replace the upper limit in the integral over t by infinity, we obtain

$$F(s)=\exp\left\{\frac{2\nu PV_0 s}{T}\int_0^1 \ln^3 t\exp\left(-\frac{2V_0 ts}{T}\right)dt\right\}$$

$$\approx\exp\left\{\frac{2\nu PV_0 s}{T}\int_0^\infty \ln^3 t\exp\left(-\frac{2V_0 ts}{T}\right)dt\right\} \quad (61)$$

$$=\exp\left\{-\nu P\left[\left(C+\ln\frac{2V_0 s}{T}\right)^3+\frac{\pi^2}{2}\left(C+\ln\frac{2V_0 s}{T}\right)-\Psi''(1)\right]\right\},$$

where C is Euler's constant and $\Psi(z)=d\ln\Gamma(z)/dz$. The formula (61) gives an explicit expression for the distribution function. From (61) it can be seen that if $|\ln s|\ll\ln(2V_0/T)\simeq\nu^{-1/3}$, then, in the lowest approximation in $\nu^{1/3}$, we again obtain (59). We can also calculate $f_0(\chi)$ in exactly the same way. In the zeroth approximation we obtain an expression of the type (43). We shall not calculate the corrections.

In conclusion the author expresses his gratitude to I. Ya. Korenblit and S. V. Maleev for discussions on the work.

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Influence of spatial dispersion and of a surface layer on the phase of light reflected from a CdS crystal

A. V. Komarov, S. M. Ryabchenko, and M. I. Strashnikova

Institute of Physics, Academy of Sciences of the Ukrainian SSR, Kiev
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Measurements were made of the change in the phase of light as a result of reflection from a CdS crystal in the region of the A absorption band at $T=4.2$ and 77°K. The experimental results did not agree with a theory ignoring spatial dispersion but were in satisfactory agreement with a theory allowing for the dispersion subject to boundary conditions giving rise to an exciton-free layer on the surface of the crystal. However, when the reflection spectrum was calculated using the layer thickness and the exciton damping parameter which agreed best with the experimental data on the change in the phase of light at 4.2°K, the results differed from the spectrum determined experimentally at 4.2°K. It was concluded that the boundary conditions should be refined within the framework of the spatial dispersion theory so that the reflection spectrum and the spectrum of the change in the phase would be described by the same set of parameters.

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Pekar[1] demonstrated theoretically the importance of the spatial dispersion effects in interpreting the low-temperature exciton spectra of crystals. The most convincing argument in support of the need to allow for the spatial dispersion effect in CdS crystals at 4.2°K is the ability to describe the optical properties measured in transmitted and reflected light by a single complex refractive index $n^*=n+i\varkappa$, where n is the real refractive index and $\varkappa$ is the absorption coefficient.[2,3] It should be noted that the spatial dispersion theory makes it possible to find the effective quantities $n^{eff}(\lambda)$ and $\varkappa^{eff}(\lambda)$ representing the reflection (amplitude and phase) of light from a crystal and, at the same time, find the parameters describing the transmission experiments.[3]

The spectral dependence of the change in the phase of the reflected light $\Delta\varphi(\lambda)$ is a characteristic which supplements the reflection coefficient $R(\lambda)$. According to the spatial dispersion theory, the peak-to-peak amplitude of $\Delta\varphi(\lambda)$ should be considerably greater than that predicted by classical crystal optics when use is made of the values $n^t(\lambda)$ and $\varkappa^t(\lambda)$, deduced from light transmitted by a crystal. However, there have been practically no experimental studies of the reflection of light in the exciton absorption region of crystals, including measurements of $\Delta\varphi(\lambda)$.

Hopfield and Thomas[4] described the results of their measurements of $R(\lambda)$ by introducing boundary conditions leading to the idea of an exciton-free "dead" layer on the surface of a crystal. It follows from crystal optics that the presence of a dead layer may also affect

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significantly the phase of the reflected light. Thus, measurements of $\Delta\varphi(\lambda)$ in the exciton absorption region, reported below, should give new information on the role of spatial dispersion and a surface dead layer in crystals.

