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## Interaction of classical Yang-Mills charges and the problem of quark confinement

I. B. Khriplovich

Institute of Nuclear Physics, Siberian Branch, Academy of Sciences of the USSR (Submitted 5 August 1977) Zh. Eksp. Teor. Fiz. 74, 37-42 (January 1978)

Equations and boundary conditions are obtained for the field produced by two point Yang-Mills charges at rest. A nontrivial property of this static system is the existence of a magnetic field. The connection between this model and the problem of quark confinement is discussed.

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It is well known that the classical solution for the field of a point Yang-Mills charge reduces to the ordinary Coulomb form, despite the formal nonlinearity of the equations. It is helpful to establish why this happens. We write down the equations of the Yang-Mills field for the case when its source has only a time component:

$$(\delta^{\alpha_1}\partial_n - g e^{\alpha\beta_1} b_n^{\beta}) (\partial_n b_0^{\gamma_1} + \partial_0 b_n^{\gamma_1} + g e^{\gamma\delta e} b_0^{\delta} b_n^{\epsilon}) = \rho^{\alpha},$$
(1)  
$$(\delta^{\alpha_1}\partial_0 + g e^{\alpha\beta_1} b_0^{\beta}) (\partial_n b_0^{\gamma_1} + \partial_0 b_n^{\gamma_1} + g e^{\gamma\delta e} b_0^{\delta} b_n^{\epsilon})$$

$$+ \left(\delta^{\alpha}{}^{\dagger}\partial_{m} - g\epsilon^{\alpha\delta}{}^{\dagger}b_{m}{}^{\beta}\right) \left(\partial_{n}b_{m}{}^{\dagger} - \partial_{m}b_{n}{}^{\dagger} + g\epsilon^{\dagger\delta\epsilon}b_{m}{}^{\delta}b_{n}{}^{\epsilon}\right) = 0.$$
<sup>(2)</sup>

All the terms on the left-hand side of the time equation

(1) except  $\Delta b_0^{\alpha}$  contain the spatial components of the vector potential  $b_n^{\alpha}$ . In the spatial equation (2) in the static case, the only term which does not vanish when  $b_n^{\alpha} = 0$  has the form  $g \in {}^{\alpha \beta \gamma} b_0^{\beta} \partial_n b_0^{\gamma}$ . It is therefore clear that if the direction of the field  $b_0^{\alpha}$  in the isotopic space does not depend on the coordinates (and this is obviously the situation in the case of a single charge when  $\rho^{\alpha} = gt^{\alpha} \delta(\mathbf{r})$ ), this term vanishes, so that  $b_n^{\alpha} = 0$  serves as a solution of Eq. (2). Simultaneously, Eq. (1) for  $b_0^{\alpha}$  reduces to the ordinary Poisson form, and the problem as a whole to the trivial Coulomb problem.

At the first glance, it would seem from this to be an inescapable conclusion that to obtain nontrivial static

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solution it really is necessary to mix the isotopic and spatial indices, i.e., seek a solution, say, in the form  $b_0^{\alpha}(\mathbf{r}) = r^{\alpha} f(r)$ . In fact, there is a simpler way: It is sufficient to consider the field produced by, not one, but two centers. It is easy to see that in this case the direction of the field  $b_0^{\alpha}$  in isotopic space depends, in general, nontrivially on the coordinates, so that the problem becomes genuinely nonlinear.

Thus, let us consider the field produced by two point Yang-Mills charges at rest. To achieve maximal simplification of the problem, we assume further that the isotopic spins of the sources, and also their geometric sum, are so large that both isospins can be regarded as classical vectors. Then the source on the right-hand side of Eq. (1) can be represented in the form

$$\rho^{\alpha} = gt[\tau_1^{\alpha}\delta(\mathbf{r}-\mathbf{r}_1) + \tau_2^{\alpha}\delta(\mathbf{r}-\mathbf{r}_2)], \qquad (3)$$

where t is the modulus of the isospin vector of the source (for simplicity we assume that it is the same for both particles) and  $\tau_{1,2}^{\alpha}$  are the unit vectors along these vectors.

