Check on the heavy-lepton hypothesis by means of the hadron spectra of the reaction $e^+e^-\to L^+L^-$

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Exact expressions are obtained for the hadron spectra of few-particle decays of the heavy leptons $L^\pm$ produced in the reaction $e^+e^-\to L^+L^-$. The possibility of verifying the hypothetical existence of heavy leptons are discussed and the values of their parameters are established in a study of the hadron spectra in e^+e^- annihilation.

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1. INTRODUCTION

By now, investigations of e^+e^- annihilation at energies $3.6\ \text{GeV}\leqslant\sqrt{s}\leqslant 7.8$ GeV has made it possible to accumulate a large aggregate of experimental data on anomalous $e^\pm\mu^\mp$ events[1–3] and anomalous inclusive muon production.[3–5] The experimentally observed properties of these anomalous events agree best with the hypothesis that their source is the production and subsequent decay of a pair of heavy leptons $\tau^\pm$ (with mass $M\sim 1.8$–2.0 GeV)[2)]

$$e^++e^-\to\tau^++\tau^-. \quad (1)$$

At present there is no other simple hypothesis capable of providing a unified explanation for the entire aggregate of the data.

In addition, all the data on the total cross section of the reaction $e^+e^-\to$ hadrons,[6,7] on the energy dependence of two-track events,[7] and on the inclusive yield of the K_S^0 and $K^\pm$ mesons[8,9] are reasonably well described precisely under the assumption that $\tau^+\tau^-$ pairs are produced in addition to the charmed hadrons at $\sqrt{s}\geqslant 4$ GeV.[10]

The apparent lack of any manifestations of the $\tau^\pm$ in neutrino experiments (e.g.,[11,12]) as well as data on the anomalous e^+e^- and $\mu^+\mu^-$ events[13] are simplest to reconcile with the hypothesis that the $\tau^\pm$ lepton is of the sequential type,[14] i.e., it has a new lepton number and is connected with a new neutrino ν_τ.[3)]

We emphasize that it can now be regarded as reliably established that the sources of the anomalous events are multiparticle decays of heavy-particle pairs with fixed mass ($M\approx 1.6$–2.0 GeV) (e.g.,[3,16]). However, the question of identifying the particles with just the $\tau^\pm$ leptons calls undoubtedly for additional detailed investigations.

This situation is made complicated here by the fact that in e^+e^- annihilation it is precisely in this mass region that production of pairs of charmed hadrons have been observed with substantial semileptonic decays.[17,18] In addition, a direct verification of the heavy-lepton hypothesis is made difficult by the fact that a nondetectable neutrino is always present in lepton decays.

To exclude reliably a possible connection between the anomalous events and the charmed hadrons, it is necessary first to obtain more stringent experimental constraints on the hadron and photon accompaniment of $e\mu$ events, as well as on the absence of any correlations whatever between the energy dependence of the anomalous-lepton yield and the structures in $\sigma(e^+e^-\to$ hadrons).[6,7] [4)]

To confirm the heavy-lepton hypothesis, the primary problems are a check on the pointlike character of the $(\tau^+\tau^-\gamma)$ vertex (e.g., by determining the angular distributions of the secondary leptons[3,19]), measurement of the spin of this particle, as well as a thorough study of the characteristics of the decays[5)]

$$\tau^\pm\to\bar\nu_\tau(\nu_\tau)+e^\pm+\nu_e(\bar\nu_e), \quad (2a)$$

$$\tau^\pm\to\bar\nu_\tau(\nu_\tau)+\mu^\pm+\nu_\mu(\bar\nu_\mu), \quad (2b)$$

$$\tau^\pm\to\bar\nu_\tau(\nu_\tau)+\pi^\pm(K^\pm), \quad (3)$$

$$\tau^\pm\to\bar\nu_\tau(\nu_\tau)+\rho^\pm(K^{*\pm}), \quad (4)$$

$$\tau^\pm\to\bar\nu_\tau(\nu_\tau)+A_1^\pm(Q^\pm). \quad (5)$$

Hereafter we assume for simplicity that $\tau^\pm$ is a sequential lepton. The properties of such leptons $L^\pm$ for a common V,A structure of the current $(\bar L\nu_L)$ and arbitrary ν_L neutrino mass have been investigated in great detail (e.g.,[14,20–23]). Detailed calculations of the characteristics of the inclusive spectra of secondary leptons from $L^\pm$ decays of type (2) have been carried out in many papers (e.g.,[19,22–25]). These spectra have a number of characteristic kinematic features,[19,23] and this makes it possible in principle, by comparison with the experimental data, to determine the masses of $L^\pm$ and ν_L as well as draw conclusions concerning the structure of the current $(\bar L\nu_L)$, the vertex $(L^+L^-\gamma)$, and the spin of $L^\pm$.

Of particular interest for the confirmation of the $\tau^\pm$ hypothesis are the fundamental hadron decays (3) and (4). Measurement of the contribution of these decays to the hadron spectra in e^+e^- collisions can be carried out, for example, by selecting events containing e or μ, or else events with two charged particles ($n_{\text{ch}}=2$) (which account for the bulk of the contribution of the $\tau^+\tau^-$ decays[16,23]). Particularly important are experiments at the maximum attainable energies, where the background situation should be most favorable for the separation of the $\tau^\pm$ contribution.

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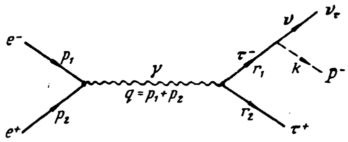

FIG. 1.

In the present paper we calculate in detail and analyze the inclusive distributions of the $\pi^\pm$ from decays (3) and (4) of the heavy lepton $\tau^\pm$ assuming a $(V \pm A)$ structure of the current $(\overline{\tau}\nu_\tau)$ at an arbitrary mass of the neutrino ν_τ. We show that the energy spectra of the pions from such decays have a distinct form that depends substantially on the values of $M(\tau^\pm)$ and $m(\nu_\tau)$. In the Appendix we present analytic formulas for the partial widths of the $L^\pm$ decays with allowance for the masses of the neutral secondary leptons.

The determination of the contribution made to the hadron spectra by the decay (5) is fraught with a number of indeterminacies and will not be considered here. We note only that the relative share of the channel (5) in the $\tau^\pm$ decay is small ($\Gamma_{A_1}/\Gamma_{\rm tot} \sim 8$–$10\%$[22,23]).

