[10]A. I. Golovashkin, I. S. Levchenko, and G. P. Motulevich, Kratk. Soobshch. Fiz. No. 12, 13 (1972).
[11]R. I. Chiao, M. J. Feldman, H. Ohta, and P. T. Parrish, Rev. Phys. Appl. 9, 183 (1974).
[12]P. W. Anderson and A. H. Dayem, Phys. Rev. Lett. 13, 195 (1964).
[13]V. Ambegaokar and A. Baratoff, Phys. Rev. Lett. 10, 486 (1963); 11, 104 (1963).
[14]Yeong-du Song and I. Rochlin, Phys. Rev. Lett. 29, 416 (1972).
[15]V. H. Gubankov, V. P. Koshelets, and P. A. Ovsyannikov, Zh. Eksp. Teor. Fiz. 71, 348 (1976) [Sov. Phys. JETP 44, 181 (1976)].
[16]A. I. Larkin and Yu. N. Ovchinnikov, Zh. Eksp. Teor. Fiz. 65, 1704 (1973) [Sov. Phys. JETP 38, 854 (1974)].
[17]M. A. Janocko, J. R. Gavaler, and C. K. Jones, IEEE Trans. Magn. MAG-11, 880 (1975).
[18]L. G. Aslamazov and A. I. Larkin, Pis'ma Zh. Eksp. Teor. Fiz. 9, 150 (1969) [JETP Lett. 9, 87, 1969)].
[19]I. R. Waldram, A. B. Pippard, and J. Clarke, Philos. Trans. A 268, 265 (1970).

Translated by J. G. Adashko

Nonequilibrium phenomena in superconductor junctions

I. É. Bulyzhenkov and B. I. Ivlev

L. D. Landau Institute of Theoretical Physics, USSR Academy of Sciences
(Submitted 12 July 1977)
Zh. Eksp. Teor. Fiz. 74, 224–235 (January 1978)

The derivation is considered of a kinetic equation that describes the asymmetry of the electronlike and holelike excitations in a superconductor having a large concentration of nonmagnetic impurities. Besides the electron-phonon interaction, the alternating field is considered as a source of additional relaxation of the electron-hole unbalance. The dependence of the shift of the chemical potential and of the energy gap on the temperature and on the injection voltage is obtained at temperatures that are low compared with the critical value.

PACS numbers: 74.50.+r

1. INTRODUCTION

A tunnel current through a junction of a superconductor and another metal has been demonstrated experimentally and theoretically[1-4] to be able to produce an equilibrium state in the junction. The particles injected into the superconductors relax in energy on the phonons, and the result is a difference between the populations of the electron and hole branches, leading in turn to a shift of the chemical potential. Whereas in a normal metal the mixing of the branches is due to spatial diffusion, a distinct mechanism of homogeneous mixing is possible in a superconductor.[4] Spatially homogeneous situations can therefore arise in flat junctions of sufficient length. Because of the electron-phonon interaction, the excess particles produce a current whose divergence in the film differs from zero, and by the same token the pattern is homogeneous in the coordinate only over very large distances.

We shall deal hereafter with the experimental situation shown in Fig. 1.[2] The particles injected into the superconductor alter both the size of the gap and the chemical potential. For this reason, to prevent tunnel current from flowing between the superconductor and the probe N_p, it is necessary to apply to the latter some compensating voltage U. We obtain here the dependence of the compensating voltage and of the energy gap on the injecting voltage V and on the temperature if the latter is small compared with the critical temperature. A similar problem was investigated by Volkov and Zaĭtsev.[5]

In the limit of large impurity concentration, the quantity $\xi = v(p - p_0)$ is a poor quantum number, so that the kinetic equation of Aronov and Gurevich,[6] for example, cannot be directly employed here. On the other hand, to calculate the collision integral and the term with the field pumping it is more convenient to use directly the kinetic-equation approximation rather than a more general approach.[7-9] For this reason we start out with Green's functions that are integrated with respect to ξ and depend on the energy variable ϵ. In these terms, we introduce a particle distribution function n_ϵ, in contrast to the quasiparticle function used in [6].

2. DERIVATION OF THE KINETIC EQUATION

We derive below a kinetic equation for a superconductor with impurities in the presence of field pumping in the asymmetrical case $n_\epsilon \pm 1 - n_{-\epsilon}$, i.e., when a shift of the chemical potential appears. In the case symmetrical in the electrons and holes, an analogous equation was derived by Éliashberg.[10] Here, however, we use Keldysh's technique,[11] in which the equations are more compact.

We write down, by way of example, one of the functions G prior to integration with respect to ξ:

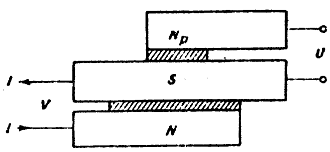

FIG. 1.

0038-5646/78/4701-0115$02.40

$$-G_+=-\frac{1}{2}n_\varepsilon\left[\left(\frac{\varepsilon}{\xi_\varepsilon^R}+1\right)\left(\xi-\xi_\varepsilon^R-\frac{i}{2\tau}\right)^{-1}+\left(\frac{\varepsilon}{\xi_\varepsilon^A}-1\right)\left(\xi+\xi_\varepsilon^A+\frac{i}{2\tau}\right)^{-1}\right]$$
$$+\frac{1}{2}(1-n_{-\varepsilon})\left[\left(\frac{\varepsilon}{\xi_\varepsilon^R}-1\right)\left(\xi+\xi_\varepsilon^R+\frac{i}{2\tau}\right)^{-1}+\left(\frac{\varepsilon}{\xi_\varepsilon^A}+1\right)\left(\xi-\xi_\varepsilon^A-\frac{i}{2\tau}\right)^{-1}\right];$$
$$\xi_\varepsilon^R=\xi_{-\varepsilon}^A=\begin{cases}(\varepsilon^2-\Delta^2)^{1/2}\operatorname{sign}\varepsilon, & |\varepsilon|\geqslant\Delta\\ i(\Delta^2-\varepsilon^2)^{1/2}, & |\varepsilon|<\Delta\end{cases}.$$

The symbol on the upper left numbers of the functions in the sense of Gor'kov, and the one on the right in the sense of Keldysh.

