to p^4 in the dispersion law, are observed only at carrier densities exceeding $10^{19}\ \text{cm}^{-3}$.[11] The six-band models of the carrier spectrum predict saturation of the carrier-density dependences of the effective masses when the density reaches $\sim 10^{20}\ \text{cm}^{-3}$. Kucherenko *et al.*[9] detected this saturation experimentally but found only a qualitative agreement between the experimental and theoretical dependences. The amount of experimental data on the degree of departure of the electron and hole dispersion laws at L from mirror-image symmetry is also insufficient.[2-4,8-12]

We investigated the galvanomagnetic effects using weak magnetic fields in the temperature range $4.2\,^\circ\text{K} \leqslant T \leqslant 300\,^\circ\text{K}$, and also the Shubnikov-de Haas effect at helium temperatures in n-type $Pb_{1-x}Sn_xSe$ ($x = 0.125 \pm 0.01$) at pressures of $1\ \text{bar} \leqslant p \leqslant 16$ kbar. A comparison of the results obtained in the present study with those deduced by us earlier from a study of the Shubnikov-de Haas effect in n-type PbSe[5] made it possible to deduce the band-gap dependences of the cyclotron mass m_{c100} in a wide range of ϵ_g: $145\ \text{meV} \geqslant \epsilon_g \geqslant -110$ meV. These dependences were strongly asymmetric relative to $\epsilon_g = 0$, as predicted by Dimmock[6] and Martinez.[4] Moreover, the electron energy surfaces became spherical at a pressure corresponding to $\epsilon_g^* \approx -100$ meV. The results obtained were used to determine the parameters of the Dimmock dispersion law.

1. MEASUREMENT METHOD. SAMPLES

The electrical resistivity ρ and the Hall coefficient R_H were determined in weak magnetic fields ($\mu H \ll 1$) for single crystals of n-type $Pb_{1-x}Sn_xSe$ ($x = 0.125$) in the temperature range $4.2\,^\circ\text{K} \leqslant T \leqslant 300\,^\circ\text{K}$. The temperature was controlled by a thermal regulation unit.[13] The galvanomagnetic effects and temperature were recorded with a two-channel digital measuring system.[14]

Quantum oscillations of the magnetoresistance (Shubnikov-de Haas effect) were investigated at temperatures $2.1\,^\circ\text{K} \leqslant T \leqslant 4.2\,^\circ\text{K}$ in the field of a superconducting solenoid $H \leqslant 55$ kOe. In investigations of the angular dependences of the period of the Shubnikov-de Haas oscillations $\Delta(1/H)$ the measurements were carried out in the field of a superconducting Helmholtz coil system ($H \leqslant 20$ kOe).

The precision of the measurements of the cyclotron masses and oscillation period was improved by suppressing the monotonic field dependence of the resistivity $\rho(H)$ with the aid of a computing device producing a signal proportional to $\alpha H \pm \beta H^2$.

Pressures $p \leqslant 16$ kbar were produced in a chamber made of heat-treated beryllium bronze.[15] The pressure-transmitting medium was a mixture of 50% pentane, 25% kerosene, and 25% transformer oil. At helium temperatures the pressure in the working channel was deduced from the shift of the superconducting transition temperature T_c of a tin sensor.[16] When the

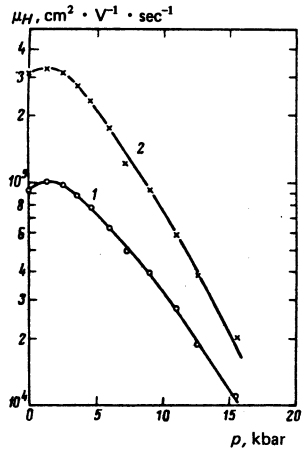

FIG. 1. Pressure dependences of the Hall mobility in samples 4-1 (curve 1) and 4-2 (curve 2) at T = 4.2 °K.

temperature was varied in the range 4.2 °K ≤ T ≤ 100 °K the pressure in the chamber remained practically constant and for $T \to 300$ °K, it increased by 2-3 kbar.[17]

Rectangular single-crystal $Pb_{1-x}Sn_xSe$ blanks were split from an ingot at liquid nitrogen temperature. Samples of 0.5×0.7×3.0 mm dimensions were then cut by spark machining. Current leads made of tinned copper wire were soldered with indium to the ends of the samples. Potential and Hall contacts were spark-welded to the samples. The dimensions of the samples and the distances between the potential contacts were determined with an MBS-1 microscope.

The concentration and homogeneity of the distribution of Sn in the alloy was determined with a JXA-50A x-ray microprobe analyzer. The Sn content in the alloy was determined to within ~1 at. %.