EXPERIMENTS

The phase changes in the light reflected from a CdS crystal may be investigated by recording the phase shift between the light-wave components $E_\parallel$ and $E_\perp$ with the projections of the vector E along and at right-angles to the optic axis c. The phase shift $\Delta\varphi(\lambda)$ in the region of the A exciton band is governed only by the contribution $\Delta\varphi_\perp$ due to changes in the absorbed component $E_\perp$, because, in the case of the A excitons, there is no absorption of $E_\parallel$ and the phase shift $\Delta\varphi_\parallel(\lambda)=\pi$ is independent of λ. The first attempt to carry out such measurements was made by Solov'ev and Babinskiĭ.[5] In their investigation, a crystal was illuminated with linearly polarized light from a static polarizer; the reflected elliptically polarized light was applied to a Babinet compensator and then to an analyzer. The resultant spectra were photographed. In the exciton spectrum region, there were dark bands, which were interpreted as the phase compensation points.

The instantaneous values of the intensities $E_\parallel$ and $E_\perp$ of a light wave reflected from a crystal and transmitted by a compensator (in the case when one of the compensator axes coincides with the c axis) are given by the expressions:

$$\left.\begin{aligned} E_\parallel &= 2^{-1/2}E_0R_\parallel^{1/2}\cos(\omega t+\delta\varphi_\parallel(h)+\Delta\varphi_\parallel),\\ E_\perp &= 2^{-1/2}E_0R_\perp^{1/2}\cos(\omega t+\delta\varphi_\perp(h)+\Delta\varphi_\perp), \end{aligned}\right\} \quad (1)$$

where E_0 is the amplitude of the light oscillations emerging from a polarizer; $R_\parallel$ and $R_\perp$ are the reflection coefficients for $E_\parallel$ and $E_\perp$; ω is the frequency of light; $R_\parallel$ and $R_\perp$ are the phase shifts produced by the Babinet compensator at a given height h in the case of light polarized in the directions $E_\parallel$ and $E_\perp$; $\Delta\varphi_\parallel$ and $\Delta\varphi_\perp$ are the phase shifts resulting from the reflection of the components $E_\parallel$ and $E_\perp$. In the phase compensation case, $\delta\varphi_\parallel(h)-\delta\varphi_\perp(h)=\Delta\varphi_\perp-\Delta\varphi_\parallel$, these components interfere to produce a linearly polarized wave. The direction of oscillations α is then defined by $\tan\alpha=E_\parallel/E_\perp=(R_\parallel/R_\perp)^{1/2}$.

In the region of the A exciton band, the coefficient $R_\perp$ depends strongly on the wavelength. Therefore, α varies with the wavelength λ and if a static polarizer, compensator, and analyzer are used, complete darkening cannot be achieved at the phase compensation points for all the wavelengths. We can also show that the maxima of relative darkening may not coincide with the phase compensation points and, consequently, there may be errors in the determination of $\Delta\varphi(\lambda)$.

We used a compensation method free of these shortcomings. We employed apparatus shown as a block diagram in Fig. 1. The optical part of the apparatus consisted of a light source O, a polarizer P_1, a quarter-wave mica plate $\lambda/4$, a CdS sample placed in a cryostat,

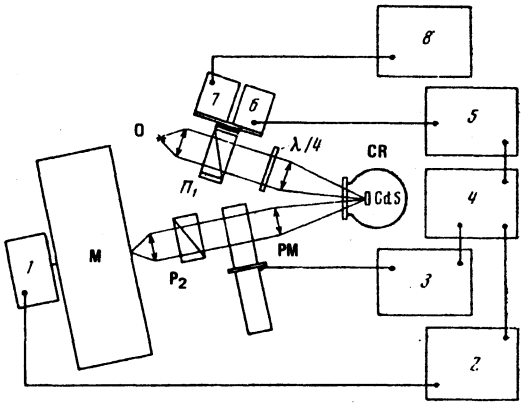

FIG. 1. Schematic diagram of the apparatus used in measurements of the change in phase of a light wave as a result of reflection from a crystal. The explanations are given in the text.

a phase modulator PM, a polarizer P_2, and a grating monochromator M. Before the measurements, the c axis of the crystal was oriented either in the plane of incidence of light or at right-angles to it. The axes of the $\lambda/4$ phase plate were oriented at 45° with respect to the c axis, and the polarization vector of the light transmitted by P_1 coincided with one of the axes of the $\lambda/4$ plate. The sample was illuminated with linearly polarized light, whose vector E was directed at 45° relative to the optic axis c of the crystal. The light reflected from the sample became elliptically polarized when a phase shift $\Delta\varphi=\Delta\varphi_\parallel-\Delta\varphi_\perp\neq 0$ appeared between the components $E_\parallel$ and $E_\perp$.

