The application of an external magnetic field increases the degree of the long-range order in a crystal and automatically reduces the activation energy in the region of T_c and weakens the scattering of carriers by the magnetic moments of the ferrons. Therefore, a giant negative magnetoresistance is observed in the region of T_c and the maximum of $\rho(T)$ is shifted toward higher temperatures, as observed in our study.

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The capture cross section for holes of a charged dislocation in a semiconductor

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The temperature dependence of the cross section for hole capture by a negatively charged dislocation in an n-type semiconductor has been found. It is shown that because of the special character of the filling of the dislocation with the electrons that saturate it, this dependence is stronger than in the case of point centers. The magnitude of the cross section is in agreement with the experimental data.

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According to the Shockley model[1] an edge dislocation in an n-type semiconductor behaves like an infinitely long negatively charged line of acceptors. As a result, dislocations act as recombination centers for excess carriers. In this situation the minority carriers, which are acted on by the electrostatic attraction, are the most effectively captured. To be captured by a dislocation it is necessary for a hole to reduce its energy by an amount sufficient for it to undergo transition to the bound state. One of the energy-removal mechanisms which ensure experimentally observable large cross sections[2] is interaction with acoustic phonons. Since at sufficiently high temperatures the scattering of holes by phonons has a quasielastic nature and the levels associated with a charged dislocation are quite deeply situated in the forbidden band, the most probable process is Lax's[3] cascade capture mechanism.

In a recent paper by Abakumov and Yassievich[4] a convenient method was proposed for calculating the cross section for capture by attracting centers in semiconductors. This method was first used by Pitaevskiĭ[5] to obtain the electron recombination coefficient in a weakly ionized plasma.

In the present paper we shall make use of the method developed in[4] to find the capture cross section for holes by a unit length of a negatively charged dislocation. Let us note that because of the special character of the electrostatic field of the dislocation as a linear imperfection in the crystal lattice, the temperature dependence of the capture cross section is stronger than in the case of point centers.[4]

1. THE ELECTROSTATIC POTENTIAL OF A DISLOCATION

It is well known that an edge dislocation can capture additional electrons on the broken bonds which arise at the edge of the extra half-plane, thereby building up a positively charged cylinder of ionized impurities around itself. In this situation the degree to which the dislocation is filled with the electrons that saturate it can be characterized by the ratio $\zeta = a/c$, where a and c are the distances between broken and saturated bonds respectively. However, in calculating the electrostatic potential we shall consider the dislocation as a uniformly charged filament of infinite length. It is evident that this idealized approximation is valid only at not too small distances from the dislocation axis, where the discrete nature of the filament charge becomes significant.

Let us introduce the cylindrical coordinates (r, θ, z) with the dislocation axis as the polar one. To find the potential $\varphi(r)$ of the charged dislocation with allowance for the screening of the electric field by free charges and by the ionized impurity we shall make use of the Poisson equation

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where Δ is the two-dimensional Laplace operator, $\varkappa$ is the dielectric constant, and the charge density ρ is determined by the expression

$$\rho=\rho_D+e(p-n+n_d^+-n_a^-), \tag{1}$$

where n and p are the electron and hole densities, n_d^+ and n_a^- are the concentrations of ionized donors and acceptors, while

$$\rho_D=-\frac{e}{c}\frac{\delta(r)}{2\pi r}$$

is the δ-function charge density of electrons which have attached themselves to the dislocation. In an n-type semiconductor with $n_d \gg n_a$, the hole density may be neglected in the temperature range in which the impurities are fully ionized, while the electron density is determined by the expression

$$n=n_d \exp(e\varphi/kT),$$

where n_d is the total donor concentration. Under these conditions we have for the charge density ρ

$$\rho=\rho_D+en_d[1-\exp(e\varphi/kT)]. \tag{1'}$$

In the general case Eq. (1) with the boundary conditions

$$\varphi(\infty)=\varphi'(\infty)=0 \tag{2}$$

cannot be solved analytically and it becomes necessary to analyze the limiting cases corresponding to small ($|e\varphi/kT| \gg 1$) and to large ($|e\varphi/kT| \ll 1$) distances from the dislocation, with subsequent merging of the solutions obtained at a certain point R at which

$$\left|\frac{e\varphi(R)}{kT}\right|=1. \tag{3}$$

The physical meaning of the cylindrical region of radius R is obvious. Inside such a cylinder the electrostatic energy of a carrier is large compared with its thermal energy and consequently there are no free carriers. At the same time the problem of the screening potential of a charged dislocation involves another characteristic length, viz., the Debye screening radius

