

$$\left(\frac{1}{T_1}\right)_{\text{eff}} = \frac{4}{3\tau_{s0}} \left[1 - \frac{(1 - T/T_c(H))S_1(t)}{4\pi\sigma D \{ [2k_z^2(t) - 1] \beta_A + n \}} \right] \times \left(1 + \frac{g_e^2 \chi_s^0}{g_s^2 \chi_e^0} \right)^{-1}. \quad (6)$$

Since $S_1(t)$ is a positive-definite function for all t , the conclusion can be drawn from Eq. (6) that, under electron-bottleneck conditions, the linewidth of the magnetic resonance of the paramagnetic impurities decreases on going from the normal to the superconducting phase. Such a behavior of the magnetic-resonance linewidth is exactly contrary to the behavior of the linewidth in the case (a). If in case (a) broadening of the line occurs on going to the superconducting phase, then in case (b) the existence in the system of electron-bottleneck conditions leads to the narrowing of it.

The experimentally observed narrowing of the electron paramagnetic resonance line for the magnetic moments of Er and La on going from the normal to the superconducting phase is partially explained apparently by the dynamical character of the interaction between the magnetic impurities and the conduction electrons.

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Certain effects related to the appearance of a temperature superlattice in a semiconductor with hot electrons

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Certain transport phenomena arising in a semiconductor with a temperature superlattice as a result of the heating of the electron gas by radiation are studied. Three effects are predicted which are due to free convection in the electron gas in the presence of a weak electric field perpendicular to the luminous flux: the variation of the effective electrical conductivity in the direction of the field, the appearance of a potential difference between the illuminated and shady sides of the sample, and the appearance of a current in the direction perpendicular to the luminous flux and the field.

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§1. INTRODUCTION

It has been shown earlier^[1,2] that the heating of the electron gas during the illumination of a sample can lead to the electronic analog of the well-known Bénard effect in hydrodynamics: under certain conditions there should arise steady convection of the carriers at a rate, u , that is a periodic function of the coordinates x and y (the direction of the luminous flux is chosen as the z axis). In this case the electron-temperature and (with a smaller amplitude) the electron-density distributions also become periodic. The constant of the resulting superlattice depends on the intensity of the heating light. In other words, there should arise in the sample a dis-

tinctive diffraction grating with a controllable spacing (diffraction can be experienced by other electro-magnetic waves, as well as by the heating light itself when it has the appropriate wavelength and the nonlinear effects are taken into consideration). It would be interesting, however, to ascertain what other consequences admitting of experimental verification the appearance of convection in the electron liquid leads to. Some of these consequences are studied in the present paper.

As before,^[1,2] we shall consider a material with unipolar conductivity, all the formulas being written out for positively charged particles (which, however, does not prevent us from calling them electrons). This ap-

proach is also justified in the presence of both negative and positive carriers if their mobilities are markedly different. Since the conditions for the appearance of a superlattice are eased appreciably if a constant voltage potential, V , is applied to the sample (see Fig. 1), we shall consider precisely this case on the basis of the results of Ref. 2. To avoid misunderstandings, let us emphasize that the voltage potential V is included in the sense of a field effect, and does not by itself generate a continuous current through the sample. It, like the gravitational field in the hydrodynamic Bénard effect, just increases the pressure of the electron gas near the "lower" (illuminated) surface, thereby facilitating the appearance of an eddy convection current of strength enu . However, in contrast to the gravitational field, the voltage potential V is concentrated almost completely in narrow space-charge layers localized near the sample surfaces (at $z=0$ and $z=l$).

§2. PHENOMENOLOGICAL EXPRESSION FOR THE CURRENT DENSITY

We shall be interested in certain transport phenomena occurring in a sample with a temperature superlattice. To wit, let us consider the current density that arises in a sample when, besides the voltage potential V , another potential difference is established between electrodes located at $x = \pm L_1/2$ by connecting them to an external load. Under the indicated conditions, the electrons in the sample are acted on by, besides the field localized near the $z=0$ and $z=l$ planes, a field of intensity

$$E_z = E_1^0 + \delta E_1. \quad (1)$$

Here E_1^0 and δE_1 denote respectively the intensity of the "primary" homogeneous field produced directly by the battery and that of the "secondary" field due to the possible redistribution of the free charges in the sample. The direction and magnitude of the vector δE_1 are determined by the setup of the experiment (in particular, it can be equal to zero; see below). On the other hand, the vector E_1^0 is, by agreement, directed along the x axis, i.e., along a direction perpendicular to the lumin-

