Observation of magnon interaction in an antiferromagnet

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Magnons of frequency $\nu_{k1} = 1/2\nu_p$ were parametrically excited in antiferromagnetic $CsMnF_3$ by microwave pumping at frequency $\nu_p = 26$–36 GHz at temperature $T = 1.6$ K. The number of parametrically excited magnons greatly exceeds the thermal level at this frequency. This leads to a change of the thresholds for parametric excitation of magnons of frequency ν_{k2} by microwave pumping at frequency $\nu_m = 2\nu_{k2}$. The change of the threshold for parametric excitation occurs in consequence of a change of the relaxation of magnons because of their interaction with parametrically excited magnons. It is shown that the change of relaxation is due to processes of fusion of magnons with frequencies ν_{k1} and ν_{k2} into a phonon. The value of the spin-lattice relaxation is estimated as $\Delta\nu_{sl} \approx 0.01$ MHz, and the superheating of the spin system of the specimen by parametric excitation of magnons as $\Delta T \approx 0.1$ K.

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1. INTRODUCTION

The object of the research was to observe the interaction of two magnons with definite values of the frequencies and of the wave vectors. The idea for solving this problem experimentally was based on the possibility of parametric excitation of magnons in specimens of antiferromagnetic $MnCO_3$ and $CsMnF_3$ by microwave pumping. The phenomenon of parametric excitation of spin waves in antiferromagnets has been studied both experimentally and theoretically[1–3] and consists of the following.

When the magnetic microwave pumping field h, with frequency ν_p, exceeds the threshold value h_1 (which is determined by relaxation of magnons), a parametric instability begins to develop in the specimen: the number of magnons with frequency $\nu_k = \nu_p/2$ increases. When the density of parametrically excited magnons (PM) reaches a value exceeding the thermal background by two or three orders of magnitude, there occurs a discontinuous increase of the rate of growth of the number of PM, due apparently to saturation of one of the mechanisms of relaxation and turning off of a part of the relaxation. The nature of this turning off is not yet clear. Together with growth of the number of PM, the power absorbed by them also grows. After this jump, the absorbed power reaches a value that can be recorded experimentally (see Ref. 1). Thereafter, the PM system reaches a stationary state with a constant value of the absorbed power.

As was shown in Ref. 4, the limitation of the number of PM and the establishment of a stationary state occur as a result of a shift of phase of the PM with respect to the phase of the pumping. The stationary state is characterized by a new value of the threshold field, $h_2 < h_1$. (When $h < h_2$, parametric excitation of magnons stops.) Parametric excitation entailing the presence of two threshold fields is called "hard."[1, 2]

From the values of the threshold fields, one can calculate the corresponding magnon relaxation frequencies $\Delta\nu_1$ and $\Delta\nu_2$.[3] For $CsMnF_3$, a specimen of which was studied in the present paper, under the condition $\nu_p \ll \nu_{20} = 115$ GHz, this relation is

$$\Delta\nu_{1,2} = 2\gamma^2 H h_{1,2}/\nu_p, \tag{1}$$

where H is the external magnetic field and γ the gyromagnetic ratio. The values of $\Delta\nu_1$ and $\Delta\nu_2$ differ by 5 to 50% (depending on the static magnetic field and the temperature); their order of magnitude is 0.1 MHz, which corresponds to a magnon lifetime $\tau = 1/2\pi\Delta\nu_1 \approx 1$ μsec. The occupancy number of the PM in the stationary state exceeds the occupancy number of thermal magnons at $T = 1$ to 2 K by five or six orders of magnitude. The wave vector of the PM is determined by the condition of parametric resonance

$$\nu_k = \nu_p/2 \tag{2}$$

and the spin-wave dispersion law

$$(\nu_k/\gamma)^2 = H^2 + H_\Delta^2 + \alpha^2 k^2, \tag{3}$$

where $H_\Delta^2 = 6.3/T$ [kOe]^2 is the gap caused by hyperfine interaction, and where $\alpha = 0.95 \cdot 10^{-5}$ kOe cm (the numerical values are taken from Ref. 2). By varying H over a range that permits satisfaction of (2) with allowance for (3), one can excite PM with wave vector from 0 to $\sim 10^5$ cm^{-1} by use of pumping in the centimeter range.

