Electromagnetic radiation of particles channeled in a crystal

V. A. Bazylev and N. K. Zhevago

(Submitted 12 May 1977)
Zh. Eksp. Teor. Fiz. 73, 1697–1709 (November 1977)

A quantum theory is developed of the radiation from channeled particles in a crystal, with account taken of the spatial and frequency dispersions of the electromagnetic field. The influence of the interplanar potential on the spectral-angular distribution of the radiation is investigated. It is shown that the polarization of the medium and the momentum exchange between the photon and the crystal as a whole during the course of the radiation lead to a number of new effects in the radiation emitted by channeled particles.

PACS numbers: 61.80.Mk, 41.10.Hv

INTRODUCTION

The theory of radiation from fast charged particles in crystals has been the subject of many studies. The interest in the problem is due to the fact that the spectral and angular distributions of the radiation and its polarization in the crystal depend to a considerable degree on the direction of particle entry into the crystal, in contrast to an amorphous substance.

This feature of emission of fast particles in a crystal was first pointed out by Ferretti[1] and Ter-Mikaelyan,[2] who calculated the bremsstrahlung spectrum of a relativistic electron in a crystal. It was shown in[2] that the medium influences the electromagnetic radiation of relativistic particles because the radiation is formed over a rather long path on the particle trajectory (over the coherence length), and in particular over the coherence length of the radiation the interaction of the emitted photon with the crystal can become substantial. This interaction can cause, for example, x-ray photons to be emitted even from a relativistic particle uniformly moving in the crystal.[3] The resultant radiation has properties close to those of Cerenkov radiation and will henceforth be called quasi-Cerenkov radiation. A detailed exposition of the foregoing effects is contained in Ter-Mikaelyan's monograph.[3]

The theory of electromagnetic radiation of a uniformly moving particle in a crystal of arbitrary thickness was considered by Garibyan and Yang Shi.[4] It was shown that besides the quasi-Cerenkov radiation there is produced in the crystal radiation propagating at Bragg angles in a forward direction relative to the particle motion. The cause of this radiation is coherent Thomson scattering of virtual photons of the particle field.[1)]

The spectral and angular distributions of the radiation by a relativistic particle in the case of planar channeling in the crystal (see, e.g.,[5]) was theoretically investigated by Kumakhov[6] in the case when the potential of the interplanar field could be regarded as parabolic (the particle trajectory was a sinusoid) and the angles of particle deflection by the field were small in comparison with the emission angles. In this case the radiation by the channeled particle[6] coincides with the dipole radiation of a uniformly moving oscillator (see, e.g.,[7]) whose oscillation frequency depends parametrically on the longitudinal velocity. This dependence distinguishes, as noted in[6], the radiation of a channeled particle from the undulatory radiation.

As already mentioned, an important role in the radiation by relativistic particles is played by the interaction of the photons with the medium. This statement pertains, generally speaking, also to radiation by channeled particles.

We develop in this paper a quantum theory of the radiation by channeled particles in a crystal, with account taken of the spatial and frequency dispersions of the electromagnetic field of the radiation. For an arbitrary form of the interplanar potential and at an arbitrary ratio of the photon emission angles and the particle-scattering angles, we obtain the spectral–angular distribution of the radiation. It is shown that allowance for the dispersion of the electromagnetic field in the crystal leads to a number of new effects in the radiation by channeled particles. We determine the influence of the interplanar field on the spectral–angular distribution of the radiation and obtain the condition under which radiation from different sections of the channeled-particle trajectory is coherently summed when the radiation has a non-dipole character.

1. GENERAL EXPRESSION FOR THE RADIATION PROBABILITY IN A CRYSTAL

In the calculation of the probability of particle radiation in a crystal we shall use the general method developed by Yakimets[8] and Zhevago[9] for the calculation of the radiative losses in inhomogeneous media.

According to Zhevago,[9] the probability $d^2W/d\omega\, dt$ of emission of a photon of energy ω from a particle with charge e and with unity mass per unit time, summed over the final states of the particle, is given by[2)] $(\omega > 0)$

$$\frac{d^2W}{d\omega dt}=\frac{e^2}{2^5\pi^7}\operatorname{Im}\int\int\int_0^\infty L^{\mu\nu}(\tau,\mathbf{k},\mathbf{k}')D_{\mu\nu}(\mathbf{k},\mathbf{k}',\omega)\,d\tau d^3k d^3k'. \tag{1}$$

The tensor $L^{\mu\nu}$ is connected with the matrix elements of the operator $j^\mu(x)$ of the particle current for a transition from the initial state I to a final state F by the relation

$$L^{\mu\nu}(\tau,\mathbf{k},\mathbf{k}')=e^{-i\omega\tau}\sum_F[j^\mu(x)e^{i\mathbf{kr}}]_{IF}[j^\nu(x')e^{-i\mathbf{k}'\mathbf{r}'}]_{FI}. \tag{2}$$

 0038-5646/77/4605-0891$02.40

The remaining quantities in (1) have the following meaning:

$$D_{\mu\nu}(\mathbf{k},\mathbf{k}',\omega)=\int D_{\mu\nu}(x,x')\,e^{i\omega\tau-i(\mathbf{k}\mathbf{r}-\mathbf{k}'\mathbf{r}')}d^3r\,d^3r'\,d\tau$$

is the photon Green's function in the crystal in the momentum representation; $\mathbf{k}$ is the momentum of the radiated photon; $\tau=t-t'$, $x=(\mathbf{r},t)$.

In the case of interest to us the general expression (1) for the radiation probability density can be simplified. First, we calculate the radiation probability in a sufficiently thick crystal the effects of whose boundaries on the radiation can be neglected. Next, using translational invariance, we can represent the photon Green's function in the form

$$D_{\mu\nu}(\mathbf{r},\mathbf{r}',\tau)=\sum_{\mathbf{K}_h}D_{\mu\nu}^{(h)}(\mathbf{r}-\mathbf{r}',\tau)\,e^{-i\mathbf{K}_h\mathbf{r}'}, \tag{3}$$

where K_h is an arbitrary vector of the reciprocal lattice of the crystal.

