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# Electromagnetic field absorption in superconducting films.

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The low-frequency dispersion of electromagnetic field absorption, connected with the relaxation of the order parameter and the excitation distribution function, is investigated. It is shown that on passage of a direct current, a sharp peak of the absorption of the high-frequency field near the threshold frequency  $\omega = 2\Delta$  appears at a current density much lower than the critical density.

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## 1. INTRODUCTION

The energy relaxation time  $\tau_\epsilon$  in superconductors is large. Therefore, inelastic collisions can usually be neglected in an approximation that is linear in the alternating field. However, at low frequencies, such neglect is not always possible and can lead, in the dynamic limit as  $\omega \rightarrow 0$  to a difference from the static limit obtained at  $\omega = 0$ . In superconducting films, in the presence of a direct current, these two limiting values, for example for the correction to the order parameter, differ from one another by a value of  $\Delta/T$  close to the transition temperature.<sup>[1]</sup> As will be shown below, such a difference leads to strong dispersion in the absorption at frequencies  $\omega \sim \Delta/T\tau_\epsilon$ . Strong dispersion arises also at  $\omega\tau_\epsilon \sim 1$ .

The low-frequency dispersion is connected with processes of relaxation of the excitation distribution function. Energy relaxation is effected in films only because of interaction of the electrons with phonons.

One more peculiarity arises when current flows in a film. It is known that there is a kink in the absorption at the frequency  $\omega = 2\Delta$ . At low temperatures, the presence of a weak current  $j \ll j_c$  leads to the appearance of a sharp maximum in the absorption at a frequency close to  $2\Delta$ .

## 2. DISPERSION OF THE ABSORPTION AT LOW FREQUENCIES

We limit ourselves below to superconductors with small free path length  $l \ll v/T, d$  ( $d$  is the film thickness,  $v$  is the velocity on the Fermi surface). We also assume that the film thickness  $d$  satisfies the condition  $d < \xi(T)$ , where  $\xi(T)$  is the correlation length of the superconductor.

At low frequencies ( $\omega \ll \Delta$ ) it is convenient to describe the properties of the superconductor by means of the

kinetic equations.<sup>[2,3]</sup> Below, we choose a special gauge, in which the scalar potential  $\varphi = 0$ ; the vector potential and the order parameter have the form

$$A(r, t) = A(r) + A_1 e^{-i\omega t}, \quad \Delta(r, t) = \Delta + \Delta_1 e^{-i\omega t},$$

respectively. Of the two distribution functions  $f$  and  $f_1$ , only the function  $f$  differs from zero in the chosen gauge:

$$f = \text{th}(\epsilon/2T) + \tilde{f} e^{-i\omega t},$$

where

$$\begin{aligned} & -\frac{i\omega}{2}(g^R - g^A)\tilde{f} + I_1^{\text{ph}} \left( \text{th} \frac{\epsilon}{2T} + \tilde{f} \right) \\ = & -i\omega \frac{\partial}{\partial \epsilon} \text{th} \left( \frac{\epsilon}{2T} \right) \left\{ \Delta_1 \frac{(F^R - F^A)}{2} - ie^2 (AA_1) D((g^R)^2 - (g^A)^2) \right\}. \end{aligned} \quad (1)$$

In Eq. (1),  $g^{R,A}$  and  $F^{R,A}$  are the static Green's functions, satisfying the set of equations

$$\begin{aligned} (g^{R,A})^2 &= (F^{R,A})^2 + 1, \\ 2e^2 A^2 D g^{R,A} F^{R,A} + i\Delta g^{R,A} - ie F^{R,A} &= 0, \end{aligned} \quad (2)$$

where  $D = \nu l_{\text{sp}}/3$  is the diffusion coefficient.