Parametric excitation of magnons is recorded experimentally on the basis of absorption of microwave power in the specimen. For observation of the interaction of magnons of different frequencies, use was made of action on the specimen by two microwave pumpings, from generators operating at different frequencies. The first pumping, at frequency ν_p, excited parametrically, in the specimen, magnons of frequency $\nu_{k1} = \nu_p/2$ in a number greatly exceeding (by a factor of about 10^6) the level of thermal magnons at this frequency. Interaction with these PM changes the relaxation frequency of the other magnons. The change of relaxation frequency of magnons of frequency ν_{k2} can be detected by measuring the threshold for parametric excitation of them. For this

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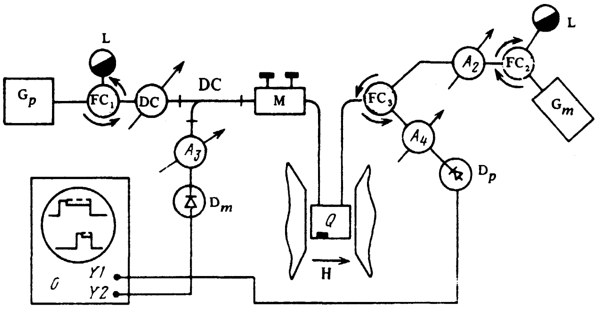


FIG. 1. Schematic diagram of the experiment. G_p and G_m, klystrons; FC, ferrite circulators; A_1, A_3, and A_4, attenuators; A_2, precision attenuator; L, absorbing loads; M, matcher; DC, directional coupler; Q, resonator containing the specimen; D_p and D_m, microwave signal detectors; O, oscilloscope. Shown by dashes is a schematic representation of the oscillograms of the signals from detectors D_p and D_m in the absence of parametric excitation of spin waves.

purpose, use is made of a second microwave pumping (at frequency ν_m), tuned to the frequency of parametric resonance with spin waves of the second frequency: $\nu_m = 2\nu_{k2}$. The threshold fields at which the development of parametric instability begins for the first and second pumps we shall denote by h_{p1} and h_{m1}; to them correspond relaxation frequencies $\Delta\nu_{p1}$ and $\Delta\nu_{m1}$ in accordance with (1). The threshold fields characteristic of the stationary PM mode we shall denote by h_{p2} and h_{m2}. To these fields correspond relaxation frequencies $\Delta\nu_{p2}$ and $\Delta\nu_{m2}$. The turned off part of the relaxation we shall denote by $\Delta\nu^* \equiv \Delta\nu_{m1} - \Delta\nu_{m2}$.

A similar investigation has been made on specimens of yttrium-iron garnet[5]; but the difference in the spin-wave spectra of ferro- and antiferromagnets and the performance of the experiments at low temperatures give reason to expect manifestations of other processes in the interactions of spin waves.

2. METHOD

A schematic of the experimental setup is shown in Fig. 1. The specimen was placed at a loop of the magnetic field of the H_{012} mode of a cylindrical resonator with $Q \approx 5000$. The resonator was located in a bath with liquid helium in the superfluid state. The microwave pumping fields with frequencies ν_p and ν_m were produced in the resonator by the klystron generators G_p and G_m. Radiation from the klystron G_p excites in the specimen PM with frequency ν_{k1}; the resulting absorption of microwave power is recorded on the basis of the magnitude of the signal of detector D_p. Parametric excitation of spin waves with frequency ν_{k2}, which occurs under the influence of radiation from klystron G_m, is recorded on the basis of the signal of detector D_m, and the threshold value of the power is determined with the precision attenuator A_2. From the scale of A_2 is read the relative change of the threshold field, $\delta = (h'_{m1} - h^0_{m1})/h^0_{m1}$, that occurs after turning on of klystron G_p (h'_{m1} is the threshold field under the action of the first pumping, h^0_{m1} without it). The ferrite circulators FC_1 and FC_2 insure decoupling of the waveguide lines and the klystrons. The circulator FC_3 is used to decouple the resonator and the line in which, in the course of the experiment, the attenuator A_2 is adjusted, and also to isolate the signal from the first pump, passing through the resonator, from the line that connects G_m with the resonator, and to direct this signal to the detector D_p. The line connecting G_p with the resonator is connected to an aperture with coupling coefficient $\beta \approx 1$ (almost all the signal generated by G_p goes into the resonator). The residual reflection is observed with the detector D_m and removed with the matcher M. As complete removal as is possible of the reflected signal is necessary for observation at the detector D_m, at the moment of measurement of h'_{m1}, of a signal from klystron G_m alone, not distorted by a signal from G_p. The klystron G_m was tuned to the frequency of the H_{012} mode; G_p was tuned either to some other mode of the resonator or to the limits of the H_{012} resonance curve. The set of modes of the resonator determines the set of possible frequencies ν_p.