The modulator PM and the polarizer P_2 acted as a modulation analyzer of the elliptic polarization of the reflected light. The phase modulator PM, similar to that described elsewhere,[6] was a phase shifter utilizing the piezooptic birefringence which appeared in a fused silica slab on the excitation of a longitudinal acoustic wave. The amplitude of the acoustic vibrations was varied by an oscillator 3 in such a way that the phase shift at the standing-wave antinodes in the slab was $+\pi/2$ during one half-period and $-\pi/2$ during the next. The elliptically polarized light passed through PM and P_2, oriented with the polarization axis at 45° with respect to the direction of propagation of sound in the slab, was then amplitude-modulated at the frequency of the acoustic vibrations in PM ($f_0=40$ kHz).

An alternating electric signal $s(f_0)$ from a photomultiplier 1 was applied to a narrow-band amplifier 2, tuned to the frequency f_0, and then to a synchronous detector 4. The magnitude and sign of the detected voltage were related to the degree of ellipticity and the sign of the predominant circular polarization of the reflected light. This voltage was passed through an amplifier 5 and it controlled a reversible motor 6 which rotated the polarizer P_1. The rotation of P_1 relative to the axes of $\lambda/4$ phase plate produced a phase shift between the components $E_\parallel$ and $E_\perp$ of the incident light. The direction of rotation of P_1 was selected so that the deliberately introduced phase shift compensated the shift resulting from the reflection of light by the sample. At the moment of compensation, the light reflected from the sam-

ple was converted to the linearly polarized form. The signal $s(f_0)$ then disappeared and the rotation of P_1 by the motor 6 was stopped. The angle of rotation ψ of P_1 relative to one of the axes of the $\lambda/4$ plate was recorded with an angle transducer 7. The instantaneous values of the intensities $E_\parallel$ and $E_\perp$ of the reflected light wave were then described by Eq. (1) with

$$\delta\varphi_\parallel = -\delta\varphi_\perp = \psi. \tag{2}$$

Since, in this case, the condition of compensation was the linear polarization of the reflected light, irrespective of the direction of its oscillations, the following relationships were true at the compensation point:

$$(\psi+\Delta\varphi_\parallel)-(-\psi+\Delta\varphi_\perp)=0, \quad 2\psi=\Delta\varphi_\perp-\Delta\varphi_\parallel. \tag{3}$$

The signal from the transducer 7 was applied to an X-Y chart recorder 8 and plotted in the course of continuous tuning of the monochromator in the region of the A exciton band.

The measurements were carried out at $T=4.2$ and 77 °K with light incident almost normally on a crystal. These measurements yielded the phase of the reflected light (Figs. 2a and 2b). A very important aspect was the correct selection of the spectral widths of the monochromator slits. At 77 °K, results were obtained using a slit whose width was such that its further reduction did not alter $\Delta\varphi=\Delta\varphi_\perp-\Delta\varphi_\parallel$. At 4.2 °K, a reduction in the slit width increased the peak-to-peak amplitude of the $\Delta\varphi$ curve (Fig. 2a). The whole phase curve of Fig. 2a, apart from its upper extremum, was measured under conditions such that further narrowing of the slit did not affect the results and the upper extremum continued to rise in intensity until the minimum spectral slit width, compatible with the sensitivity of the apparatus (~2.5 cm^{-1}), was reached. Consequently, the true change in the phase at the point of the upper extremum of the curve in Fig. 2b was greater than or equal to the measured value and the rest of the curve could then be treated with confidence.

