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Effect of collisions between carriers on the dissipative conductivity

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The existence of dissipative conductivity due to the mutual scattering of carriers is considered. It is shown that in a d.c. field such a conductivity exists only in a carrier system for which the total charge is zero. No magnetoresistance of this type exists. In a high frequency field the conductivity due to the mutual scattering of carriers exists in a system of carriers having different e/m.

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INTRODUCTION

There exists two types of scattering of carriers in a solid: scattering by the lattice, i.e., by phonons, impurities and defects, and scattering of the carriers by one another. These two types of scattering are basically different—in scattering by the lattice, the momentum obtained by the system of carriers from the external electric field is transferred to the lattice, while in scattering of carriers by one another, this momentum remains inside the carrier system. Here we must immediately make two stipulations. We shall assume that the phonons are in equilibrium and form a thermostat; therefore, the transfer of momentum to the lattice is equivalent to momentum dissipation. Further, we shall not consider the scattering of carriers by one another with participation of the lattice, i.e., umklapp processes and processes of the transfer of a carrier from one valley to another (in such processes, the momentum is also transferred to the lattice). Moreover, it is assumed that the system is spatially homogeneous, i.e., such situations as, for example, the anomalous skin effect and thin plates are excluded.

Since the two scattering types mentioned above have different characters, they are not "additive." This means that if the scattering by the lattice is characterized by a relaxation time τ_L and the scattering of the carriers by one another by another time τ_{ee}, then there does not exist an effective relaxation time τ^* which would be determined by the relation

$$1/\tau^* = 1/\tau_L + 1/\tau_{ee}. \tag{1}$$

This is seen even from the simplest example for conductivity in a static electric field. If there is no electron-electron scattering, then the conductivity is

$$\sigma = \frac{ne^2}{m}\langle \tau_L \rangle. \tag{2}$$

Here n is the concentration of the electrons, m is their effective mass, and $\langle \ldots \rangle$ denotes averaging over the energy ϵ. If the interelectron scattering predominates, i.e., $\tau_{ee} \ll \tau_L$ then, as is well known,[1]

$$\sigma = \frac{ne^2}{m}\left\langle \frac{1}{\tau_L} \right\rangle^{-1}, \tag{3}$$

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and nowhere is there

$$\sigma=\frac{ne^2}{m}\langle\tau_{ee}\rangle, \tag{4}$$

as might be inferred from (1).

If we exclude the case of a strong dependence of τ_L on ϵ, then the quantities $\langle\tau_L\rangle$ and $\langle 1/\tau_L\rangle^{-1}$ have the same order and, regardless of the pace of the interelectron scattering, the order of magnitude of the conductivity is determined only by the scattering from the lattice.[2,3] In this connection, the following question arises: does there exist generally such a dissipative conductivity whose order of magnitude is determined by the mutual scattering of the carriers? This question is important, for example, in connection with the interpretation of experiments on the magnetoresistance of bismuth which, in the opinion of several authors,[4] is determined by the electron-hole scattering.

When the conductivity is determined by the mutual scattering of the carriers, it is naturally possible to calculate it by neglecting the scattering of the carriers by the lattice. Under these conditions, no momentum is transferred to the lattice and a stationary state of the system of carriers is possible only in the case in which the momentum is not transferred from the field to this system (at least when averaged over the time).

In a static electric field, the momentum from the field will not be transferred to the carrier system only if it is compensated, i.e., the total charge is equal to zero (as for example in bismuth). In an alternating electric field or in the presence of a magnetic field, we might think that the total force acting on the system averages out (over the period of vibration of the field and/or the period of cyclotron rotation). We therefore initially study the case of conductivity in a stationary field, and then the magnetoconductivity and the conductivity in a high-frequency field. Cyclotron resonance is the natural generalization of the last two cases.