2. DISTRIBUTION OF PSEUDOSCALAR MESONS FROM THE DECAYS $L^\pm \to \overline{\nu}_L(\nu_L) + \pi^\pm(K^\pm)$

The differential cross section for the production of a pseudoscalar meson $P = \pi^\pm$ or $K^\pm$ as a result of decays of the type (3) is represented in the form (the designations of the 4-momenta are shown in Fig. 1)

$$\frac{\omega d^3\sigma_P}{d^3k} = \beta_P \frac{\alpha^2 M}{32E^3kk^*}\left[1 + \frac{M^2}{E^2} + \frac{r^2(1-z^2)}{2E^2} - \frac{(\omega E - M\omega^*)^2}{2k^2E^2}(1-3z^2)\right]. \quad (6)$$

Here β_P is the relative fraction of the $L^\pm$ decays via channel (3) (see Appendix I), $E = \sqrt{s}/2$ is the energy of the initial $e^\pm$, r is the $L^\pm$ momentum in the c.m.s., ω and k are the energy and momentum of the P meson, $z = \cos\theta_P$, θ_P is the angle of the emission of P relative to the e^+ and e^- collision axis, M is the $L^\pm$ mass, m is the $\nu_L(\overline{\nu}_L)$ mass, μ_P is the P mass, $\omega^* = (M^2 + \mu_P^2 - m^2)/2M$ is the P energy in the rest system of $L^\pm$, and $k^* = (\omega^{*2} - \mu_P^2)^{1/2}$.

The values of ω and θ_P vary in the following ranges

$$(\omega^* - v_L k^*)/M \leqslant 2\omega/\sqrt{s} \leqslant (\omega^* + v_L k^*)/M, \quad 0 \leqslant \theta_P \leqslant \pi, \quad (7)$$

where $v_L = r/E$ is the $L^\pm$ velocity.

In the cases of practical interest, the decays $\tau^\pm \to \overline{\nu}_\tau(\nu_\tau) + \pi^\pm$ and $\tau^\pm \to \overline{\nu}_\tau(\nu_\tau) + \rho^\pm$, to enhance the relative contribution of the $\tau^+\tau^-$ scattering to the observed e^+e^--annihilation cross section, it may be convenient to measure the pion spectra in events with $e^\pm(\mu^\pm)\pi^\mp$ correlations or with $n_{\rm ch} = 2$ (and (or) in events that contain no K mesons). Then it is necessary to add in formula (6) and in formulas (11) and (12) below a factor $\gamma_{P,V}$ corresponding to the restrictions imposed on the appropriate decay channel of the second lepton.

According to (6), in the nonrelativistic limit ($(E-M)/M \ll 1$) the P are produced isotropically, and at $E \gg M$ the angular distribution of the fast particles coincides with the L^+L^- distribution, i.e., it takes the form $d\sigma/d\Omega \sim 1 + \cos^2\theta_P$ if the $L^\pm$ spin is equal to 1/2. The distribution of $\pi^\pm(K^\pm)$ from the decay (3) with respect to ω has a steplike form, a result of the two-particle character of the decay.

It is interesting that in the approximation $\mu_P = 0$ the expression for the angular distribution, integrated with respect to ω, can be represented in the form

$$\frac{d\sigma_P}{d\Omega_P}\frac{1}{\sigma(e^+e^- \to L^+L^-)}\frac{1}{\beta_P} = \frac{1}{4\pi}\left[1 + \frac{3\alpha(v_L)}{3+\alpha(v_L)}F(v_L)\left(\cos^2\theta_P - \frac{1}{3}\right)\right], \quad (8)$$

where

$$F(v_L) = \frac{3-2v_L^2}{v_L^2} - \frac{3(1-v_L^2)}{2v_L^3}\ln\frac{1+v_L}{1-v_L}. \quad (9)$$

The quantity $\alpha(v_L) = v_L^2/(2 - v_L^2)$ at a spin 1/2 characterizes the angular distribution of $\tau^+\tau^-$ in process (1):

$$\frac{d\sigma_{L^+L^-}}{d\Omega_L} = \frac{3}{4\pi}(1+\alpha(v_L)\cos^2\theta_L)\frac{\sigma(e^+e^- \to L^+L^-)}{3+\alpha(v_L)}. \quad (10)$$

Using the approaches developed in[20,23] it is easily shown that an expression of the type (8) is valid for the case of an arbitrary $L^\pm$ decay (if we neglect the mass of the detected particle) and is determined only by the $L^\pm$ spin.[6] Thus, the angular distributions of the secondary particles (in particular, from the decay (2)) can yield in principle information on the $L^\pm$ spin.

Formula (8) can be easily generalized to the case of transverse polarizations ζ_+ and ζ_- of the initial e^+ and e^-, respectively. In the case of transverse antiparallel polarization, which is of practical interest, it is necessary to make in (8) the substitution (see, e.g.,[26])

$$\cos^2\theta_P \to \cos^2\theta_P + |\zeta_+||\zeta_-|\sin^2\theta_P\cos 2\Phi,$$

where Φ is the azimuthal angle between the direction of the polarization ζ_- and the plane of the reaction.

The distribution of P from the decays (3) with respect to the variable $\eta = \omega/E = 2\omega/\sqrt{s}$ is given by

$$\frac{1}{\sigma_{\mu\mu}}\frac{d\sigma_P}{d\eta} = \beta_P\frac{M}{2k^*}\left(1 + \frac{M^2}{2E^2}\right). \quad (11)$$

Here $\sigma_{\mu\mu} = 4\pi\alpha^2/3s$ is the cross section of the process $e^+e^- \to \mu^+\mu^-$ at $\sqrt{s} \gg m_\mu$. The height of the steplike distribution (11) depends on the parameters β_P, M^2/E^2, and m^2/M^2 (at $\mu_P \ll m$).

For the lepton τ, the value of $\beta_\tau(M, m)$ has been calculated quite reliably (e.g.,[20–23]), and this makes it possible in principle to determine the masses M and m from the observed height of the plateau and from the extreme values of the $\pi^\pm$ energy (7). Interest attaches here to the region of the upper boundary of the spectrum, inasmuch as the largest $\eta_{\max}$ of all the hadron decays of $\tau^\pm$ are obtained in the decay (3). In addition, in contrast to the case (3), the spectra of π from all other $\tau^\pm$ decays (and also from the decays of charmed mesons) decrease steeply enough with increasing η as $\eta \to \eta_{\max}$.

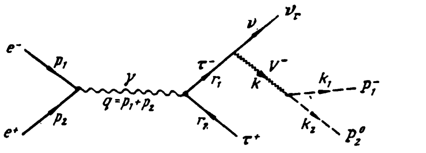

FIG. 2.

3. DISTRIBUTIONS OF THE PSEUDOSCALAR MESONS FROM THE CASCADE

$$L^{\pm}\to\bar{\nu}_\tau(\nu_\tau)+\rho^{\pm}(K^{*\pm}),\ \rho^{\pm}\to\pi^{\pm}+\pi^0$$
$$(K^{*\pm}\to K^{\pm}+\pi^0,\ K^{*\pm}\to K^0(\bar{K}^0)+\pi^{\pm})$$

The expression for the differential cross section for the production of $\pi^{\pm}(K^{\pm})$ via decay of the vector mesons $V^{\pm}=\rho^{\pm}, K^{*\pm}$ in the process $e^+e^-\to L^{\pm}(\to\bar{\nu}_L+V^+,\ \nu_L+V^-)+L^{\pm}$ is quite cumbersome and is given in Appendix II in the approximation $\Gamma_V\ll m_V$, where Γ_V is the width of V (the particle-4-momenta are designated on Fig. 2).