The threshold fields h_{m2} and h_{p2} were measured according to a method developed in Ref. 1: while the specimen was subject to a pumping field with slowly decreasing amplitude, the value of the microwave field was registered at the moment of cessation of the parametric excitation of magnons.

3. RESULTS AND ANALYSIS OF THEM

It was found that excitation in the specimen of PM of frequency ν_{k1} leads to a change of the threshold for parametric excitation of magnons with frequency ν_{k2}. This effect of certain spin waves on others depends importantly on the difference of their frequencies, $\nu_{k1} - \nu_{k2} = (\nu_p - \nu_m)/2$. When $|\nu_p - \nu_m| < 3$ GHz, a decrease of the threshold occurs ($h'_{m1} < h^0_{m1}$), whereas for large differences of the frequencies the threshold increases ($h'_{m1} > h^0_{m1}$). The change of the threshold field amounts to 10 to 40% and depends on the amplitude h_p of the first pumping field and on the static magnetic field. The change of the threshold field h_{m1} is noticeable only after the threshold of the first pumping has been exceeded: $h_p > h_{p2}$.

It is natural to present the results of an experiment on change of the relaxation of magnons when they interact with PM as functions of the number N_p of these magnons. The number of magnons N_p can be determined as follows, on the basis of the easily measured value of the excess over the threshold power, $(h_p/h_{p2})^2$. The power absorbed from the pump at frequency ν_p is connected with N_p by a relation that expresses the equality of the absorbed and dissipated powers:

$$P_{\text{abs}} = 2\pi\hbar\nu_{k1}N_p\Delta\nu_{p2}. \qquad (4)$$

At the same time, it is usually expressed in terms of the imaginary part of the high-frequency susceptibility, χ'':

$$P_{\text{abs}} = 4\pi h_p^2\nu_p\chi'' V_{\text{spec}}. \qquad (5)$$

It has been shown experimentally[6] that χ'' follows the theoretically predicted[7] dependence on $(h_p/h_{p2})^2$:

$$\chi'' = \frac{A(H)\left[(h_p/h_{p2})^2 - 1\right]^{1/2}}{(h_p/h_{p2})^2} \qquad (6)$$

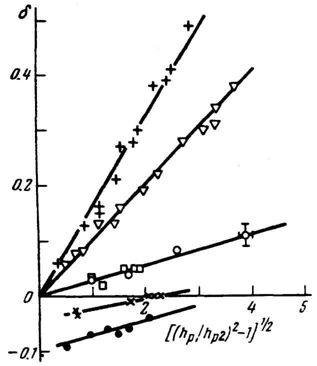


FIG. 2. Relative change of threshold field, $\delta=(h'_{m1}-h^0_{m1})/h^0_{m1}$, with excitation of parametric spin waves with frequency $\nu_{k1}=\nu_p/2$, $\nu_m=35.73$ GHz. The experimental points correspond to (H is in kOe, $\Delta\nu=\nu_m-\nu_p$ in MHz): ●, $H=2.4$, $\Delta\nu=10$; ×, $H=2.4$, $\Delta\nu=90$; □, $H=1.2$, $\Delta\nu=5900$; ∇, $H=2.4$, $\Delta\nu=5900$; +, $H=3.36$, $\Delta\nu=5900$; o, $H=4.8$, $\Delta\nu=5900$.

and does not depend explicitly on temperature. Then from (4)–(6) and (1) follows

$$N_p \propto \frac{\Delta\nu_{p2}}{H^2} A(H) \left[\left(\frac{h_p}{h_{p2}} \right)^2 - 1 \right]^{1/2}. \quad (7)$$

The coefficient of proportionality in (7) for $CsMnF_3$ depends only on the pumping frequency. Formula (7) enables us to calculate N_p from the value of $(h_p/h_{p2})^2$ measured in the course of the experiment. The results of the experiment—the relative changes of threshold field $\delta=(h'_{m1}-h^0_{m1})/h^0_{m1}$—were plotted as a function of $[(h_p/h_{p2})^2-1]^{1/2}\propto N_p$. This dependence is shown in Fig. 2 for several combinations of the pumping frequencies ν_p and ν_m.