2. FORMULA FOR THE CONDUCTIVITY

We shall not be interested in the effect of scattering between carriers on the method of averaging of the lattice relaxation, since this effect does not determine the order of magnitude of the conductivity but affects only a numerical factor of the order of unity in the rate of lattice relaxation. Therefore, in place of the kinetic equation, we can use the equations of motion. We shall assume that there are two groups of carriers with charges e_1 and e_2, isotropic masses m_1 and m_2, lattice relaxation times τ_1 and τ_2, and a coefficient of mutual friction η. Then the equation of motion for the particles of group 1 will be

$$\frac{d\mathbf{p}_1}{dt}=e_1\mathbf{E}e^{-i\omega t}+\frac{e_1}{c}[\mathbf{v}_1\times\mathbf{H}]-\frac{\mathbf{p}_1}{\tau_1}-\eta n_2(\mathbf{v}_1-\mathbf{v}_2); \tag{5}$$

there will be a similar equation for particles of group 2.

Setting $\mathbf{E}=0$, $\mathbf{H}=0$, and $\tau_1^{-1}=\tau_2^{-1}=0$, we can, by subtracting the equations of motion from one another, find that the relative velocity $\mathbf{v}_1-\mathbf{v}_2$ relaxes to zero like $e^{-t/\tau}$, where

$$\frac{1}{\tau}=\frac{n\bar{m}}{m_1m_2}\eta=\gamma. \tag{6}$$

Here and below, the bar above indicates averaging over the groups of carriers:

$$\bar{m}=(n_1m_1+n_2m_2)/n, \quad n=n_1+n_2. \tag{7}$$

It is natural to take the time τ as the relaxation time for scattering between carriers.

We direct **H** along z and introduce complex variables according to the rule

$$p=p_x+jp_y, \quad \sigma=\sigma_{xx}+j\sigma_{xy}. \tag{8}$$

The geometric imaginary unit j should not be confused with the variable i. We then find the conductivity from the equations of motion:

$$\sigma=G/D, \tag{9}$$

where

$$D=(\nu_1-i\omega)(\nu_2-i\omega)+j\omega_1(\nu_2-i\omega)+j\omega_2(\nu_1-i\omega)-\omega_1\omega_2-i\omega\gamma+\gamma\bar{\nu}+j\bar{\omega}\gamma, \tag{10}$$

$$G=\bar{\Omega}^2\gamma+\Omega_1^2(\nu_2-i\omega)+\Omega_2^2(\nu_1-i\omega)+j(\omega_2\Omega_1^2+\omega_1\Omega_2^2). \tag{11}$$

In (10) and (11) we have introduced the cyclotron and plasma frequencies of the individual groups 1 and 2:

$$\omega_1=\frac{e_1H}{m_1c}, \quad \Omega_1^2=\frac{n_1e_1^2}{m_1}, \tag{12}$$

and also the averaged quantities

$$\bar{\nu}=\frac{m\nu}{\bar{m}}, \quad \nu_1=\tau_1^{-1}, \quad \bar{\omega}=\frac{\bar{e}H}{\bar{m}c}, \quad \bar{\Omega}^2=\frac{n\bar{e}^2}{\bar{m}}. \tag{13}$$

We note the following: if we assume the particles of the different groups to be identical, i.e., set $e_1=e_2=e$, $m_1=m_2=m$ and $\nu_1=\nu_2=\nu$, then the factor $\nu+\gamma-i\omega+\gamma\bar{\omega}$ can be eliminated from G and D and there remains

$$\sigma=\frac{ne^2}{m}\frac{1}{\nu-i\omega+j\bar{\omega}}, \quad n=n_1+n_2, \quad \bar{\omega}=\frac{eH}{mc}. \tag{14}$$

Actually, we have only the one group of carriers in this case, and the fact that γ does not enter into the result (14) means that collisions between the identical carriers do not affect the conductivity.