The distribution in the variable $\eta=\omega_1/E$ (ω_1 is the $\pi^{\pm}(K^{\pm})$ energy in the e^+e^- c. m. s.) is represented in the form

$$\frac{1}{\sigma_{\mu\mu}}\frac{d\sigma_V}{d\eta}=\frac{3\beta_V}{M^3}\frac{B_V m_V^3}{k^*(k_1^*)^3}\left(1+\frac{M^2}{2E^2}\right)\left[\left(1-\frac{m^2}{M^2}\right)^2+\frac{m_V^2}{M^2}\left(1+\frac{m^2}{M^2}\right)-2\frac{m_V^4}{M^4}\right]^{-1}$$
$$\times\left\{\left[\frac{M^2}{m_V^2}\omega_1^{*2}\omega^{*2}+\frac{1}{2}\omega_1^{*2}M^2-\frac{1}{2}M\omega^*k_1^{*2}\right]\ln X+\frac{1}{2}\left[x_m(x_m^2-m_1^2M^2)^{1/2}-x_M(x_M^2-m_1^2M^2)^{1/2}\right]\right.$$
$$\left.-2\frac{M}{m_V}\omega^*\omega_1^*\left[(x_m^2-m_1^2M^2)^{1/2}-(x_M^2-m_1^2M^2)^{1/2}\right]\right\}. \qquad (12)$$

Here ω^* and k^* are the energy and the momentum V in the $L^{\pm}$ rest system, ω_1^* and k_1^* are the energy and momentum of $\pi^{\pm}(K^{\pm})$ in the V rest system, m_V is the V mass, m_1 is the mass of the detected $\pi(K)$, β_V is the relative fraction of L decays via channel (4), and B_V is the relative fraction of the $\rho\to2\pi(K^*\to K\pi)$ decays. The values of ω^* and ω_1^* are

$$\omega^*=(M^2+m_V^2-m^2)/2M,$$
$$\omega_1^*=(m_V^2+m_1^2-m_2^2)/2m_V, \qquad (13)$$

where m_2 is the mass of the second (neutral) meson from the V decay. The values of X, x_m, and x_M in (12) are respectively

$$X=[x_m+(x_m^2-m_1^2M^2)^{1/2}]/[x_M+(x_M^2-m_1^2M^2)^{1/2}],$$
$$x_m=\omega_1E-k_1rz_m,\quad x_M=\omega_1E-k_1rz_M, \qquad (14)$$

where

$$z_m=\max\left\{-1,\frac{\omega_1E-M\omega_{1M}}{k_1r}\right\},\quad z_M=\min\left\{\frac{\omega_1E-M\omega_{1m}}{k_1r},1\right\}, \qquad (15)$$

k_1 is the $\pi^{\pm}(K^{\pm})$ momentum in the e^+e^- c. m. s., $\omega_{1M}(\omega_{1m})$ is the maximal (minimal) energy of the $\pi^{\pm}(K^{\pm})$ in the $L^{\pm}$ rest system:

$$\omega_{1M,1m}=(\omega_1^*\omega^*\pm k_1^*k^*)/m_V. \qquad (16)$$

Formula (12) was obtained in the approximation with $\Gamma_V\ll m_V$. The corresponding expression, with a finite value of Γ_V taken into account, is given in Appendix III. The zero-width approximation is convenient for the analysis of the behavior of the spectra and serves as a good guideline when it comes to determining the characteristic features of these spectra.

Tables I and II give the values of x_m and x_M for different intervals of ω_1 as functions of the velocity $v_L=r/E$; the following symbols are used:

$$k_{1M}=(\omega_{1M}^2-m_1^2)^{1/2},\quad k_{1m}=(\omega_{1m}^2-m_1^2)^{1/2},$$
$$\bar{\omega}_{2,1}=E[\omega_{1m}\pm v_Lk_{1m}]/M,\quad \bar{\omega}_{4,3}=E[\omega_{1M}\pm v_Lk_{1M}]/M,$$
$$v_{L1}=\frac{\omega_{1M}-\omega_{1m}}{k_{1M}+k_{1m}},\quad v_{L2}=\frac{k_{1m}}{\omega_{1m}},\quad v_{L3}=\frac{\omega_{1M}-\omega_{1m}}{k_{1M}-k_{1m}},\quad v_{L4}=\frac{k_{1M}}{\omega_{1M}}. \qquad (17)$$

Analyzing (12) in various regions of ω_1 and using Tables I and II, we can determine the characteristic features of this distribution. At relatively small v_L

$$v_L\leqslant v_{L1} \qquad (18)$$

in the energy region

$$\bar{\omega}_2\leqslant\omega_1\leqslant\bar{\omega}_3 \qquad (19)$$

the distribution (12) has a minimum at the point

$$\omega_0=M\omega^*\omega_1^*/m_VE. \qquad (20)$$

The distribution (12) decreases monotonically in the region $\bar{\omega}_2<\omega_1<\omega_0$ and increases monotonically in the region $\omega_1<\bar{\omega}_2$. It also increases monotonically in the region $\omega_0<\omega_1<\bar{\omega}_3$ and decreases monotonically at $\omega_1>\bar{\omega}_3$. Consequently, if the condition (18) is satisfied, the dis-

TABLE I.

ω_1	x_m	x_M
	$0\leqslant v_L\leqslant v_{L1}$	
$\bar{\omega}_1\leqslant\omega_1\leqslant\bar{\omega}_2$	$E\omega_1+rk_1$	$M\omega_{1m}$
$\bar{\omega}_2\leqslant\omega_1\leqslant\bar{\omega}_3$	$E\omega_1+rk_1$	$E\omega_1-rk_1$
$\bar{\omega}_3\leqslant\omega_1\leqslant\bar{\omega}_4$	$M\omega_{1M}$	$E\omega_1-rk_1$
	$v_{L1}\leqslant v_L\leqslant v_{L2}$	
$\bar{\omega}_1\leqslant\omega_1\leqslant\bar{\omega}_3$	$E\omega_1+rk_1$	$M\omega_{1m}$
$\bar{\omega}_3\leqslant\omega_1\leqslant\bar{\omega}_2$	$M\omega_{1M}$	$M\omega_{1m}$
$\bar{\omega}_2\leqslant\omega_1\leqslant\bar{\omega}_4$	$M\omega_{1M}$	$E\omega_1-rk_1$
	$v_{L2}\leqslant v_L\leqslant v_{L3}$	
$m_1\leqslant\omega_1\leqslant\bar{\omega}_1$	$E\omega_1+rk_1$	$E\omega_1-rk_1$
$\omega_1\leqslant\omega_1\leqslant\bar{\omega}_3$	$E\omega_1+rk_1$	$M\omega_{1m}$
$\bar{\omega}_3\leqslant\omega_1\leqslant\bar{\omega}_2$	$M\omega_{1M}$	$M\omega_{1m}$
$\bar{\omega}_2\leqslant\omega_1\leqslant\bar{\omega}_4$	$M\omega_{1M}$	$E\omega_1-rk_1$
	$v_{L3}\leqslant v_L\leqslant v_{L4}$	
$m_1\leqslant\omega_1\leqslant\bar{\omega}_3$	$E\omega_1+rk_1$	$E\omega_1-rk_1$
$\bar{\omega}_3\leqslant\omega_1\leqslant\bar{\omega}_1$	$M\omega_{1M}$	$E\omega_1-rk_1$
$\bar{\omega}_1\leqslant\omega_1\leqslant\bar{\omega}_2$	$M\omega_{1M}$	$M\omega_{1m}$
$\bar{\omega}_2\leqslant\omega_1\leqslant\bar{\omega}_4$	$M\omega_{1M}$	$E\omega_1-rk_1$
	$v_{L4}\leqslant v_L$	
$\bar{\omega}_3\leqslant\omega_1\leqslant\bar{\omega}_1$	$M\omega_{1M}$	$E\omega_1-rk_1$
$\bar{\omega}_1\leqslant\omega_1\leqslant\bar{\omega}_2$	$M\omega_{1M}$	$M\omega_{1m}$
$\bar{\omega}_2\leqslant\omega_1\leqslant\bar{\omega}_4$	$M\omega_{1M}$	$E\omega_1-rk_1$