In order to understand the observed phenomenon, it was necessary to explain whether the observed change of the threshold field h_{m1}, which according to (1) is proportional to the relaxation frequency $\Delta\nu_{m1}=\Delta\nu_{m2}+\Delta\nu^*$, is determined by the change of the part $\Delta\nu_{m2}$ of the relaxation, of the part $\Delta\nu^*$, or of both of these quantities. The change of the value of $\Delta\nu_{m2}$ upon action of the first pump upon the crystal can be independently determined by measurement of the "second" threshold h_{m2}.

As the result of measurement of the threshold field h_{m2} with simultaneous measurement of h_{m1}, it was clarified that the decrease of h_{m1} under the influence of PM when $|\nu_p-\nu_m|<3$ GHz, at small excesses over the threshold field of the first pump, is caused by change of $\Delta\nu^*$. When $|\nu_p-\nu_m|>3$ GHz, the threshold fields h_{m1} and h_{m2} increase equally under the influence of PM, within the limits of measurement error, ±2%. Thus when $|\nu_p-\nu_m|>3$ GHz, the result of excitation of PM with frequency ν_{k1} is that only the relaxation frequency $\Delta\nu_{m2}$ changes, while $\Delta\nu^*$ remains constant. The change of the value of δ with increase of the power of the first pump when $|\nu_p-\nu_m|<3$ GHz (see Fig. 2) is obviously caused by both effects: a decrease of h_{m1} because of decrease of $\Delta\nu^*$ and an increase of h_{m1} because of increase of $\Delta\nu_{m2}$.

The threshold field h_{m2} is measured by observation of the signal from detector D_m on the oscilloscope screen and can be measured with accuracy no better than 2%. Therefore the determination of the value of δ by measurement of h_{m2} is made with accuracy ±0.05%. This worsening of the accuracy does not affect the qualitative deductions that follow from this investigation.

As is seen from Fig. 2, when $|\nu_p-\nu_m|>3$, over the whole accessible range of power of the first pump, there was observed a linear increase of the relaxation frequency of spin waves at frequency $\nu_{k2}=\nu_m/2$ with the number of PM:

$$\Delta\nu'=\Phi N_p. \quad (8)$$

The quantity $2\pi\Phi$ is the probability per unit time of an interaction of magnons with frequencies ν_{k1} and ν_{k2}.

To illustrate the dependence of the effect on the static magnetic field, values of δ at various magnetic fields, but at a single value of $(h_p/h_{p2})^2$, are plotted in Fig. 3.

In order to treat the dependence of Φ on temperature and magnetic field, however, it is necessary to take into account that the curves of Fig. 3 require rescaling, because the coefficient of proportionality between N_p and $[(h_p/h_{p2})^2-1]^{1/2}$ contains $\Delta\nu_{p2}/H^2$ and depends on temperature and magnetic field (see (7)). The quantity obtained from experiment is δ, which is connected with $\Delta\nu'$ in accordance with (1):

$$\Delta\nu'=\delta\Delta\nu_{m1}. \quad (9)$$

From (7)–(9) we have

$$\Phi=\frac{\delta\Delta\nu_{m1}}{N_p}\propto\frac{\delta\Delta\nu_{m1}H^2}{\Delta\nu_{p2}[(h_p/h_{p2})^2-1]^{1/2}A(H)}, \quad (10)$$

that is, for fixed h_p/h_{p2} (the experimental data shown in Fig. 3 were obtained under this condition)

$$\Phi\propto\delta\frac{\Delta\nu_{m1}H^2}{\Delta\nu_{p2}A(H)}\equiv\delta F(H,T). \quad (11)$$

For the specimen under investigation, the field and temperature dependences of $A(H)$, $\Delta\nu_{m1}$, and $\Delta\nu_{p2}$ were measured. From them, values of the function $F(H,T)$ were calculated and graphs of the function $\Phi(H)$ were plotted (Fig. 4).