We seek a solution of the system (1)-(3) in the form

$$b_0^{\alpha} = \frac{1}{g} [\tau_1^{\alpha}(t) b^1(\mathbf{r}) + \tau_2^{\alpha}(t) b^2(\mathbf{r})], \qquad (4)$$

$$b_{n}^{\alpha} = \frac{1}{g} \varepsilon^{\alpha\beta} \tau_{1}^{\beta}(t) \tau_{2}^{\gamma}(t) b_{n}(\mathbf{r}).$$
(5)

The equations of motion for the vectors  $\tau_{1,2}^{\alpha}(t)$  are obvious:

$$\tau_{i,2}^{\alpha} = -g \varepsilon^{\alpha i} b_{\nu}^{\beta}(\mathbf{r}_{i,2}) \tau_{i,2}^{\gamma}, \qquad (6)$$

in other words, the isotopic spin precesses around the vector  $b_0^{\alpha}$ . It is easy to see that Eq. (6) guarantees vanishing of the generalized divergence of the external current:

$$\partial_{\mu}j_{\mu}{}^{\alpha}+g\varepsilon^{\alpha\beta\gamma}b_{\mu}{}^{\beta}j_{\mu}{}^{\gamma}=0.$$
(7)

Equation (7) is, as is well known, a consequence of the field equations (1) and (2), so that Eq. (6) is necessary for self-consistency of the problem. Substituting (4) in (6), we obtain

$$\tau_1^{\alpha} = b^2(\mathbf{r}_1) \varepsilon^{\alpha\beta\gamma} \tau_1^{\beta} \tau_2^{\gamma}, \quad \tau_2^{\alpha} = -b^1(\mathbf{r}_2) \varepsilon^{\alpha\beta\gamma} \tau_1^{\beta} \tau_2^{\gamma}.$$
(8)

It is obvious that in the degenerate case when the isospins of the sources are parallel or antiparallel the problem again becomes trivial.

In the general case though, substitution of (4), (5), and (8) in (1) and (2) leads to the equations

$$\partial_n \Phi_{in} + (\theta \Phi_{in} + \Phi_{2n}) b_n = 4\pi\beta\delta(\mathbf{r} - \mathbf{r}_i), \qquad (9)$$

$$\partial_n \Phi_{2n} - (\Phi_{1n} + \Theta \Phi_{2n}) b_n = 4\pi\beta \delta(\mathbf{r} - \mathbf{r}_2);$$
  
$$\partial_m (\partial_m b_n - \partial_n b_m) + \phi_2 \Phi_{1n} - \phi_1 \Phi_{2n} = 0,$$
 (10)

where

$$\begin{split} \Phi_{in} = &\partial_n \varphi_i + (\theta \varphi_i + \varphi_2) b_n, \ \Phi_{2n} = &\partial_n \varphi_2 - (\varphi_i + \theta \varphi_2) b_n, \\ \theta = &\tau_i^{\alpha} \tau_2^{\alpha}, \qquad \beta = g^2 t / 4\pi, \end{split}$$

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$$\varphi_{1}(\mathbf{r}) = b^{1}(\mathbf{r}) - b^{1}(\mathbf{r}_{2}), \quad \varphi_{2}(\mathbf{r}) = b^{2}(\mathbf{r}) - b^{2}(\mathbf{r}_{1}).$$
 (11)

It is remarkable that the field equations (9) and (10) do not contain the time component of the vector potential itself (through  $b^{1,2}(\mathbf{r})$  but rather the quantities  $\varphi_{1,2}(\mathbf{r})$  determined by Eqs. (11). This can be pictured as follows: The potential produced by one of the sources at the position of the other is expended, by virtue of Eqs. (8), on rotating the isospin of this other source, and it must therefore be subtracted from the dynamical variable which characterizes the field in ordinary space. The function  $\varphi_{1,2}(\mathbf{r})$ , simply by virtue of their definition (11), satisfy the boundary conditions

$$\varphi_1(\mathbf{r}_2) = \varphi_2(\mathbf{r}_1) = 0.$$
 (12)

It should be emphasized that these conditions are unquestionably a specific feature of the Yang-Mills nature of the field.

With regard to the point sources on the right-hand side of Eqs. (9), we shall assume, as usual, that they determine a singularity of the functions  $\varphi_{1,2}$ , i.e., they lead to the boundary conditions

$$\varphi_{i,z}(\mathbf{r})|_{\mathbf{r}\to\mathbf{r}_{i,z}} = -\frac{\beta}{|\mathbf{r}-\mathbf{r}_{i,z}|}.$$
 (13)

The behavior of the remaining functions as  $\mathbf{r} - \mathbf{r}_{1,2}$  must be sufficiently good to ensure that the terms  $\Delta \varphi_{1,2}$ really are the most singular ones on the left-hand side of Eqs. (9).