*The data in the table are valid at

$$m+m_V\leqslant M\leqslant m_V\left[\frac{\omega_1^*+m_1}{2m_1}\right]^{1/2}+\left[\frac{m_V^2(\omega_1^*-m_1)}{2m_1}+m^2\right]^{1/2};$$
$$M>m_V\frac{\omega_1^{*2}+k_1^{*2}}{m_1^2}+\left(4\frac{m_V^2}{m_1^4}\omega_1^{*2}k_1^{*2}+m^2\right)^{1/2}.$$

TABLE II.

ω_1	x_m	x_M
	$0 \leqslant v_L \leqslant v_{L2}$	
$\bar{\omega}_1 \leqslant \omega_1 \leqslant \bar{\omega}_2$	$E\omega_1 + rk_1$	$M\omega_{1m}$
$\bar{\omega}_2 \leqslant \omega_1 \leqslant \bar{\omega}_3$	$E\omega_1 + rk_1$	$E\omega_1 - rk_1$
$\bar{\omega}_3 \leqslant \omega_1 \leqslant \bar{\omega}_4$	$M\omega_{1M}$	$E\omega_1 - rk_1$
	$v_{L2} \leqslant v_L \leqslant v_{L1}$	
$m_1 \leqslant \omega_1 \leqslant \bar{\omega}_1$	$E\omega_1 + rk_1$	$E\omega_1 - rk_1$
$\bar{\omega}_1 \leqslant \omega_1 \leqslant \bar{\omega}_2$	$E\omega_1 + rk_1$	$M\omega_{1m}$
$\bar{\omega}_2 \leqslant \omega_1 \leqslant \bar{\omega}_3$	$E\omega_1 + rk_1$	$E\omega_1 - rk_1$
$\bar{\omega}_3 \leqslant \omega_1 \leqslant \bar{\omega}_4$	$M\omega_{1M}$	$E\omega_1 - rk_1$
	$v_{L1} \leqslant v_L \leqslant v_{L3}$	
$m_1 \leqslant \omega_1 \leqslant \bar{\omega}_1$	$E\omega_1 + rk_1$	$E\omega_1 - rk_1$
$\bar{\omega}_1 \leqslant \omega_1 \leqslant \bar{\omega}_3$	$E\omega_1 + rk_1$	$M\omega_{1m}$
$\bar{\omega}_3 \leqslant \omega_1 \leqslant \bar{\omega}_2$	$M\omega_{1M}$	$M\omega_{1m}$
$\bar{\omega}_2 \leqslant \omega_1 \leqslant \bar{\omega}_4$	$M\omega_{1M}$	$E\omega_1 - rk_1$
	$v_{L3} \leqslant v_L \leqslant v_{L4}$	
$m_1 \leqslant \omega_1 \leqslant \bar{\omega}_3$	$E\omega_1 + rk_1$	$E\omega_1 - rk_1$
$\bar{\omega}_3 \leqslant \omega_1 \leqslant \bar{\omega}_1$	$M\omega_{1M}$	$E\omega_1 - rk_1$
$\bar{\omega}_1 \leqslant \omega_1 \leqslant \bar{\omega}_2$	$M\omega_{1M}$	$M\omega_{1m}$
$\bar{\omega}_2 \leqslant \omega_1 \leqslant \bar{\omega}_4$	$M\omega_{1M}$	$E\omega_1 - rk_1$
	$v_{L4} \leqslant v_L$	
$\bar{\omega}_3 \leqslant \omega_1 \leqslant \bar{\omega}_1$	$M\omega_{1M}$	$E\omega_1 - rk_1$
$\bar{\omega}_1 \leqslant \omega_1 \leqslant \bar{\omega}_2$	$M\omega_{1M}$	$M\omega_{1m}$
$\bar{\omega}_2 \leqslant \omega_1 \leqslant \bar{\omega}_4$	$M\omega_{1M}$	$E\omega_1 - rk_1$

*The data in the table are valid at

$$m_V \left(\frac{\omega_1^* + m_1}{2m_1}\right)^{1/2} + \left(\frac{m_V^2}{2m_1}(\omega_1^* - m_1) + m^2\right)^{1/2} \leqslant M \leqslant m_V \frac{\omega_1^{*2} + k_1^{*2}}{m_1^2} + \left(4\frac{m_V^2}{m_1^4}\omega_1^{*2}k_1^{*2} + m^2\right)^{1/2}.$$

tribution (12) has two maxima at the points $\omega_1 = \bar{\omega}_2$ and $\omega_1 = \bar{\omega}_3$ and a minimum at the point (20), i.e., it has a double hump.

At sufficiently high velocities of $L^{\pm}$, when

$$v_L \geqslant v_{L1}, \tag{21}$$

the distribution of $\pi^{\pm}(K^{\pm})$ with respect to η acquires a plateau, to the left and right of which the distribution decreases rapidly. If the velocity v_L is in the range

$$v_{L1} \leqslant v_L \leqslant v_{L3}, \tag{22}$$

then this plateau lies in the following range of the energies ω_1:

$$\bar{\omega}_3 \leqslant \omega_1 \leqslant \bar{\omega}_2. \tag{23}$$

On the other hand if

$$v_L \geqslant v_{L3}, \tag{24}$$

then the plateau lies in the interval

$$\bar{\omega}_1 \leqslant \omega_1 \leqslant \bar{\omega}_2. \tag{25}$$

This characteristic behavior of the spectra of the hadrons from the decay (4) can be qualitatively illustrated by analyzing the region of integration with respect to the variables η and y (where My is the pion energy in the rest system of the heavy lepton), a region considered in detail in[23]. The specifics of the decay (4), which makes in particular the double hump possible, is connected with the additional constraint (16), as well as with the fact that in this case the distribution of $(y^2 - m_1^2/M^2)^{1/2}G(y)$ has a characteristic minimum at $y = \omega^*\omega_1^*/2Mm_V$ (corresponding to kinematic suppression of the contribution of the transverse polarization V).