Second, the motion of the radiating channeled particle in the crystal turns out to be "almost linear." The tensor $L^{\mu\nu}(\tau,\mathbf{k},\mathbf{k}')$ can therefore be approximately represented in the form

$$L^{\mu\nu}(\tau,\ \mathbf{k},\ \mathbf{k}')=L^{\mu\nu}(\tau,\ \mathbf{k},\ \mathbf{k})\,\delta(\mathbf{k}-\mathbf{k}'). \tag{2'}$$

Substituting in (1) photon Green's function (3) in the momentum representation and the tensor $L^{\mu\nu}(\tau,\mathbf{k},\mathbf{k}')$ in the form (2′) we obtain

$$\frac{d^2W}{d\omega dt}=\frac{e^2}{4\pi^4}\,\mathrm{Re}\int_0^\infty\int L^{\mu\nu}(\tau,\mathbf{k},\mathbf{k})\,\mathrm{Im}\,D^{(h=0)}(\mathbf{k},\omega)\,d^3k\,d\tau, \tag{4}$$

where

$$D_{\mu\nu}^{(h=0)}(\mathbf{k},\omega)=\int D_{\mu\nu}^{(h=0)}(\boldsymbol{\rho},\omega)\,e^{-i\mathbf{k}\boldsymbol{\rho}}d^2\rho$$

is the Fourier component of the zeroth term of the expansion of the total photon Green's function (3) in the reciprocal-lattice vectors.

The result (4) for the probability density of photon emission by a particle in a crystal agrees with the analogous result (see formula (6) of[9]) obtained for a one-dimensional periodic structure. The representation of the tensor $L^{\mu\nu}(\tau,\mathbf{k},\mathbf{k}')$ in the form (2′) corresponds here in the classical limit to averaging of the probability density of the radiation (1) over the time of flight of the particle through a unit cell of the crystal.

2. MOTION OF CHARGED PARTICLE IN A CRYSTAL

We consider for simplicity the case when a positively charged relativistic particle with charge e enters at an angle $\psi\leqslant\psi_c$ through one of the planes of the crystal. Here $\psi_c\approx[U_0/(E-1)]^{1/2}$ is the critical angle for planar channeling (see, e.g.,[5]), E is the particle energy, and U_0 is the characteristic value of the particle interaction energy with the channeling planes.

A particle moving in the interplanar potential $\Phi(x)$ (x is the particle displacement from the median plane) has, besides, energy, a definite projection $\mathbf{p}^{\parallel}$ of the generalized momentum on the channeling plane. Let the momentum $\mathbf{k}$ of the emitted photon be small compared with the particle momentum $\mathbf{p}^{\parallel}$. We can then neglect the quantum recoil in the radiation, and the longitudinal motion relative to the crystal plane turns out to be quasiclassical. The transverse motion of channeled particles always has nonrelativistic velocities $v^{\perp}\lesssim\psi_c$.

Thus, the interaction of the particle spin with the electric field of the planes can be neglected and it can be assumed that the particle is characterized by a wave function $\Psi(\mathbf{r},t)$ satisfying the Klein-Gordon equation

$$\left[\left(i\frac{\partial}{\partial t}-e\Phi(x)\right)^2+\Delta-1\right]\Psi(\mathbf{r},t)=0.$$

We seek the solution of the equation in the form

$$\Psi(\mathbf{r},\ t)=(2E^{\parallel})^{-1/2}e^{-iEt+i\mathbf{p}^{\parallel}\boldsymbol{\rho}}\varphi(x), \tag{5}$$

where $\boldsymbol{\rho}=(y,z)$ is the radius vector in the median plane; $E=E^{\parallel}+E^{\perp}$, $E^{\parallel}=[\mathbf{p}^{\parallel 2}+1]^{1/2}$ is the particle energy connected with the longitudinal motion.

The energy $E^{\perp}$ connected with the transverse motion of the channeled particle takes on discrete values ($E_n^{\perp}\lesssim U_0$). The kinetic energy $E^{\parallel}-1$ of the longitudinal motion of the relativistic particles always exceeds their potential energy in the interplanar field ($E^{\parallel}-1\gg U_0$). We can therefore neglect in the Klein-Gordon equation the term quadratic in $\Phi(x)$, as well as the quantity $E^{\perp}$ in comparison with $E^{\parallel}$. As a result we obtain for the function $\varphi(x)$ the equation

$$\left[-\frac{1}{2E^{\parallel}}\frac{d^2}{dx^2}+e\Phi(x)\right]\varphi(x)=E^{\perp}\varphi(x), \tag{6}$$

which has the same form as the Schrödinger equation for a particle with a relativistic "mass" $E^{\parallel}$.