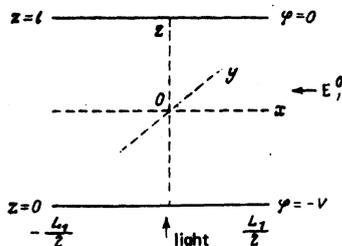


FIG. 1. The coordinate system and the orientation of the electric fields. E_1^0 denotes the "primary" (see text) intensity of the weak field generating the current (electrodes are located at $x = \pm L_1/2$); φ is the potential of the electric field in the sample for $E_1 = 0$. There is no through current along the z axis. The sample is a rectangular parallelepiped with dimensions L_1 , L_2 , and $L_3 = l$. The sample may be either closed or open along the y axis.

ous flux. For simplicity, we shall assume the field E_1 is weak, limiting ourselves to the linear approximation with respect to it.

The expression for the current density could, under the conditions in question, have been found by solving the kinetic equation. It is, however, simpler to write down at once a phenomenological formula that follows from obvious dimensional and vectorial considerations. Restricting ourselves to cubic crystals, we have (the lower Greek indices are vector indices):

$$j_\alpha = \sigma \left\{ E_{1\alpha} + a_{\alpha\beta\mu\nu} \frac{u_\beta u_\nu}{v_0^2} E_{1\beta} \right\} + enu_\alpha. \quad (2)$$

Here n is the carrier concentration (which differs little from its value, n_0 , in the homogeneous sample), σ is the ohmic conductivity, v_0 is the thermal or Fermi (depending on the degree of degeneracy) velocity of the carriers, $a_{\alpha\beta\mu\nu}$ is a dimensionless tensor that is symmetric with respect to the interchange of the third and fourth indices. In the formula (2) we have dropped the terms of higher order in the velocity u (near the threshold for superlattice formation $u \ll v_0$).

The quantities σ and $a_{\alpha\beta\mu\nu}$ may depend on the electron temperature T , which varies in space. Nevertheless, the writing down of (1) in the approximation of localized quantities is justified; for under the conditions in question the mean free path in momentum space is significantly smaller than the other characteristic lengths figuring in the problem (in particular, the cooling-off length). It should only be borne in mind that σ and $a_{\alpha\beta\mu\nu}$ are connected by the equation of continuity. In particular, under steady-state conditions,

$$E_1 \nabla \sigma + n^2 u_\mu u_\nu \frac{\partial}{\partial x_\alpha} \left(\frac{\sigma a_{\alpha\beta\mu\nu}}{n^2} \right) E_{1\beta} = 0. \quad (3)$$

Below it will be sufficient to restrict ourselves to the values of σ and $a_{\alpha\beta\mu\nu}$ obtained when the heating of the electrons is neglected. In this case σ and $a_{\alpha\beta\mu\nu}$ are constants and, under conditions of quasineutrality, the relation (3) is fulfilled automatically.

The explicit computation of the components of the tensor $a_{\alpha\beta\mu\nu}$ is quite tedious. For our purposes, however, it is sufficient to know only its symmetry properties, which are well known.^[3] In particular, in a cubic crystal, only the components

$$a_{xxxx} = a_1, \quad a_{xyxy} = a_2, \quad a_{xyyz} = a_3 \quad (4)$$

and their equivalents are different from zero in the system of principal axes. Notice, however, that the coordinate axes in Fig. 1 (picked out by the conditions of the experiment) need not coincide with the principal axes of the crystal.