When account is taken of the $\rho(K^*)$ width, the plateau turns into an asymmetrical maximum with a steeper decrease of the distribution on the left than on the right. We call attention to one circumstance: the width of the plateau or of the maximum, when account is taken of the finite width of the resonance in the region (24), is $\bar{\omega}_2 - \bar{\omega}_1 = 2rk_{1m}/M$ and increases with increasing energy, and this may be important for its observation. In term of the variable η the width of this plateau (maximum) $\Delta\eta = 2v_L k_{1m}/M$ is practically constant (at $E \gg M$).

Figure 3 shows by way of example the distribution with respect to η for pions from ρ-meson decay at various initial energies E, at $M = 1.8$ GeV, and at $m = 0$; these illustrate the described behavior of the inclusive energy spectra of the pions from the decay (4). The dashed curves on Fig. 3 were calculated with allowance for the ρ-meson width. It is seen from Fig. 3a that the positions of the maxima and minima of the function (12) at low heavy-lepton velocities (18) do not change in comparison with the zero-width approximation, although the shape of the curve is somewhat altered.

The characteristic double hump of the energy spectrum of the pions at low velocities v_τ (18) and its transformation into a single maximum at relatively higher velocities (21), with a width determined by formulas (23) and (25), is an important feature of the cascade of the decays $\tau \to \nu_\tau + \rho$ and $\rho \to \pi + \pi$, which can be used to detect the conversion of a heavy lepton by the decay mode (4) jointly with the decays (2) and to determine the masses of τ and ν_τ from the characteristic singularities of the pion spectrum. We note that when the finite width of ρ is taken into account the steep fall-off of the spec-

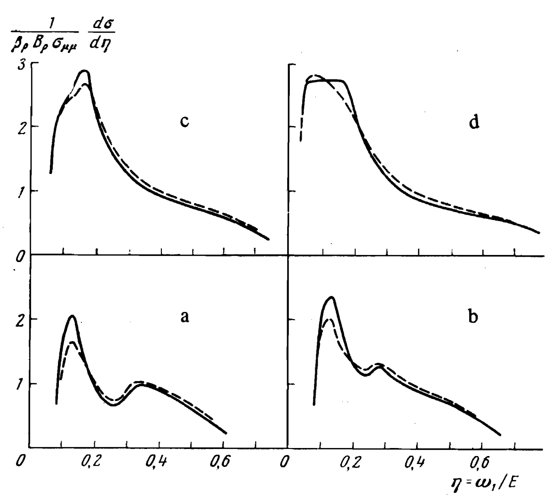

FIG. 3. Distribution of pions from the decay of a ρ meson produced in process (4) relative to the variable $\eta = \omega_1/E$: a) $E = 1.9$ GeV, b) $E = 2.0$ GeV c) $E = 2.4$ GeV, d) $E = 4.3$ GeV. The heavy lepton mass is $M = 1.8$ GeV. Solid curve—without allowance for the width of ρ, dashed—assuming a width $\Gamma_\rho = 0.152$ GeV.

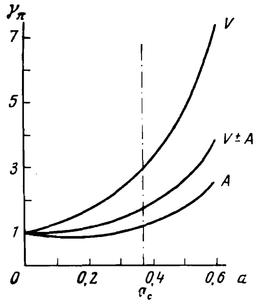

FIG. 4. Dependence of the quantity $\gamma_\tau=(\Gamma_\pi/\Gamma_\pi^{(0)})/(\Gamma_l/\Gamma_l^{(0)})$ on $a=m/M$ at various structures of the current $(\bar{L}\nu_L)$. It is assumed that $M=1.8$ GeV; $a_c=0.39$ corresponds to the restriction $m<0.7$ GeV.

trum on the left of the maximum begins near a point corresponding to the start of the plateau if the approximation $\Gamma_\rho\to 0$ is used (see Fig. 3d).

We note that at fixed s and $\omega_1>\bar{\omega}_3$ the cross section $d\sigma_\rho/d\eta$ decreases with increasing η no faster than $\eta_{\max}-\eta$, whereas it appears that one should expect a much steeper fall-off, $\sim(\eta_{\max}-\eta)^{1+n}$ for the contribution of the many-hadron decays of $\tau^\pm$ and of the heavy mesons to the pion spectra.[23] Thus, the contribution of the decays (3) and (4) to the spectra may turn out to be decisive at large η, particularly at high $\sqrt{s}$.

The contribution to the pion spectra from the light quark-partons (for which a decrease no slower than $1-\eta$ is usually assumed as $\eta\to 1$) can be suppressed by selection rules (e.g., $n_{\text{ch}}=2$ and (or) $\pi-e(\mu)$ correlations).

At relatively low velocities v_τ there can appear in the pion spectrum a distinct maximum at $\eta\approx\frac{1}{2}(1-m^2/M^2)\times(1-v_\tau)$, corresponding mainly to the inclusion of the decay (3) ($m_\pi/M\ll 1$) and (at $m_\rho^2/M^2\ll 1$) to the right-hand hump in the distribution of the pions from the decay (4).

4. CONCLUSION

The experimental data on events with anomalous leptons[1–5] and a number of inclusive characteristics of e^+e^- annihilation[10] offer quite weighty evidence in favor of the existence of the heavy lepton $\tau^\pm$. A final confirmation of the heavy-lepton hypothesis, however, calls for the performance of a number of precision measurements with sufficient statistics.

In particular, this question can be solved in experiments on the measurement of the angular and energy spectra of secondary leptons and hadrons from the decays (2)–(4). The predicted simple qualitative behavior of the energy spectra of the hadrons from the decays (3) and (4), namely: the presence of a plateau between definite values of the energies, and also the double-hump and single-hump distributions, can be quite useful in the determination of the parameters of $\tau^\pm$. The problem of determining the structure of the current $(\bar{\tau}\nu_\tau)$ can also be solved in principle (if $m\neq 0$) by measuring the ratios of the partial widths of the $\tau^\pm$ decays (see Appendix I and Fig. 4).

We have recently learned of a paper[28] likewise devoted to a discussion of hadron spectra produced in e^+e^- annihilation by $\tau^+\tau^-$ decays. That paper, however, cites only numerical results for the spectra of the hadrons from the decays (3)–(5) at $m=0$. It presents also analytic expressions for the spectra of the hadrons from the decay (4) at arbitrary values of m, but no detailed analysis of the properties of these spectra in all the kinematic configurations. In addition, a number of our results do not agree with those of[28], in particular the heights of the humps in the limit of zero width of the vector meson.