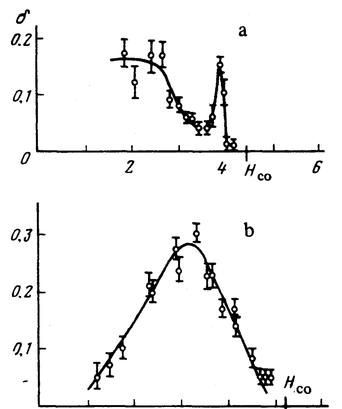


FIG. 3. Dependence of relative change of threshold δ on magnetic field at a fixed value of $(h_p/h_{p2})^2=5$: a, $\nu_p=26.30$ GHz; b, $\nu_p=29.83$ GHz; $\nu_m=35.73$ GHz.

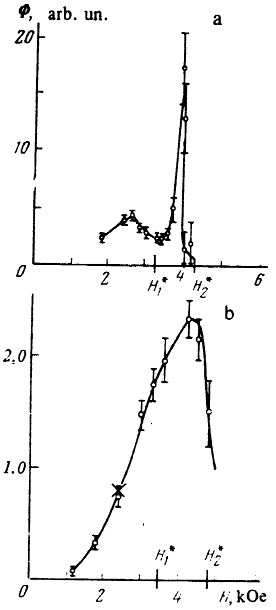


FIG. 4. Probability of interaction of two magnons of different frequencies as a function of magnetic field ($\nu_m = 35.73$ GHz): a, $\nu_p = 26.30$ GHz, $H_1^* = 3.25$ kOe, $H_2^* = 4.27$ kOe; b, $\nu_p = 29.83$ GHz, $H_1^* = 3.50$ kOe, $H_2^* = 4.88$ kOe; $T = 1.6$ K (the cross shows a value at $T = 2.1$ K).

A control measurement at $T = 2.1$ K and $H = 2.41$ kOe showed that Φ is independent of temperature, with accuracy ±20%, on change of T from 1.6 to 2.1 K.

Thus the principal results of the experiment for $|\nu_p - \nu_m| > 3$ GHz reduce to the following:

1. As a result of excitation of PM with frequency ν_{k1}, there occurs an increase of the relaxation frequencies $\Delta\nu_{m1}$ and $\Delta\nu_{m2}$ of magnons with frequency ν_{k2}, by an amount $\Delta\nu'$ that is proportional to the number of PM. The increase $\Delta\nu'$ of the relaxation amounts to about $0.1\Delta\nu_{m1}$; that is, to about 0.01 MHz.

2. The coefficient of proportionality Φ between $\Delta\nu'$ and N_p is independent of temperature over the interval 1.6 to 2.1 K.

3. The coefficient Φ varies with magnetic field as is shown in Fig. 4.

4. DISCUSSION OF RESULTS

1) We consider the case $|\nu_p - \nu_m| < 3$ GHz.

The decrease of the threshold h_{m1} under the influence of the first pump, at small frequency differences $|\nu_p - \nu_m|$, has an appreciable value even at small excesses over the threshold field: $h_p/h_{p2} - 1 \lesssim 0.1$. When $|\nu_p - \nu_m| < 10$ MHz, $h'_{m1} \approx h^0_{m2}$: that is, the presence of magnons of the second frequency (differing from the frequency of the magnons under study by no more than 5 MHz) leads to practically complete turning off of the part of the relaxation that is subject to turnoff; and when the frequency difference of the magnons is 50–1000 MHz, to a partial turning off. The variation of the turned off part $\Delta\nu^*$ of the relaxation with the frequency of the additional (in relation to the thermal distribution) magnons, for fixed h_p/h_{p2}, is shown in Fig. 5.

The decrease of the "first" threshold for parametric instability, at frequency ν_m, under the influence of PM of frequency $\nu_{k1} = \nu_p/2$, when $|\nu_p - \nu_m| < 3$ GHz, is similar to the decrease of the threshold in the course of the nonstationary stage of parametric excitation of magnons, described in the Introduction: with increase of the number of spin waves, the relaxation decreases, and with it the threshold field also decreases, from h^0_{m1} to h^0_{m2}. In the present situation, the threshold field decreases from h^0_{m1} to h'_{m1} because of the presence of magnons of the second frequency. In Ref. 1, characteristic magnon occupancy numbers were determined, at which the process of turning off of a part of the relaxation begins and ends: $n_{k1} \approx 10^2$ and $n_{k2} \approx 10^5$.