Explicit expressions for the components of the vector u were obtained in our previous paper.^[2] Let us introduce the following notation: μ for the mobility of the carriers; T_0 the lattice temperature; $\mathbf{k} = \{k_x, k_y\}$ the two-dimensional wave vector; $\mathbf{r} = \{x, y\}$ the two-dimensional radius vector; $\mathbf{u}_r = \{u_x, u_y\}$ the two-dimensional velocity of the electron flux; $\varphi_s(z)$ the potential of the electric

field in the one-dimensional static problem; γ the light-absorption coefficient (assumed to be T independent); λ_0^{-1} the cooling-off length; r_0 the screening radius; I_m the light intensity at $z=+0$; I_{cr} the critical value of the light intensity at which convection begins. The relative superheat of the electron liquid is

$$(T-T_0)T_0^{-1}=f_1(z) \cos(kr).$$

According to Ref. 2,

$$u_x = -\mu \{f_1(0)\varphi_1'(0)e^{-kx} - f_1(z)\varphi_1'(z)\} \cos(kr), \quad (5)$$

$$u_y = -(k/k_y)\mu f_1(0)\varphi_1'(0)e^{-kx} \sin(kr). \quad (6)$$

The function $f_1(z)$ was found earlier^[2] up to a multiplicative constant (as which we can simply take $f_1(0)$). It is impossible to determine this constant within the framework of the linear theory.^[2] But since we are dealing with weak conditions for the appearance of convection, it is to be expected that $f_1(0)$ will be proportional to the square root of the supercriticality:

$$f_1(0) \approx C(I_m/I_{cr} - 1)^{1/2}, \quad (7)$$

where C is a dimensionless constant.

For $k \neq 0$ the formulas (5) and (6) are valid in the case of volume absorption of light in a thick sample in which the screening radius is sufficiently small:

$$l \gg \gamma^{-1} \gg \lambda_0^{-1} \gg r_0. \quad (8)$$

Such a situation is in fact realized in the intraband absorption of electromagnetic waves in a sample with a not too low concentration of the free carriers. As can be seen from (5), the boundary condition $u_x=0$ is satisfied exactly at $z=0$ and up to quantities of the order of e^{-kl} and $\exp(-l/r_0)$ at $z=l$. The quantity $\exp(-l/r_0)$ is always negligibly small; e^{-kl} is also small, excluding the region near the convection threshold: in a regime of a "high voltage potential" ($\varphi_1'(0) \gg T_0^2 \gamma^2 / e \lambda_0 |F_0|$, where F_0 is the Fermi level, measured from the zone edge, under equilibrium conditions)^[2]

$$k = \lambda_0(I_m/I_{cr} - 1). \quad (9)$$

Near the threshold (i.e., for $kl \lesssim 1$), the first term in the curly brackets on the right-hand sides of (5) and (6) should be replaced by the expression

$$\frac{f_1(0)\varphi_1'(0)}{1 - e^{-2kl}} (e^{-kx} - e^{-2kl+kx}). \quad (5')$$

The dimensions of the sample along the x and y axes are also assumed to be sufficiently large—in the sense of the inequalities (8), in which l should be replaced by L_1 or L_2 . The exact form of the boundary conditions at $x = \pm L_1/2$ and $y = \pm L_2/2$ do not then play a role, it being only important that they allow the flow of current in the appropriate directions. We shall use the usual periodicity conditions for T as a function of x and y .

§3. THREE EFFECTS

As can be seen from the formula (2), free convection of the carriers can lead to three effects that admit of direct observation in experiment. First, the effective conductivity in the direction of the transverse (to the luminous flux) E_1^0 field turns out to be different from σ : it contains an extra term that is quadratic in the u components. Secondly, the field E_1^0 , which is parallel to the x axis, can lead to the appearance of a current not only along the x axis, but also along the y axis. Thirdly, the transverse field can lead to the appearance of an additional— V —longitudinal potential difference between the $z=0$ and $z=l$ surfaces.

The calculations in all the three cases reduce to the investigation of the formula (2). It is only necessary to take two circumstances into consideration.

First, the current density j determined in experiment—from the readings of an instrument in the load circuit—is given not directly by the expression (2), but by equalities of the type

$$\langle j_x \rangle = \frac{1}{L_x l} \int_{-L_x/2}^{L_x/2} dy \int_0^l dz j_x, \quad (10)$$

etc. It follows, in particular, from this that the terms $em u_x$ on the right hand side of (2) do not make a contribution to the experimentally observable current density, which was to be expected: they describe the carriers' eddy motion, during which the carriers remain in the sample.