After integration with respect to ξ, the Keldysh equations for the functions g of the superconductors assume, in matrix notation, the form

$$(\omega-\mathbf{vk})\,{}^-g_{\varepsilon,\varepsilon-\omega}=\frac{e}{c}\mathbf{v}\{{}^-g\mathbf{A}-\mathbf{A}{}^-g\}_{\varepsilon,\varepsilon-\omega}$$
$$+i\{\sigma_z{}^-\Sigma{}^-g-\sigma_z{}^-\Delta{}^+f-{}^-g{}^-\Sigma\sigma_z+{}^-f{}^+\Delta\sigma_z\}_{\varepsilon,\varepsilon-\omega}+{}^-I_{\varepsilon,\varepsilon-\omega}^{imp},$$
$$(2\varepsilon-\omega+\mathbf{vk})\,{}^+f_{\varepsilon,\varepsilon-\omega}=-\frac{e}{c}\mathbf{v}\{{}^+f\mathbf{A}+\mathbf{A}{}^+f\}_{\varepsilon,\varepsilon-\omega}$$
$$+i\{\sigma_z{}^+\Sigma{}^+f-\sigma_z{}^+\Delta{}^-g+{}^+f{}^-\Sigma\sigma_z-{}^+g{}^+\Delta\sigma_z\}_{\varepsilon,\varepsilon-\omega}+{}^+K_{\varepsilon,\varepsilon-\omega}^{imp},\tag{1}$$
$$(\omega+\mathbf{vk})\,{}^+g_{\varepsilon,\varepsilon-\omega}=\frac{e}{c}\mathbf{v}\{{}^+g\mathbf{A}-\mathbf{A}{}^+g\}_{\varepsilon,\varepsilon-\omega}$$
$$+i\{\sigma_z{}^+\Sigma{}^+g-\sigma_z{}^+\Delta{}^-f-{}^+g{}^+\Sigma\sigma_z+{}^+f{}^-\Delta\sigma_z\}_{\varepsilon,\varepsilon-\omega}+{}^+I_{\varepsilon,\varepsilon-\omega}^{imp},$$
$$(2\varepsilon-\omega-\mathbf{vk})\,{}^-f_{\varepsilon,\varepsilon-\omega}=-\frac{e}{c}\mathbf{v}\{{}^-f\mathbf{A}+\mathbf{A}{}^-f\}_{\varepsilon,\varepsilon-\omega}$$
$$+i\{\sigma_z{}^-\Sigma{}^-f-\sigma_z{}^-\Delta{}^+g+{}^-f{}^+\Sigma\sigma_z-{}^-g{}^-\Delta\sigma_z\}_{\varepsilon,\varepsilon-\omega}+{}^-K_{\varepsilon,\varepsilon-\omega}^{imp}.$$

The last terms in the equations of the system (1), which are connected with elastic scattering by impurities, determine the angular parts of the Green's functions and vanish after averaging over the angles. We write down, by way of example, two of them:

$$-I_{\varepsilon,\varepsilon-\omega}^{imp}=\frac{1}{2\pi\tau}\{\langle{}^-g\rangle\sigma_z{}^-g-{}^-g\sigma_z\langle{}^-g\rangle+{}^-f\sigma_z\langle{}^+f\rangle-\langle{}^-f\rangle\sigma_z{}^+f\}_{\varepsilon,\varepsilon-\omega},\tag{2}$$

$$^+K_{\varepsilon,\varepsilon-\omega}^{imp}=\frac{1}{2\pi\tau}\{\langle{}^+g\rangle\sigma_z{}^+f-{}^+g\sigma_z\langle{}^+f\rangle+{}^+f\sigma_z\langle{}^-g\rangle-\langle{}^+f\rangle\sigma_z{}^-g\}_{\varepsilon,\varepsilon-\omega}.\tag{3}$$

The second terms in the right-hand sides of (1) describe the interaction of the electrons with the phonons, both inelastic scattering by the phonons and processes that lead to Cooper pairing.

To simplify the derivation, we leave out of the Green's functions for the time being the self-energy parts due to electron tunneling. The angle brackets $\langle\cdots\rangle$ in (2) and (3) mean averaging over the angles. We use here the abbreviated notation

$$\{AB\}_{\varepsilon,\varepsilon-\omega}=\int A_{\varepsilon,\varepsilon-\omega_1}B_{\varepsilon-\omega_1,\varepsilon-\omega}\frac{d\omega_1}{2\pi}.$$

The Green's functions and the self-energy parts in (1)–(3) should be taken to mean Keldysh matrices of the following form:

$$g=\begin{pmatrix}g_c & g_-\\ g_+ & \tilde{g}_c\end{pmatrix},$$

whose elements are connected by the Keldysh identities[11] both in the case of the Green's functions and for the self-energy parts.