2. RESULTS OF MEASUREMENTS

At helium temperatures the Hall coefficient R_H and the Shubnikov-de Haas oscillation period $\Delta_{100}(1/H)$ of the investigated $Pb_{1-x}Sn_xSe$ ($x = 0.125$) n-type samples remained independent of pressure (within the limits of the experimental error). The 4.2 °K electrical resistivity ρ passed through a minimum and the Hall mobility $\mu_H = R_H/\rho$ passed through a maximum at the pressure $p^* = 1.5 \pm 0.5$ kbar (Fig. 1). The maximum of the pressure dependence of the carrier mobility μ_H was due to the passage of the cyclotron mass $m_c(\epsilon_F)$ through a minimum as a result of band inversion under pressure.

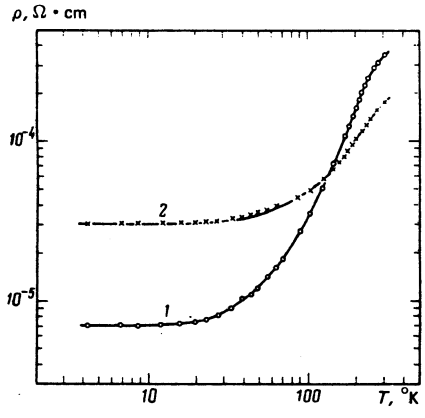

FIG. 2. Temperature dependences of the electrical resistivity of sample 4-2 at pressures p (kbar): 1) 6.0; 2) 12.5.

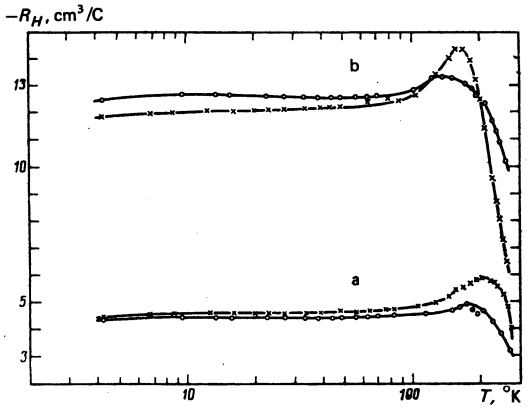

FIG. 3. Temperature dependences of the Hall coefficient: a) sample 4-1; b) sample 4-2. Pressure p (kbar): ○) 2.5; X) 12.5.

The temperature dependences of the electrical resistivity $\rho(T)$ were "metallic" throughout the investigated range of pressures because of the high carrier density in our samples (Fig. 2). After band inversion under pressure at $T = 4.2$ °K, i.e., at pressures $p > p_0$, there was a return transition to the zero-gap state when the temperature was increased. Therefore, at helium temperatures the electrical resistivity rose with increasing pressure ($\epsilon_g < 0$) but at room temperature it fell with rising pressure ($\epsilon_g > 0$) (Fig. 2).

The temperature dependences of the Hall coefficient $R_H(T)$ exhibited a small maximum at about 200 °K (Fig. 3). An increase in the pressure p made this maximum more pronounced and shifted it toward higher temperatures.

Oscillations of the transverse magnetoresistance $\rho(H)$ were usually investigated in a magnetic field $\mathbf{H} \parallel \langle 100 \rangle$. The extremal sections of the four electron Fermi surfaces were identical and the oscillations were monochromatic. The cyclotron masses at the Fermi level $m_{100}(\epsilon_F)$ were deduced from the temperature dependences of the amplitudes of the Shubnikov-de Haas

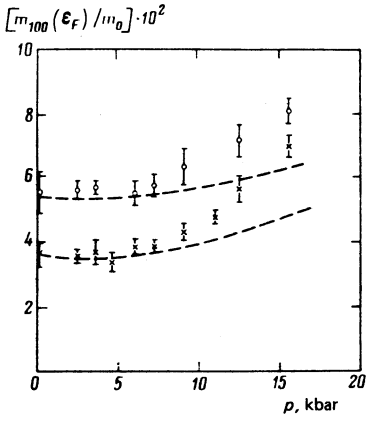

FIG. 4. Pressure dependences of the cyclotron mass at the Fermi level in $\mathbf{H} \parallel \langle 100 \rangle$ for n-type $Pb_{1-x}Sn_xSe$ ($x = 0.125$): ○) sample 4-1; X) sample 4-2. The dashed curves are plotted in accordance with the Kane model using the parameters given in the text.