3. STATIC CONDUCTIVITY

Setting $H=0$, $\omega=0$, and $\nu_1=\nu_2=0$ in (10) and (11), we obtain

$$D=0 \quad G=\bar{\Omega}^2\gamma. \tag{15}$$

If the system is uncompensated, i.e., its total charge $n\bar{e}\neq 0$, then $\bar{\Omega}^2\neq 0$ and $\sigma=\infty$. This means that in such a system there cannot be a static conductivity which would

be determined only by mutual scattering of the particles. For a compensated system, where $\bar{e}=0$, $\bar{\Omega}^2=0$ and an indeterminacy is obtained for the conductivity: $\sigma=0/0$. To remove this, it is necessary to take into account the finite scattering by the lattices. We do this for the compensated system with

$$n_1=n_2=n_0, \quad e_1=-e_2=e, \tag{16}$$

which simulates bismuth. For such a system we find from (9)–(11)

$$\sigma=n_0e^2\left(\frac{\tau_1}{m_1}+\frac{\tau_2}{m_2}\right)\left[1+\left(\frac{\tau}{m^*}\right)^{-1}\left(\frac{\tau_1}{m_1}+\frac{\tau_2}{m_2}\right)\right]^{-1}, \tag{17}$$

where, in correspondence with (6),

$$1/\tau=\eta n_0/m^*, \quad 1/m^*=1/m_1+1/m_2. \tag{18}$$

It is seen from (17) that if $\tau \ll \tau_1, \tau_2$, i.e., if the mutual scattering of carriers predominates, then the conductivity is completely determined by this scattering:

$$\sigma=(n_0e^2/m^*)\tau=e^2/\eta. \tag{19}$$

However, it is useful to emphasize that, although σ does not depend on τ_1 and τ_2 in the considered limit, the state of the system does depend on these quantities. The ratio of the momenta $\mathbf{p}_1$ and $\mathbf{p}_2$ does depend on τ_1 and τ_2, because the total momentum transfer to the lattice should be equal to zero:

$$\mathbf{p}_1/\tau_1+\mathbf{p}_2/\tau_2=0. \tag{20}$$

The momenta $\mathbf{p}_1$ and $\mathbf{p}_2$ are directed oppositely, but the total momentum $\mathbf{P}=n_0(\mathbf{p}_1+\mathbf{p}_2)$ is generally not equal to zero in spite of the fact that the forces acting on the carriers of the different groups are equal in magnitude and opposite in direction (see the drawing).

4. MAGNETORESISTANCE

Setting $\omega=0$ and $\nu_1=\nu_2=0$ in (10) and (11), we get from (9)

$$\sigma=-j(n\bar{e}c/H). \tag{21}$$

This means that if there is only mutual scattering, and there is no scattering by the lattices, then the dissipative conductivity is lacking in a magnetic field. The result (21) is evident when scattering within the system of carriers is excluded; it is important that it does not depend on this scattering. One can give the following illustrative interpretation to this assertion. If the charged particle acquires the momentum $\Delta\mathbf{p}$ in the magnetic field, then the center of its orbit is shifted by $\Delta\mathbf{R}=(c/eH^2)\Delta\mathbf{p}\times\mathbf{H}$. If several particles collide, then the total charge transfer is

$$\sum e\,\Delta\mathbf{R}=(c/H^2)\left[\sum\Delta\mathbf{p}\mathbf{H}\right]=0, \tag{22}$$

since the total change of momentum of all the colliding particles is $\Sigma\Delta p=0$. Therefore, the inclusion of mutual scattering does not change the current and the conductivity in any essential way. In other words, there can be no magnetoresistance whose value would be determined by the mutual scattering of the carriers.

The situation seems especially paradoxical for a compensated system, for example (16). In this case, we can write down the general formula for the conductivity tensor in a magnetic field:

$$\sigma=n_0e^2\left(\frac{\tau_1}{m_1}+\frac{\tau_2}{m_2}\right)\left[1+\left(\frac{\tau}{m^*}\right)^{-1}\left(\frac{\tau_1}{m_1}+\frac{\tau_2}{m_2}\right)+j(\omega_1\tau_1+\omega_2\tau_2)-\omega_1\tau_1\omega_2\tau_2\right]^{-1} \tag{23}$$

From Eq. (23), as $H\to\infty$, we get

$$\mathrm{Re}\,\sigma=\frac{n_0c^2}{H^2}\left(\frac{m_1}{\tau_1}+\frac{m_2}{\tau_2}\right). \tag{24}$$

Thus, while for $H=0$ the conductivity at $\tau \ll \tau_1, \tau_2$ was entirely determined by the electron-hole scattering (see (19)), for $H\to\infty$, it is determined by scattering from the lattice, regardless of the relations between τ and τ_1 or τ_2.