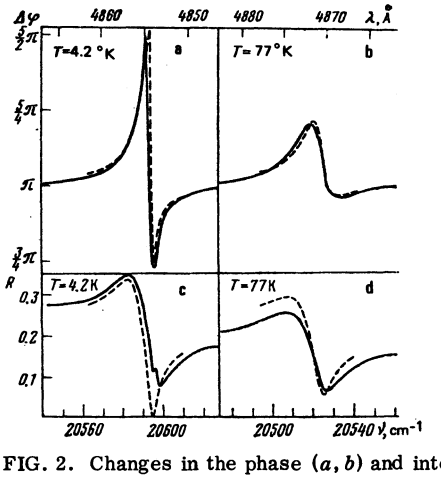

FIG. 2. Changes in the phase (a, b) and intensity (c, d) of light due to reflection from a CdS crystal in the region of the A exciton band, recorded for the $E\perp c$ component at temperatures of 4.2°K (a, c) and 77°K (b, d). The continuous curves are experimental and the dashed curves are calculated theoretically, allowing for spatial dispersion and assuming the following parameters: $\gamma_{4.2}=1.37$ cm^{-1}, $d_{4.2}=83.5$ Å, $\gamma_{77}=4.3$ cm^{-1}, and $d_{77}=76.5$ Å. The scale of R applies to the calculated values. The experimental curves are plotted in relative units.

In addition to the measurements of $\Delta\varphi(\nu)$ (where ν is the wave number representing the energy of the light quanta in reciprocal centimeters), we used the same apparatus (but without the compensation system) to measure $R(\nu)$ photoelectrically except that the $R(\nu)$ was not recorded at 4.2 °K for minimum slit widths. Instead, the $R(\nu)$ spectrum (Fig. 2c) was determined photographically using a spectrograph with a spectral slit width of 1 cm^{-1} and the results were then analyzed photometrically.

We also recorded spectra photographically by the method of Solov'ev and Babinskiĭ [5] for fixed positions of the polarizer and Babinet compensator. The results were completely identical (and hence are not given here) with those reported by Solov'ev and Babinskiĭ,[5] whereas our measurements of $\Delta\varphi(\nu)$ by the compensation method differed basically from their results.[5]

DISCUSSION

The boundary conditions of Hopfield and Thomas[4] give a model of a dead layer on the surface with $n_2=\epsilon_0^{1/2}$ and $\varkappa_2=0$, where ϵ_0 is the background value of the permittivity of the crystal. The bulk of the crystal is characterized by the constants n_3 and $\varkappa_3$. In this case (all subsequent formulas and discussions apply to the reflection properties of a crystal for $E\perp c$, so that the index "$\perp$" can be omitted), we have

$$\operatorname{tg}\Delta\varphi=\frac{\rho_{23}(1-r_{12}^2)\sin(\varphi_{23}+2\beta)}{r_{12}(1+\rho_{23}^2)+\rho_{23}(1+r_{12}^2)\cos(\varphi_{23}+2\beta)}, \tag{4}$$

$$R=\frac{r_{12}^2+\rho_{23}^2+2r_{12}\rho_{23}\cos(\varphi_{23}+2\beta)}{1+r_{12}^2\rho_{23}^2+2r_{12}\rho_{23}\cos(\varphi_{23}+2\beta)}, \tag{5}$$

where

$$r_{12}=-\frac{n_2-1}{n_2+1}=\rho_{12}\exp(i\varphi_{12})$$

is the reflection coefficient for the amplitude of a light wave at the 1–2 (air–dead layer) interface, and ρ_{23} and φ_{23} are the modulus of the amplitude of the reflection coefficient and the phase change due to reflection at the 2–3 (dead layer—crystal) interface,

$$\rho_{23}^2=\frac{(n_2-n_3)^2+\varkappa_3^2}{(n_2+n_3)^2+\varkappa_3^2}, \quad \operatorname{tg}\varphi_{23}=\frac{-2\varkappa_3 n_2}{-(n_3^2+\varkappa_3^2-n_2^2)},$$

$\beta=2\pi n_2 d/\lambda_0$ is the phase advance due to propagation across a dead layer of thickness d, and λ_0 is the wavelength at the exciton absorption maximum of the A band ($\Delta\lambda/\lambda_0\ll 1$).