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APPENDIX I

The general expression for the partial width of the decay of L^- via the channel

$$L^-\to\nu_L+X \tag{I.1}$$

at arbitrary (V,A) structure of the current $(\bar{L}\nu_L)$[31]

$$J_L{}^i=\bar{L}\gamma^i[\cos\alpha(1+\gamma_5)+\sin\alpha(1-\gamma_5)]\nu_L \tag{I.2}$$

is of the form

$$\Gamma_X\equiv\Gamma(L^-\to\nu_L+X)=\frac{G^2M^2}{8\pi}\int_{t_{min}}^{t_{max}}dt|\mathbf{l}_X|[F_1{}^X(a^2,t)+atF_2{}^X(a^2,t)\sin 2\alpha], \tag{I.3}$$

where

$$F_1{}^X(a^2,t)=\rho_1{}^X(q^2)[(1-a^2)^2+t(1+a^2)-2t^2]+\rho_2{}^X(q^2)[(1-a^2)^2-t(1+a^2)],$$
$$F_2{}^X(a^2,t)=-6\rho_1{}^X(q^2)+2\rho_2{}^X(q^2), \tag{I.4}$$
$$a=m/M,\quad t=q^2/M^2,\quad q^2\equiv m_X{}^2,$$

$|\mathbf{l}_X|$ is the momentum of ν_L in the rest system of L^-:

$$|\mathbf{l}_X|={}^1/_2M[(1-a^2-t)^2-4a^2t]^{1/2}, \tag{I.5}$$

$\rho_1^X(q^2)$ and $\rho_2^X(q^2)$ are the spectral functions of the weak current J_i^W.[20,21]

It is easily seen that the values of all the partial widths Γ_X decrease with increasing m/M, and from the ratio of these values for different $L^\pm$ decays we can in principle, at $m\neq 0$, still obtain information on the structure of the current $(\bar{L}\nu_L)$ (for example if the masses of M and m were determined beforehand from the form of the spectra of the secondary leptons and (or) hadrons).

The most sensitive to the interference of the left-hand and right-hand currents (corresponding to the coefficient of $\sin 2\alpha$ in (I.3)) are decays with large effective values of m_X, particularly the decays (2).

1. The decay $L^\pm\to\bar{\nu}_L(\nu_L)+l^\pm+\nu_l(\bar{\nu}_l)$

If $m_{l^\pm}/M\ll 1$ and the current $(\bar{l}\nu_L)$ has a $(V-A)$ structure, then the lepton width $\Gamma_l=\Gamma(L^\pm\to\bar{\nu}_L(\nu_L)+l^\pm+\nu_l(\bar{\nu}_l))$ is given by formulas (I.3) and (I.4), where

$$\rho_1{}^l=1/6\pi^2,\quad\rho_2{}^l=0, \tag{I.6}$$

and can be reduced to the form

$$\Gamma_l=\Gamma_l{}^{(0)}[N_1(a)+aN_2(a)\sin 2\alpha]; \tag{I.7}$$

here

$$N_1(a)=(1-a^4)(1-8a^2+a^4)+24a^4\ln(1/a), \qquad \text{(I. 8a)}$$

$$N_2(a)=-2[(1-a^2)(1+10a^2+a^4)-12a^2(1+a^2)\ln(1/a)], \qquad \text{(I. 8b)}$$

$$\Gamma_l^{(0)}=\Gamma_l(m, m_i=0)=G^2M^5/192\pi^3. \qquad \text{(I. 9)}$$

Expressions (I. 7) and (I. 8) coincide with the results of[22]. We note that at fixed values of M and m

$$\Gamma_l(A)>\Gamma_l(V\pm A)>\Gamma_l(V). \qquad \text{(I. 10)}$$

If we do not neglect the muon mass, then in the case of a $(V\pm A)$ structure of the current $(\bar{L}\nu_L)$ a calculation of the exact expression of the width Γ_μ can be found in[19].

2. The decay $L^\pm \to \bar{\bar{\nu}}_L(\nu_L)+\pi^\pm(K^\pm)$

In this case (e.g., [20]) we have

$$\rho_1^{\pi,K}=0, \quad \rho_2^{\pi,K}(q^2)=(g^{\pi,K})^2\delta(q^2-m_{\pi,K}^2),$$
$$g^\pi=f_\pi\cos\theta_C, \quad g^K=f_\pi\sin\theta_C; \quad f_\pi\approx 0.9m_\pi \qquad \text{(I. 11)}$$

and

$$\Gamma_\pi=\frac{G^2f_\pi^2\cos^2\theta_C}{8\pi}M^2|\mathbf{l}_\pi|\left\{(1-a^2)^2-\frac{m_\pi^2}{M^2}(1+a^2)+2a\frac{m_\pi^2}{M^2}\sin 2\alpha\right\}. \qquad \text{(I. 12)}$$

Thus, Γ_π is practically independent of $\sin 2\alpha$.

Figure 4 illustrates the behavior of the quantity

$$\gamma_\pi=(\Gamma_\pi/\Gamma_\pi^{(0)})/(\Gamma_l/\Gamma_l^{(0)})$$

as a function of a for a lepton $L^\pm$ with mass $M=1.8$ GeV $(\Gamma_\pi^{(0)}=\Gamma_\pi(m=0))$. We note that Γ_π/Γ_l increases noticeably with increasing (a).

3. The decay $L^\pm \to \bar{\bar{\nu}}_L(\nu_L)+\rho^\pm$

For this decay we have[20]

$$\rho_1^\rho(q^2)=\frac{2m_\rho^2}{f_\rho^2}\delta(q^2-m_\rho^2)\cos^2\theta_C, \quad \rho_2^\rho=0; \quad \frac{f_\rho^2}{4\pi}\approx 2.5 \qquad \text{(I. 13)}$$

and

$$\Gamma_\rho=\frac{G^2M^2m_\rho^2\cos^2\theta_C}{4\pi f_\rho^2}|\mathbf{l}_\rho|\left\{(1-a^2)^2+\frac{m_\rho^2}{M^2}(1+a^2)-2\frac{m_\rho^4}{M^4}-6a\frac{m_\rho^2}{M^2}\sin 2\alpha\right\}. \qquad \text{(I. 14)}$$

Comparing (I. 7)–(I. 9) with (I. 14) we see easily that the value of Γ_ρ/Γ_l (at $0<a<0.39$; see Fig. 4) depends much more weakly on the current structure than Γ_π/Γ_l. The increase of Γ_ρ/Γ_l with increasing a is also slower.

4. Many-hadron decay $L^\pm \to \bar{\bar{\nu}}_L(\nu_L)+$hadron continuum

The theoretical investigation of many-hadron $L^\pm$ decays is fraught with very large uncertainties. Usually[20,21] (at $m_h^2 \geqslant 1$ GeV^2) one uses arguments based on asymptotic chiral symmetry and the quark model, and the contribution to $\Gamma_{\rm cont}$ from these states to the total width of the $L^\pm$ decay is determined by formulas (I. 3)–(I. 5) at

$$\rho_1^{\rm cont}(q^2)=R(q^2)/8\pi^2, \quad \rho_2^{\rm cont}=0,$$
$$\rho_1^{\rm cont}=\rho_1^V+\rho_1^A\approx 2\rho_1^V. \qquad \text{(I. 15)}$$

Here

$$R(q^2)=\sigma(e^+e^-\to\text{hadrons})/\sigma_{\mu\mu}. \qquad \text{(I. 16)}$$

This procedure, however, is not fully justified in the mass region $M\lesssim 2$ GeV ($\langle m_h^2\rangle \lesssim 1.5$ GeV^2), where a substantial role is played by few-particle and resonant states. It is quite possible, for example, that $\rho_1^A \ll \rho_1^V$, inasmuch as at such $\langle m_h^2\rangle$ the overwhelming part of ρ_1^A can be due to the A_1 mesons.[23] In addition, there are at present no reliable measurements of $R(q^2)$ in the region of interest to us.