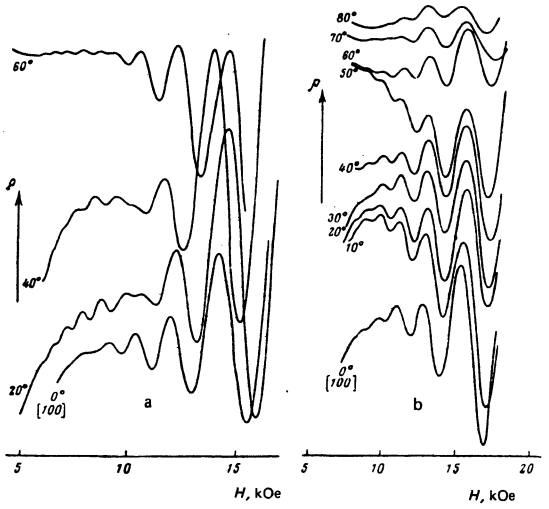

FIG. 5. Oscillations of the magnetoresistance $\rho(H)$ on rotation of the field H in the (100) plane, obtained for sample 4-2 at $T=4.2$ °K: a) at pressure of 6.0 kbar; b) at pressure of 15.5 kbar (the angle was measured from the direction of $\mathbf{H} \| \langle 100 \rangle$).

oscillations. At pressures $p > p_0$ the cyclotron masses rose with rising pressure (Fig. 4), and this happened much more rapidly than predicted by the two-band Kane model.[5]

One of the samples (4-2) was used to investigate the angular dependences of the Shubnikov-de Haas oscillation period $\Delta(1/H)$ when the field was rotated in the (100) plane and this was done throughout the selected pressure range. At low pressures there were beats of two oscillation periods from pairwise coincident extremal sections of the four electron ellipsoids and this was observed over a wide pressure range (Fig. 5a). An increase in the pressure altered considerably the beat pattern and at $p \approx 15$ kbar the oscillations became completely monochromatic (Fig. 5b). The oscillation period was independent of the angle, which was evidence of the spherical shape of the Fermi surface in this case.

3. DISCUSSION OF RESULTS

The band gap ϵ_g of the investigated samples of the $Pb_{1-x}Sn_xSe$ ($x=0.125$) alloy was $\epsilon_g \approx +24$ meV at atmospheric pressure and helium temperature. This gap was deduced from[18]

$$\epsilon_g(x)|_{T=4.2\text{K}} = 145 - 967x \text{ [meV]}. \quad (1)$$

The pressure of the transition to the zero-gap state p_0 did not correspond to the maximum in the pressure dependence of the Hall mobility (Fig. 1) and could be found by using the reliably established pressure coefficient $\partial\epsilon_g/\partial p = -8.5$ meV/kbar.[4,19] In our case, this pressure was $p_0 = 2.8 \pm 0.2$ kbar and in the range $p > p_0$ the negative band gap ϵ_g increased in the absolute sense right up to $\epsilon_g \approx -110$ meV at the maximum pressure employed.

The $T=4.2$ °K value of the Hall coefficient remained constant (within the limits of the experimental error) throughout the investigated range of pressures, indicating constancy of the electron density in our samples. However, the anisotropy of the Fermi surface at L decreased monotonically with rising pressure until the Fermi surface became spherical when the band gap reached $\epsilon_g^* \approx -100$ meV (Fig. 5). The extremal section of the Fermi surface S_{100} corresponding to the $\mathbf{H} \| \langle 100 \rangle$ orientation did not change greatly under pressure. This was due to the proximity of S_{100} to the average section of the Fermi surface.

The nonparabolicity of the dispersion law of carriers at the point L resulted in a strong rise of the effective masses on increase of the absolute value of the band gap ϵ_g (Fig. 4), which reduced the Hall mobility μ_H in the range $p > p_0$ (Fig. 1).

A characteristic feature of the $Pb_{1-x}Sn_xSe$ samples with the inverted spectrum at $T=4.2$ °K was a change in the sign of $\partial\rho/\partial p$ with rising temperature (Fig. 2). This was the qualitative difference from PbSe and from $Pb_{1-x}Sn_xSe$ alloys with the normal spectrum.[5] This effect provided an additional confirmation of the band inversion in $Pb_{1-x}Sn_xSe$ ($x=0.125$) under pressure.

It was worth noting the nontrivial nature of the temperature dependences of the Hall coefficient R_H of the investigated samples under pressure (Fig. 3). The maximum in the dependences $R_H(T)$ was most likely due to indirect transitions of thermally excited electrons from the L to other extrema located higher in the conduction band.[1] However, the available experimental data were insufficient to estimate the indirect gap.