Figures 3a and 3b show the curves $\Delta\varphi(\lambda)$ calculated using Eq. (4) and various values of d on the assumption that n_3 and $\varkappa_3$ can be replaced with $n^t(\nu)$ and $\varkappa^t(\nu)$, deduced from the transmission of light at temperatures of 4.2 °K[8,9] and 77 °K.[10] The values of n_{77}^t and $\varkappa_{77}^t$ apply to an optical contact with a crystal and are corrected for spectral shift due to deformation. We can see that the

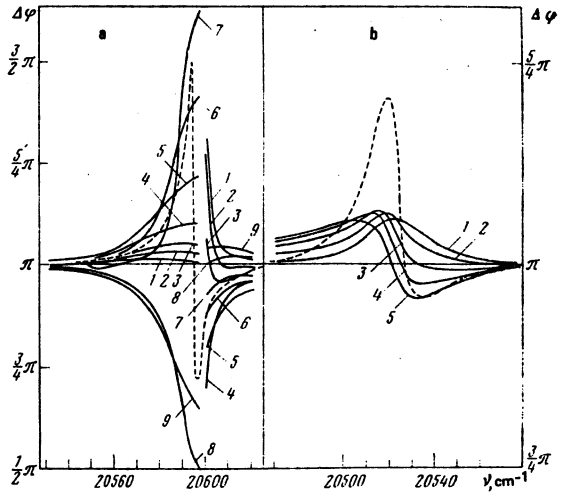

FIG. 3. Changes in the phase in the exciton A band region, calculated using Eq. (4) in conjunction with the values of n_3 and n_3 found by transmission measurements. The thicknesses d of the dead layer (Å) assumed for the curves in Fig. 3a were as follows: (1) 0; (2) 28; (3) 56; (4) 111; (5) 222; (6) 333; (7) 389; (8) 500; (9) 555. The corresponding values assumed for the curves in Fig. 3b were (Å): (1) 0; (2) 50; (3) 111; (4) 167; (5) 222. The experimental dependences $\Delta\varphi(\nu)$ are shown dashed.

$\Delta\varphi(\nu)_{77}$ curves (Fig. 3b), calculated for $d\neq 0$, are qualitatively similar to those obtained experimentally and differ only from the latter by a smaller peak-to-peak amplitude $\Delta\varphi(\nu)$, whereas the $\Delta\varphi(\nu)_{4.2}$ curves (Fig. 3a), calculated for reasonable hypothetical thicknesses of the dead layer ($d\lesssim 60$–200 Å) differ greatly in the qualitative and quantitative senses from the experimental results. In a narrow range of layer thicknesses, $d\approx 360\pm 40$ Å, which can be regarded as a singularity (curves 6 and 7), the peak-to-peak amplitude and the calculated phase curve are close to the experimental results. Nevertheless, such details of the qualitative nature of $\Delta\varphi(\nu)$ as the ratio of the positive and negative components cannot be matched with the experiment even in this range of d. Considerable discrepancies between the calculated and experimental results noted for $d=0$[2,3] also apply to the dependences $R(\nu)$ calculated for $T=4.2$ °K, which we shall not give here. Thus, the model which allows for spatial dispersion cannot explain the experimental data on the reflection of light in the A exciton band of a CdS crystal at $T=4.2$ °K and only gives a moderate description of the experimental results obtained at 77 °K.

We shall now carry out calculations for a model which allows for spatial dispersion and use the formulas derived by Pekar.[1] In this case, we have $n_3=n^{\text{eff}}$ and $\varkappa_3=\varkappa^{\text{eff}}$, given by

$$n^{\text{eff}}=\operatorname{Re} n^*,\quad \varkappa^{\text{eff}}=\operatorname{Im} n^*,\quad n^*=\frac{\varepsilon_0+n_+n_-}{n_++n_-},\qquad (6)$$

where

$$n_\pm^2=\frac{1}{2}(\mu+\varepsilon_0)\pm\left(\frac{1}{4}(\mu-\varepsilon_0)^2+b\right)^{1/2},\quad \mu=\mu'+i\mu'',$$

$$\mu'=\frac{2Mc^2}{\hbar\omega_0^2}(\omega-\omega_0),\quad \mu''=\frac{2Mc^2}{\hbar\omega_0^2}\gamma,$$

$$b=\frac{4\pi c^2e^2M}{\hbar\omega_0^2m}Nf,$$

where M is the effective mass of an exciton, f is the oscillator strength, N is the number of unit cells per unit volume, $\hbar\omega_0$ is the energy at the bottom of the exciton band, and γ is the exciton damping parameter. Apart from unimportant coefficients of the ω/ω_0 type, the above formulas are identical with those obtained by Hopfield and Thomas.[4] The damping parameter Γ, introduced by Hopfield and Thomas,[4] is related to our γ by $\gamma=\Gamma/2$.