The collision integral  $I_1^{\text{ph}}$  is determined by the expression<sup>[2]</sup>

$$\begin{aligned} I_1^{\text{ph}}(f) &= \frac{ivg^2}{16} \int \frac{d\epsilon_1}{2\pi} \int d\Omega_p [ (g_{\epsilon_1}^R - g_{\epsilon_1}^A) (g_{\epsilon-\epsilon_1}^R - g_{\epsilon-\epsilon_1}^A) - (F_{\epsilon_1}^R - F_{\epsilon_1}^A) (F_{\epsilon-\epsilon_1}^R - F_{\epsilon-\epsilon_1}^A) ] \\ & \times \{ (f(\epsilon) - f(\epsilon_1)) D_{p-p_1}(\epsilon_1 - \epsilon) + (1 - f(\epsilon)) f(\epsilon_1) [ D_{p-p_1}^R(\epsilon_1 - \epsilon) - D_{p-p_1}^A(\epsilon_1 - \epsilon) ] \}. \end{aligned} \quad (3)$$

where  $\nu = mp/2\pi^2$  is the density of states on the Fermi surface. At thermodynamic equilibrium, the photon Green's functions are

$$\begin{aligned} D_k^R(\omega) &= D_k^A(\omega) = -\omega^2(k) / [\omega^2(k) - (\omega + i\delta)^2], \\ D_k(\omega) &= \text{cth}(\omega/2T) [D_k^R(\omega) - D_k^A(\omega)]. \end{aligned} \quad (4)$$

We limit ourselves below to the region of tempera-

tures close to critical ( $T_c - T \ll T_c$ ), inasmuch as the dispersion in this region turns out to be very strongly expressed. The values  $\epsilon \sim \Delta \ll T$  are important in Eq. (1). For such values of  $\epsilon$  we find from Eq. (3)

$$I_1^{R,A}(f) = \tau_c^{-1} \left( f - \text{th} \frac{\epsilon}{2T} \right) \frac{g^R - g^A}{2}, \quad (5)$$

where

$$\tau_c^{-1} = 7\zeta(3) \pi v g^2 T^2 / 2(sp)^2; \quad (6)$$

$s$  is the sound velocity in the normal metal and  $\zeta$  is the Riemann zeta function.

At sufficiently weak anisotropy, the inelastic scattering processes of electrons on phonons also determine the relaxation time  $\tau_Q$  with mixing of the branches of the spectrum. Near the transition temperature,

$$\tau_Q(T) = \tau_Q(0) \Delta(0) / \Delta(T).$$

The connection between the relaxation times  $\tau_e$  and  $\tau_Q(0)$  was obtained by Schmid and Schön:<sup>[4]</sup>  $\tau_e = (\pi^2/4\gamma) \tau_Q(0)$ , where  $\gamma$  is the Euler constant.

The relaxation time  $\tau_Q$  can be obtained experimentally from measurements of the damping of the quasiparticle current on the boundary of the superconducting metal. Such measurements, performed by M. L. Yu and J. E. Mercereau<sup>[5]</sup> for tantalum films, give a value of  $0.9 \times 10^{-10}$  sec. In the work of Clarke<sup>[6]</sup> the value of  $5.5 \times 10^{-10}$  sec was obtained for the  $\tau_e$  of dirty tin. The estimate of  $\tau_e$  for aluminum, made by Keck and Schmid,<sup>[7]</sup> leads to the value  $\tau_e(\text{Al}) = 0.25 \times 10^{-8}$  sec. The large value of  $\tau_e$  for aluminum is evidently due to the relatively high Debye temperature.