The results obtained in the present study for the $Pb_{1-x}Sn_xSe$ ($x=0.125$) alloy may be compared with our earlier data on the transition to the zero-gap state in n-type PbSe under pressure.[5] Table I lists the properites determined at atmospheric pressure and helium temperature for our samples and those investigated earlier.[5] In the case of samples 4-2 ($x=0.125$) and 1-1 ($x=0$) the electron densities N agreed to within a few percent, which made it possible to plot a single band-gap dependence of the cyclotron mass at the Fermi level $m_{100}(\epsilon_F)$ in the band-gap range $+145 \text{ meV} \geq \epsilon_g \geq -110$ meV. These band-gap dependences of the cyclotron masses are calculated using Eq. (1) and assuming that $\partial\epsilon_g/\partial p = -8.5$ meV/kbar is independent of the pressure p and composition x.[4,19] It is clear from Fig. 6 that the dependence of $m_{100}(\epsilon_F)$ on ϵ_g for the $N = 5.0 \times 10^{17}$ cm^{-3} case (samples 4-2 and 1-1) is asymmetric relative to $\epsilon_g = 0$. The dashed curves in Fig. 6 are plotted in accordance with the Kane model for the parameters $E_\perp = 3.8$ eV and $E_\parallel = 2.9$ eV.[5] The Kane dependences, symmetric relative to $\epsilon_g = 0$, are in poor agreement with the experimental points after the inversion.

TABLE I.

Sample (n-type)*	x	$\Delta_{100}(1/H)$, 10^{-5} Oe^{-1}	$\frac{m_{100}(\epsilon_F)}{m_0} \cdot 10^2$	$N \times 10^{-17}$, cm^{-3}	$\mu_H \times 10^{-5}$, cm$^2 \cdot$V$^{-1} \cdot$sec^{-1}
4-1	0.125	0.59	5.6	16.0	0.93
4-2	0.125	1.25	3.7	5.0	3.20
1-1 [5]	0	1.25	5.7	5.0	2.46
1-2 [5]	0	4.40	4.10	0.8	1.13
2-2 [5]	0	4.40	4.45	0.78	1.70

*The results are given for $T=4.2$ °K and $p=1$ bar.

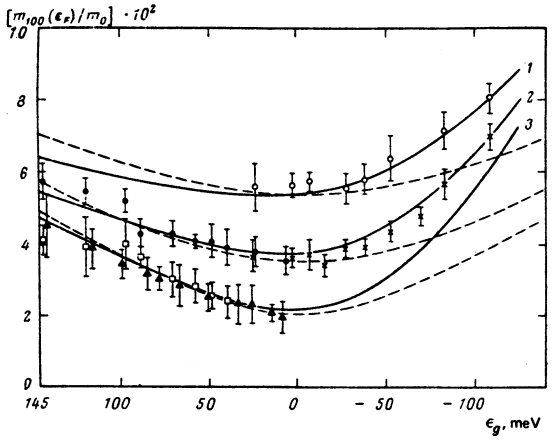

FIG. 6. Band-gap dependences of the cyclotron mass at the Fermi level in H || ⟨100⟩ obtained at helium temperatures for n-type samples of PbSe: ○) 1-1, □) 1-2, △) 2-2, and n-type samples of $Pb_{1-x}Sn_xSe$ ($x=0.125$): ○) 4-1, X) 4-2. The dashed curves are plotted on the basis of the Kane model and the continuous ones on the basis of the Dimmock model. The parameters used in the calculations are given in the text.

It should be noted that the variation of $E_\perp$ and $E_\parallel$ makes it possible to obtain agreement with the experimental data either for $\epsilon_g > 0$ or for $\epsilon_g < 0$ but not for all the values of the band gap. The asymmetric band-gap dependences of the cyclotron masses and transformation of the Fermi surface to the spherical shape under pressure, a similar transformation of the Fermi surface in $Pb_{1-x}Sn_xSe$ alloys with rising x,[8,9] the difference between the anisotropies of the electron and hole constant-energy surfaces,[12,20] and the dependence of the Fermi surface anisotropy on the carrier density[11,12,20,21] all indicate the need to use models more complex[4,6] than the Kane model for the carrier spectrum at the point L. The large number of parameters occurring in the six-band Dimmock model[6] and particularly in the Martinez model[4] are known to cause difficulties in comparisons of the theory and experiment. We shall consider the Dimmock model,[6] which—as indicated by estimates reported by Martinez[4]—gives practically identical results with the Martinez model[4] in the range of band gaps of interest to us.