In the approximation corresponding to the kinetic equation, when

$$^+g_{ij}(n_\varepsilon)={}^-g_{ij}(1-n_{-\varepsilon}),\quad {}^+f_{ij}={}^-f_{ij},$$

the Green's functions diagonal in the energy, which satisfy equations (1), take the form

$$^-g_+=-\pi i[u_\varepsilon(1+\beta_\varepsilon)+\alpha_\varepsilon],\quad {}^-g_c={}^-g_++i\pi u_\varepsilon,$$
$$^-g_-=-\pi i[u_\varepsilon(\beta_\varepsilon-1)+\alpha_\varepsilon],\quad {}^-\tilde{g}_c={}^-g_--i\pi u_\varepsilon;\tag{4}$$
$$f_+=-\pi iv_\varepsilon(\beta_\varepsilon+1),\quad f_c=f_++i\pi v_\varepsilon,$$
$$f_-=-\pi iv_\varepsilon(\beta_\varepsilon-1),\quad \tilde{f}_c=f_--i\pi v_\varepsilon.\tag{5}$$

Here and below we use the notation

$$u_\varepsilon=\frac{\varepsilon}{\Delta}v_\varepsilon=\frac{|\varepsilon|}{(\varepsilon^2-\Delta^2)^{1/2}}\theta(\varepsilon^2-\Delta^2),$$
$$\alpha_\varepsilon=n_\varepsilon+n_{-\varepsilon}-1,\quad \beta_\varepsilon=n_\varepsilon-n_{-\varepsilon}.$$

For the self-energy parts connected with the electron-phonon interaction we have, according to Keldysh,[11]

$$\Sigma_{ij}=g\int\frac{d\varepsilon'}{2\pi}\int\frac{dO_p}{4\pi}\gamma_{ii'}^k g_{i'j'}\Gamma_{j'j}^{k'}D_{k'k}.\tag{6}$$

The phonon propagator is of the form

$$D_\pm=2\pi i\omega_k[N_{\omega_k}\delta(\omega\mp\omega_k)+(N_{\omega_k}+1)\delta(\omega\pm\omega_k)],$$
$$D_{c(\tilde{c})}=D_\pm\mp{}^1/_2\omega_k[(\omega-\omega_k+i\delta)^{-1}-(\omega+\omega_k+i\delta)^{-1}].\tag{7}$$

Integrating in (6) over the angles by using the relation

$$\int\frac{dQ_p}{4\pi}=\int\frac{\omega_k\,d\omega_k}{2(sp_0)^2},$$

we get, for example, and expression for ${}^-\Sigma_+$:

$$^-\Sigma_+=-\frac{g\pi}{2(sp_0)^2}\int d\varepsilon'(\varepsilon'-\varepsilon)^2[u_{\varepsilon'}(\beta_{\varepsilon'}+1)$$
$$+\alpha_{\varepsilon'}][(N_{\varepsilon'-\varepsilon}+1)\theta(\varepsilon'-\varepsilon)+N_{\varepsilon-\varepsilon'}\theta(\varepsilon-\varepsilon')].\tag{8}$$

It is similarly easy to verify that

$$\Delta=i(\tilde{\Delta}_c-\Delta_c)/2.\tag{9}$$

By virtue of gauge invariance, we assume that Δ is a real quantity.

It is convenient next to write down the equations that follow from the system (1) for the Green's functions diagonal in the energy, $g_\epsilon=g_++g_-$ and $f_\epsilon=f_++f_-$, so as to retain the direct connection with the Eliashberg case which is symmetric in the electrons and the holes:[10]

$$\left\langle\left(i\frac{\partial}{\partial t}-\mathbf{vk}\right){}^-g_\varepsilon\right\rangle=\left\langle\frac{e}{c}\mathbf{v}\{{}^-g\mathbf{A}-\mathbf{A}{}^-g\}_{\varepsilon\varepsilon}\right\rangle\tag{10}$$
$$+2i\{{}^-\Sigma_-{}^-g_+-{}^-g_-{}^-\Sigma_++{}^-f_-{}^-\Delta_+-{}^-\Delta_-{}^-f_+\}_{\varepsilon\varepsilon}+\langle\Delta({}^+f_\varepsilon-{}^-f_\varepsilon)\rangle+I_\varepsilon^T,$$
$$\langle 2\varepsilon({}^+f_\varepsilon-{}^-f_\varepsilon)\rangle=2i\{f_+({}^+\Sigma_--{}^-\Sigma_-)+({}^+g_+-{}^-g_+)\Delta_--f_-({}^+\Sigma_+-{}^-\Sigma_+)$$
$$-({}^+g_--{}^-g_-)\Delta_+\}_{\varepsilon\varepsilon}-\langle\mathbf{vk}({}^+f_\varepsilon+{}^-f_\varepsilon)\rangle-\left\langle\frac{e\mathbf{v}}{c}\{\mathbf{A}({}^+f-{}^-f)$$
$$+({}^+f-{}^-f)\mathbf{A}\}_{\varepsilon\varepsilon}\right\rangle+K_\varepsilon^T,\tag{11}$$

where I_ϵ^T and K_ϵ^T are due to the tunneling.