Calculations based on Eqs. (4) and (5) in conjunction with Eq. (6) were carried out by us for a range of values of γ and d. The other parameters were taken from the paper by Pekar and Strashnikova.[3] The solutions for $\Delta\varphi(\nu)$ depended on the parameters γ and d. Figure 4 shows the ranges of the parameters γ and d, the corresponding typical calculated curves, and the points at which the calculations were carried out. The solutions for regions I and II, separated by a continuous curve in Fig. 4 (the solutions for region I were also obtained by Solov'ev and Babinskiĭ[5] for $\gamma=0$), were found to differ basically. These regions were separated by a boundary corresponding to the condition $\varphi_{23}+2\beta=2\pi$ for $|r_{12}|=\rho_{23}$. In region II, the positive and negative components of the calculated curve $\Delta\varphi(\nu)$ depended on γ and d.

An analysis of the calculations gave the families of the $\Delta\varphi_{\max}$ and $\Delta\varphi_{\min}$ curves for different values of γ and d (Fig. 5). Using the results of Fig. 5, we were able to associate specific values of γ_T and d_T with our experimental results, which were $\gamma_{77}=4.3$ cm^{-1}, $d_{77}=76.5$ Å, $\gamma_{4.2}=1.37$ cm^{-1}, and $d_{4.2}=83.5$ Å. The curves calculated for these values of γ and d (shown dashed in Figs. 2a

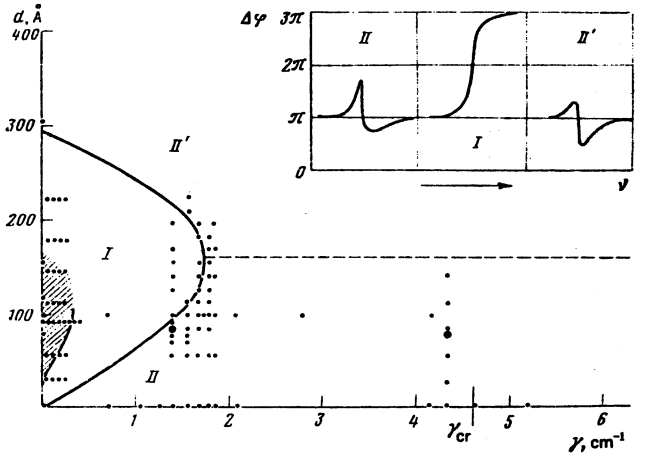

FIG. 4. Ranges of the parameters γ and d corresponding to different types of solution for $\Delta\varphi(\nu)$. The points are the values of γ and d used in the calculations of $\Delta\varphi(\nu)$ and $R(\nu)$. The best agreement between the calculated and experimental curves at 4.2 and 77°K is obtained for the encircled points. The line separating the shaded part of the figure corresponds to $\gamma_s(d)$. For a given value of d and $\gamma\leqslant\gamma_s(d)$, the calculated dependence $R(\nu)$ has a spike [the number of extrema is $R(\nu)>2$], but such a spike is not obtained for $\gamma>\gamma_s(d)$. The value of γ_{cr} is the "critical" value of γ,[15] below which the spatial dispersion effects are important. The inset shows typical $\Delta\varphi(\nu)$ dependences obtained for different ranges of the parameters γ and d.