The Dimmock dispersion model[6] can be written in a form used by Abrikosov:[22]

$$\left[E+\frac{p_\perp^2}{2m_t^*}+\frac{p_\parallel^2}{2m_l^*}\right]^2=\left[\frac{\epsilon_g}{2}+\frac{p_\perp^2}{2m_t^{**}}+\frac{p_\parallel^2}{2m_l^{**}}\right]^2+E_\perp\frac{p_\perp^2}{2m_0}+E_\parallel\frac{p_\parallel^2}{2m_0}; \qquad (2)$$

here, the energy E is measured from the midpoint of the gap, ϵ_g is the band gap after the inversion ($\epsilon_g < 0$),

$$E_\perp=\frac{2\mathcal{P}_\perp^2}{m_0}, \quad E_\parallel=\frac{2\mathcal{P}_\parallel^2}{m_0},$$

$\mathcal{P}_\parallel$ and $\mathcal{P}_\perp$ are the longitudinal and transverse matrix elements, m_0 is the free electron mass,

$$\frac{1}{m_t^*}=\frac{1}{2}\left(\frac{1}{m_t^+}-\frac{1}{m_t^-}\right) \quad \frac{1}{m_t^{**}}=\frac{1}{2}\left(\frac{1}{m_t^+}+\frac{1}{m_t^-}\right),$$
$$\frac{1}{m_l^*}=\frac{1}{2}\left(\frac{1}{m_l^+}-\frac{1}{m_l^-}\right) \quad \frac{1}{m_l^{**}}=\frac{1}{2}\left(\frac{1}{m_l^+}+\frac{1}{m_l^-}\right),$$

$m_t^\pm$ and $m_l^\pm$ are the transverse and longitudinal correction masses representing the interaction with the more distant bands (the plus sign corresponds to holes and the minus to electrons). If the terms containing $1/m_t^*$, $1/m_l^*$, $1/m_t^{**}$, and $1/m_l^{**}$ can be neglected, the dispersion law (2) becomes of the Kane type. The terms with $1/m_t^*$ and $1/m_l^*$ on the left-hand side of Eq. (2) govern the degree of departure of the electron and hole spectra at l from the mirror symmetry.

At the bottom of the conduction band the ratio of the maximal and minimal Fermi surface sections S_{max}/S_{min} is equal to the ratio of the maximal and minimal cyclotron masses m_{cmax}/m_{cmin} ($S_{min}\equiv S_{111}$, $m_{cmin}\equiv m_{c111}$) and it is given by[1)]

$$\frac{S_{max}}{S_{min}}=\frac{m_{c\,max}}{m_{c\,min}}=\left[E_\perp+\epsilon_g\left(\frac{m_0}{m_t^{**}}-\frac{|\epsilon_g|}{\epsilon_g}\frac{m_0}{m_t^*}\right)\right]^{1/2}$$
$$\times\left[E_\parallel+\epsilon_g\left(\frac{m_0}{m_t^{**}}-\frac{|\epsilon_g|}{\epsilon_g}\frac{m_0}{m_t^*}\right)\right]^{-1/2}. \qquad (3)$$

If the electron and hole spectra do not have the mirror symmetry, the band-gap dependences of S_{max}/S_{min} and m_{cmax}/m_{cmin} at the bottom of the conduction band [see Eq. (3)] have a kink at $\epsilon_g = 0$ (in the zero-gap state) when the bottom of the conduction band and the top of the valence band at L "interchange" their correction masses. In the zero-gap state the anisotropy of the Fermi surface sections and cyclotron masses at the bottom of the band is governed solely by the parameters $E_\perp$ and $E_\parallel$. The values of the ratios S_{max}/S_{min} and m_{cmax}/m_{cmin}, determined in the Dimmock approximation, depend not only on ϵ_g but also on the carrier density in the bands, and away from the bottom of the band the values of S_{max}/S_{min} and m_{cmax}/m_{cmin} corresponding to a given carrier density are, strictly speaking, unequal. When the spectra do not have the mirror symmetry, the carrier-density dependences of S_{max}/S_{min} and m_{cmax}/m_{cmin} are different for the conduction and valence bands.

The expression for the minimal electron cyclotron mass m_{cmin} (H || ⟨111⟩) in the Dimmock approximation has (in contrast to m_{cmax}) the relatively simple form:[8]

$$\frac{m_0}{m_{c\,min}}=-\frac{m_0}{m_t^*}+\left\{E_\perp+\frac{m_0}{m_t^{**}}\left[\epsilon_g+2S'\frac{m_0}{m_t^{**}}\right]\right\}$$
$$\times\left[\left(\epsilon_g+2S'\frac{m_0}{m_t^{**}}\right)^2+4E_\perp S'\right]^{-1/2}, \qquad (4)$$

where $S'=S_{min}/2\pi m_0$.