Secondly, the components k_x and k_y enter separately into the formulas (5) and (6) for the convection velocity u . Moreover, the condition for the existence of convection^[2] determines only the quantity $k = (k_x^2 + k_y^2)^{1/2}$, the relation between k_x and k_y remaining arbitrary. Upon the application of the weak field E_1^0 this degeneracy is not removed. Consequently, the experimentally observable result is obtained by "averaging over the phases." The latter operation is defined by the equality

$$\langle \dots \rangle_\varphi = \frac{1}{2\pi} \int_0^{2\pi} (\dots) d\varphi, \quad (11)$$

where the dots stand for the expression being averaged, while $\varphi = \arctg(k_y/k_x)$; the average (in the sense of (11)) values of the products $u_\mu u_\nu$ are computed in the Appendix.

Let us, first of all, determine the additional field intensity δE_1 . It is to be expected that (at least with the above-adopted degree of accuracy with respect to the parameter u^2/v_0^2) δE_1 will be parallel to the z axis. Further, on account of the assumed uniformity of the E_1^0 field, the quantity δE_1 should not depend on x and y . Thus, the condition $\langle j_x \rangle = 0$ assumes the form

$$\delta E_{1z} + (a_{11\mu\nu} \delta E_{1\mu} + a_{12\mu\nu} E_1^0) \frac{1}{S} \left\langle \int dx dy \frac{u_\mu u_\nu}{v_0^2} \right\rangle_\varphi = 0. \quad (12)$$

Here we have taken into consideration the fact that E_1^0 is parallel to the x axis; S denotes the area of the cross section perpendicular to E_1^0 . Summation over μ and ν

here should be limited to the values x and y . Indeed, on account of the boundary conditions imposed on u_x at $z=0$ and $z=l$, the terms with u_x do not make a contribution to the electron flux through the respective surfaces.

The integral figuring in the formula (12) can be easily computed. Averaging it over the phases according to the formula (11), we obtain with the accepted degree of accuracy

$$\delta E_{1z} = -a_{zxy} \frac{f_1^2(0) \varphi_1'^2(0) \mu^2}{4v_0^2} E_1^0 e^{-2kz}. \quad (13)$$

As can easily be verified, the additional space-charge density due to this field is small compared to en_0 , being smaller by a factor given by the parameter

$$kr_0 \frac{eE_1^0 r_0}{W_0} \frac{f_1^2(0) \varphi_1'^2(0) \mu^2}{v_0^2},$$

where $W_0 = T_0$ or $W_0 = \hbar^2 n_0^{2/3} / 2m$, according as the electron gas is non-degenerate or degenerate. Thus, the quasineutrality condition is not violated.

Integrating (13) over the sample thickness, we find the longitudinal potential difference

$$\Delta V = - \int_0^l \delta E_{1z} dz = \frac{1-e^{-2kl}}{8k} a_{zxy} \frac{f_1^2(0) \varphi_1'^2(0) \mu^2}{v_0^2} E_1^0. \quad (14)$$

As can be seen from the formulas (7) and (9), the right-hand side of (14) depends appreciably on the light intensity only in the near-threshold region where $kl \lesssim 1$. In particular, for $kl \ll 1$,

$$\Delta V \approx la_{zxy} f_1^2(0) \varphi_1'^2(0) \mu^2 E_1^0 / 4v_0^2. \quad (14')$$

Since $l\lambda_0 \gg 1$, it is to be expected that, upon the appearance of free convection, the quantity ΔV as a function of I_m will vary rapidly and reach saturation. Under the latter conditions, the value of ΔV is given by the formula (14) with the factor $(1 - e^{-2kl})k^{-1}f_1^2(0)$ replaced by $C^2\lambda_0^{-2}$. In this case we need not go outside the framework of the linear theory,^[2] since the limits of its applicability are determined by parameters different from $l\lambda_0$.