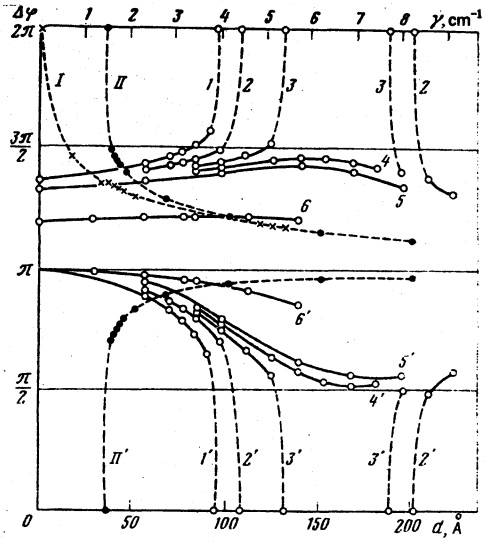

FIG. 5. Families of calculated dependences of $\Delta\varphi_{max}$ and $\Delta\varphi_{min}$ on γ and d: (I) $\Delta\varphi_{max}(\gamma)_{d=0}$ [$\Delta\varphi_{min}(\gamma)_{d=0}=\pi$]; (II), (II′) $\Delta\varphi_{max}(\gamma)_{d=98}$ and $\Delta\varphi_{min}(\gamma)_{d=98Å}$, respectively. The other curves, denoted by Arabic numbers, represent $-\Delta\varphi_{max}(d)_\gamma$ and $\Delta\varphi_{min}(d)_\gamma$ (the latter are identified by primes), calculated for the following values of γ (cm^{-1}): (1) 1.37; (2) 1.54; (3) 1.66; (4) 1.77; (5) 1.83; (6) 4.3.

and 2b) agreed well with the experimental results. We have pointed out above that the upper extremum of the phase curve (Fig. 2a) at 4.2 °K was not measured by us accurately enough and, therefore, the true value of $\Delta\varphi_{max}$ could have been higher than the measured value. Bearing this point in mind, we deduced from Fig. 5 that the experimental values should have been somewhat smaller than the values $\gamma_{4,2}$ and $d_{4,2}$ adopted by us (in region II of Fig. 4 near the boundary separating regions I and II). Nevertheless, the good agreement between the calculated and experimental curves indicated that the likely deviations of $\gamma_{4,2}$ and $d_{4,2}$ from the values given above were small.

We shall now consider the values of γ and d obtained for 4.2 and 77 °K. Our values of d_{77} and $d_{4,2}$ are practically identical, which is in good agreement with the main features of the dead layer model. However, in the case of γ, we have to point out that it may vary from crystal to crystal, depending on the impurity content or deformation. Therefore, we may conclude that our values are in satisfactory agreement with those given by Pekar and Strashnikova[3] ($\gamma_{4,2}\approx 2$ cm^{-1}) and those obtained by Brodin, Davydova, and Strashnikova[11] for very thin "free" platelets. The crystals deformed by optical contact at the same temperatures are characterized by much higher (by a factor of 3–4) values of γ than free crystals, as pointed out by Brodin *et al.*[11]; this explains Fig. 3b.

The situation in the case of the calculated $R(\nu)$ dependences (obtained for the same values of γ_T and d_T and shown dashed in Figs. 2c and 2d) is slightly different. At $T=77$ °K, there is satisfactory agreement with the experimental results but the discrepancy is quite considerable at $T=4.2$ °K. The experimental $R(\lambda)$ curve has an additional structure (a "spike"), attributed by Hopfield and Thomas[4] to interference because of the presence of a dead layer under spatial dispersion conditions. However, it follows from several other investigations,[12] that, in full agreement with our calculations, a spike appears when Eqs. (5) and (6) are used only for very small values $\gamma<\gamma_s(d)$. We can see from Fig. 4 that the range $\gamma<\gamma_s(d)$ does not extend outside region I, whereas our phase curve shows that the set of γ and d, selected for $T=4.2$ °K, applies to region II.