Substituting expression (5) for the collision integral in Eq. (1), we get

$$f = -\frac{\omega}{\omega + i\tau_c^{-1}} \frac{\partial}{\partial \epsilon} \text{th} \left( \frac{\epsilon}{2T} \right) \left\{ \Delta_1 \frac{F^R - F^A}{g^R - g^A} - 2ie^2(AA_1)D(g^R + g^A) \right\}. \quad (7)$$

For the calculation of the correction  $\Delta_1$  to the order parameter, we also need corrections to the regular Green's functions  $F^{R,A(1)}$  and  $g^{R,A(1)}$ . These corrections can be found from the set of equations (2):

$$\begin{aligned} F^{R,A(1)} &= (g^R, A)^2 [\Delta_1 - 4ie^2(AA_1)DF^{R,A}] \\ &\times \{ \epsilon g^{R,A} - \Delta F^{R,A} + 2ie^2 A^2 D((g^R, A)^2 + (F^R, A)^2) \}^{-1}, \\ g^{R,A(1)} &= F^{R,A(1)} F^{R,A} / g^{R,A}. \end{aligned} \quad (8)$$

The correction to the order parameter is expressed in terms of the distribution function  $f$  and the Green's function  $F^{R,A(1)}$  by the formula<sup>[2]</sup>

$$\begin{aligned} \int \frac{d\epsilon}{2\pi} \left\{ \frac{\Delta_1}{\Delta} \text{th} \frac{\epsilon}{2T} (F^R - F^A) - f \left[ (F^R - F^A) + \omega \frac{\partial}{\partial \epsilon} (F^R + F^A) \right] \right. \\ \left. - \text{th} \frac{\epsilon}{2T} (F^{R(1)} - F^{A(1)}) + \omega \frac{\partial}{\partial \epsilon} \text{th} \frac{\epsilon}{2T} (F^{R(1)} + F^{A(1)}) \right\} = 0. \end{aligned} \quad (9)$$

Substituting the expressions (1) and (7) in Eq. (9) for the distribution function and in expression (8) for the functions  $F^{R,A(1)}$ , we obtain (after uncomplicated calculations)

$$\Delta_1 = 2e^2(AA_1)D \left( \frac{\omega}{\omega + i\tau_c^{-1}} - 2 \right) / \left( \frac{7\zeta(3)\Delta}{\pi^2 T} + \frac{\omega}{\omega + i\tau_c^{-1}} \right). \quad (10)$$

The current density is expressed in terms of the distribution function and the corrections to the functions  $F^{R,A(1)}$  according to the formula<sup>[2]</sup>

$$\begin{aligned} \frac{j^{(1)}}{\sigma_0} &= -\frac{i}{8} \int_{-\infty}^{\infty} d\epsilon \left\{ 2A_1 \left[ \left( \text{th} \frac{\epsilon + \omega/2}{2T} - \text{th} \frac{\epsilon - \omega/2}{2T} \right) (g^R g^A + F^R F^A) \right. \right. \\ &\quad \left. \left. + \text{th} \frac{\epsilon - \omega/2}{2T} ((g^R)^2 + (F^R)^2) - \text{th} \frac{\epsilon + \omega/2}{2T} [(g^A)^2 + (F^A)^2] \right] \right. \\ &\quad \left. + 8A \left[ \text{th} \frac{\epsilon - \omega/2}{2T} F^R F^{R(1)} - \text{th} \frac{\epsilon - \omega/2}{2T} F^A F^{A(1)} \right] + 4A f [(g^R)^2 - (g^A)^2] \right\}, \end{aligned} \quad (11)$$

where  $\sigma_0 = e^2 p^2 l_{tr} / 3\pi^2$  is the conductivity of the metal in the normal state. We find from Eqs. (7), (8), (10), and (11)

$$\begin{aligned} \frac{j^{(1)}}{\sigma_0} &= -A_1 \left[ \frac{\pi \Delta^2}{2T} - i\omega \left( 1 + \frac{\Delta}{3T} \ln \frac{8\Delta}{\Gamma} \right) \right] \\ &- e^2 A(AA_1) \frac{\pi \Delta}{T} D \left\{ \frac{\omega}{\omega + i\tau_c^{-1}} \left( \frac{\Delta}{\Gamma} \right)^{1/2} \frac{\pi \cdot 3^{3/2} \cdot 2^3}{10\Gamma^{2/3}} \right. \\ &\quad \left. - \left( \frac{\omega}{\omega + i\tau_c^{-1}} - 2 \right)^2 \left[ \frac{7\zeta(3)\Delta}{\pi^2 T} + \frac{\omega}{\omega + i\tau_c^{-1}} \right]^{-1} \right\}, \end{aligned} \quad (12)$$

where  $\Gamma = \sigma_0 A^2 / \nu$  and  $\Gamma(2/3)$  is the Euler gamma function.