The current-density operator for scalar particles is known to be of the form

$$j^{\nu}(x)=i\left(\frac{\overleftarrow{\partial}}{\partial x_\nu}-\frac{\overrightarrow{\partial}}{\partial x_\nu}\right).$$

If we use a photon Green's-function gauge with zero scalar potential (see Sec. 3), then we need calculate only the spatial component of the tensor $L^{\mu\nu}(\tau,\mathbf{k},\mathbf{k})$. Using the particle wave functions in the form (5), we obtain for the spatial components of the transition current

$$(\mathbf{j}(x)e^{i\mathbf{k}\mathbf{r}})_{IF}=\mathbf{v}A_{if}(k_x)\exp\{i\omega_{if}t+i(E_i^{\parallel}-E_f^{\parallel})t\}(2\pi)^2\delta(\mathbf{p}_i^{\parallel}-\mathbf{p}_f^{\parallel}-\mathbf{k}^{\parallel}),$$

where

$$A_{if}(k_x)=\int\varphi_i^*(x)\,e^{ik_xx}\varphi_f(x)\,dx, \tag{7}$$

where $\mathbf{v}$ is a three-dimensional vector with components $v^x=\omega_{if}/k_x$, $(v^y,v^z)=\mathbf{v}^{\parallel}$, $\mathbf{v}^{\parallel}=\mathbf{p}_i^{\parallel}/E_i^{\parallel}\approx\mathbf{p}_f^{\parallel}/E_f^{\parallel}$ is the longitudinal velocity of the particle, $\omega_{if}=E_i^{\perp}-E_f^{\perp}$ is the difference between the energies of the particles in the states $\varphi_i(x)$ and $\varphi_f(x)$ connected with the transverse motion in the interplanar field, and k_x and $\mathbf{k}^{\parallel}$ are the photon momentum components perpendicular and parallel to the channeling plane, respectively. In the calculation of the tensor $L^{\mu\nu}(\tau,\mathbf{k},\mathbf{k})$ we can sum over the final projections $\mathbf{p}_f^{\parallel}$ of the particle momentum on the channeling plane. The tensor then takes the form

$$L^{jk}(\tau,\mathbf{k},\mathbf{k})=e^{-i(\omega-\mathbf{k}\|\mathbf{v}\|)\tau}\sum_f|A_{if}(kx)|^2e^{i\omega_{if}\tau}v^jv^k,\quad j,k=1,2,3. \tag{8}$$

3. PHOTON GREEN'S FUNCTION IN A CRYSTAL

To calculate the photon Green's function in a medium we use Dyson's equation (see, e.g., [10] Chap VI):

$$D_{ik}(\mathbf{r},\mathbf{r}',\omega)=D_{ik}^{(0)}(\mathbf{r}-\mathbf{r}',\omega)$$
$$+\ D_{il}^{(0)}(\mathbf{r}-\mathbf{r}_1,\omega)\Pi_{lm}(\mathbf{r}_1,\mathbf{r}_2,\omega)D_{mk}(\mathbf{r}_2,\mathbf{r}',\omega)\,d^3r_1d^3r_2, \tag{9}$$

where $D_{ik}^{(0)}(\mathbf{r}-\mathbf{r}',\omega)$ is the photon vacuum Green's function, and $\Pi_{lm}(\mathbf{r}_1,\mathbf{r}_2,\omega)$ is the polarization and is determined by the interaction of the photons with the medium (see below). We apply to both sides of (9) the operator $R_{ji}^{(0)}=\omega^2\delta_{ij}-\mathrm{curl}_{jn}\,\mathrm{curl}_{ni}$. We take into account the fact that in a gauge with zero scalar potential the function $D_{il}^{(0)}(\mathbf{r}-\mathbf{r}_1,\omega)$ satisfies the equation $R_{ji}^{(0)}D_{il}^{(0)}=4\pi\delta_{jl}\delta(\mathbf{r}-\mathbf{r}_1)$, while the polarization operator, by virtue of translational invariance, can be represented in the form

$$\Pi_{jm}(\mathbf{r},\mathbf{r}_2,\omega)=\sum_{\mathbf{K}_h}\Pi_{jm}^{(h)}(\mathbf{r}-\mathbf{r}_2,\omega)e^{i\mathbf{K}_h\mathbf{r}}.$$

We then obtain for the photon Green's function in a crystal, in the momentum representation, the following functional equation:

$$R_{ji}^{(0)}(\mathbf{k})D_{ik}(\mathbf{k},\mathbf{k}',\omega)=4\pi\delta_{jk}(\mathbf{k}-\mathbf{k}')$$
$$+4\pi\sum_{\mathbf{K}_h}\Pi_{jm}^{(h)}(\mathbf{k},\omega)D_{mk}(\mathbf{k}+\mathbf{K}_h,\mathbf{k}',\omega), \tag{10}$$

where

$$R_{ji}^{(0)}(k)=(\omega^2-k^2)\delta_{ji}-k_jk_i,$$
$$\Pi_{jm}^{(h)}(\mathbf{k},\omega)=\int\Pi_{jm}^{(h)}(\rho,\omega)e^{-i\mathbf{k}\rho}d^3\rho.$$

Consider for simplicity the case (see (11) and (14)) when the tensor structure of the polarization operator is determined by the relation

$$\Pi_{jm}^{(h)}(\mathbf{k},\omega)=\Pi_{(tr)}^{(h)}(k,\omega)\left(\delta_{jm}-\frac{k_jk_m}{k^2}\right)+\Pi_{(l)}^{(h)}(k,\omega)\frac{k_jk_m}{k^2},$$

where $\Pi_{(tr)}^{(h)}(k,\omega)$, $\Pi_{(l)}^{(h)}(k,\omega)$ are the respective amplitudes of the scattering of transverse and longitudinal virtual photons with momentum $\mathbf{k}$ and energy ω by a unit cell of the crystal, with transfer of a momentum $\mathbf{K}_h$ (the subscript $h=0$ corresponds to the forward-scattering amplitude).