The magnetoacoustic resistance in complex form is calculated simply as $\rho=1/\sigma$. It is then seen from (23) and (17) that the difference $\rho(H)-\rho(0)$, generally does not depend on the electron-hole scattering although $\rho(0)$ does depend on it. For this reason, the temperature dependence $\rho(H)-\rho(0)\propto T^2$, observed in an experiment on bismuth,[4] cannot serve as an argument in support of electron-hole scattering.

5. HIGH-FREQUENCY CONDUCTIVITY

Setting $H=0$ and $\nu_1=\nu_2=0$ in (10) and (11), we find the dissipative component of the conductivity from (9):

$$\mathrm{Re}\,\sigma=\frac{\gamma}{\gamma^2+\omega^2}[\Omega_1^2+\Omega_2^2-\bar{\Omega}^2]=\frac{\gamma}{\gamma^2+\omega^2}\frac{n_1m_1n_2m_2}{n_1m_1+n_2m_2}\left(\frac{e_1}{m_1}-\frac{e_2}{m_2}\right)^2. \tag{25}$$

Thus, if not all the carriers have the same ratio e/m, then there exists a dissipative conductivity whose value is determined by the scattering between carriers. It is not difficult to understand this result physically.

When there is no scattering between carriers, their velocities oscillate in the field with amplitudes $\mathbf{v}=(-e\mathbf{E}/i\omega)(e/m)$. If all the e/m are identical, then there is no relative motion of the carriers and the inclusion of scattering between them changes nothing. If there are carriers with different e/m, then the inclusion of such scattering leads to the appearance of a frictional force in phase with the velocity; the action of this force produces the dissipation.

Simplifying (25) in the case of the conditions (16), we can see that the correct static limit (19) follows from (25) at $\omega=0$. Therefore, we can expect that, at all frequencies down to $\omega=0$, Eq. (25) is valid in the case of a compensated system at $\bar{\nu}\ll\gamma$. We can establish this by writing the formula for σ with account taken of ν_1 and ν_2 for the system (16):

$$\mathrm{Re}\,\sigma=\frac{n_0e^2}{m}\frac{\bar{\nu}(\gamma\bar{\nu}-\omega^2)+\omega^2\gamma}{(\gamma\bar{\nu}-\omega^2)^2+\omega^2\gamma^2}. \tag{26}$$

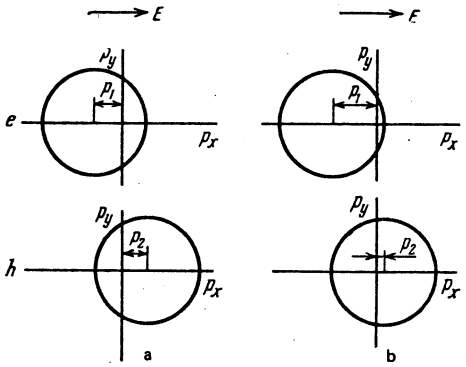

FIG. 1. Two different states of a compensated system ($e_1=-e_2$, $n_1=n_2$; 1—electrons, 2—holes) with one and the same current (the circles are regions of occupied states) a—$\tau_1=\tau_2\gg\tau_{ee}$, $P=0$; b—$\tau_1>\tau_2\gg\tau_{ee}$, $P\neq 0$.

A similar formula for the uncompensated system shows that we can neglect the scattering from the lattice only at $\omega\gg(\overline{\nu}\gamma)^{1/2}$. At lower frequencies, the conductivity depends essentially on the scattering from the lattice.

We note that Eq. (25) can be applied to electrons in a nonparabolic band, by regarding the band as a system of carriers with different e/m. We can therefore expect that in this case, in a high-frequency field, there exists non-zero dissipative conductivity determined by electron-electron scattering. That this is actually so is easily established by solving the kinetic equation with the model derived electron-electron collision term used in Ref. 1. We then have the factor

$$1-\langle pv\rangle^2/\langle p^2\rangle\langle v^2\rangle \tag{27}$$

in (25) in place of the expression in the square brackets. This factor is equal to zero only for a parabolic band (the angle brackets denote averaging over the equilibrium distribution).