We also give the dependence of the order parameter  $\Delta$  and of the current density  $j$  on the vector potential  $A$ :

$$1 - \frac{T}{T_c} - \frac{7\zeta(3)\Delta^2}{8\pi^2 T^2} - \frac{\pi e^2 A^2 D}{2T} = 0, \quad j = -\sigma_0 \frac{\pi \Delta^2}{2T} A. \quad (13)$$

It follows from Eq. (12) that at low frequencies  $\omega \sim \Delta/T\tau_e$  strong dispersion develops. In currents of the order of critical, the real and imaginary parts of the kernel, which gives the connection of the current with the vector potential, is of the same order of magnitude at these frequencies. Upon increase in the frequency, the second term in the curly brackets falls off rapidly and becomes unimportant. At  $\omega \sim \tau_e^{-1}$  the dispersion is connected with the first term in the curly brackets. Thus, there are two maxima in the dependence of the absorption on the frequency: at  $\omega = 7\zeta(3)\Delta/\pi^2 T\tau_e$  and  $\omega = \tau_e^{-1}$ . We note that near the transition temperature the quantity  $\Gamma$  is always small in comparison with  $\Delta$ .

### 3. ABSORPTION OF HIGH FREQUENCY ELECTROMAGNETIC FIELD NEAR THE THRESHOLD FREQUENCY

Experiments<sup>[8]</sup> have shown that a distinct peak near the threshold frequency  $\omega = 2\Delta$  arises in the imaginary part of the impedance at low temperatures against the background of smooth variation if a static magnetic field is applied together with a weak alternating field along the surface of the sample. The peak appears only if the magnetic field of the wave is directed along the static magnetic field.

We investigate below this phenomenon in thin films. The results are applicable also to bulk superconductors

if the static magnetic field  $H$  satisfies the condition

$$eH\lambda^2 \gg 1, \quad (14)$$

where  $\lambda$  is the penetration depth. The condition (14) means that the gradient terms in the equation for the static Green's functions  $g^{R,A}$  and  $F^{R,A}$  are small.

If current flows in thin films, or upon satisfaction of condition (14) in bulk superconductors, the connection of the current with the potential has the form

$$j^{(1)}/\sigma_0 = -\{Q_1(\omega)A_1 + Q_2(\omega)A(AA_1)/A^2\}. \quad (15)$$

Here  $\sigma_0$  is the conductivity of the metal in the normal state. For thin films, the quantity  $\sigma_0$  is a rather complicated function of the ratio of the film thickness to the path length and also depends on the form of the boundary conditions on the surface of the sample.<sup>[1]</sup>

Expressions for the kernels  $Q_1$  and  $Q_2$  were obtained in [1]. We investigate now these expressions near the threshold frequency  $\omega = 2\Delta$ .

The kernels  $Q_1(\omega)$  and  $Q_2(\omega)$  are equal to [1]

$$Q_1(\omega) = -i\omega - \frac{i}{2} \int_{-\infty}^{\infty} d\varepsilon \operatorname{th} \frac{\varepsilon}{2T} (1 - F_{\varepsilon+\omega}^R F_{\varepsilon}^R - g_{\varepsilon+\omega}^R g_{\varepsilon}^R) + \frac{i}{4} \int_{-\infty}^{\infty} d\varepsilon \left( \operatorname{th} \frac{\varepsilon+\omega}{2T} - \operatorname{th} \frac{\varepsilon}{2T} \right) (1 + F_{\varepsilon+\omega}^R F_{\varepsilon}^A + g_{\varepsilon+\omega}^R g_{\varepsilon}^A),$$