The quantity ΔV depends on, besides other factors, the orientation of the luminous flux relative to the principal axes of the crystal. Thus, according to (4), in cubic crystals the effect under consideration will generally be absent if the axes chosen by us coincide with the principal axes. This is also the case in other materials of not too low symmetry. The effect could, however, be observed in the case of a different orientation of the axes. Let, for example, the y axis be a principal axis, while the z and x axes are turned (in the $y=0$ plane) through an angle χ from the principal axes of the cubic crystal. Then, as can easily be verified, $a_{xyz\mu} \neq 0$ if $\mu = x$, it being given by

$$a_{xxx} = \frac{1}{2}(a_2 - a_1 + 2a_3) \sin 4\chi. \quad (4')$$

As was to be expected, in the isotropic case, when $a_2 - a_1 + 2a_3 = 0$, the right-hand side of (4') vanishes. In computing the effective conductivity along the x axis,

we should neglect the field δE_1 : its allowance would give a correction of higher order in smallness. Thus, according to (2), (10), and (11),

$$\langle j_x \rangle = \sigma_{\text{eff}} E_1^0, \quad \sigma_{\text{eff}} = \sigma \left\{ 1 + \frac{a_{zxy}}{v_0^2 l L_2} \int_{-L/2}^L dy \int_0^l dz \langle u_x u_x \rangle \right\}. \quad (15)$$

Using the formulas (A.1) and (A.5), we obtain in the principal-axis system of the cubic crystal^[1]

$$\frac{\sigma_{\text{eff}} - \sigma}{\sigma} = \frac{1}{L_2 l v_0^2} \int_{-L/2}^L dy \int_0^l dz \left\{ \frac{a_1 + a_3}{4} f_1^2 [1 - J_0(2kr)] + \frac{1}{2} a_2 f_2^2 [1 + J_0(2kr)] \right\}. \quad (16)$$

Here $r = (y^2 + L_1^2/4)^{1/2}$.

For $kL_1 \gg 1, kL_2 \gg 1$ we can drop the terms containing the Bessel functions. Further, in evaluating the integral over z , we can neglect the second term on the right-hand side of (5): it makes a contribution of order not more than $\lambda_0 r_0$. Thus, with allowance for (5), (7), and (9), we obtain (for $kl \gg 1$)

$$\frac{\sigma_{\text{eff}} - \sigma}{\sigma} = \frac{C^2 \varphi_1'^2(0) \mu^2}{8v_0^2 \lambda_0 l} (a_1 + 3a_3). \quad (17)$$

On the other hand, near the threshold, where the numbers kl, kL_1 , and kL_2 are not large, we had no right to discard the Bessel functions in the formula (16). It is evident, however, that for $kL_1 \ll 1, kL_2 \ll 1$ the first and second terms of the integrand vanish, while the third term becomes $a_3 f_2^2$. Thus, near the threshold

$$\frac{\sigma_{\text{eff}} - \sigma}{\sigma} \approx \frac{\mu^2 f_1^2(0) \varphi_1'^2(0)}{3v_0^2} a_3. \quad (17')$$

Finally, let us turn to the computation of the current density in the direction of the y axis. Here we should also neglect the field δE_1 . The formula (2), with allowance for (10) and (11), then yields

$$\langle j_y \rangle = \sigma a_{yxy} \frac{E_1^0}{L_1 l} \int_{-L/2}^L dx \int_0^l dz \langle u_y u_y \rangle. \quad (18)$$

As can be seen from the formulas (A.5), nonzero contributions to the integral over x are made here by only the terms with $\mu = \nu$. Taking this into consideration, and using the formulas (5), (6), and (5'), we obtain near the threshold

$$\langle j_y \rangle = \sigma \frac{f_1^2(0) \mu^2 \varphi_1'^2(0)}{3v_0^2} a_{yxx} E_1^0. \quad (19)$$

On the other hand, for $kl \gg 1, kL_1 \gg 1$, and $kL_2 \gg 1$, the formula (18) leads to the saturation of $\langle j_y \rangle$ as a function of I_m :

$$\langle j_y \rangle \approx \sigma \frac{\mu^2 \varphi_1'^2(0) C^2}{8v_0^2 \lambda_0 l} a_{yxx} E_1^0. \quad (19')$$

Here we have taken into consideration the fact that

$$a_{yxx} + a_{yyx} + 2a_{yxx} = a_{yxx},$$

since the sum of the $a_{y\alpha\alpha}$ is an invariant of an ortho-

gonal transformation.