In attempts to explain the presence of the spike at values of γ higher than those predicted by the Hopfield and Thomas model,[4] we carried out calculations for the following cases: 1) the presence not only of a dead layer but also an additional nonabsorbing layer (for example, an oxide or a disturbed stoichiometric layer) with $n\gtrless\epsilon_0^{1/2}$; 2) the deviation of the refractive index of the dead layer from $\epsilon_0^{1/2}$. In case 1), we observed no reduction in the discrepancy between the calculated and experimental $R(\nu)$ curves. In case 2), we found that, for certain values of $n_2<\epsilon_0^{1/2}$, a spike appeared at somewhat higher values of γ than in the Hopfield and Thomas model,[4] but we were unable to obtain solutions giving a spike at sufficiently high values of γ and, moreover, the parameters for which the spike appeared failed to give agreement with the experimental dependences $R(\nu)$ and $\Delta\varphi(\nu)$. In analyzing the reason for the appearance of the spike, we noted that it was quite sensitive to any change in the dead layer model or to possible dependences $d(\omega-\omega_0)$ and $\gamma(\omega-\omega_0)$. The solutions for $\Delta\varphi(\nu)$ were somewhat less affected by small changes in the dead layer model.

The observed discrepancies between the calculated and experimental $R(\nu)$ spectra demonstrate the need to define the boundary conditions and, consequently, the dead layer model. Moreover, these results suggest that the data on the reflection of light in the exciton bands of crystals should be interpreted with caution when an interference structure (spike) is present. There have been theoretical investigations[13,14] based on less idealized models of the influence of the surface on the reflection spectra of crystals and, in these cases, a spike appears at higher values of γ than in the Hopfield and Thomas model.[4] However, these treatments do not deal with the change in the phase of light on reflection, so that we shall not compare them with our experimental results.

Summarizing, we may say that the calculations allowing for spatial dispersion ensure good agreement between the theory and experiment at $T=77$°K, explain well the behavior of $\Delta\varphi(\nu)$ at 4.2°K, but fail to account for the discrepancies between the calculated and experimental $R(\nu)$ dependences at $T=4.2$°K, which are sensitive to the fine details of the model. Calculations without allowance for spatial dispersion and for the presence of a dead layer give results which disagree almost totally with the experimental data at 4.2°K and differ quantitatively from the data obtained at 77°K.

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Effect of collisions between carriers on the dissipative conductivity

V. F. Gantmakher

Institute of Solid State Physics, USSR Academy of Sciences

I. B. Levinson

Institute of Theoretical Physics, USSR Academy of Sciences

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The existence of dissipative conductivity due to the mutual scattering of carriers is considered. It is shown that in a d.c. field such a conductivity exists only in a carrier system for which the total charge is zero. No magnetoresistance of this type exists. In a high frequency field the conductivity due to the mutual scattering of carriers exists in a system of carriers having different e/m.

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INTRODUCTION

There exists two types of scattering of carriers in a solid: scattering by the lattice, i.e., by phonons, impurities and defects, and scattering of the carriers by one another. These two types of scattering are basically different—in scattering by the lattice, the momentum obtained by the system of carriers from the external electric field is transferred to the lattice, while in scattering of carriers by one another, this momentum remains inside the carrier system. Here we must immediately make two stipulations. We shall assume that the phonons are in equilibrium and form a thermostat; therefore, the transfer of momentum to the lattice is equivalent to momentum dissipation. Further, we shall not consider the scattering of carriers by one another with participation of the lattice, i.e., umklapp processes and processes of the transfer of a carrier from one valley to another (in such processes, the momentum is also transferred to the lattice). Moreover, it is assumed that the system is spatially homogeneous, i.e., such situations as, for example, the anomalous skin effect and thin plates are excluded.

Since the two scattering types mentioned above have different characters, they are not "additive." This means that if the scattering by the lattice is characterized by a relaxation time τ_L and the scattering of the carriers by one another by another time τ_{ee}, then there does not exist an effective relaxation time τ^* which would be determined by the relation

$$1/\tau^* = 1/\tau_L + 1/\tau_{ee}. \tag{1}$$

This is seen even from the simplest example for conductivity in a static electric field. If there is no electron-electron scattering, then the conductivity is

$$\sigma = \frac{ne^2}{m}\langle \tau_L \rangle. \tag{2}$$

Here n is the concentration of the electrons, m is their effective mass, and $\langle \ldots \rangle$ denotes averaging over the energy ϵ. If the interelectron scattering predominates, i.e., $\tau_{ee} \ll \tau_L$ then, as is well known,[1]

$$\sigma = \frac{ne^2}{m}\left\langle \frac{1}{\tau_L} \right\rangle^{-1}, \tag{3}$$

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