In the absence of a phase of the order parameter and at equilibrium we have $\langle {}^+f_\epsilon - {}^-f_\epsilon \rangle = 0$. A deviation of this quantity from zero is due to the suppression of the even-in-energy increment to the distribution function. The procedure of obtaining the kinetic equation consists of determining the difference between the functions from (11) and substituting it in (10). The right-hand side of (10) is then the effective integral for the collisions with the phonons. In addition, this substitution cancels out the London part in the term $\mathbf{v}\cdot\mathbf{k}g_\epsilon$, which corresponds to divergence of the total current. What is left then is only the diffusion part.[12]

To continue the solution of the system (10), (11) it is necessary to find the angular parts, which depend on the field A_ω, of the nondiagonal Green's functions. This can be done by solving the system (1) prior to averaging over the angles, when the intergals of the collisions with the impurities are given by (2) and (3). This cumbersome procedure can be avoided, however, by using, in analogy with the technique developed by Gor'kov and Éliashberg,[12,10] the fact that in the approximation linear to the field the nondiagonal Green's functions take the form:

$$ {}^-G_{\varepsilon\,\varepsilon-\omega} = \frac{e}{c}\mathbf{A}_\omega({}^-G_\varepsilon{}^-G^A_{\varepsilon-\omega} + {}^-F_\varepsilon{}^+F^A_{\varepsilon-\omega} + {}^-G_\varepsilon^R{}^-G_{\varepsilon-\omega} + {}^-F_\varepsilon^R{}^+F_{\varepsilon-\omega}), \qquad (12) $$

$$ {}^+F_{\varepsilon\,\varepsilon-\omega} = -\frac{e}{c}\mathbf{A}_\omega({}^+F_\varepsilon{}^-G^A_{\varepsilon-\omega} - {}^+G_\varepsilon{}^+F^A_{\varepsilon-\omega} + {}^+F_\varepsilon^R{}^-G_{\varepsilon-\omega} - {}^+G_\varepsilon^R{}^+F_{\varepsilon-\omega}), \qquad (13) $$

where

$$ {}^-G_\varepsilon = \frac{1}{2}\Big\{(1-2n_\varepsilon)\Big[\Big(\frac{\varepsilon}{\xi_\varepsilon^R}+1\Big)\Big(\xi-\xi_\varepsilon^R-\frac{i}{2\tau}\Big)^{-1} + \Big(\frac{\varepsilon}{\xi_\varepsilon^A}-1\Big)\Big(\xi+\xi_\varepsilon^A+\frac{i}{2\tau}\Big)^{-1}\Big] + (1-2n_{-\varepsilon})\Big[\Big(\frac{\varepsilon}{\xi_\varepsilon^R}-1\Big)\Big(\xi+\xi_\varepsilon^R+\frac{i}{2\tau}\Big)^{-1} + \Big(\frac{\varepsilon}{\xi_\varepsilon^A}+1\Big)\Big(\xi-\xi_\varepsilon^A-\frac{i}{2\tau}\Big)^{-1}\Big]\Big\}, $$

$$ {}^+F_\varepsilon = -\frac{1}{2}\Big\{(1-2n_\varepsilon)\Big[\frac{\Delta}{\xi_\varepsilon^R}\Big(\xi+\xi_\varepsilon^R+\frac{i}{2\tau}\Big)^{-1} + \frac{\Delta}{\xi_\varepsilon^A}\Big(\xi-\xi_\varepsilon^A-\frac{i}{2\tau}\Big)^{-1}\Big] + (1-2n_{-\varepsilon})\Big[\frac{\Delta}{\xi_\varepsilon^R}\Big(\xi-\xi_\varepsilon^R-\frac{i}{2\tau}\Big)^{-1} + \frac{\Delta}{\xi_\varepsilon^A}\Big(\xi+\xi_\varepsilon^A+\frac{i}{2\tau}\Big)\Big]\Big\}. $$

At this stage of the calculations it is necessary to make some concrete assumptions concerning the electron mean free path. We shall be henceforth interested, just as in [10] in the "high contamination" case. In this limiting case we integrate with respect to ξ the nondiagonal Green's functions (12) and (13) and get

$$ {}^-g_{\varepsilon\,\varepsilon-\omega} = -\frac{\pi\tau A_\omega ev}{c}\Big\{(1-2n_\varepsilon)\Big[\frac{\varepsilon(\varepsilon-\omega)+\Delta^2}{\varepsilon\xi^A_{\varepsilon-\omega}}u_\varepsilon + \frac{\varepsilon-\omega}{\xi^A_{\varepsilon-\omega}}\Big] - (1-2n_{-\varepsilon})\Big[\frac{\varepsilon(\varepsilon-\omega)+\Delta^2}{\varepsilon\xi^A_{\varepsilon-\omega}}u_\varepsilon - \frac{\varepsilon-\omega}{\xi^A_{\varepsilon-\omega}}\Big] + (1-2n_{\varepsilon-\omega})\Big[\frac{\varepsilon(\varepsilon-\omega)+\Delta^2}{(\varepsilon-\omega)\xi_\varepsilon^R}u_{\varepsilon-\omega} + \frac{\varepsilon}{\xi_\varepsilon^R}\Big] - (1-2n_{\omega-\varepsilon})\Big[\frac{\varepsilon(\varepsilon-\omega)+\Delta^2}{(\varepsilon-\omega)\xi_\varepsilon^R}u_{\varepsilon-\omega} - \frac{\varepsilon}{\xi_\varepsilon^R}\Big]\Big\}, \qquad (14) $$

$$ {}^+f_{\varepsilon\,\varepsilon-\omega} - {}^-f_{\varepsilon\,\varepsilon-\omega} = \frac{4\pi\tau evA_\omega\Delta}{c}\Big\{\frac{\alpha_{\varepsilon-\omega}}{\xi_\varepsilon^R} - \frac{\alpha_\varepsilon}{\xi^A_{\varepsilon-\omega}}\Big\}. \qquad (15) $$