$$r_D=(\varkappa kT/4\pi n_d e^2)^{1/2}. \tag{4}$$

Depending on the ratio of these two radii, which is a function of the filling of the dislocation with electrons, two limiting cases must be distinguished; these correspond to strong (low temperatures) and weak (high temperatures) filling. At low temperatures ($R \gg r_D$) the solution of Eq. (1) is

$$\varphi(r)=-\frac{kT}{e}\begin{cases}1-\frac{1}{4}\frac{R^2-r^2}{r_D^2}-\frac{2e^2}{\varkappa ckT}\ln\frac{r}{R}, & c<r\leqslant R\\ \sqrt{\frac{R}{r}}\exp\left(\frac{R-r}{r_D}\right), & r\geqslant R\end{cases}. \tag{5}$$

We have introduced a cutoff parameter c here, since it must be remembered that the model of a dislocation as a uniformly charged filament is valid only, generally speaking, at distances $r \gg c$ from the dislocation axis.

In the opposite, high-temperature limiting case, $R \ll r_D$ and

$$\varphi(r)=-\frac{2e}{\varkappa c}K_0\left(\frac{r}{r_D}\right), \tag{6}$$

where $K_0(x)$ is the MacDonald function. In this case we have for the radius R

$$R=r_D\begin{cases}(4\alpha)^{1/2}, & \alpha\gg 1\\ \frac{2}{\gamma}\exp\left(-\frac{1}{2\alpha}\right), & \alpha\ll 1\end{cases}, \tag{7}$$

where $\ln\gamma = C = 0.577$ is Euler's constant, while

$$\alpha=e^2/\varkappa ckT \tag{8}$$

is a parameter which characterizes the ratio of the Coulomb interaction energy of neighboring electrons which have attached themselves to the dislocation to their thermal energy. The case of strong filling, when the radius of the cylinder deprived of free carriers is sharply defined, corresponds to values $\alpha \gg 1$. In this case screening is due only to ionized impurities and the concentration of free carriers may be neglected. The existence of such a cylinder was first pointed out by Read.[6] In the opposite case of weak filling ($\alpha \ll 1$), the boundary of the region depleted of carriers is strongly smeared out, and the dislocation potential is screened mainly by free carriers.

2. THE CROSS SECTION FOR HOLE CAPTURE BY A DISLOCATION

To find the cross section for hole capture by a dislocation we shall make use, following[4], of the kinetic equation for the distribution function $f(\mathbf{r}, \mathbf{p}, t)$ with a collision integral which allows for the interaction of holes with acoustic phonons. At temperatures which are not unduly low, scattering by phonons has a quasielastic character and the fraction of energy lost by a hole in each collision is $\Delta\varepsilon = \varepsilon(8ms^2/\varepsilon)^{1/2}$, where $\varepsilon = p^2/2m$ is the kinetic energy of the hole, m is its effective mass, and s is the velocity of sound in the crystal. For a hole to be captured by a dislocation it is necessary that, in a cascade of collisions, it be in a bound state with an energy $\lesssim -kT$, where T is the temperature of the lattice. In other words, it is necessary for the hole to be in the region of the cylinder of radius R in which its electrostatic energy $e\varphi$ exceeds its thermal energy kT. Since $\Delta\varepsilon \ll kT$, the condition for the cascade model to be applicable is

$$l \ll r_D, \tag{9}$$

where l is the momentum relaxation length for phonon collision. On the other hand, the smallness of l allows the distribution function to be considered as dependent not on the momentum, but on the kinetic energy ε. In this case, in the absence of degeneracy, the kinetic equation is[7]

$$\frac{\partial f}{\partial t}+v\nabla f+e\frac{\partial\varphi}{\partial r}\frac{\partial f}{\partial p}=\frac{\delta}{\rho(\varepsilon)}\frac{\partial}{\partial\varepsilon}\left[\varepsilon\rho(\varepsilon)v(\varepsilon)\left(f+kT\frac{\partial f}{\partial\varepsilon}\right)\right], \quad (10)$$

where $\delta=2ms^2/kT\ll 1$ is the quasielastic scattering parameter,

$$\rho(\varepsilon)=\frac{8\sqrt{2}\pi m^{3/2}V}{(2\pi\hbar)^3}\varepsilon^{1/2} \quad (11)$$

is the density of states in the energy interval $(\varepsilon, \varepsilon+d\varepsilon)$, and

$$v(\varepsilon)=\frac{\sqrt{2}E_c^2m^{3/2}kT}{\pi\hbar^4\rho s^2}\varepsilon^{1/2} \quad (12)$$

is the frequency of collisions between holes and acoustic phonons which determines the momentum relaxation length l. Here E_c is the deformation-potential constant, ρ is the density, and V is the volume of the crystal.