The theory predicts an asymmetric dependence of the cyclotron mass on ϵ_g in the case of band inversion and a constant carrier density.[2] Such a dependence m_{c100} (ϵ_F) is exhibited by our samples (Fig. 6). The theory predicts also a shift of the minimum in the energy-gap dependences relative to $\epsilon_g = 0$, in the direction of positive values of the gap away from the bottom of the band. In principle, this shift may explain the discrepancy between the values of p_0 and of the pressure corresponding to the maximum of the Hall mobility exhibited by samples 4-1 and 4-2 of the investigated alloy (Fig. 1).

The experimental results obtained in the present study and those reported elsewhere were used by us in

the determination of the parameters of the Dimmock dispersion law. The calculations were carried out on a computer. In these calculations we used: 1) the experimental dependences of the electron cyclotron masses m_{c100} on the gap ϵ_g (Fig. 6); 2) the dependence of the electron cyclotron mass m_{c100} on the carrier density in PbSe;[23] 3) the value of the band gap ϵ_g^* at which the Fermi surface became spherical in the investigated $x = 0.125$ alloy with $N = 5\times10^{17}$ cm^{-3}; 4) the anisotropies of the electron and hole Fermi surfaces at the bottom (top) of the band of PbSe;[12,21] 5) the dependence of the anisotropy of the hole Fermi surface at L on the hole density in PbSe at atmospheric pressure.[8,11,12,20,21]

We recall that simple analytic dependences of the extremal sections and cyclotron masses are obtained from the dispersion law (2) only in the $\mathbf{H}\|\langle111\rangle$ case. Therefore, in computer calculations of the Fermi surface anisotropy and carrier density at given values of the gap parameter ϵ_g and energy E we used the numerical integration method. The anisotropy of the cyclotron masses was determined using approximate relationships obtained by Foley and Langenberg[24] which were sufficiently accurate in the range of electron densities of interest to us ($N \leq 2\times10^{18}$ cm^{-3}). We also assumed that in the investigated range of pressures p and compositions x the changes in the correction masses and in the parameters $E_\perp$ and $E_\parallel$ were negligible.[2]

Variation of the parameters in the dispersion law (2) gave us the following set:

$$E_\perp = 2.85\pm0.2 \text{ eV}, \quad E_\parallel = 2.2\pm0.2 \text{ eV},$$
$$m_0/m_t^+ = 9.5\pm1, \quad m_0/m_t^- = 7.5\pm1,$$
$$m_0/m_l^+ = 0.7\pm0.5, \quad m_0/m_l^- = 1.0\pm0.5,$$

satisfying all the experimental data given above. The degree of agreement between the experiment and theory can be judged on the basis of Fig. 6, where the continuous curves are plotted in accordance with the Dimmock model.

Figure 7 gives the theoretical dependences of the anisotropy coefficient $K = (S_{max}/S_{min})^2$ of the electron Fermi surface at L on the band ϵ_g for several electron densities. It is worth noting the kink in the dependence

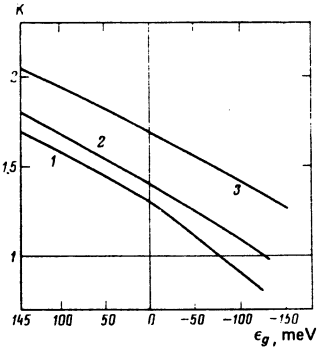

FIG. 7. Dependences of the anisotropy coefficient of the electron Fermi surface $K = (S_{max}/S_{min})^2$ on the band gap ϵ_g at helium temperatures: 1) at the bottom of the conduction band; 2), 3) for electron densities of $N = 1\times10^{19}$ cm^{-3} and $N = 5\times10^{19}$ cm^{-3}. The calculations are based on the Dimmock model and the parameters are given in the text.

TABLE II.

x	Type	$N\times10^{-18}$, cm^{-3}	K_{exp}	K_{theor}	Ref.
0	n	0.3	1.67±0.2	1.70	[21]
0	»	0.6	1.70±0.03	1.71	[12]
0	»	1.3	1.70±0.03	1.72	[12]
0	»	1.1	1.70±0.2	1.72	[20]
0	p	1.0	1.80±0.2	1.91	[21]
0	»	1.3	1.87±0.03	1.92	[12]
0	»	6	2.00±0.2	2.06	[20]
0	»	7	2.10±0.2	2.08	[11]
0	»	3.4	2.34±0.2	2.43	[11]
0.08	»	1.4	1.70±0.1	1.67	[8]

$K(\epsilon_g)$ which occurs at $\epsilon_g = 0$ for the curve representing the bottom of the conduction band [see also Eq. (3)]. Similar calculations were also carried out for p-type materials. For all values of ϵ_g the carrier density dependence of K was much stronger for the valence band than for the conduction band. Hence, we concluded that the plotting of the dependence $K(\epsilon_g)$ on the basis of the experimental data obtained for $Pb_{1-x}Sn_xSe$ alloys with different impurity concentrations[9] was an incorrect procedure.