Substituting (14) and (15) in (10) and (11), we can write the equations for the Green's function $g_\epsilon = g_\epsilon^+ + g_\epsilon^-$. We note, according to (4), that

$$ u_\varepsilon\frac{\partial n_\varepsilon}{\partial t} = i\frac{\partial}{\partial t}[g_\varepsilon - g_{-\varepsilon} + u_\varepsilon(g_\varepsilon + g_{-\varepsilon})], \qquad (16) $$

and present the kinetic equations directly for the function n_ϵ, making the corresponding symmetrization

$$ u_\varepsilon\frac{\partial n_\varepsilon}{\partial t} - D\nabla^2 n_\varepsilon = \lambda\{U_-(n_{\varepsilon-\omega} - n_\varepsilon - n_{\omega-\varepsilon} + n_{-\varepsilon}) + U_+(n_{\varepsilon+\omega} - n_\varepsilon - n_{-\varepsilon-\omega} + n_{-\varepsilon}) + \alpha_{\varepsilon-\omega} + \alpha_{\varepsilon+\omega} - \alpha_\varepsilon(U_- + U_+)\} + I_\varepsilon^{ph} + u_\varepsilon\Big(\frac{\partial n}{\partial t}\Big)^T. \qquad (17) $$

Here

$$ \lambda = D(e/c)^2 A_\omega A_{-\omega}, \qquad U_\pm = u_\varepsilon u_{\varepsilon\pm\omega} + v_\varepsilon v_{\varepsilon\pm\omega}. $$

We see therefore that the current is an additional source of relaxation of the potentials in the superconductor. In the case of direct current this effect was discussed by Galaiko.[7] If the alternating field is strong enough, it can serve as the main cause of the relaxation and determine, as a result of the frequency dispersion, the form of the distribution function.

The phonon collision integral is obtained from Eqs. (10), (11), and (16) with allowance for (4)–(7):

$$ I_\varepsilon = \frac{\pi g}{4(sp_0)^2}\Big\{\int_\varepsilon^\infty(\varepsilon'-\varepsilon)^2(uu'-vv'+1)[(1-n)n'(N+1) - n(1-n')N]d\varepsilon' + \int_{-\infty}^{\varepsilon}(\varepsilon-\varepsilon')^2(uu'-vv'+1)[(1-n)n'N - n(1-n')(N+1)]d\varepsilon' + \int_{-\infty}^{-\varepsilon}(\varepsilon+\varepsilon')^2(uu'+vv'-1)[(1-n)(1-n')(N+1) - nn'N]d\varepsilon' + \int_{-\varepsilon}^{\infty}(\varepsilon+\varepsilon')^2(uu'+vv'-1)[(1-n)(1-n')N - nn'(1+N)]d\varepsilon'\Big\}. \qquad (18) $$

The obtained collision integral is of the same type as the well known integral obtained in the quasistatic description, when the variable is the quantity ξ. The first two terms of (18) conserve the total number of electron excitations, just as in the normal metal. The last two terms describe the nonconservation of the particle number. It is they which lead to relaxation of the population difference between the electron and hole branches.[3,4] It is seen that (17) coincides with the Éliashberg equation[10] if the chemical-potential shift is disregarded.

As already noted, it is necessary to add to the right-hand side of the kinetic equation (17) a non-equilibrium source, which leads in fact to a change of the chemical potential. We take this source to be, just as in,[1-4] the tunneling of electrons from the normal metal into the superconductor. In the derivation of the expression for the tunnel source, we note that the corresponding term in the right-hand side of (17) is equivalent formally to impurity collision integrals of the type (2) and (3), except that the functions averaged over the direction are replaced by functions pertaining to the other metal. We assume the superconductor to be at zero voltage, and the normal metal at a voltage V. In the normal metal, by virtue of the time dependence of the phases of the Green's functions

$$^{\mp}g_V(t-t')={}^{\mp}g(t-t')\exp[\mp iV(t-t')]$$

their Fourier components take the form

$$^{\mp}g={}^{\mp}g(\varepsilon\mp V).$$

Introducing a quantity w proportional to the tunneling probability, we obtain from with account taken of the symmetrization rule (16), the tunnel source of quasiparticles in the kinetic equation (17):

$$u_\varepsilon\left(\frac{\partial n_\varepsilon}{\partial t}\right)^T=2w[u_\varepsilon(n_{\varepsilon-V}-n_{-V-\varepsilon}+1-2n_\varepsilon)+(n_{\varepsilon-V}-n_{-V-\varepsilon}-1)\theta(\varepsilon^2-\Delta^2)]. \tag{19}$$

The difference between (19) and the corresponding expressions of [4,5] is that the distribution functions that enter in this expression are not assumed to be in equilibrium. The source (19) in the kinetic equation describes tunnel injection of normal excitations, whereas the total tunnel current receives contributions also from the direct transition of the particles into the superfluid current, according to Tinkham.[4]

To obtain the tunnel current it is necessary to calculate the right-hand side of the continuity equation, which is obtained by integrating Eq. (10). In the right-hand side of this equation a nonzero contribution to the integration is made only by the tunnel source:

$$\frac{mp_0e}{\pi^2}\left[\frac{eD}{c}\operatorname{div}(\beta_\Delta\Delta\mathbf{A})-\int_\Delta^\infty u_\varepsilon D\nabla^2\alpha_\varepsilon d\varepsilon\right]$$

$$=4w\int_\Delta^\infty\frac{mp_0e}{\pi^2}[u_\varepsilon(n_{\varepsilon-V}+n_{-\varepsilon-V}-1)+(1-n_\varepsilon-n_{-\varepsilon})\theta(\varepsilon^2-\Delta^2)]d\varepsilon. \tag{20}$$

The first term on the left is the divergence of the London current, and the second is the divergence of the normal current. Since we are considering a stationary problem, there is not time derivative of the charge density. The quantity on the right-hand side of (20) is the total tunnel current given in a more general form than in [4], since account is taken here, in principle, also of the shift of the chemical potential in the normal metal. We recall that the function $n_{\epsilon-V}$ pertains to the normal metal and the function n_ϵ to the superconductor.