$$Q_2(\omega) = \frac{\Gamma}{\omega} \left[ I_2 + \frac{I_1}{\omega I_3} \right],$$

where

$$\Gamma = \sigma_0 A^2 / v; \quad (16)$$

$$I_1(\omega) = -i \int_{-\infty}^{\infty} d\varepsilon \operatorname{th} \frac{\varepsilon}{2T} [F_{\varepsilon+\omega}^R (g_{\varepsilon}^R - g_{\varepsilon}^A) - g_{\varepsilon+\omega}^R (F_{\varepsilon}^R - F_{\varepsilon}^A)],$$

$$I_2(\omega) = \int_{-\infty}^{\infty} d\varepsilon \operatorname{th} \frac{\varepsilon}{2T} [g_{\varepsilon+\omega}^R ((F_{\varepsilon}^R)^2 - (F_{\varepsilon}^A)^2) - (F_{\varepsilon+\omega}^R)^2 (g_{\varepsilon}^R - g_{\varepsilon}^A)], \quad (17)$$

$$I_3(\omega) = - \int_{-\infty}^{\infty} d\varepsilon \operatorname{th} \frac{\varepsilon}{2T} \left\{ \frac{1}{2\Delta} (F_{\varepsilon}^R - F_{\varepsilon}^A) + \frac{(g_{\varepsilon+\omega}^R + g_{\varepsilon}^R)(F_{\varepsilon+\omega}^R - F_{\varepsilon}^R)}{\omega(F_{\varepsilon+\omega}^R + F_{\varepsilon}^R)} - \frac{(g_{\varepsilon+\omega}^R + g_{\varepsilon}^A)(F_{\varepsilon+\omega}^R - F_{\varepsilon}^A)}{\omega(F_{\varepsilon+\omega}^R + F_{\varepsilon}^A)} \right\}.$$

TABLE I.

$v$	$x_1$	$x_2$	$\alpha_1$	$\alpha_2$	$\alpha_3$	$\beta_1$	$\beta_2$	$\beta_3$
0	0.081	0	0.09	0.543	2.39	0	0	0
0.2	0.155	0.022	0.097	0.649	2.89	0.0005	0.0157	0.127
0.4	0.254	0.088	0.107	0.787	3.446	0.0024	0.07	0.509
0.6	0.365	0.194	0.119	0.943	3.953	0.0068	0.178	1.161
0.8	0.48	0.339	0.132	1.097	4.291	0.0155	0.364	2.112
1	0.594	0.517	0.142	1.178	4.267	0.0286	0.609	3.225
1.2	0.703	0.725	0.158	1.265	3.938	0.0556	1.037	4.823
1.4	0.805	0.961	0.159	1.067	2.615	0.098	1.456	5.98
1.6	0.898	1.222	0.143	0.68	0.812	0.121	1.78	6.524
1.8	0.983	1.505	0.115	0.2	-0.945	0.141	1.877	6.15
2	1.057	1.809	0.085	-0.214	-2.157	0.147	1.765	5.176
2.2	1.12	2.132	0.055	-0.558	-2.92	0.147	1.56	3.978
2.4	1.17	2.473	0.042	-0.65	-2.953	0.134	1.334	3.121
2.6	1.191	2.83	0.03	-0.73	-2.915	0.125	1.135	2.36
2.8	1.233	3.202	0.022	-0.765	-2.774	0.116	0.972	1.777
3	1.246	3.586	0.017	-0.769	-2.57	0.107	0.832	1.327
3.2	1.253	3.981	0.013	-0.762	-2.375	0.101	0.724	0.995
3.4	1.227	4.387	0.01	-0.746	-2.178	0.095	0.633	0.74
3.6	1.197	4.8	0.008	-0.726	-1.994	0.089	0.558	0.543
3.8	1.154	5.221	0.0066	-0.706	-1.827	0.085	0.496	0.391
4	1.098	5.647	0.0055	-0.684	-1.675	0.0806	0.444	0.271