The photon Green's function has in this case an analogous tensor structure. We are interested in the emission of transverse photons in the crystal. The transverse part of the photon Green's function satisfies according to (10) the equation

$$R(\mathbf{k})D_{tr}(\mathbf{k},\mathbf{k}',\omega)=4\pi\delta(\mathbf{k}-\mathbf{k}')+4\pi\sum_{\mathbf{K}_h\neq0}\Pi_{tr}^{(h)}(k,\omega)$$
$$\times D_{tr}(\mathbf{k}+\mathbf{K}_h,\mathbf{k}',\omega), \tag{10'}$$

where

$$R(k)=\omega^2\varepsilon^{(0)}(k,\omega)-k^2,$$
$$\varepsilon^{(0)}(k,\omega)=1-\frac{4\pi}{\omega^2}\Pi_{tr}^{(h=0)}(k,\omega).$$

We solve Eq. (10′) by successive approximations. In the zeroth approximation, neglecting spatial dispersion, we obtain

$$D_{tr}(\mathbf{k},\mathbf{k}',\omega)=4\pi\delta(\mathbf{k}-\mathbf{k}')/R(\mathbf{k}). \tag{11}$$

We can confine ourselves to this approximation, for example, in the calculation of the spectrum of radiation with wavelengths greatly exceeding the lattice period of a cubic crystal far from the resonant absorption lines. For shorter radiation wavelengths, it is necessary, generally speaking, to take the spatial dispersion into account.

The probability of emission by a particle in a crystal is determined by that part of the Green's function $D_{tr}(\mathbf{k},\mathbf{k}',\omega)$ which is singular in $\mathbf{k}-\mathbf{k}'$ and is the zeroth term of the expansion of $D_{tr}(\mathbf{r},\mathbf{r}',\omega)$ in the crystal reciprocal-lattice vectors (see (4)). In the next application, which takes the spatial dispersion into account, we get

$$D_{tr}^{(h=0)}(\mathbf{k},\omega)=\frac{4\pi}{R(\mathbf{k})}+(4\pi)^3\sum_{\mathbf{K}_h\neq0}\frac{|\Pi_{tr}^{(h)}(k,\omega)|^2}{R^2(\mathbf{k})R(\mathbf{k}-\mathbf{K}_h)}. \tag{12}$$

For a sufficiently thick crystal and for emission wavelengths comparable with the length of the lattice period, the applicability of expression (12) obtained by successive approximation calls for an additional analysis. In particular, if several vectors of the type $\mathbf{k}-\mathbf{K}_h$ are located on the Ewald sphere $k^2=\varepsilon^{(0)}\omega^2$, then a more consistent method for the solution of (10′) is the method of dynamic theory of diffraction (see, e.g., [11]).

We shall show that even for insufficiently hard photons with wavelength much shorter than the crystal lattice period ($\omega\gg K_h$), when the successive-approximation method can be used, the spatial dispersion leads to noticeable effect in the emission by channeled particles. Photons with such wavelengths are emitted at small angles to the direction of motion of the particle (see below), and in this case the denominator $R(\mathbf{k}-\mathbf{K}_h)$ in (12) can be approximately represented in the form

$$R(\mathbf{k}-\mathbf{K}_h)\approx\omega^2\varepsilon^{(h)}(\omega)-k^2, \tag{13}$$

where $\varepsilon^{(h)}(\omega)=1-\omega_p^2/\omega^2+2K_h^z/\omega$, $\omega_p=4\pi Ne^2$ is the plasma frequency of the medium, and K_h^z is the projection of the reciprocal-lattice vector on the longitudinal velocity of the particle. When summing in (12) over the reciprocal-lattice vectors it is necessary to confine oneself to vectors $(\mathbf{K}_h)_{\max}$ such that the condition $|\varepsilon_{\max}^{(h)}-1|\ll1$ is satisfied and the photon emission angles remain small. The employed approximation is more accurate the faster the decrease of the Green's function expansion coefficient with increasing $|\mathbf{K}_h|$. In this approximation, the transfer of a momentum $\mathbf{K}_h$ from the photon to the crystal is equivalent to the change of the phase velocity of the photon, which is determined by the effective dielectric constant $\varepsilon^{(h)}(\omega)$ (cf. formula (13) of[9]).

We are interested in the x-ray spectrum at frequencies when the absorption of virtual photons is immaterial ($|\mathrm{Im}\,\Pi_{tr}^{(h)}(k,\omega)| \ll \mathrm{Re}\,\Pi_{tr}^{(h)}(k,\omega)|$), as are consequently also processes such as ionization and excitation of the atoms (nuclei) directly by the particle.[9] In the same approximation, in the calculation of the photon-scattering probability $\Pi_{tr}^{(h)}(k,\omega)$, we can neglect the difference between the virtual and real photons ($k^2=\omega^2$). Then, if the photon scattering is mainly by the quasi-free electrons of the medium, and the unit cell of the crystal contains one atom, the probability of photon scattering with allowance for the thermal vibrations of the lattice takes the rather simple form

$$|\Pi_{tr}^{(h)}(k,\omega)|^2 = \left[\frac{\omega_p^2}{4\pi}\frac{|f(\mathbf{K}_h)|}{Z}\right]^2 e^{-2W(\mathbf{K}_h)}, \tag{14}$$

where $f(\mathbf{K}_h)$ is the atomic form factor, Z is the number of electrons in the atom, and $W(\mathbf{K}_h)$ is the Debye-Waller factor.

4. PARTICLE EMISSION IN PLANAR CHANNELING

By substituting in the general expression (4) the tensor $L^{\mu\nu}(\tau,\mathbf{k},\mathbf{k})$ in the form of (8) and the photon Green's function $D_{\mu\nu}^{(h=0)}(\mathbf{k},\omega)$ in the form determined by formula (11) or formulas (12) and (13), and subsequently integrating with respect to the modulus k of the photon momentum and with respect to the time variable τ, we arrive at the following result for the spectral–angular probability density $d^2w/d\omega\,d\Omega$ of photon emission by the particle per unit time in the case of planar channeling in the crystal:

$$\frac{d^2w}{d\omega\,d\Omega} = \frac{e^2\omega}{2\pi}\sum_f |A_{if}(n_x\omega\sqrt{\varepsilon^{(0)}})|^2\Big[(n_x^{-2}-1)\frac{\omega_{if}^2}{\omega^2\varepsilon^{(0)^{1/2}}} + (1-n_z^2)v_z^2\sqrt{\varepsilon^{(0)}} - \frac{2\omega_{if}}{\omega}v_z n_z\Big]\delta[\omega(1-n_z v_z\sqrt{\varepsilon^{(0)}})-\omega_{if}], \tag{15}$$