Thus, the kinetic equation (17) with the phonon collision integral (18) and the tunnel source (19), in conjunction with the continuity equation (20), make it possible in principle to solve kinetic problems in situations with a shift of the chemical potential. The electroneutrality is satisfied here automatically because of the shift of the energy origin by the produced scalar potential, which in the static case makes only a negligible contribution of the order of φ/ϵ_F to all quantities other than the charge density.

3. TUNNEL JUNCTION AT LOW TEMPERATURES ($T\ll\Delta$)

Our task is to calculate the compensating voltage U on the probe and the change of the gap in a superconducting sample by the disequilibrium produced by the tunnel current. Various cases are possible then, depending on the injecting voltage.

a) $0<V-\Delta\ll\Delta$. The nonequilibrium particles are then accumulated directly near the thresholds, and the nonequilibrium distribution function differs from zero in the region $\Delta<|\epsilon|<V$. It is advantageous to separate in the kinetic equation the parts even and odd in ϵ:

$$\begin{aligned}I_\varepsilon+I_{-\varepsilon}&=2w\left(\operatorname{th}\frac{\varepsilon-V}{2T}-\operatorname{th}\frac{\varepsilon+V}{2T}\right),\\ I_\varepsilon-I_{-\varepsilon}&=-2w\left(2\operatorname{th}\frac{\varepsilon}{2T}-\operatorname{th}\frac{\varepsilon+V}{2T}-\operatorname{th}\frac{\varepsilon-V}{2T}\right).\end{aligned} \tag{21}$$

As seen from the expression for the collision integral (18), the terms that are quadratic in the increment and the distribution function enter at absolute zero with an energy transfer 2Δ to the phonons, whereas the linear terms enter with a transfer on the order of $V-\Delta$. For this reason, if $n_1=n-n_F$ is not too small a quantity, then the principal role in the collision integral will be played by the nonlinear terms. The condition for the applicability of this approximation will be written out below. Introducing

$$p=(V-\Delta)/\Delta,\quad x=(\varepsilon-\Delta)/\Delta,$$

we obtain from (21)

$$\begin{aligned}\gamma\frac{\alpha(x)}{x^{1/2}}\int_0^p\frac{\beta(x')}{x'^{1/2}}dx'&=w\theta(x)\theta(p-x),\\ \gamma\frac{\beta(x)}{x^{1/2}}\int_0^p\frac{\beta(x')}{x'^{1/2}}dx'&=\left(\frac{2}{x}\right)^{1/2}w\theta(x)\theta(p-x),\end{aligned} \tag{22}$$

where $\gamma=g\Delta^3/\omega_D^2$ is the reciprocal energy-relaxation time. Hence

$$\begin{aligned}\alpha(x)&=(2p)^{-1/4}(wx/2\gamma)^{1/2}\theta(x)\theta(p-x),\\ \beta(x)&=(2p)^{-1/4}(w/\gamma)^{1/2}\theta(x)\theta(p-x).\end{aligned} \tag{23}$$

The compensating voltage U, which is determined from the condition that there be no current through the probe contact, is determined according to (20) from the condition

$$2\int_\Delta^\infty\alpha_\varepsilon\,d\varepsilon=\int_\Delta^\infty u_\varepsilon\left(\operatorname{th}\frac{\varepsilon+U}{2T}-\operatorname{th}\frac{\varepsilon-U}{2T}\right)d\varepsilon. \tag{24}$$

In our case we obtain

$$\begin{aligned}\frac{U-\Delta}{\Delta}&=\frac{1}{2}\left(\int_0^p\alpha_x\,dx\right)^2=\frac{w}{9\sqrt{2}\gamma}\left(\frac{V-\Delta}{\Delta}\right)^{5/2};\\ \frac{\Delta_0-\Delta}{\Delta_0}&=\frac{1}{\sqrt{2}}\int_0^p\beta_x\frac{dx}{x^{1/2}}=\left(\frac{w}{\gamma}\right)^{1/2}\left(2\frac{V-\Delta}{\Delta}\right)^{1/4}.\end{aligned} \tag{25}$$

This enables us to write down the conditions under which the terms linear in n_1 that are discarded in the collision integrals are small, viz., $p^{13/2} \ll w/\gamma$. Combining this with the requirement $n_1 \ll 1$, we obtain the condition for the validity of (25):

$$\left(\frac{V-\Delta}{\Delta}\right)^{13/2} \ll \frac{w}{\gamma} \ll \left(\frac{V-\Delta}{\Delta}\right)^{1/2}. \tag{26}$$

At somewhat higher voltages

$$\frac{w}{\gamma} \ll \left(\frac{V-\Delta}{\Delta}\right)^{13/2} \tag{27}$$

the linear terms are the principal ones in the collision integral. It is then impossible to obtain an exact solution, but in order of magnitude we have $I_\epsilon + I_{-\epsilon} \sim \gamma\alpha p^3$ and $I_\epsilon - I_{-\epsilon} \sim \gamma\beta p^3$, whence, using the kinetic equations (21), we get

$$\frac{U-\Delta}{\Delta} \sim \left(\frac{w}{\gamma}\right)^2 \left(\frac{\Delta}{V-\Delta}\right)^2, \quad \frac{\Delta_0-\Delta}{\Delta_0} \sim \frac{w}{\gamma}\left(\frac{\Delta}{V-\Delta}\right)^3. \tag{28}$$