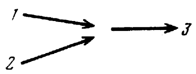
By use of these data and also of the results shown in Fig. 5, a frequency characteristic of this type can be established for the mechanism of turning off of part of the relaxation. For threshold excesses $h_p/h_{p2} \sim 1$, the number N_p of PM in the stationary state corresponds to occupancy numbers $n_p \approx 10^6$. [1] With such a number of PM of frequency $\nu_p/2$ when $|\nu_p - \nu_m| \approx 3$ GHz, a part of the relaxation of magnons of frequency $\nu_m/2$ begins to be turned off. For $\nu_p = \nu_m$ (the case of a single pump), this occurs, as follows from Ref. 1, when $n_{k1} \approx 10^2$. From this it may be concluded that in the presence of detuning of the magnon frequencies by ~1 GHz, the same decrease of relaxation requires 10^4 as many magnons as when the magnon frequencies coincide.

2) We now consider the results obtained when $|\nu_p - \nu_m| > 3$ GHz, and enumerated at the end of Sec. 3.

The increase of relaxation frequency when a second microwave pump acts on the crystal cannot be caused by superheating of the spin system of the specimen, for the following reason. In the case of superheating, there would occur a sharp increase of $\Delta\nu'$ with increase of magnetic field (for fixed power $P_{abs} \sim N_p \Delta\nu_{p2}$ absorbed by the spin system), because with increase of the magnetic field a stronger temperature dependence of $\Delta\nu_{m1}$ is observed. [2] But according to the data presented in Fig. 5 and the $\Delta\nu_{p2}(H)$ dependence, a decrease rather than an increase of the value of $\Delta\nu'|_{N_p \Delta\nu_{p2}}$ occurs in the high-field range. In this case an increase of the relaxation must be caused by interaction of magnons with frequency ν_{k2} and PM with frequency ν_{k1}.

We shall consider two types of possible processes:

a) These two magnons fuse into a third quasiparticle:



(12a)

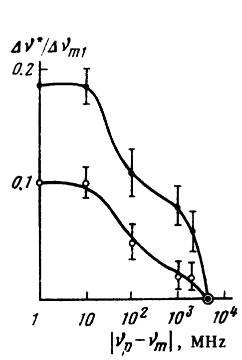
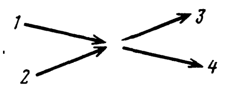


FIG. 5. Variation of the turned off part of the relaxation, $\Delta\nu^*$, with the difference of pumping frequencies: $\nu_m = 35.73$ GHz, $h_p/h_{p2} = 1.05$ to 1.25; ○, $H = 2.4$ kOe; ●, $H = 1.2$ kOe.

b) These two magnons are transformed into two quasiparticles:



(12b)

Processes involving participation of a larger number of particles are less probable. Consideration of the probabilities of direct and inverse transitions for the processes (12a) and (12b) gives the following dependences on the number of particles for the frequency $\Delta\nu'$ of relaxation occurring because of these processes[8]:

$$\text{a)}\ \Delta\nu' = \frac{4}{\hbar}\sum_{2,3}|\Phi_{12;3}|^2(n_p-n_3)\delta(h\nu_1+h\nu_2-h\nu_3) \propto N_p,$$

since the thermal occupancy numbers n_3 are much smaller than n_p;

$$\text{b)}\ \Delta\nu' = \frac{48}{\hbar}\sum_{2,3,4}|\Phi_{12;34}+\Phi^*_{34;12}|^2 n_p(n_3+n_4+1)\delta(h\nu_1+h\nu_2-h\nu_3-h\nu_4) \propto N_p$$

for the same reason. Here $\Phi_{12;3}$, $\Phi_{12;34}$, and $\Phi_{34;12}$ are the amplitudes of the corresponding interactions.

Of processes of the type (12a), only fusion of two magnons into a phonon is allowed by the conservation laws. Of processes of the type (12b), the ones allowed are transformations of two initial magnons into two magnons of the low-frequency branch or into two phonons, or into a phonon and a magnon.

Relaxation caused by any of the processes of type (12b) should depend substantially on temperature, since it depends on the thermal occupancy numbers $n_3, n_4 \sim 1$ at $T=1-2$ K. The observed constancy of the effect on change of temperature indicates that the main contribution to the phenomenon apparently comes from fusion of two magnons into a phonon.