If it is assumed that the energy relaxation length for phonon collision $l_\varepsilon = l\delta^{-1}$ is much greater than the Debye radius, then the second and third terms on the left-hand side of the kinetic equation (10) will be large compared to the other terms. From the fact that the sum of these terms, which is a classical Poisson bracket, is equal to zero, it follows that the distribution function depends only on the total energy

$$E=\varepsilon-e\varphi(r) \quad (13)$$

and on time. Averaging of the remainder of the equation in the phase space between the hypersurfaces $E=\text{const}$ and $E+\Delta E=\text{const}$[5] leads to the equation

$$\rho(E)\frac{\partial f}{\partial t}=\frac{\partial}{\partial E}\left[B(E)\left(f+kT\frac{\partial f}{\partial E}\right)\right], \quad (14)$$

where

$$\rho(E)=\int_0^{r_{max}}\rho(\varepsilon)\frac{2\pi r\,dr}{S}, \quad (15)$$

$$B(E)=\delta\int_0^{r_{max}}\varepsilon\rho(\varepsilon)v(\varepsilon)\frac{2\pi r\,dr}{S}. \quad (16)$$

Here S is the area of the surface perpendicular to the dislocation axis, while r_{max} corresponds to the turning point of the finite motion which is determined according to Eq. (13) from the condition

$$e\varphi(r_{max})=|E|. \quad (17)$$

To find the total hole current j flowing into the dislocation in stationary conditions in the negative-total-energy region it is necessary to solve the differential equation

$$j=B(E)\left[f(E)+kT\frac{df(E)}{dE}\right] \quad (18)$$

with the boundary conditions

$$\lim_{E\to E_0}f(E)=0, \quad \lim_{E\to 0}f(E)=f_B(E=0), \quad (19)$$

where E_0 is the energy of the ground state. The first of these conditions means that a particle which has entered the ground state "goes out of play," while the second corresponds to the smooth merging of the distribution function in the negative total energy region with the Boltzmann continuous spectrum function of the valence band. Taking account of what has been said, we have for the current j

$$j=\frac{(2\pi\hbar)^3p}{2(2\pi mkT)^{3/2}}\left[\int_{-|E_0|}^{0}\frac{\exp(\varepsilon/kT)}{B(\varepsilon)}\frac{d\varepsilon}{kT}\right]^{-1}. \quad (20)$$

Inasmuch as $|E_0|\sim|e\varphi(c)|\gg kT$ and the integral converges at values of $\varepsilon\sim -kT$, it is easy to see that the lower integration limit may be replaced by $-\infty$. According to Eqs. (3) and (17) we then have for the turning point r_{max}

$$r_{max}\approx R, \quad (21)$$

where R is defined by expression (7). From this we obtain for the coefficient $B(\varepsilon)$ in the limiting low- and high-temperature cases

$$B(|\varepsilon|)=\frac{4m^2E_c^2(kT)^2}{\pi^2\hbar^7\rho}R^2\begin{cases}(1-|\varepsilon|/kT+\alpha/2)^2+{}^7/_{12}\alpha^2, & \alpha\gg 1\\ (1-|\varepsilon|/kT+\alpha)^2, & \alpha\ll 1\end{cases}. \quad (22)$$

On dividing the total current j determined from Eqs. (20) and (22) by the hole density p and the average hole velocity $\langle v\rangle=(8kT/m)^{1/2}$, we obtain the cross section for hole capture by a unit length of a negatively charged dislocation

$$\sigma=\sigma_0\begin{cases}\alpha^3, & \alpha\gg 1\\ A\exp(-1/\alpha), & \alpha\ll 1\end{cases}, \quad \sigma_0=\frac{20}{3\pi^{3/2}}\frac{m^3E_c^2}{\rho\hbar^4}r_D^2, \quad (23)$$

where A is a number of the order of unity. From Eq. (23) it is evident that the cross section for hole capture by a negatively charged dislocation is significantly larger at low temperatures than at high temperatures. Because of the special features of the filling of the dislocation with electrons, the temperature dependence of the cross section turns out to be larger than in the case of point centers of attraction. At low temperatures this dependence is a power function, while at high temperatures it is exponential (see (4), (7), and (8)). The transition from the one type of dependence to the other takes place at a temperature T_0:

$$kT_0=e^2/\varkappa c(T_0), \quad (24)$$

at which $\alpha=1$ and the electrostatic interaction energy of neighboring electrons on the dislocation becomes of the order of their thermal energy.