We limit ourselves to the study of the regions of low temperatures  $T \ll \Delta$  and  $\Gamma \ll \Delta$ , which correspond to a current density much less than critical. Under these conditions, it is convenient to represent the kernels  $Q_{1,2}$  in the form

$$Q_1(y) = 2\Delta - \Delta(\Gamma/\Delta)^{1/2} \{ (y/2-1) \ln(\Delta/\Gamma) + \chi_1(y) + i\chi_2(y) \},$$

$$Q_2(y) = -\Delta(\Gamma/\Delta)^{1/2} \{ [\alpha_1(y) \ln^2(\Delta/\Gamma) + \alpha_2(y) \ln(\Delta/\Gamma) + \alpha_3(y)] + i[\beta_1(y) \ln^2(\Delta/\Gamma) + \beta_2(y) \ln(\Delta/\Gamma) + \beta_3(y)] \}, \quad (18)$$

where the dimensionless variable  $y$  is connected with the frequency  $\omega$  by the relation

$$y = 2 + \frac{2}{3\Delta} \left( \frac{\Delta}{\Gamma} \right)^{1/2} (\omega - 2\Delta). \quad (19)$$

At  $y = 0$ , the frequency  $\omega$  is equal to twice the gap in the excitation spectrum. At low temperatures and a current density much less than critical, the current density is connected with the vector potential by the relation

$$j/\sigma_0 = -\pi\Delta A.$$

The value of the functions  $\chi, \alpha, \beta$  are given in the table. It follows from the table that the imaginary part of the kernel  $Q_2(y)$  has a pronounced maximum. The location of this maximum depends weakly on the parameter  $\Gamma/\Delta$ . This is connected with the fact that all three functions  $\beta(y)$  have maxima at almost the same point.

The imaginary parts of the kernels  $-Q_1(y)(\Delta/\Gamma)^{2/3}\Delta^{-1}$  and  $-(Q_1(y) + Q_2(y))(\Delta/\Gamma)^{2/3}\Delta^{-1}$  are shown in the drawing as an illustration of the frequency dependence, for the three values of the parameter  $\Gamma/\Delta$ : 0.05, 0.02, and 0.01. Upon decrease in the parameter  $\Gamma/\Delta$  the position of the maximum of the quantity  $-\operatorname{Im}(Q_1 + Q_2)(\Delta/\Gamma)^{2/3}\Delta^{-1}$  shifts very slowly toward higher values of  $y$ . For values of the parameter  $\Gamma/\Delta = 0.05, 0.02, 0.01$ , the maximum lies at the points  $y = 1.757, 1.774, \text{ and } 1.785$ , respectively. Upon increase in the frequency, the imaginary part of the kernel  $-(Q_1 + Q_2)$  passes through a minimum and begins to increase.

#### 4. CONCLUSION

When a static current flows in the film, a correction to the modulus of the order parameter appears in the approximation that is linear in the alternating electro-

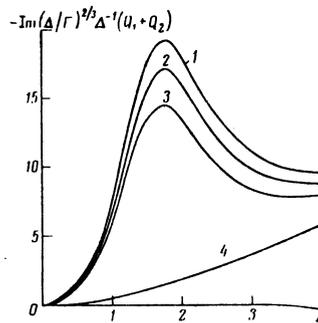


FIG. 1. Frequency dependence of the imaginary part of the kernel  $-(Q_1 + Q_2)(\Delta/\Gamma)^{2/3}\Delta^{-1}$  at  $\Gamma/\Delta = 0.01, 0.02, 0.05$  for the curves 1, 2, 3, respectively; curve 4 is the frequency dependence of the kernel  $-\operatorname{Im}(\Delta/\Gamma)^{2/3}\Delta^{-1} Q_1$ .