In the mass region $M\gtrsim 2$ GeV there are opened thresholds connected with the production of new hadrons, particularly mesons: $L^+\to\bar{\nu}_L(F^+, F^{*+}, \dots)$ $(F^+, F^{*+}=c\bar{s}$, where c and s are the charmed and strange quark), and with increasing M the contribution to the widths of the many-particle decays, for example from the weak current $(\bar{c}s)$, become significant. At very large M this contribution can become comparable with the contribution of ordinary hadrons $(\Gamma_{\rm hadr})_q$ (in the colored-quark model we have $(\Gamma_{\rm hadr})_q\approx 3\Gamma_l$ [21,23]).

At sufficiently large values of M, the quantity $B_l=\Gamma_l/\Gamma_{\rm tot}$, which determines the yield of anomalous $e\mu$ events in e^+e^- annihilation, may therefore turn out to be much less than the value $B_l\approx 0.2$ at $M\approx 1.8$–2.0 GeV. In addition, at large M there can appear new lepton or quark currents, and the width $\Gamma_{\rm tot}$ can acquire a contribution from the cascade decays of $L^\pm$ into lighter leptons or (and) new hadrons.

APPENDIX II

The differential cross sections for $\pi(K)$ production in the process $e^+e^-\to L^++L^-(\to\nu_L+\rho^-(K^{*-}))$ is given by the formula

$$\frac{\omega_1 d^3\sigma_V}{d^3k_1}=\frac{3\beta_V B_V\alpha^2 m_V^3}{16E^3M^3k^*(k_1^*)^3k_1}\left[\left(1-\frac{m^2}{M^2}\right)^2+\frac{m_V^2}{M^2}\left(1+\frac{m^2}{M^2}\right)-2\frac{m_V^4}{M^4}\right]^{-1}$$
$$\times\left\{\left[1+\frac{M^2}{E^2}+\frac{r^2}{2E^2}(1-z^2)+\frac{\omega_1^2}{2k_1^2}(3z^2-1)\right]\right.$$
$$\times\left[\left(\frac{M^2}{m_V^2}\omega_1^{*2}\omega^{*2}+\frac{1}{2}M(M-\omega^*)k_1^{*2}\right)\ln X\right.$$
$$-2\frac{M}{m_V}\omega_1^*\omega^*[(x_m^2-m_1^2M^2)^{1/2}-(x_M^2-m_1^2M^2)^{1/2}]$$
$$\left.+\frac{1}{2}[x_m(x_m^2-m_1^2M^2)^{1/2}-x_M(x_M^2-m_1^2M^2)^{1/2}]+\frac{1}{2}m_1^2M^2\ln X\right]$$
$$+\frac{3z^2-1}{k_1^2E^2}\left[\frac{M^2}{m_V^2}\omega_1^{*2}\omega^{*2}+\frac{1}{2}M(M-\omega^*)k_1^{*2}\right]$$
$$\times\left[\frac{1}{4}[x_m(x_m^2-m_1^2M^2)^{1/2}-x_M(x_M^2-m_1^3M^2)^{1/2}]\right.$$
$$\left.+\frac{1}{4}m_1^2M^2\ln X-\omega_1E[(x_m^2-m_1^2M^2)^{1/2}-(x_M^2-m_1^2M^2)^{1/2}]\right]$$
$$+\frac{3z^2-1}{k_1^2E^2}\left[\frac{M}{m_V}\omega_1^*\omega^*\omega_1E[x_m(x_m^2-m_1^2M^2)^{1/2}-x_M(x_M^2-m_1^2M^2)^{1/2}+m_1^3M^2\ln X]\right.$$
$$-\left(\omega_1E+\frac{M}{m_V}\omega_1^*\omega^*\right)\left(\frac{1}{3}(x_m^2-m_1^2M^2)^{3/2}-\frac{1}{3}(x_M^2-m_1^2M^2)^{3/2}\right.$$
$$\left.+m_1^2M^2[(x_m^2-m_1^2M^2)^{1/2}-(x_M^2-m_1^2M^2)^{1/2}]\right)$$
$$+\frac{1}{8}\left(x_m(x_m^2-m_1^2M^2)^{3/2}-x_M(x_M^2-m_1^2M^2)^{3/2}+\right.$$

$$+\frac{5}{2}m_1^2M^2[x_m(x_m^2-m_1^2M^2)^{1/2}-x_M(x_M^2-m_1^2M^2)^{1/2}]+\frac{3}{2}m_1^4M^4\ln X\Big)\Big]\Big\}.$$

Here $z=\cos\theta$, θ is the $\pi(K)$ emission angle, and the other symbols are defined in the text.

APPENDIX III

When the width of the V meson is taken into account, formula (12) with $m\neq 0$ is replaced by

$$\frac{1}{\sigma_{\mu\mu}}\frac{d\sigma_V}{d\eta}=\frac{1}{N}\frac{48\beta_V B_V}{M^4}\left(1+\frac{M^2}{2E^2}\right)$$
$$\times\int_{x_1}^{x_2}\frac{dx}{(x^2-m_1^2M^2)^{1/2}}\Big\{\Big[x^2-\frac{1}{2}x(M^2-m^2+m_1^2-m_2^2)$$
$$+\frac{1}{16}(M^2-m^2)^2+\frac{1}{8}m^2(m_2^2-3m_1^2)-\frac{1}{8}M^2(3m_2^2-m_1^2)\Big]I_1(x)$$
$$+\frac{m_V^2}{16}(3M^2-m^2+4m_1^2-8x)I_2(x)$$
$$+\frac{(m_1^2-m_2^2)}{8m_V^2}\Big[(M^2-m^2)^2+\frac{1}{2}(3M^2-m^2)(m_1^2-m_2^2)-4x(M^2-m^2)\Big]I_3(x)$$
$$+\frac{1}{16m_V^4}(M^2-m^2)^2(m_1^2-m_2^2)^2I_4(x)\Big\},$$