3. CONCLUDING REMARKS

In deriving the kinetic equation it was assumed that the scattering of holes by phonons has a quasielastic character, the momentum relaxation length being much less than the Debye length. The latter assumption permitted restriction of the analysis to the isotropic part of the distribution function only. However, it should be noted that this condition is much more important inasmuch as the Lax cascade mechanism ceases to be meaningful if

it is violated. On the other hand, for $r_D \ll l_e$ the distribution function depends only on the total energy E. It is thus necessary to satisfy the chain of inequalities

$$1 \ll r_D/l \ll \delta^{-1}, \quad l = \frac{\pi\hbar^4\rho s^2}{m^2 E_c^2 kT}, \tag{25}$$

which leads to a lower temperature bound on the range of applicability of the results obtained.

Let us note that in finding the potential of a charged dislocation we regarded the donors as fully ionized and in doing so restricted ourselves to temperatures such that $kT \gg \varepsilon_d$, where ε_d is the activation energy of the donors. However, our results hold qualitatively even at lower temperatures, when screening is provided by electrons moving among the donors without their prior activation. This occurs until quasicontinuity of the donor distribution in the volume of the semiconductor is ensured, i.e., until the distance between the donors $n_d^{-1/3}$ is much less than the characteristic scale of variation of the electrostatic potential, given by the Debye radius, i.e.

$$T > \frac{4\pi e^2}{\varkappa k} n_d^{1/3}. \tag{26}$$

For germanium in which $m \sim 10^{-28}$ g, $\varkappa = 16$, $n_d \sim 10^{13}$ cm^{-3}, and $10^{-2} < \zeta < 10^{-1}$, a combined analysis of the inequalities (25) and (26) shows that the results obtained in this work are qualitatively valid down to temperatures $T \sim 30$ K. For the temperature at which the dependence of the cross section on temperature goes over from a power function to an exponential one we obtain $T_0 \approx 130$ K. The absolute magnitude of the capture cross section (radius) in the temperature range $30\ \text{K} < T < 300\ \text{K}$ proves to be of the order of $3 \cdot 10^{-7}\ \text{cm} < \sigma < 2 \cdot 10^{-4}\ \text{cm}$, which is in agreement with the experimentally observed[2] large cross sections.

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The electronic thermal conductivity of clean superconductors

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The temperature dependence of the electron-phonon thermal conductivity is obtained by numerical solution of the kinetic equation and compared with experiment.

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1. In very clean superconductors the electronic thermal conductivity $\varkappa_e$ is determined entirely by the scattering of electrons by phonons ($\varkappa_{eph}$) and by the crystal boundaries, while the phonon thermal conductivity $\varkappa_{ph}$ is determined entirely by the scattering of the phonons by electrons ($\varkappa_{phe}$) and also by the crystal boundaries. The solution for the nonequilibrium correction φ to the electron distribution function f has the form[1,2]

$$\varphi = -\frac{f-f_0}{f_0(1-f_0)} = \left[\varphi_2(\varepsilon, T) + \frac{\xi}{|\xi|}\varphi_3(\varepsilon, T)\right]\cos(\widehat{\mathbf{p}\nabla T}), \tag{1}$$

where $\varepsilon = (\xi^2 + \Delta^2)^{1/2}$. Under the condition[2]

$$\left(\frac{T}{\Theta_D}\right)^4 \frac{e_F}{\Theta_D} e^{\Delta/T} \ll 1 \tag{2}$$

the decisive role in the calculation of the electronic thermal conductivity, as was noted in the work of Gurevich and Krylov,[1] is played by the function φ_3. In Ref. 3 an integral equation for φ_3 was obtained. In this article we report the results of a numerical calculation of the electronic thermal conductivity, based on the solution of this equation, and compare the results with the experimental data. We neglect the influence of the nonequilibrium character of the phonons on the electron distribution function, which is permissible for temperatures that are not too low[1,2]:

$$\frac{T^2}{\varepsilon_F \Theta_D} e^{\Delta/T} \ll 1.$$

2. For the electronic thermal conductivity in a normal metal we have, using Matthiessen's rule,

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