magnetic field. The relaxation of the modulus to the equilibrium value occurs very slowly, with a characteristic frequency  $\omega \sim \Delta/T\tau_e$ . Therefore, a sharp peak appears at this frequency in the imaginary part of the kernel describing the coupling of the current with the potential. At a current of the order of the critical value, the imaginary part becomes of the same order as the real one. Upon further increase in the frequency, dispersion develops in the kernel at  $\omega\tau_e \sim 1$ , because the excitation distribution function relaxes to equilibrium with a characteristic frequency  $\omega \sim \tau_e^{-1}$ .

There is a kink in the absorption of the high frequency field at a frequency of  $\omega = 2\Delta$ . When the static current is turned on, a maximum is produced if the current is directed along the magnetic field of the wave. The maximum in the absorption near the frequency  $\omega = 2\Delta$  is connected with the large density of states near

the threshold of single-particle excitations. The width of the peak in the absorption at low current density is proportional to  $\Delta(j/j_c)^{4/3}$ .

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## Echo effect in metallic powders at low temperatures

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The response of metallic powder to two-pulse excitation by an RF field at low temperatures is investigated theoretically. The mechanism that produces the echo signals is assumed to be the anharmonicity of the sound oscillations generated by the electromagnetic field in the metal. The theoretical expressions derived for the echo-signal parameters are in good agreement with the experimental data of Kupca and Searle [Can J. Phys. 53, 2622 (1975)].

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Experimental observation of echo signals induced by a sequence of electromagnetic field pulses in powdered metallic samples in the presence of a constant external magnetic field has been reported in a number of papers.<sup>[1-3]</sup> A number of the properties of the observed signals (the presence of the effect only in a narrow interval of powder-particle dimensions, vanishing of the echo when the powder is placed in a viscous dielectric medium etc.) have made it possible for the authors of these papers to state that the echo is due to resonant excitation of acoustic oscillations in the metallic particles. However, no theoretical calculation was made of the echo response for such systems.

In the present paper we calculate theoretically the echo signals in powders of normal metals at low temperatures, i.e., in the case when  $\omega_c\tau \gg 1$ , where  $\omega_c$  is the electron cyclotron frequency and  $\tau$  is the electron free path time. The onset of the echo is attributed to nonlinearity of the acoustic oscillations generated in the metal by the external RF fields. This nonlinearity is assumed to be due to the lattice-vibration anharmonicity, which is described by the fourth-order terms of the expansion of the free energy in the ion displacements  $\xi$ . The calculations are performed in the "local" limit  $qv_F/\omega_c \ll 1$ , where  $q$  is the wave vector of the sound wave and  $v_F$  is the Fermi velocity.

The metallic powder is a set of particles of irregular shape passed through a sieve to make their linear dimensions distributed, about a certain characteristic  $l_0$ . The spectrum of the natural frequencies of the acoustic oscillations of each particles cannot be analytically predicted, since the particles are of irregular shape. For an aggregate of such particles, however, it can be assumed that the set of powder frequencies covers in continuous fashion an interval  $\Delta\omega$ , near some frequency  $\omega_0$  which is connected with the dimension  $l_0$ . The electromagnetic field of the pulse, if its carrier frequency  $\omega$  falls in the interval  $\Delta\omega$ , excites intense oscillations of those particles whose natural frequency coincides with  $\omega$ . The echo response produced in the powder by the natural vibrations of the particles after turning on the external RF pulse is therefore also concentrated near the frequency  $\omega$ .

The irregularity of the particle shapes makes it impossible to determine the analytic form of the vibrational modes, which is needed for the description of the echo. This makes it necessary to use a simple geometrical approximation of the powder particles. The metal powder will hereafter be regarded as an aggregate of parallelepipeds whose dimensions are randomly distributed about a characteristic dimension  $l_0$ .