Satisfaction of the conservation laws for this process is possible when

$$k_1+k_2 \geqslant k_3 = \frac{\nu_{k1}+\nu_{k2}}{C_i}\cdot 2\pi \tag{13}$$

(C_i is the velocity of sound waves of a definite type); this is satisfied in the magnetic-field range $H<H_1^*$ for fusion into a transverse phonon, and in the range $H<H_2^*$ for fusion into a longitudinal phonon. The velocities of sound are $C_1=2.31\cdot 10^5$ cm/sec and $C_2=4.16\cdot 10^5$ cm/sec for transverse and longitudinal phonons propagated in the easy plane of the crystal.[9] Analysis of the magnon-phonon interaction[2] shows that magnons can interact only with these acoustic oscillations. The fields H_1^* and H_2^* are shown on the graphs of Fig. 4. The values of H_1^* and H_2^* are determined with accuracy $\pm 10\%$, since experimental values of α and of $C_{1,2}$ are used. In the graphs of Fig. 4 we notice a drop of the effect when $H>H_2^*$.

At magnetic field 3.9 kOe, when $\nu_m=35.73$ GHz, $\nu_p=26.30$ GHz, and $T=1.6$ K, a sharp peak is observed in the probability of interaction of magnons with frequencies ν_{k1} and ν_{k2}. This value of H is characteristic of the interaction of the magnons under study when phonons participate, since at this magnetic field the intersection of the magnon spectrum with the spectrum of transverse phonons occurs at frequency ν_{k1}, and with the spectrum of longitudinal phonons at frequency ν_{k2}; that is, at the frequencies of the magnons under study. This peak is not observed at another combination of ν_p and ν_m (Fig. 4b), when the spectrum intersections mentioned correspond to the frequencies ν_{k1} and ν_{k2} at different values of H, or for the conditions of Fig. 4a with $T=2.1$ K, when these intersections correspond to frequencies ν_{k1} and ν_{k2} at different values of H because of the temperature dependence of the magnon spectrum (3). Evidently under the conditions when a peak is observed (Fig. 4a), the probability of four-particle processes in which phonons participate has a resonance maximum.

On the basis of the analysis given and of the data of the experiment described, one can estimate the square of the amplitude of interaction of two magnons when they fuse into a phonon. Since $\Delta\nu' \approx 0.01$ MHz and in 1 cm^3 of the specimen $N_p \approx 10^{16}$, we get for a specimen of volume 1 cm^3

$$|\Phi_{12;3}|^2 \approx 10^{-39}\ \text{erg}^3\ .$$

The results obtained enable us to estimate $\Delta\nu(m_1+m_2\to p)$, the contribution to magnon relaxation from the process of fusion of two magnons into a phonon ($m_1+m_2\to p$), and also the value of the spin-lattice relaxation $\Delta\nu_{sl}$. Here it should be mentioned that PM of frequency ≈ 20 GHz possess an energy of the order of the mean energy of thermal magnons, and their number is of the order of the total number of thermal magnons, at $T\approx 1-2$ K. Hence the relaxation frequency of magnons in the processes $m_1+m_2\to p$, with participation by PM in the role of m_2, must be of the order of the contribution to relaxation from these processes with participation of thermal magnons. Thus

$$\Delta\nu(m_1+m_2\to p) \sim \Delta\nu' \sim 0.01\ \text{MHz}\ .$$

In its turn, the spin-lattice relaxation $\Delta\nu_{sl}$ is not less than $\Delta\nu(m_1+m_2\to p)$, since this process is one of the processes that insure transfer of energy from the magnons to the lattice. At the same time, $\Delta\nu_{sl}$ does not exceed the total relaxation of magnons $\Delta\nu\approx 0.1$ MHz. Therefore the quantity 0.01 MHz correctly characterizes $\Delta\nu_{sl}$ in order of magnitude.

The estimate of the contribution to magnon relaxation from processes of fusion of two magnons into a phonon, obtained from the experiment described, is in agreement with a theoretical calculation[10] in which the probabilities of various magnon-phonon processes in an antiferromagnet and their contribution to the magnon relaxation are considered.

By use of $\Delta\nu_{sl}$, one can estimate the superheating of the spin system in the excitation of PM. For the excesses over the threshold field used, $h/h_1\sim 2$, at $T=1.6$ K, the superheating of the spin system is of order 0.1 K.