High-frequency absorption of a similar type takes place also when the mass difference is connected with anisotropy of the energy spectrum of the carriers, as, for example, for electrons in germanium and silicon. To establish this fact, it suffices to consider the equations of motion of type (5) with effective mass tensors for the two identical ellipsoidal valleys, one turned relative to the other. Here, for $\omega\gg\gamma$, we obtain

$$\mathrm{Re}\,\hat{\sigma}=(n_e e^2/\omega^2)n_c\eta(\hat{m}_1^{-1}-\hat{m}_2^{-1})^2, \tag{28}$$

where $\hat{\sigma}$ is the conductivity tensor, and $\hat{m}_1$ and $\hat{m}_2$ are the effective-mass tensors; η is assumed to be a scalar. It is seen from this formula that the absorption takes place also in the case in which the field is oriented symmetrically relative to both valleys and the ohmic masses are the same (for example, $\mathbf{E}\|[100]$ in germanium). This absorption is connected with the fact that, in contrast to the isotropic case, there exists relative motion of the carriers from the different valleys in a direction perpendicular to the field.

6. CYCLOTRON RESONANCE

When $\omega\neq 0$ and $H\neq 0$, the formulas are too complicated to write down even at $\nu_1=\nu_2=0$. We therefore give the result only for the resonance value of the dissipative conductivity under the conditions $\nu_1=\nu_2=0$, $\omega_1,\omega_2\gg\gamma$. We have

$$\mathrm{Re}\,\sigma|_{\omega=\omega_i}=\frac{1}{2\gamma}\Omega_i^2\left(1+\frac{n_1m_1}{n_2m_2}\right)=\frac{1}{2}\frac{n_1}{n_2}e_i^2\frac{1}{\eta}. \tag{29}$$

Thus, if there are two groups of carriers, then their mutual scattering assures a finite value of the resonance absorption at the resonance frequency of each group. It is natural that the value of the absorption is proportional to the concentration of carriers of the resonance group, and is inversely proportional to the concentration of carriers of the nonresonance group, which assume the role of scatterers.

For the compensated system (16) we have

$$\mathrm{Re}\,\sigma|_{\omega=\omega_i}=\frac{1}{2}\frac{e^2}{\eta}=\frac{1}{2}\sigma(\omega=0,H=0), \tag{30}$$

i.e., as is usual for a single group of carriers, the resonance conductivity is equal to one half the static conductivity. For a noncompensated system there is no such relation.

7. CONCLUSION

It follows from the results of Secs. 3–6 that the dissipative conductivity, due to scattering between particles, exists under the following conditions:

(1) static conductivity in a system of carriers whose total charge is equal to zero (compensated system);

(2) high-frequency conductivity (in a magnetic field or without a magnetic field) in a system of carriers with different e/m; if the system is compensated, then such conductivity exists at all frequencies down to $\omega=0$, if there is no compensation, it exists only at frequencies $\omega\gg(\tau_h\tau_{ee})^{-1/2}$.

Magnetoresistance due to scattering between carriers does not exist. Thus, the analogy between high-frequency conductivity and conductivity in a magnetic field disappears here.

The existence of dissipative conductivity without scattering by the lattice does not contradict the law of energy conservation, as it might seem at first glance. The situation here is entirely analogous to the case of static conductivity due to elastic scattering by impurities. In the latter case, the seeming paradox is resolved in the following fashion: although the ohmic conductivity depends only on the momentum relaxation time τ_L and does not depend on the energy relaxation time $\bar{\tau}_L$, the range of fields E in which the Ohm's law is valid does depend on $\bar{\tau}_L$; this interval is proportional to $\bar{\tau}_L^{-1/2}$.[5] In mutual scattering of the carriers, the ohmic conductivity cannot depend on the mechanism of energy dissipation, i.e., on the scattering by the lattice, but the region of applicability of this ohmic conductivity is determined by the scattering by the lattice.