$$\frac{d^2w}{d\omega\,d\Omega} = \frac{e^2\omega}{2\pi}\sum_f\sum_{K_h}|A_{if}(\omega\theta\cos\varphi)|^2 P^{(h)}(\omega)\left(\frac{\omega_{if}^2}{\omega^2\theta^2\cos^2\varphi} - 2\frac{\omega_{if}}{\omega} + \theta^2\right) \times\delta\left(\frac{K_h^z+\omega_{if}}{\omega} - \frac{E^{-2}+\omega_p^2/\omega^2}{2} - \frac{\theta^2}{2}\right). \tag{15'}$$

Expression (15) is valid here for relatively soft emission frequencies, while (15′) holds for x-ray frequencies and small emission angles ($\theta \ll 1$); $\mathbf{n}$ is a unit vector in the emission direction, and the coordinates x and z are chosen respectively perpendicular to the channeling plane and along the longitudinal velocity v_z; θ and φ are the polar and azimuthal emission angles; $d\Omega = \sin\theta\,d\theta\,d\varphi$ is the differential solid angle. The matrix element A_{if} of the radiative transition are determined by expression (7); $\omega_{if}=\omega_{if}(E^{\parallel})$ is the difference between the transverse-motion level energies and depends on the longitudinal-motion energy $E^{\parallel}$. The coefficient $P^{(h)}(\omega)$ is the probability of transfer of a longitudinal momentum K_h^z from the photon to the crystal in the radiation process, and is given by

$$P^{(h\neq0)}(\omega) = \frac{(4\pi|\Pi_{tr}^{(h)}|)^2}{\omega^2(K_h^z)^2} \ll 1, \quad P^{(0)}(\omega) = 1 - \sum_{\mathbf{K}_h\neq0}P^{(h)}\omega, \tag{16}$$

where $|\Pi_{tr}^{(h)}|^2$, in particular, is determined by relation (14).

In the sums over the final states f in (15) and (15′) we can separate a term with $f=i$, which determines the probability of the Cerenkov radiation at the soft frequencies (15) or of quasi-Cerenkov radiation[3] at the hard frequencies (15′).

On the other hand, at $f\neq i$ and $\mathbf{K}_h=0$, the terms in (15′) determine the probability of emission by a channeled particle in the interplanar field in a medium with frequency dispersion. The terms in (15′) with $f\neq i$ and $K_h^z\neq0$ describe more complicated processes, in which account is taken of the spatial dispersion, so that when it emits a hard photon the particle can exchange a longitudinal momentum K_h^z with the crystal as a unit.

Since the long-wave dielectric constant of the crystal $\varepsilon^{(0)}(\omega)$ is usually greater than unity, it follows that, according to (15), a process is possible in which the transverse energy of the channeled particle increases ($\omega_{if}<0$) and the particle emits a soft photon (anomalous Doppler effect). For complicated particles such as atoms, this effect was predicted by Frank.[12] For x-ray frequencies, the dielectric constant, as a rule, is smaller than unity.[3)] However, as follows from our result, in a crystal with allowance for spatial dispersion the energy of the transverse motion can increase upon emission of an x-ray photon. The analogy of this effect with the anomalous Doppler effect becomes complete if it is recognized that the particle channeled by the planes of the crystal is a uniformly moving "one-dimensional atom" (see (6)), and at hard frequencies the crystal, with allowance for spatial dispersion, is characterized by the effective dielectric constant (13), which can exceed unity.

The possibility of the onset of the anomalous Doppler effect in the case of radiation by channeled particles at frequencies close to an absorption line of a Mössbauer nucleus was investigated theoretically by Baryshevskiĭ and Dubovskaya.[15] Such an investigation, however, should in general be carried out with account taken of the virtual photons, since $|1-\varepsilon'|\sim\varepsilon''$ near an absorption line.

Let us analyze the spectral–angular distribution of the spontaneous emission of a channeled particle (terms with $\mathbf{K}_h=0$) in the region of relatively hard frequencies ($\omega \gg \omega_p$). We consider first effects that do not depend on the concrete form of the interplanar potential.

According to (15′), a photon with fixed energy ω is emitted when a particle goes over to a level f at a definite angle

$$\theta^{(f)} = \sqrt{\frac{2\omega_{if}}{\omega} - \left(E^{-2}+\frac{\omega_p^2}{\omega^2}\right)}. \tag{17}$$

This result is the consequence of neglecting the finite path of the particle in the crystal, the metastability of the transverse-motion levels ($\mathrm{Im}\,\omega_{if}$), and the absorption of the virtual photons in the course of the radiation ($\mathrm{Im}\,\Pi_{tr}(k,\omega)$) (allowance for these photons leads to replacement of the δ function by a function with finite "width").

The x-ray frequency band obtained in spontaneous transition of a particle from the level i to the level f is determined by the condition that the radicand in (17) be positive:

$$\omega_{min}^{(f)} \leqslant \omega \leqslant \omega_{max}^{(f)},$$
$$\omega_{\substack{max,\\min}}^{(f)} = E^2[\omega_{if} \pm (\omega_{if}^2 - \omega_p^2 E^{-2})^{1/2}]. \tag{18}$$

When the particle energy E^{II} decreases to a value $E_{cr}^{(f)}$ defined by the relation

$$E_{cr}^{(f)} = \omega_p / \omega_{if}(E_{cr}), \tag{19}$$

the x-ray band emitted in the $i \to f$ transition contracts towards the frequency $\omega_{cr}^{(f)} = \omega_p^2/\omega_{if}(E_{cr}^{(f)})$. At particle energies $E < E_{cr}^{(f)}$ the emission of x-ray frequencies on going to the level f becomes impossible because of the influence of the polarization of the medium on the radiation by the channeled particle.