Combining (25) and (28) we see that the nonequilibrium properties of the junction in this voltage region depend on the voltage nonmonotonically. The maximum is reached when

$$\left(\frac{U-\Delta}{\Delta}\right)_{max} \sim \left(\frac{w}{\gamma}\right)^{22/13},$$
$$\left(\frac{\Delta_0-\Delta}{\Delta_0}\right)_{max} \sim \left(\frac{w}{\gamma}\right)^{7/13} \frac{V-\Delta}{\Delta} = \frac{V_1-\Delta}{\Delta} \approx \left(\frac{w}{\gamma}\right)^{2/13}. \tag{29}$$

b) $\Delta \ll V$. In this case the situation recalls the problem of finding the nonequilibrium distribution function produced by radiation of high frequency.[13] When high-energy excitations relax on phonons, they are gathered in an energy region above a gap on the order of the temperature. The fast process in this case is the establishment of the form of the distribution function, and the "bottleneck" is the relaxation of the total number of quasiparticles (quasi-electrons plus quasiholes), in contrast to the symmetrical case, when this pertains to one of the excitation branches. Processes in which the total number of quasiparticles is conserved, are linear at low temperature in a small correction to the distribution function, and are shown in Fig. 2. Process 2 corresponds to nonconservation of the number of particles, but conserves the number of quasiparticles.

We find the form of the distribution function from the condition that the principal linear part of the collision integral vanish. Then the energy-odd part of the distribution function $\beta(\epsilon)$, just as in [13], is of the Boltzmann type

$$\beta(\varepsilon) \sim \exp\left(-\frac{\varepsilon-\Delta}{T}\right) = b\left(\frac{\varepsilon-\Delta}{T}\right), \tag{30}$$

and the normalization condition for it is obtained by integrating the second equation in (21). We present directly the result for the change of the gap:

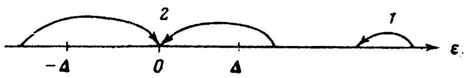

FIG. 2.

$$\frac{\Delta_0-\Delta}{\Delta_0} = \frac{1}{(2\Delta)^{1/2}} \int_\Delta^\infty \frac{\beta(\varepsilon)\,d\varepsilon}{(\varepsilon-\Delta)^{1/2}}$$
$$= \begin{cases} \dfrac{1}{4}\dfrac{w}{\gamma}\dfrac{V}{\Delta}\left(\dfrac{\Delta}{2\pi T}\right)^{1/2} e^{\Delta/T}, & \dfrac{wV}{\gamma T} \ll e^{-2\Delta/T} \\ \left(\dfrac{wV}{\gamma\Delta}\right)^{1/2}, & e^{-2\Delta/T} \ll \dfrac{wV}{\gamma T} \ll e^{-\Delta/T} \end{cases}. \tag{31}$$

The energy-even increment $\alpha_\epsilon = a((\epsilon - \Delta)/T)$ is determined from the equation

$$\int_0^\infty dy' |y'-y| (y-y') \left[a(y) \frac{y+y'}{2(yy')^{1/2}} \left(\operatorname{cth}\frac{y'-y}{2} - 1\right) - a(y')\left(\operatorname{cth}\frac{y'-y}{2} + 1\right)\right] = 0. \tag{32}$$

To find the normalization of this function it is necessary to integrate the first equation of (21) with a certain weighting function $f(\epsilon)$, which, unlike in the preceding case, is not equal identically to unity. The form of the weighting function f is determined by setting equal to zero the integrated ϵ-even principal part of the collision integral, after which the remaining term give the normalized function α.

We seek the function f at $T \ll \epsilon - \Delta$, and then

$$\int_\Delta^\infty f(\varepsilon)\,d\varepsilon \left[\alpha(\varepsilon) m_\varepsilon - \int_\varepsilon^\infty \alpha(\varepsilon')(\varepsilon'-\varepsilon)^2 d\varepsilon\right] = \int_\Delta^\infty \alpha(\varepsilon)\,d\varepsilon\, Q(\varepsilon) = 0,$$

$$m_\varepsilon = \int_\Delta^\varepsilon d\varepsilon' (\varepsilon-\varepsilon')^2 u_\varepsilon u_{\varepsilon'} (1-\Delta^2/\varepsilon\varepsilon') = \begin{cases} {}^{64}/_{105}(\varepsilon-\Delta)^3, & \varepsilon-\Delta \ll \Delta \\ {}^{1}/_{3}\varepsilon^3, & \Delta \ll \varepsilon \end{cases}$$

From the vanishing of $Q(\epsilon)$ we have

$$Q(\varepsilon) = m(\varepsilon) f(\varepsilon) - \int_\Delta^\varepsilon f(\varepsilon')(\varepsilon-\varepsilon')^2 d\varepsilon' = 0. \tag{33}$$

Whence

$$f(\varepsilon) = \begin{cases} 1, & \Delta \ll \varepsilon \\ [\Delta/(\varepsilon-\Delta)]^q, & (\varepsilon-\Delta) \ll \Delta \end{cases}; \tag{34}$$

for the exponent q we obtain the algebraic equation

$$(q-3)(q-2)(q-1) + {}^{105}/_{32} = 0,$$

the solution of which

$$q = 2 - \left\{\frac{105}{64} + \left[\left(\frac{105}{64}\right)^2 - \frac{1}{27}\right]^{1/2}\right\}^{1/3} - \left\{\frac{105}{64} - \left[\left(\frac{105}{64}\right)^2 - \frac{1}{27}\right]^{1/2}\right\}^{1/3} \approx 0.3.$$

It is seen from (34) that the presence of the weighting function manifests itself only at low energies; we assume therefore

$$f = \left(\frac{\Delta}{T}\right)^q \varphi\left(\frac{\varepsilon-\Delta}{T}\right).$$

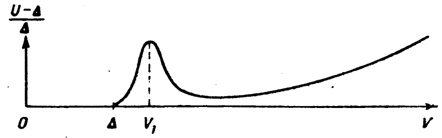

FIG. 3.