Let us now discuss the relation of our equations to the isotopic gauge invariance of the theory. Our equations (9) and (10) follow from the basic Eqs. (1) and (2)only if the vectors  $\boldsymbol{\tau}_1, \, \boldsymbol{\tau}_2, \,$  and  $\boldsymbol{\tau}_1 \times \boldsymbol{\tau}_2$  are linearly independent, i.e., if the isospins of the sources are not parallel. It is however clear that such a condition is not in general gauge invariant; for by virtue of gauge invariance the vectors  $\boldsymbol{\tau}_1$  and  $\boldsymbol{\tau}_2$  at different spatial points can each be rotated through an arbitrary angle; in particular, they may be made parallel, and in this case the solution must, it would seem, have the trivial Coulomb form. But in fact the choice of the solution in the form (4) and (5) already determines the gauge to a considerable extent, and the angle between  $\tau_1$  and  $\tau_2$  is essentially fixed by means of Gauss's theorem by the specification of the total isospin T of the system and the coefficients of  $\mathbf{r}^{-1}$  in the asymptotic expressions for the functions  $\varphi_1$  and  $\varphi_2$  if, of course, the asymptotic behavior of these functions is reasonable. Thus, despite the isotopic gauge invariance of the basic equations, our formulation of the problem appears reasonable.

In fact, the gauge invariance can be used to eliminate the precession of the isospins described by Eqs. (8), i.e., to make  $\tau_{1,2}$  constant, keeping, of course, the angle between them fixed. Then the boundary conditions (12) arise simply from the condition of compatibility of the system of equations (9) and (10). This can be readily seen by substituting Eqs. (9) in the divergence of the vector equation (10).

It should also be noted that the system of equations (9) and (10) is invariant under transformations whose infinitesimal form is

$$\delta \varphi_1 = \delta \alpha (\varphi_2 + \theta \varphi_1), \quad \delta \varphi_2 = -\delta \alpha (\varphi_1 + \theta \varphi_2), \quad \delta b_n = -\partial_n \delta \alpha.$$
(14)

An additional requirement imposed on these transformations is that the boundary conditions (12) and (13) should remain unchanged. Because of this invariance, the vector potential  $b_n$  can be chosen differently. It is convenient to take the Coulomb gauge  $\partial_n b_n = 0$ , and this is what we shall use in what follows.

We investigate the solution near one of the centers, for example, the first. Since the function  $\varphi_1$  here has a singularity:  $\varphi_1 = -\beta/\rho, \rho = |\mathbf{r} - \mathbf{r}_1|$  the problem linearizes with respect to the other quantities. To solve the problem, it is convenient to introduce the magnetic field vector  $\mathbf{h} = \nabla \times \mathbf{b}$ , which in spherical coordinates has only one component, with azimuthal direction. In this linear approximation, we readily obtain for it from (10) the equation

$$[\nabla \rho^2 \times [\nabla \times \mathbf{h}]] - \beta^2 \mathbf{h} = 0. \tag{15}$$

The general solution is

$$h(\rho,\theta) = \sum_{l>1} P_l^{t}(\theta) \rho^{-\frac{n}{2}} [c_l^{t} \rho^{l(l+\frac{n}{2})^2 - \beta^2]^{\frac{n}{2}}} + c_l^{2} \rho^{-l(l+\frac{n}{2})^2 - \beta^2]^{\frac{n}{2}}}].$$
(16)

From the same Eq. (10) in the Coulomb gauge, when  $\partial_n b_n = 0$ , we obtain

$$\Delta(\rho\varphi_2) = \beta^{-1} \nabla[\rho^2 \nabla \mathbf{h}]. \tag{17}$$

From this we readily find that the condition (12) is satsified provided all  $c_t^2 = 0$ , and  $c_t^1$  are nonzero only for

$$\lambda = [(l+1/2)^2 - \beta^2]^{\frac{1}{2}} > 1/2.$$
(18)

Further, standard calculations show that the complete solution in the neighborhood of the point  $r_1$  is

$$\begin{aligned}
\varphi_{1} &= \beta \left( a - 1/\rho \right), \\
\varphi_{2} &= 2\beta b l \left( l+1 \right) \rho^{\lambda - l_{2}} P_{l}(\theta), \\
b_{1} &= -\beta^{2} b l \left( l+1 \right) \rho^{\lambda - l_{2}} P_{l}(\theta), \\
b_{0} &= \beta^{2} b \left( \lambda + \frac{3}{2} \right) \rho^{\lambda - l_{2}} P_{l}^{1}(\theta), \\
h &= \beta^{2} b \left( 2\lambda + 1 - \beta^{2} \right) \rho^{\lambda - l_{2}} P_{l}^{1}(\theta).
\end{aligned}$$
(19)

The coupling constant  $\beta$  has been separated from the constants of integration a and b in such a way as to make clear which is the perturbation order in  $\beta$  in which the corresponding function arises.