It follows from (17) that the end-point frequencies $\omega_{min}^{(f)}$ and $\omega_{max}^{(f)}$ are emitted at a zero polar angle θ. The maximum emission angle possible for hard frequencies is given by

$$\theta_{max}^{(f)} = (\omega_{if}^2/\omega_p^2 - E^{-2})^{1/2}. \tag{20}$$

At this angle there is emitted a single (at a fixed transition $i \to f$) frequency $\omega(\theta_{max}) = \omega_p^2/\omega_{if}$. Each fixed emission angle $\theta \neq \theta_{max}$ corresponds to two emission frequencies[4] in the allowed band $\omega_{min}^{(f)} \leqslant \omega \leqslant \omega_{max}^{(f)}$ ("complex" Doppler effect).

These are the spectral and angular characteristics of the radiation by a channeled particle, and follow essentially from the condition for the energy and momentum conservation in the radiation.

The spontaneous-emission probability is determined by the values of the matrix elements A_{if} of the radiative transition between the levels i and f:

$$A_{if}(n_x\omega) = \int \varphi_i^*(x)\varphi_f(x) e^{i\omega\theta x \cos\varphi} dx. \tag{21}$$

Using (17), we can easily show that the phase δ of the exponential factor in the integrand of (21) reaches its maximum value at the emission frequency $\omega = \omega_{if}(E)E^2$ and is determined (at $\cos\varphi = 1$) by the equation

$$\delta_{max}^{(f)} = E\omega_{if}(E)(1 - \omega_p^2/\omega_{if}^2 E^2)^{1/2} x. \tag{22}$$

If we use for the transition frequencies the estimate (see (25) below) $\omega_{if} \sim (U_0/E)^{1/2} d^{-1}$ and use the value d for the effective x, then we find that the phase is small ($\delta \ll 1$) if[5] $(U_0 E)^{1/2} \ll 1$ (U_0 is the characteristic value of the interplanar potential). In this case the radiation has a dipole character:

$$A_{ii} \approx 1, \quad A_{if}(n_x\omega) \approx i\omega\theta(\cos\varphi)x_{if}, \tag{23}$$

where

$$x_{if} = \int \varphi_i^*(x) x \varphi_f(x) dx.$$

The frequency dependence of the spectral distribution $dw^{(f)}/d\omega$ of the probability of emission when the particle goes from the state i to the state f of transverse motion has a universal form that does not depend on the type of interplanar potential

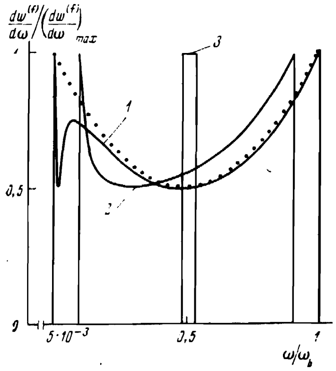

FIG. 1. Emission spectrum of a particle of unit mass, channeled in a crystal, at different particle energies. The abscissa is the ratio ω/ω_b of the emission frequency to the frequency $\omega_b = 2\omega_{if}E^2$, and the ordinate is the ratio of the emission spectral probability density per unit time to its maximum value $(dw^{(f)}/d\omega)_{max} = e^2\omega_{if}^2 |x_{if}|^2$. Solid curves—emission spectrum with allowance for polarization of the medium, dotted—without allowance for the polarization. Curves 1–3 correspond to particles with energies $E = 50E_{cr}$, $3E_{cr}$, and $1.001E_{cr}$, respectively.

$$\frac{dw^{(f)}}{d\omega} = e^2\omega_{if}^2 |x_{if}|^2 \left[1 - \frac{\omega}{\omega_{if}}\left(E^{-2} + \frac{\omega_p^2}{\omega^2}\right) + \frac{\omega^2}{2\omega_{if}^2}\left(E^{-2} + \frac{\omega_p^2}{\omega^2}\right)^2\right][\eta(\omega_{min}^{(f)}) - \eta(\omega_{max}^{(f)})], \tag{24}$$

where $\eta(\omega)$ is the Heaviside unit function,

$$dw/d\omega = \sum_{f<i} \frac{dw^{(f)}}{d\omega}.$$

The emission spectrum of a channeled particle in the dipole approximation (24) is illustrated in Fig. 1. The ordinates represent the ratio

$$\frac{dw^{(f)}}{d\omega} \Big/ \left(\frac{dw^{(f)}}{d\omega}\right)_{max}$$

and the abscissas the ratio $\omega/2\omega_{if}E^2$ (cf.[6]). The solid lines show the transformation of the emission spectrum with increasing energy of the positron (electron), starting with $E_{cr}^{(f)}$, below which the emission of hard quanta ($\omega \gg \omega_p$) is impossible,[6] for different values of the energy ($E = 1.001\,E_{cr}^{(f)}$, $E = 3E_{cr}^{(f)}$, $E = 50\,E_{cr}^{(f)}$).

For comparison, the figure shows the spectrum without allowance for the polarization of the medium[6] (dotted line).

With further increase of the particle energy ($E \gg E_{cr}^f$), the effect of the polarization reduces in fact to an abrupt termination of the emission-spectrum curve at $\omega \approx \omega_p^2(dE/8U_0)^{1/2} \sim 10^2 E^{1/2}$ eV, and the maximum differs by a factor of two from the dotted curve in the region of $\omega \sim \omega_p^2(dE/2U_0)^{1/2}$.