where

$$I_1(x)=\mathrm{arctg}\frac{y_2-m_V^2}{m_V\Gamma_V}+\mathrm{arctg}\frac{m_V^2-y_1}{m_V\Gamma_V},\quad I_2(x)=I_1(x)+\frac{\Gamma_V}{2m_V}Y,$$
$$I_3(x)=\left(1+\frac{\Gamma_V^2}{m_V^2}\right)^{-1}\left[I_1(x)+\frac{\Gamma_V}{2m_V}\left(2\ln\frac{y_2}{y_1}-Y\right)\right],$$
$$I_4(x)=\left(1+\frac{\Gamma_V^2}{m_V^2}\right)^{-2}\Big[\left(1-\frac{\Gamma_V^2}{m_V^2}\right)I_1(x)$$
$$+m_V\Gamma_V\left(1+\frac{\Gamma_V^2}{m_V^2}\right)\left(\frac{1}{y_1}-\frac{1}{y_2}\right)+\frac{\Gamma_V}{m_V}\left(2\ln\frac{y_2}{y_1}-Y\right)\Big],$$
$$Y=\ln\frac{(y_2-m_V^2)^2+m_V^2\Gamma_V^2}{(m_V^2-y_1)^2+m_V^2\Gamma_V^2};$$
$$y_{2,1}=m^2+M^2-\frac{2Mu_0v_0}{u}\pm\frac{2M|\mathbf{u}||\mathbf{v}|}{u},$$
$$u_0=\frac{M^2-x}{M},\quad |\mathbf{u}|=\frac{1}{M}(x^2-m_1^2M^2)^{1/2},\quad u=(M^2+m_1^2-2x)^{1/2},$$
$$v_0=\frac{u^2+m^2-m_2^2}{2u},\quad |\mathbf{v}|=(v_0^2-m^2)^{1/2};$$
$$N=m_V\Gamma_V\int_{(m_1+m_2)^2}^{(M-m)^2}\frac{ds}{(s-m_V^2)^2+m_V^2\Gamma_V^2}\left[\left(1+\frac{m^2-s}{M^2}\right)^2-\frac{4m^2}{M^2}\right]^{1/2}$$
$$\times\left[\left(1-\frac{m_1^2+m_2^2}{s}\right)^2-\frac{4m_1^2m_2^2}{s^2}\right]^{1/2}\Big[\left(1-\frac{m^2}{M^2}\right)^2$$
$$+\frac{s}{M^2}\left(1+\frac{m^2}{M^2}\right)-2\frac{s^2}{M^4}\Big].$$

The limits x_1 and x_2 in the integral depend on the pion energy ω_1:

a) at $m_1\leqslant\omega_1\leqslant(\tilde\omega_1^*E-r\tilde k_1^*)/M$

$$E\omega_1-rk_1\leqslant x\leqslant E\omega_1+rk_1,$$

b) at $(\tilde\omega^*E-r\tilde k_1^*)/M\leqslant\omega_1\leqslant(\tilde\omega_1^*E+r\tilde k_1^*)/M$

$$E\omega_1-rk_1\leqslant x\leqslant M\tilde\omega_1^*.$$

The values of $\tilde\omega^*$ and $\tilde k^*$ are

$$\tilde\omega_1^*=[M^2+m_1^2-(m+m_2)^2]/2M,$$
$$\tilde k_1^*=(\tilde\omega_1^{*2}-m_1^2)^{1/2}.$$

[1)] Leningrad Institute of Nuclear Physics, USSR Academy of Sciences.

[2)] Following Perl, we shall call this the $\tau^\pm$ lepton, and leave the symbol $L^\pm$ for the general case without reference to reaction (1).

[3)] Contemporary experimental data do not exclude other possibilities. For example, as discussed in detail in[15], in principle one could identify ν_τ with a right-hand electronic or muonic neutron[$(\nu_e)_R$, $(\nu_\mu)_R$]. Of critical importance for the determination of the nature of ν_τ are searches for the radiative decays $\tau^\pm\to e^\pm(\mu^\pm)+\gamma$.

[4)] It must be borne in mind, however, that in principle the τ can appear also as products of the decay of new heavy mesons (e.g., charmed ones: $F^+\to\nu_\tau+\tau^+$).

[5)] We recall (e.g., [20–23]) that single production of strange mesons in the $\tau^\pm$ decays (3) has a low probability because of the factor $\tan^2\theta_C$(θ_C is the Cabibbo angle), and pair production is suppressed, in particular, because the phase volume is small. For leptons with $M>2$ GeV (when charmed hadrons can be produced), this suppression may not occur. We note that the observation of the decay (5) makes it possible, in particular, to confirm reliably the existence of A_1.

[6)] Our attention was recently called to an article by Pais and Treiman,[27] whose conclusion that the integrated angular distributions of the decay particles have a universal character if their mass is neglected agree with our conclusions.

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Asymptotic estimates of high order perturbation theory approximations in scalar electrodynamics

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The asymptotic behavior of the expansion coefficients of the Green's functions in a perturbation theory series is found with an accuracy up to a factor preceding the exponential. In order to obtain the form of the solutions of classical equations with finite action for which a saddle exists in the functional integral for the Green's functions a specialized perturbation theory is used, the parameter of which is the ratio of the orders of perturbation theory in terms of the coupling constants e and g.

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1. INTRODUCTION

In earlier papers of one of the authors (L.N.L.) a method was proposed for estimating high orders of perturbation theory for the Green's functions based on the saddle-point method of calculating corresponding functional integrals.[1] Although in these papers only scalar field theory models were considered, with the interaction $H_{\rm int}=g\varphi^n/n!$ in $D=2n/(n-2)$-dimensional Euclidean space-time, it was shown that the method of calculating can be transferred to other more interesting field models. In the present paper we obtain asymptotic formulas for high orders of perturbation theory in scalar electrodynamics. We regard this result as an intermediate stage preceding the main problem—obtaining asymptotic estimates in spinor electrodynamics. The solution of this problem would certainly be of interest both from the point of view of an approximate calculation of higher radiative corrections to observable quantities of the type of the anomalous magnetic moment of the electron,[2] and also from a purely theoretical point of view as a method of going beyond the framework of perturbation theory in the problem of internal selfconsistency of quantum electrodynamics.[3]

The first indications of the divergence of the perturbation theory series in quantum electrodynamics were obtained by Dyson.[4] He gave qualitative arguments in favor of the Green's functions having a singularity at the origin in terms of the fine structure constant α. This singularity arises as a consequence of the instability of the theory for $\alpha<0$. Within the framework of quantum mechanics of the anharmonic oscillator the nature of the singularity in terms of the coupling constant was investigated in the papers by Vaĭnshteĭn[5] and Langer.[6] The exact asymptotic formula for the expansion coefficient for the energy of the ground state of an anharmonic oscillator was obtained in the paper by Bender and Wu.[7] Until very recently, there existed in quantum field theory only rough estimates of high orders of perturbation theory (cf., for example, Ref. 8). The saddle-point method of calculating the functional integral utilized in Ref. 1 enables one to construct asymptotic formulas in principle of arbitrary accuracy in the form of an expansion in inverse powers of the order of perturbation theory. This method was generalized by the authors of Ref. 9 to the case when the scalar field has an internal symmetry group. They also showed that an analogous approach can be formulated for the problem of estimating high orders in the ε-expansion due to Wilson. This provided the possibility of calculating with greater confidence critical indexes in the theory of second-order phase transitions[9] and in Regge theory.[10]

Asymptotic estimates in perturbation theory enable one to calculate the nature of the singularity in the case of small coupling constants.[11] In Ref. 12 the question is discussed of the summability in the Borel sense of the perturbation theory series under the condition that the theory does not contain solutions of classical equations appropriate to the physical sign of the coupling constant. Summation of the perturbation theory series according to Borel is equivalent to replacing it by a Watson-Sommerfeld integral under the conditions that the coefficients are analytic in the index. The evaluation of the sum of the perturbation theory series with the aid of the Watson-Sommerfeld transformation was utilized in Refs. 1 and 13.

Bogomol'nyĭ[14] and Parisi[15] generalized to field the-

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