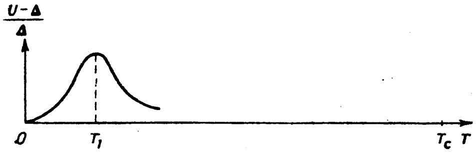

FIG. 4.

This yields for the even increment to n_ϵ the normalization condition

$$\int_0^\infty \frac{\varphi(y)}{(y)^{1/2}} a(y)\,dy \int_0^\infty \frac{dy'}{(y')^{1/2}}\left[2\exp\left(-\frac{\Delta}{T}-y'\right)+b(y')\right]=\frac{w}{\gamma}\frac{V}{\Delta}\left(\frac{\Delta}{T}\right)^{1-q}. \tag{35}$$

We can then write down directly the expressions for the voltage on the probing contact:

$$\frac{U-\Delta}{\Delta}=\frac{1}{2}\left(\frac{T}{\Delta}\int_0^\infty a(y)\,dy\right)^2$$

$$\approx\begin{cases}\left(\dfrac{wV}{\gamma\Delta}\right)^2\left(\dfrac{T}{\Delta}\right)^{2q}e^{2\Delta/T}, & \dfrac{wV}{\gamma\Delta}\ll\dfrac{T}{\Delta}e^{-2\Delta/T}.\\ \left(\dfrac{wV}{\gamma\Delta}\right)\left(\dfrac{T}{\Delta}\right)^{1+2q}, & \dfrac{T}{\Delta}e^{-2\Delta/T}\ll\dfrac{wV}{\gamma\Delta}\ll\dfrac{T}{\Delta}e^{-\Delta/T}\end{cases} \tag{36}$$

The second inequality takes the condition $n_1 \ll 1$ into account.

Thus, the dependence of the experimentally observed quantities U and Δ on the applied voltage V is determined by formulas (25), (28), (31), and (36) for different intervals of the variation of V. The dependence is shown schematically on Fig. 3, and V_1 is determined from (29). We note that calculations at small V were made under the condition that the number of temperature excitations is much smaller than the number of the nonequilibrium excitations; this is equivalent to the inequality

$$\exp(-{}^{13}/{}_{6}\Delta/T)\ll w/\gamma.$$

Interest attaches also to the temperature dependence at large values of the injecting voltage $\Delta \ll V$; this dependence is given by formula (36) and is shown in Fig. 4. The maximum is reached when

$$\left(\frac{U-\Delta}{\Delta}\right)_{\max}\sim\frac{wV}{\gamma\Delta}\left(\ln\frac{\gamma\Delta}{wV}\right)^{1+2q}, \quad \frac{2\Delta}{T_1}\sim\ln\frac{\gamma\Delta}{wV}. \tag{37}$$

The characteristic temperature dependence (31) and (36) is connected with the fact that at low temperatures the relaxation of the total number of quasiparticles (quasi-electrons plus quasi-holes) has an exponential character. The most convenient object for the observation of these low-temperature anomalies seems to be lead.

We thank A. F. Volkov, A. I. Larkin, Yu. N. Ovchinnikov, and G. M. Éliashberg for interesting discussions.

[1]J. Clarke, Phys. Rev. Lett. **28**, 1363 (1972).
[2]J. Clarke and J. Paterson, J. Low Temp. Phys. **15**, 491 (1974).
[3]M. Tinkham and J. Clarke, Phys. Rev. Lett. **28**, 1336 (1972).
[4]M. Tinkham, Phys. Rev. B **6**, 1747 (1972).
[5]A. F. Volkov, Zh. Eksp. Teor. Fiz. **68**, 756 (1975) [Sov. Phys. JETP **41**, 376 (1975)]; A. F. Volkov and A. V. Zaĭtsev, Zh. Eksp. Teor. Fiz. **69**, 2222 (1975) [Sov. Phys. JETP **42**, 1130 (1975)].
[6]A. G. Aronov and V. L. Gurevich, Fiz. Tverd. Tela **16**, 2656 (1974) [Sov. Phys. Solid State **16**, 1727 (1975)].
[7]V. P. Galaĭko, Zh. Eksp. Teor. Fiz. **66**, 379 (1974) [Sov. Phys. JETP **39**, 181 (1974)].
[8]A. Schmid and G. Schön, J. Low Temp. Phys. **20**, 207 (1975).
[9]A. I. Larkin and Yu. N. Ovchinnikov, Zh. Eksp. Teor. Fiz. **73**, 299 (1977) [Sov. Phys. JETP **46**, 155 (1977)].
[10]G. M. Éliashberg, Zh. Eksp. Teor. Fiz. **61**, 1254 (1971) [Sov. Phys. JETP **34**, 668 (1972)].
[11]L. V. Keldysh, Zh. Eksp. Teor. Fiz. **47**, 1515 (1964) [Sov. Phys. JETP **20**, 1018 (1965)].
[12]L. P. Gor'kov and G. M. Éliashberg, Zh. Eksp. Teor. Fiz. **54**, 612 (1968) [Sov. Phys. JETP **46**, 155 (1977)].
[13]C. S. Owen and D. J. Scalapino, Phys. Rev. Lett. **28**, 1559 (1972); R. A. Vardanyan and B. I. Ivlev, Zh. Eksp. Teor. Fiz. **65**, 2315 (1973) [Sov. Phys. JETP **38**, 1156 (1974)].

Translated by J. G. Adashko