The experimental and theoretical values of the anisotropy coefficient K of n- and p-type PbSe and of p-type $Pb_{0.92}Sn_{0.08}Se$ samples with different carrier densities at $T = 4.2$°K are compared in Table II. The agreement between the theory and experiment can be regarded as fully satisfactory.

The band gap ϵ_g^* corresponding to the transformation of the Fermi surface to the spherical shape shifts toward higher absolute values of this gap when the electron density N is increased (Fig. 7). For $N = 5\times10^{17}$ cm^{-3} the Fermi surface becomes spherical at $\epsilon_g^* = -85$ meV [our experimental value is $\epsilon_g^* = -(100\pm20)$ meV]. The universality of our parameters of the Dimmock model was checked by calculating the cyclotron masses at the Fermi level for different carrier densities in n- and p-type samples of PbSe and $Pb_{1-x}Sn_xSe$ alloys ($0 < x \leq 0.2$). A comparison of the calculated values of m_{100} and m_{111} with those found experimentally by various authors[8,20,21,25] showed a good agreement between the theory and experiment (the discrepancy between the calculated and experimental data did not, on the average, exceed 10%).

The reported results indicate that the Dimmock model is in satisfactory agreement with the available experimental data for PbSe and $Pb_{1-x}Sn_xSe$ alloys in the band gap range 145 meV $\geq \epsilon_g \geq -110$ meV. However, we may expect the model to become unsatisfactory in the case of the energy spectra of carriers in $Pb_{1-x}Sn_xSe$ beyond the point at which the Fermi surface becomes spherical.[4]

The authors take this opportunity to thank S. D. Beneslavskiĭ for extremely valuable discussions.

[1)] There are errors in the expression for the anisotropy of the Fermi surface at L obtained in the Dimmock approximation by Kucherenko *et al.*[9]

[1] P. J. Lin and L. Kleinman, Phys. Rev. 142, 478 (1966).
[2] J. Melngailis, T. C. Harman, and J. A. Kafalas, in: Physics of IV–VI Compounds and Alloys (ed. by S. Rabii), Gordon

and Breach, New York, 1974, p. 59.
[3]G. Martinez, in: Physics of IV–VI Compounds and Alloys (ed. by S. Rabii), Gordon and Breach, New York, 1974, p. 215.
[4]G. Martinez, Phys. Rev. B 8, 4678 (1973).
[5]N. B. Brandt, O. N. Belousova, V. P. Zlomanov, Ya. G. Ponomarev, and E. P. Skipetrov, Fiz. Tverd. Tela (Leningrad) 19, 437 (1977) [Sov. Phys. Solid State 19, 250 (1977)].
[6]J. O. Dimmock, in: Proc. Intern. Conf. on Physics of Semimetals and Narrow-Gap Semiconductors, Dallas, 1970 (ed. by D. L. Carter and R. T. Bate), in: J. Phys. Chem. Solids 32, Suppl. 1, 319 (1971).
[7]E. O. Kane, J. Phys. Chem. Solids 1, 249 (1957).
[8]J. Melngailis, T. C. Harman, and W. C. Kernan, Phys. Rev. B 5, 2250 (1972).
[9]I. V. Kucherenko, V. N. Moiseenko, and A. P. Shotov, Fiz. Tekh. Poluprovodn. 11, 162 (1977) [Sov. Phys. Semicond. 11, 90 (1977)].
[10]V. N. Ivanov, I. V. Kucherenko, V. N. Moiseenko, M. S. Taktakishvili, and A. P. Shotov, Fiz. Tekh. Poluprovodn. 9, 690 (1975) [Sov. Phys. Semicond. 9, 453 (1975)].
[11]O. Beckman and Y. Shapira, Phys. Lett. 21, 282 (1966).
[12]P. Kästner, Phys. Status Solidi B 53, 753 (1972).
[13]Ya. G. Ponomarev, Prib. Tekh. Eksp. No. 6, 218 (1966).
[14]O. N. Belousova, Kand. diss. (Thesis for Candidate's Degree), Moscow State University, 1975.
[15]N. B. Brandt and Ya. G. Ponomarev, Zh. Eksp. Teor. Fiz. 55, 1215 (1968) [Sov. Phys. JETP 28, 635 (1969)].
[16]L. D. Jennings and C. A. Swenson, Phys. Rev. 112, 31 (1958).
[17]N. B. Brandt, S. V. Kuvshinnikov, N. Ya. Minina, and E. P. Skipetrov, Prib. Tekh. Eksp. No. 6, 160 (1973).
[18]T. C. Harman, in: Proc. Intern. Conf. on Physics of Semimetals and Narrow-Gap Semiconductors, Dallas, 1970 (ed. by D. L. Carter and R. T. Bate), in: J. Phys. Chem. Solids 32, Suppl. 1, 363 (1971).
[19]A. P. Shotov, Proc. Fourth Conf. on Solid State Devices, Tokyo, 1972, publ. by Japanese Society of Applied Physics, Tokyo (1973), in: Oyo Butsuri 42, Suppl., 282 (1973).
[20]K. F. Cuff, M. R. Ellett, C. D. Kuglin, and L. R. Williams, Proc. Seventh Intern. Conf. on Physics of Semiconductors, Paris, 1964, Vol. 1, Physics of Semiconductors, publ. by Dunod, Paris; Academic Press, New York (1964), p. 677.
[21]S. Bermon, Phys. Rev. 158, 723 (1967).
[22]A. A. Abrikosov, J. Low Temp. Phys. 8, 315 (1972).
[23]O. N. Belousova and E. P. Skipetrov, Tezisy dokladov 1-ĭ Vsesoyuznoĭ konf. "Poluchenie i svoĭstva poluprovodnikovykh soedineniĭ $A^{II}B^{VI}$ i $A^{IV}B^{VI}$ i tverdykh rastvorov na ikh osnove" (Abstracts of Papers presented at First All-Union Conf. on Preparation and Properties of II–VI and IV–VI Semiconductor Compounds and Solid Solutions Based on Them), M., 1977, p. 10.
[24]G. M. T. Foley and D. N. Langenberg, Preprint, University of Pennsylvania, Philadelphia, 1976.
[25]J. C. Ramage, R. A. Stradling, A. Aziza, and M. Balkanski, J. Phys. C 8, 1731 (1975).