Sometimes one encounters in the literature the assertion that the cause of the finite static conductivity, due

to electron-electron scattering, can be the difference in the masses of carriers of a single sign.[6,7] The logic is probably as follows: If the masses of all the particles are the same, then conservation of momentum in the collision leads to current conservation:

$$\Delta \mathbf{J} = e\,\Delta \mathbf{v}_1 + e\,\Delta \mathbf{v}_2 = \frac{e}{m}(\Delta \mathbf{p}_1 + \Delta \mathbf{p}_2) = 0. \tag{31}$$

If the masses are different, then the current is not conserved but "relaxes":

$$\Delta \mathbf{J} = e\left(\frac{\Delta \mathbf{p}_1}{m_1} + \frac{\Delta \mathbf{p}_2}{m_2}\right) \neq 0. \tag{32}$$

However, such a "relaxation of the current," if important at all, matters only in the alternating field. In a static field, in a system of carriers of the same sign, a stationary state cannot exist (see also Ref. 1).

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Kinetics of nucleation and stratification of dilute He^3–He^4 solutions under pressure at low temperatures

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It is possible to obtain under pressure, in the absence of a free liquid-vapor surface, a metastable solution of liquid He^3 in liquid He^4 in the stratification region in the phase diagram. The rate of formation of nucleating centers in the volume of such a solution and its dependence on temperature and concentration are computed. Classical and quantum conditions for nucleation are analyzed. The cases of the normal and the superfluid states of the Fermi component in the solution are considered in the region of the quantum regime. It is shown that the stratification of the supersaturated solution under the indicated conditions should begin not on the phase-equilibrium line, but on the rapid-nucleation line, T_q. The T_q line is constructed.

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It is well known that the solubility of He^3 in liquid He^4 at $T < 0.87$ K is finite.[1] Liquid solutions of the helium isotopes with He^3 contents in the range $x_l(T,P) < x < x_u(T,P)$, where $x = x_l(T,P)$ and $x = x_u(T,P)$ are the branches of the phase diagram, separate into two equilibrium phases with molar concentrations $x_l(T,P)$ and $x_u(T,P)$. At $T < 0.15$ K the upper phase is virtually pure He^3 ($x_u \approx 1$), while the bottom phase is a solution with a He^3 content within the limits $0.0637 < x_l < 0.094$, depending on the pressure.[2]

As has been shown by Andreev,[3,4] a film of impurity He^3 atoms exists at the liquid solution-vapor interface. As the conditions approach the conditions for phase equilibrium a He^3-rich phase develops continuously out of the surface layer.[5,6] The inverse effect is observed at the liquid-solid boundary: owing to the van der Waals forces, the vessel walls and dust particles get covered with He^4 films,[6,7] and these surfaces cannot serve as effective nucleation centers. Thus, in the absence of a liquid-vapor boundary (such a situation arises in experiments under pressure when the vessel is filled with the solution at a temperature higher than the stratification temperature and then cooled), the stratification of the supersaturated solution should occur through the formation and growth of nuclei of the stable phase in the volume of the metastable medium.

The appearance of stable nuclei is connected with the surmounting of an energy barrier by the system. At temperatures close to absolute zero, a supercritical nucleus can arise only as a result of quantum penetration through the barrier.[8] A nucleating center is a macroscopic formation, and can be described in the quasiclassical approximation. In the same way as was done in Ref. 8, we should write down the classical Hamiltonian and then quantize it.

If the degree of metastability of the medium is not high, then the optimal forms of the density and concentration distributions correspond to a spherical nucleus of the new phase, the thickness of the transition layer being small compared to the dimensions of the nucleus, i.e., $d/R \ll 1$. This circumstance allows us to describe the transition region in terms of surface tension. On account of the slowness of the motion of the interphase boundary ($\ddot{R}/u \ll 1$, u is the speed of sound), energy dissipation can be neglected, i.e., it can be assumed that, in the course of the growth of a nucleus, new excitations do not arise and all the parameters of the system adia-

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