Thus in this work there has been observed an interaction of two magnons of different frequencies, consisting of fusion of them into a phonon. An estimate has been made of the value of the spin-lattice relaxation and of the superheating of the spin system in parametric excitation of the waves.

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[1]V. V. Kveder, B. Ya. Kotyuzhanskiĭ, and L. A. Prozorova, Zh. Eksp. Teor. Fiz. **63**, 2205 (1972) [Sov. Phys. JETP **36**, 1165 (1973)].
[2]B. Ya. Kotyuzhanskiĭ and L. A. Prozorova, Zh. Eksp. Teor. Fiz. **65**, 2470 (1973) [Sov. Phys. JETP **38**, 1233 (1974)].
[3]V. I. Ozhogin, Zh. Eksp. Teor. Fiz. **58**, 2079 (1970) [Sov. Phys. JETP **31**, 1121 (1970)].
[4]L. A. Prozorova and A. I. Smirnov, Zh. Eksp. Teor. Fiz. **67**, 1952 (1974) [Sov. Phys. JETP **40**, 970 (1974)].
[5]G. A. Melkov, Fiz. Tverd. Tela (Leningrad) **17**, 1728 (1975) [Sov. Phys. Solid State **17**, 1123 (1975)].
[6]V. V. Kveder and L. A. Prozorova, Pis'ma Zh. Eksp. Teor. Fiz. **19**, 683 (1974) [JETP Lett. **19**, 353 (1974)].
[7]V. A. Kolganov, V. S. L'vov, and M. I. Shirokov, Pis'ma Zh. Eksp. Teor. Fiz. **19**, 680 (1974) [JETP Lett. **19**, 351 (1974)].
[8]A. I. Akhiezer, V. G. Bar'yakhtar, and M. I. Kaganov, Usp. Fiz. Nauk **72**, 3 (1960) [Sov. Phys. Usp. **3**, 661 (1961)].
[9]M. H. Seavey, Phys. Rev. Lett. **23**, 132 (1969).
[10]V. S. Lutovinov, Fiz. Tverd. Tela (Leningrad) **20**, in press (1978) [Sov. Phys. Solid State **20**, in press (1978)].

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Interaction between vortices and the surface of a type II superconductor and the field of a vortex in a cavity

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The response of a hollow, thin-walled Pb+30 at.% In superconducting cylinder in the mixed state to an external weak alternating magnetic field is investigated experimentally. It is found that vortices located near the inner surface of the cylinder produce a magnetic field in the cavity of the cylinder. In response to an external alternating field, the vortices adjacent to the outer surface of the cylinder execute reversible oscillations until their amplitude reaches a certain critical value, which is proportional to the period of the vortex lattice. This state corresponds to the flow of a critical current along the outer surface of the cylinder. The vortex oscillation amplitude in this case is of the order of several angstroms for vortices adjacent to the inner surface of the cylinder. The interaction between the vortices and the surface of the superconductor depends on the modulus of rigidity k, in accord with Eq. (3). The proportionality coefficient between the oscillation amplitudes for the outer and inner vortices is proportional to the external magnetic field.

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1. INTRODUCTION

The purpose of the present work is twofold. First, we want to investigate experimentally the interaction of vortices with the surface of a superconductor; second, we wish to find the magnetic field created in a macroscopic cavity by a vortex located near this cavity. The second is by no means trivial. Actually, it is well known[1] that a vortex located near the open surface of a superconductor does not create a magnetic field outside the superconductor. This is illustrated in Fig. 1a, where a vortex is shown, whose core is located near ($x_0 < \lambda$) the plane surface of a superconductor. If the superconductor occupies the halfspace $x > 0$ then the magnetic field is equal to zero in the region $x < 0$. Furthermore, the total magnetic flux which is created in the superconductor by such a vortex is not equal to the flux quantum Φ_0 but is equal to [1]

$$\Phi_v = \Phi_0 (1 - e^{-x_0/\lambda}),$$

where x_0 is the coordinate of the core of the vortex, λ is the penetration depth of the weak magnetic field. If now we close the open surface of the superconductor at a distance from the vortex that is large in comparison with λ (Fig. 1b), the picture changes radically. A superconducting current immediately passes along the inner surface of the resulting macroscopic cavity, and a magnetic field induced by the vortex develops inside the cavity. This follows both from the result of the calcula-

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