The two constants of integration a and b are sufficient to specify in the neighborhood of the point  $\mathbf{r}_1$  the values of the functions and their partial derivatives of first order. (Since the function  $\varphi_1$  is here singular, we should speak of the values of  $\rho \varphi_1$  and the partial derivatives of  $\rho \varphi_1$ .) Therefore, the solution in other regions obtained by continuation from this neighborhood is also completely determined by the two parameters a and b. The values of these constants are fixed by means of the two remaining boundary conditions (12) and (13) at the point  $\mathbf{r}_2$ . Thus, Eqs. (9) and (10) in conjunction with the boundary conditions (12) and (13) completely determine the solution of the problem. We emphasize that, as follows from the condition (18), the minimal multipole order of the solution near the source, and, therefore, the qualitative behavior of the solution as a whole as well, depends on the magnitude of the coupling constant.1)

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There is an attractive physical analogy for this problem. By means of the substitution

$$\varphi_{1} = \frac{1}{2r_{+}r_{-}} [\psi_{1}(r_{+}+r_{-}) - \psi_{2}(r_{+}-r_{-})],$$

$$\varphi_{2} = \frac{1}{2r_{+}r_{-}} [-\psi_{1}(r_{+}-r_{-}) + \psi_{2}(r_{+}+r_{-})],$$

$$\mathbf{b} = \mathbf{a}/(r_{+}r_{-}), \quad r_{\pm} = (1\pm\theta)^{\frac{1}{2}}$$
(20)

the system of equations (9) and (10) can be reduced to the form

$$(-i\nabla - \mathbf{a})^2 \psi = 0, \quad \psi = (\psi_1 + i\psi_2)/2^{1/2},$$
 (21)

$$[\nabla \times \mathbf{H}] = -\mathbf{j}, \quad \mathbf{H} = [\nabla \times \mathbf{a}], \quad \mathbf{j} = \psi^{*}(-i\nabla - \mathbf{a})\psi + [(-i\nabla - \mathbf{a})\psi]^{*}\psi.$$
(22)

The omitted  $\delta$ -function sources on the right-hand side of Eq. (21) are taken into account by means of obvious boundary conditions imposed on the function  $\psi$ . We have thus arrived at the electrodynamics of the scalar field  $\psi$  in three-dimensional space. The only difference is the sign in the Maxwell equation (22), which is opposite to the usual one. The reason for this is easy to understand: Whereas usually the terms  $|(-i\nabla - \mathbf{a})\psi|^2$  and  $\frac{1}{2}\mathbf{H}^2$ enter the Lagrangian with the same sign, in our case, in which the first term is the transformed  $\frac{1}{2}f_{0n}^{\alpha}f_{0n}^{\alpha}$  and the second the transformed  $\frac{1}{4}f^{\alpha}_{mn}f^{\alpha}_{mn}$ , their signs in the Lagrangian are different. This difference in the sign is of no small importance. The azimuthal magnetic field generated by the currents does not, as usual, pull the currents together, but rather pushes them apart. There is, as it were, an antipinch effect. In itself, the existence of a magnetic field in the static problem of the interaction of Yang-Mills fields is, from our point of view, an extremely interesting fact. In such a situation, it is not surprising that even the qualitative picture of the phenomenon changes with increasing coupling constant.

Unfortunately, it has not been possible to solve the problem completely. It is not even obvious that a sensible solution exists at all. As we have noted above, Eqs. (9) and (10) and the boundary conditions (12) and (13) can be expected to determine the solution uniquely in a finite region containing both centers. But in general it is not clear whether this solution goes over in the limit  $r \rightarrow \infty$  into a well decreasing asymptotic solution corresponding, by virtue of Gauss's theorem, to a given value of the total isospin of the system. If there really is no sensible solution, and in addition the configuration with parallel isospins is unstable, does this not mean in the language of quantum chromodynamics, i.e., on the transition from the group SU(2) to SU(3), that only "white" states are realized in nature?<sup>21</sup>

But if a solution exists, then the interaction energy of the charges depends on the distance between them as  $|\mathbf{r}_1 - \mathbf{r}_2|^{-1}$ . This follows uniquely from dimensional considerations. It is not however clear whether this interaction will be attractive or repulsive. A certain argument in favor of repulsion for a sufficiently large coupling constant is the fact that the energy of the magnetic field, which is always positive, increases in perturba-

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tion theory with  $\beta$  faster than the electrostatic energy. Nevertheless, it cannot by any means be precluded that the interaction is attractive. If at the same time the attraction increases rapidly with increasing coupling constant, then such a situation could be an indication of quark confinement in a less simplified model of the interaction.