Like the additional voltage potential, the relative changes in the conductivity and current density along the y axis vary very rapidly with increasing supercriticality near the threshold, reaching saturation almost discontinuously. Further, in crystals of not too low symmetry the current density (j_y), like ΔV , appears only if the coordinate system chosen by us differs from the principal-axis system. Thus, in a cubic crystal we have

$$a_{yzxz} = \frac{1}{2}(a_1 - a_2 - 2a_3) \sin^2 \theta \{ \sin 2\psi [2 \cos^2 \theta - \frac{1}{2}(1 + \cos^2 \theta) \sin^2 2\varphi] + \sin 2\varphi \cos 2\psi \cos \theta \}.$$

Here θ , φ , and ψ are the Euler angles determining the positions of the chosen coordinate axes relative to the principal axes of the crystal. Let us recall that θ is the angle between the z and z' axes, while $\psi = \tan^{-1}(\alpha_{xz'}/\alpha_{yz'})$, $\varphi = -\tan^{-1}(\alpha_{xz'}/\alpha_{xy'})$. Here we have in mind the principal branches of the inverse trigonometric functions, $\alpha_{xz'}$, etc., denote the cosines of the angles between the x and z' axes, etc.; the primed coordinates are connected with the principal axes of the crystal.

The quantitative estimations of the effects under consideration are difficult; for the complete computation of the components of the tensor $a_{\alpha\beta\mu\nu}$ and the constant C inevitably involves a number of far-reaching assumptions. Since, however, they are dimensionless, it may be inferred that the nonzero values of the $a_{\alpha\beta\mu\nu}$ will not differ too much from unity. The ratio $\mu^2 \varphi_s'^2(0)/v_0^2$ can turn out to be quite large, but the velocity u is, by agreement, lower than v_0 . Thus, we cannot justifiably consider the values of $\varphi_s' \mu f_1/v_0$ exceeding unity.

Let us note in conclusion that the first two of the above-considered effects (and, probably, the third) can also be observed in the absence of an electron-temperature superlattice; the electrons only need to be heated.²⁾ The mechanism of their production investigated in the present paper can be identified on the basis of the following distinguishing features:

- 1) a unique dependence of the magnitudes of the effects on the light intensity: rapid growth to saturation near the threshold (this pertains to all the three effects);
- 2) the nonnecessity of the presence of a magnetic field⁴⁾ or of crystallographic anisotropy⁵⁾ (this pertains to the second and third effects).

APPENDIX

According to (5) and (6)

$$u_x = f_x \cos(kr), \quad u_{\perp} = \frac{k}{k} f_{\perp} \sin(kr), \quad (\text{A.1})$$

where the functions f_x and f_{\perp} are determined by these relations. Let us introduce cylindrical coordinates in the (x, y) plane by setting

$$x = r \cos \theta, \quad y = r \sin \theta, \quad k_x = k \cos \varphi, \quad k_y = k \sin \varphi. \quad (\text{A.2})$$

Then

$$kr = a \cos(\varphi - \theta), \quad a = kr, \\ u_x = f_x \cos[a \cos(\varphi - \theta)], \quad u_x + iu_y = e^{i\varphi} f_{\perp} \sin[a \cos(\varphi - \theta)]. \quad (\text{A.3})$$

Let us introduce the notation

$$A_{\mu\nu} = \frac{1}{2\pi} \int_0^{2\pi} d\varphi \langle u_{\mu} u_{\nu} \rangle = \langle u_{\mu} u_{\nu} \rangle_{\varphi}. \quad (\text{A.4})$$

Substituting into this the expressions for u_{μ} and u_{ν} , we easily find

$$A_{xx} = \frac{1}{2} f_x^2 (1 + J_0(2a)), \quad A_{xx} = \frac{1}{2} f_x f_{\perp} J_1(2a) \cos \theta, \\ A_{xy} = \frac{1}{2} f_x f_{\perp} J_1(2a) \sin \theta, \quad A_{yy} = \frac{1}{2} f_{\perp}^2 J_1^2(2a) \sin^2 \theta, \\ A_{zz} = \frac{1}{2} f_{\perp}^2 [1 - J_0(2a) + J_2(2a) \cos 2\theta], \\ A_{yy} = \frac{1}{2} f_{\perp}^2 [1 - J_0(2a) - J_2(2a) \cos 2\theta]. \quad (\text{A.5})$$

Here J_0 , J_1 , and J_2 are Bessel functions.

¹⁾We have dropped from the integrand in (16) the terms that give zero in the integration over y .

²⁾The author is grateful to the JETP referee for recommending a discussion of this circumstance.

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