At sufficiently high particle energies $E \gtrsim U_0^{-1}$ it is incorrect to calculate the probability of the x-radiation on the basis of the dipole approximation[7] and the matrix elements A_{if} must be calculated from the general formula (21). The frequency dependence of the radiation is then determined by the actual form of the interplanar potential. Since the quantities A_{if} doe not contain a small factor in the high-energy case considered here, the probability of emission turns out to be higher than the value given in the dipole approximation. The maxima of the spectral probability density occur now at frequencies that are determined by the proximity of the exponential factor in (21) to the quantities $2\pi n$ $(n=1,2,\ldots, n_{max})$ in the region where substantial changes take place in the particle wave functions. Unlike in the dipole approximation, these frequencies do not coincide, in general, with the end-point frequencies ω_{min} and ω_{max}.

5. INFLUENCE OF THE INTERPLANAR POTENTIAL ON THE EMISSION SPECTRUM

The matrix elements $A_{if}(n_x\omega)$ of the radiative transition of a channeled particle have a relatively simple analytic form for two types of interplanar potential.

For particles with a sufficiently low initial transverse energy, the interplanar potential can be regarded as almost parabolic[5]: $e\Phi(x)=(4U_0/d^2)x^2$. In this case

$$\begin{gathered}\omega_{if}=\omega_0(i-f),\\ |A_{if}(n_x\omega)|^2=2^{i-f}\frac{f!}{i!}e^{-2\xi}\xi^{i-f}|L_i^{i-f}(2\xi)|^2,\quad i>j,\\ |A_{if}|^2=|A_{fi}|^2,\end{gathered}\tag{25}$$

where $\omega_0^2=8U_0/d^2E$, $\xi=n_x^2\omega^2d/8(2U_0E)^{1/2}$, $L_i^{i-f}(\xi)$ is a Laguerre polynomial in ξ. In the quasiclassical limit, when the quantum numbers of the transverse motion are large $(i,f\gg1)$ and their difference is small $(|i-f|\ll i)$, the matrix elements are expressed in terms of Bessel functions: $|A_{if}(n_x\omega)|^2\approx J_{f-i}^2(n_x\omega a)$, where $a=(2f/\omega_0E)^{1/2}$ is the amplitude of the transverse harmonic oscillations of the particle; this agrees with the result obtained by Ginzburg and Éĭdman for a uniformly moving classical oscillator.

In the dipole approximation, both in the quantum and in the classical case, the matrix elements of the radiative transition have for a parabolic potential the form

$$|A_{if}(n_x\omega)|^2=\frac{n_x^2\omega^2a^2}{4}(\delta_{i+1,f}+\delta_{i-1,f}),\tag{26}$$

where δ_{mn} is the Kronecker symbol. Thus, the dipole-radiation spectrum of a particle channeled by a parabolic potential is determined by the only possible transition to the lower transverse-motion level closest to the initial level. Radiation with a transition to an upper level $f=i+1$ is possible with emission of anomalous Doppler frequencies (see Sec. 4). If we substitute $|A_{if}|^2$ in the form (26) in expression (24) and neglect frequency dispersion, then we obtain for the spectral density of the emission probability in the spontaneous transition $i\to f$ a result that coincides with the analogous result of Kumakhov.[6]

If the initial transverse energy is high enough (the particle enters the crystal at near-critical angle), the interplanar potential acts as a "potential box" for the particle, with walls that can be assumed for simplicity to be infinitely high. The matrix elements A_{if} and the transition frequencies are determined in this case by the relations

$$\begin{gathered}|A_{if}(n_x\omega)|^2=y^2\left|\frac{|\sin[y-\pi(f-i)/2]|}{y^2-[\pi(f-i)/2]^2}-\frac{|\sin[y-\pi(f+i)/2]|}{y^2-[\pi(f+i)/2]^2}\right|^2\\ \omega_{if}=\frac{\pi^2}{2Ed^2}(i^2-f^2),\quad y=n_x\omega d.\end{gathered}\tag{27}$$

In the quasiclassical limit $i\gg1$, $f\gg1$, and $|i-f|\ll i$, motion in this potential corresponds to a sawtooth particle trajectory, while the values of ω_{if} and $|A_{if}|^2$ become

$$\begin{gathered}\omega_{if}=\frac{\pi^2f}{Ed^2}(i-f),\\ |A_{if}(n_x\omega)|^2=y^2\frac{\sin^2[y-\pi(f-i)/2]}{\{y^2-[\pi(f-i)/2]^2\}^2}.\end{gathered}\tag{28}$$

In the dipole approximation $(y\ll1)$ the quasiclassical matrix elements A_{if} $(f\neq i)$ decrease slowly enough (like $(f-i)^{-2}$) with increasing energy-level difference. Consequently, several (partially overlapping) bands, corresponding to transitions to different final transverse-motion levels, should be observed in the dipole-radiation spectrum of a channeled particle in a non-parabolic potential. This difference between parabolic and other potentials, which is the consequence of the selection rules for the matrix element of the oscillator dipole moment, vanishes in the general case when the radiation is not of the dipole type (cf. (25) and (27)).

CONCLUSION

The results in this paper on the spectral-angular probability distribution of radiation from a particle planarly channeled in a crystal apply to a considerable degree also to radiation in axial channeling. Our investigations are of practical interest for the following reasons:

1. The spectral-angular density of the x-radiation, as shown by estimates,[6] is relatively high even when the channeling-particle radiation is of the dipole type. According to our results it is possible, by using the effect of the polarization of the medium, to concentrate all the radiation in a narrow spectral interval $\Delta\omega\lesssim10^{-2}\omega$ in the x-ray frequency region $\omega\sim100$ eV to 1 keV. On the other hand, in the channeling of positrons with realistic energies $E>10$ GeV, the angles of deflection of the particles by the interplanar field become comparable with the effective x-ray angles, and radiation is formed coherently from different section of the particle trajectory (non-dipole radiation). One should then expect a relative increase in the spectral-angular probability density of the radiation. Since the effective potential of an atomic chain is larger by at least one order of magnitude than in the case of planes, the non-dipole character of the x-radiation in axial channeling should set in already at energies $E\sim1$ GeV.