The naturalness and nontriviality of the model considered here seem to me to justify publication of this paper, despite the absence of definite conclusions.

I am very grateful to V. N. Gribov for numerous stimulating discussions, which for more than two years have maintained my interest in the present problem. I am also extremely grateful to him and to A. I. Vainshtein for a number of very important critical remarks, to L. B. Okun' for his interest in the work and valuable comments, and to B. N. Breizman and V. S. Synakh for a helpful discussion of the possibilities of solving the resulting equations.

- <sup>1)</sup>Formally, the situation recalls the one that arises in the solution of the problem of the behavior of a small deviation of the Yang-Mills field from the Coulomb solution corresponding to a single point center.<sup>[1]</sup> However, the singularity of the vector field found in <sup>[1]</sup> in the Coulomb potential for sufficiently large coupling constant, or "the fall toward the center," is by no means peculiar to the Yang-Mills problem. This phenomenon is well known in the ordinary relativistic Coulomb problem and does not depend in renormalizable theory on the spin of the particle (in this case, the problem concerns the renormalizable interaction of a vector particle). The dependence that is found in the present paper of the solution on the magnitude of the coupling constant is peculiar to the Yang-Mills situation and certainly has no electrodynamic analog.
- <sup>2)</sup>This possibility of interpreting the absence of a solution was pointed out by V. N. Gribov.

<sup>1</sup>J. E. Mandula, Phys. Lett. B 67, 175 (1977).

Translated by Julian B. Barbour

## Intracavity laser spectroscopy with continuously and quasicontinuously operating lasers

V. M. Baev, T. P. Belikova, É. A. Sviridenkov, and A. F. Suchkov

P. N. Lebedev Physics Institute, USSR Academy of Sciences, Moscow (Submitted 1 April 1977) Zh. Eksp. Teor. Fiz. 74, 43-56 (January 1978)

Ways of increasing the sensitivity of the method of intracavity laser spectroscopy (ICLS) are investigated experimentally and theoretically. Factors restricting sensitivity include the selective properties of the cavity, finite time of continuous generation in the neighborhood of the line under investigation, spontaneous emission of the active medium, and spatial inhomogeneity of inversion decay in the active medium. The selective properties of the cavity can be improved by simplifying it and reducing the area of surfaces in its interior. The optimum configuration is a cavity with a single surface separating the medium under investigation from the active medium. In contrast to most other work concerned with ICLS, it is shown that the influence of spatial inhomogeneities in inversion decay is negligible in comparison with the influence of spontaneous emission in practical lasers. Spontaneous emission restricts the sensitivity of the ICLS method to  $\sim 10^{-12}$  cm<sup>-1</sup>. A sensitivity of  $10^{-9}$  cm<sup>-1</sup> has been achieved experimentally in the range between 0.6 and 1.06  $\mu$ . This sensitivity is determined by the time of quasicontinuous generation in the neighborhood of the absorption line under investigation. A concentration sensitivity for the detection of I<sub>2</sub> and NO<sub>2</sub> of better than  $10^{-9}$  mole/mole has been achieved.

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## **1. INTRODUCTION**

Intracavity laser spectroscopy (ICLS)<sup>[1-20]</sup> is based on the availability of active media in which the amplification coefficient can be held constant in the neighborhood of a given absorption (amplification) line. The theoretical sensitivity limit of this method that can be attained in the case of continuous generation is determined by the ratio  $s_m/J_m$  of spontaneous to induced radiated power per longitudinal mode:

$$\delta k_l / k_{\rm th} = s_m / J_m, \tag{1}$$

where  $k_i$  and  $k_{th}$  are, respectively, the absorption coefficient for the given line and the threshold absorption coefficient, both averaged over the length L of the

## cavity.

Quantitative measurements are based on the time dependence of the generated spectrum. The time t of stable generation in the neighborhood of a given line, necessary for the intensity to fall by a factor of e, is related to  $\delta k_1(\omega_0)$  and the velocity of light by the following formula:<sup>(1,9)</sup>

$$t = 1/c\delta k_1(\omega_0) \quad (\delta k_1 \gg k_{th} s_m/J_m).$$
<sup>(2)</sup>

In lasers with an inhomogeneously broadened amplification band, the amplification coefficient can be held constant with a high degree of precision due to inhomogeneous saturation in a spectral interval comparable with the amplification band width. For example, in