Translated by A. Tybulewicz

Oscillations of the impedance of thin tungsten plates in a magnetic field

O. A. Panchenko, V. V. Vladimirov, P. P. Lutsishin, and M. A. Mukhtarov

Institute of Physics, Ukrainian Academy of Sciences
(Submitted 16 July 1977)
Zh. Eksp. Teor. Fiz. **74**, 658–664 (February 1978)

The derivative with respect to the magnetic field of the surface impedance of thin tungsten plates (the skin layer depth being comparable with the thickness) in circularly polarized radio-frequency fields ($f = 5$ MHz) is investigated experimentally and theoretically. The magnetic field is perpendicular to the (001) face of the sample. It is shown that in both polarizations the impedance oscillations are due to a group of holes lying at the inflection of the Fermi surface octahedron. The "–" polarization oscillation series can be attributed entirely to the Gantmakher-Kaner radio-frequency effect [Sov. Phys. JETP **21**, 1053 (1965)]. The impedance oscillations in the "+" polarization are due to both the Gantmakher-Kaner effect and to doppleron excitation [McGraddy *et al.*, Phys. Rev. **141**, 437(1966); Fisher *et al.*, Sov. Phys. JETP **33**, 410 (1971)].

PACS numbers: 72.15.Gd

1. It is known that weakly attenuating electromagnetic waves, called dopplerons, can exist in metals near the threshold of Doppler-shifted cyclotron resonance.[1,2] The dopplerons are most clearly observed in compensated metals, where the helicons cannot propagate. Tungsten is one such metal. The excitation of dopplerons can lead to an oscillatory dependence of the impedance of the plate on the magnetic field.

In the present work, we have investigated the surface resistance of thin single-crystal samples of tungsten ($D \sim \delta$, D is the thickness of the plate, δ is the skin depth) in the radiofrequency region of the spectrum. A circularly polarized exciting field was used in the experiments. A sharply delineated series of oscillations of the derivative of the impedance with respect to the magnetic field was observed in strong magnetic fields in both polarizations. The period of the oscillations in the "+" polarization (direction of rotation of the radiofrequency field corresponds to the cyclotron rotation of the holes) changes by about 5% with the magnetic field, and in the "–" polarization it is practically independent of the field. In very strong fields, the two periods coincide and correspond to the cyclotron displacements of the holes located at the inflection points of the octahedron of the Fermi surface. The conclusion has been drawn, on the basis of these results, that all the observed phenomena are connected with a single group of carriers. It will be shown that the series of oscillations with constant period ("–" polarization) is due to the Gantmakher-Kaner ef-

 0038-5646/78/4702-0346$02.40