2. A channeled particle has finite degrees of freedom and constitutes a model of a one-dimensional or two-dimensional (in the case of axial channeling) atom whose levels are quantized as functions of the longitudinal-motion energy. From this point of view, there is a realistic possibility of experimentally observing the anomalous Doppler effect in the optical band, inasmuch as, in contrast to atoms, it is possible to move a channeled particle with relativistic velocity. The foregoing applies also to the effect predicted in Sec. 4, of x-radiation from a

channeled particle whose transverse energy has been increased, an effect that appears when account is taken of the momentum exchange between the radiated photon and the crystal as a unit.

3. Spontaneous transitions to low-lying transverse-motion levels lead to damping of the classical amplitude of the transverse oscillations of the channeled particle. When account is taken of the interaction of the virtual photons with the crystal, transitions to higher levels of the transverse motion become possible, with emission of an optical or x-ray photon, and these lead to buildup of transverse oscillations. In particular, transitions to the continuous spectrum of the transverse motion in the course of the radiation lead to dechanneling of the particle. The probability of the corresponding processes can be calculated with the aid of expressions (15) and (15′) by integrating over the angles and frequencies of the emitted photons and by summing over the quantum numbers of the continuous spectrum of the transverse motion. For light high-energy channeled particles the radiative buildup and damping of the transverse oscillations may turn out to prevail over the nonradiative dechanneling processes.

4. It follows from our results that the positions of the end-point frequencies of the radiation in the case of spontaneous transition of the channeled particles between transverse levels determine the energies of these levels, while the spectral density of the radiation energy determines the dipole moments of the transitions in the interplanar field. It is thus possible to draw a number of important conclusions concerning the form of the channeling potential by investigating the spectrum of the x rays emitted by the particles.

The authors thank V. I. Glebov for the computer calculations.

1) The forward radiation is due to Thomson scattering near the boundary of the medium and should be observed also in amorphous substances.

2) Here $\hbar = m = c = 1$, and the metric is $g_{\mu\nu} = 0$ $(\mu \neq \nu)$, $g_{11} = g_{22} = g_{33} = -g_{44} = 1$.

3) Exceptions are the frequecny regions near x-ray lines[13] and near the absorption bands[14] of the medium.

4) The possibility of the appearance of the complex Doppler effect in radiation by channeled particle was pointed out in[15].

5) This condition is equivalent to the classical condition that the particle deflection angles in the field be small in comparison with the emission angles.

6) Typical values for crystals are $E_{cr} \sim 10-30$ MeV and $\omega_{cr} \sim 0.1-1$ keV; the value ω_{cr} should lie in the region where the "plasma" formula holds for $\varepsilon^{(0)}(\omega)$.

7) For positrons and for the characteristic $U_0 \approx 10$ eV, this energy is $E \approx 25$ GeV.

1. B. Ferretti, Nuovo Cimento **7**, 118 (1950).
2. M. L. Ter-Mikaelyan, Zh. Eksp. Teor. Fiz. **25**, 289, 296 (1953).
3. M. L. Ter-Mikaelyan, Vliyanie sredy na élektromagnitnye protsessy pri vysokikh energiyakh (Effect of Medium on Electromagnetic Processes at High Energies), Akad. Nauk Arm. SSSR, Erevan, 1969.
4. G. M. Garibyan and Yan Shi, Zh. Eksp. Teor. Fiz. **61**, 930 (1971); **63**, 1198 (1972) [Sov. Phys. JETP **34**, 495 (1972); **36**, 631 (1973)].
5. D. S. Gemmell, Rev. Mod. Phys. **46**, 129 (1974).
6. M. A. Kumakhov, Dokl. Akad. Nauk SSSR **230**, 1077 (1976) [Sov. Phys. Dokl. **21**, 581 (1976)].
7. V. L. Ginzburg and V. Ya. Éĭdman, Zh. Eksp. Teor. Fiz. **36**, 1823 (1959) [Sov. Phys. JETP **9**, 1300 (1959)].
8. V. V. Yakishets, Dokl. Adad. Nauk SSSR **159**, 728 (1964).
9. N. K. Zhevago, Zh. Eksp. Teor. Fiz. **72**, 428 (1977) [Sov. Phys. JETP **45**, 224 (1977)].
10. A. A. Abrikosov, L. P. Gor'kov, and I. E. Dzyaloshinskiĭ, Metody kvantovoĭ teorii polya v statisticheskoĭ fizike (Quantum Field Theoretical Methods in Statistical Physics), Fizmatgiz, 1962 [Pergamon, 1955].
11. B. W. Batterman and H. Cole, Rev. Mod. Phys. **36**, 681 (1964).
12. I. M. Frank, Izv. Akad. Nauk SSSR Ser. Fiz. **6**, 3 (1942); Usp. Fiz. Nauk **68**, 397 (1959) [Nobel Prize Lecture, 1959].
13. A. V. Kolpakov, Yad. Fiz. **16**, 1003 (1972) [Sov. J. Nucl. Phys. **16**, 554 (1973)].
14. V. A. Bazylev, V. I. Glebov, E. I. Denisov, N. K. Zhevago, and A. S. Khlebnikov, Pis'ma Zh. Eksp. Teor. Fiz. **24**, 406 (1976) [Sov. Phys. JETP **24**, 371 (1976)].
15. V. G. Baryshevskiĭ and I. Ya. Dubovskaya, Dokl. Akad. Nauk SSSR **231**, 1335 (1976) [Sov. Phys. Dokl. **21**, 741 (1976)].

Translated by J. G. Adashko