$$C=\frac{16-P_AP_B}{16(P_AP_B)^{1/2}},\quad D=\frac{P_B-P_A}{(P_AP_B)^{1/2}}.$$

In these experiments θ coincides with σ, $\eta=W_0\partial\theta/\partial E$, and W_0 is the imaginary part of the potential.

4. The proposed experiment for the detection of quasi-stationary neutron states in matter has, apart from its scientific interest, also a practical significance. A large volume of work with ultracold neutrons is planned at present,[3] and the problem arises of measuring energy spectra of neutrons with energies $\lesssim 10^{-7}$ eV. It is seen from Figs. 2 and 3 that one can use for this purpose the three-layer target described above, which transmits selectively neutrons with definite energies. Placing such a target in the path of the neutrons toward a detector, we record neutrons of definite energies. A set of calibrated three-layer targets made of various materials, with different or even variable thickness, will make possible measurements of energy spectra in the neutron energy range $\lesssim 10^{-7}$ eV. We note that the principle of operation of the proposed neutron spectrometer is analogous to that of the widely known Fabry-Pérot optical interferometer.[4]

One can hope that a neutron spectrometer utilizing the wave properties of the neutron will have better characteristics with respect to the resolution $\Delta E=\hbar\omega(P_A+P_B)/4$ and small dimensions and will prove more convenient in operation than a gravitation neutron spectrometer.

[1]Yu. M. Kagan, Pis'ma Zh. Eksp. Teor. Fiz. **11**, 235 (1970) [JETP Lett. **11**, 147 (1970)].

[2]A. V. Ignatyuk, N. S. Rabotnov, and G. N. Smirenkin, Phys. Lett. **29B**, 209 (1969).

[3]F. L. Shapiro, Soobshcheniya OIYaI R3-7135 (1073).

[4]G. S. Landsberg, Optika (Optics), Gostekhizdat, 1957.

Translated by Z. V. Chraplyvy

Theory of the temperature dependence of the muon precession frequency shift for anisotropic muonium

I. G. Ivanter

(Submitted 1 June 1977)
Zh. Eksp. Teor. Fiz. **73**, 1639–1643 (November 1977)

Equations have been obtained for the polarization density matrix for muonium for the case of anisotropic hyperfine interaction which may occur in impurity hydrogen (and muonium) in strongly doped semiconductors. Expressions have been obtained for the precession of muionium in a transverse field and it has been shown that the anisotropic characteristics of the hyperfine interaction can be obtained from the variation of the temperature dependence of the amplitudes and the frequencies of muon precession.

PACS numbers: 36.10.Dr

An experimental study of the anisotropy of the hyperfine interaction of impurity hydrogen and semiconductors has not been possible until now. In the present paper a theory has been developed of the muon method of investigating such anisotropy in those cases when it must be most strongly pronounced.

In order to have a shift in the muon precession frequency, it is necessary to have a sufficiently high density of conduction electrons.[1] In principle, this is possible both in metals and in strongly doped semiconductors. However in metals one should expect either that the muonium is ionized, or that its dimensions increase greatly and thereby weaken the hyperfine interaction constant. As will be seen from the following, this makes it practically impossible to observe effects associated with anisotropy. Therefore everywhere in the following we shall speak only of strongly doped semiconductors, having in mind at the same time that formally the theory is also applicable to metals.

We consider a strongly doped semiconductor at a low temperature and a situation in which the muonium either has not formed a diamagnetic compound at all or has entered into a chemical reaction only partially (for example during slowing down).

Strong doping gives rise to a relatively high density of conduction electrons. This leads to two consequences. Firstly, the conduction electrons will be scattered by the electrons in the muonium atoms and therefore the situation arises of "rapid electron spin exchange" in the muonium atom. Secondly, the considerable density of electrons of the medium leads to an increase in the dielectric constant which in a semiconductor, even without this occurring, differs from unity even at distances of the order of the Bohr radius. This leads to a swelling of muonium, whose dimensions become comparable with the characteristic dimensions of an atomic cell. At the same time the spherical symmetry of the hyperfine interaction of muonium and the electron disappears. We first consider the case when the hyperfine interaction preserves the symmetry of an ellipsoid of rotation but, as will be seen later, our results will be valid also for the general case.

In the case of rapid electron spin exchanges the muon precession occurs as if the muon were free. However at low temperatures in an external magnetic field the electrons of the medium are magnetized and the weak magnetization of these electrons leads to a quite significant magnetization of the electrons in the muonium $\mathscr{P} \sim \mu_e H/kT$. It is well known[1] that with this is associated the appearance of temperature dependence of the muon precession frequency. However, attempts to find such a shift in metals have not led to positive results,[1] and from this it follows that no atomic muonium exists in metals that have been investigated. Muonium exists in many pure semiconductors. It would doubtless be of interest to determine the limiting electron density for which the singly charged impurity center forms a bound state. The method of studying the temperature dependence of the frequency in principle enables us to do this, but even in order to attain such a limited goal the interpretation of experiments requires a theory which takes into account anisotropy which may not be small.

We first obtain the temperature dependence from semiquantitative arguments. If the hyperfine interaction of muonium is anisotropic, then the field produced by the electron of the muonium will be directed at a certain angle α to the external magnetic field H_{ex} since the magnetized electrons in muonium in this case are distributed over a certain nonspherical volume. Thus, the total field at the muonium will have the form

$$H_{\text{tot}} = [(H_{\text{ext}} + H_k \cos\alpha)^2 + H_k^2 \sin^2\alpha]^{1/2}, \tag{1}$$

where H_k is the field produced by the magnetized electrons. It depends on the temperature according to $1/T$. Therefore the temperature dependence of the total field (and consequently of the precession frequency) will have the form

$$H_{\text{tot}} = [(H_{\text{ext}} + C_1/T)^2 + C_2^2/T^2]^{1/2}. \tag{2}$$

If the constants C_1 and C_2 are small then we have

$$H_{\text{tot}} \approx H_{\text{ext}} + C_1/T, \tag{3}$$

and the anisotropy will not affect the nature of the temperature dependence. But in doped semiconductors one can select such a degree of doping that the value of C_1 would not be small compared to HT. In this case the dependence (1) will be realized and in order to observe it we shall not require the criterion for a strong anisotropy.

In order to make the derivation given above rigorous, it is necessary to take into account the process of rapid relaxation of the spin of the electron in the muonium. For this we shall utilize the density matrix apparatus in the following discussion.

We choose the coordinate axies (of the primed system associated with the crystal) in such a manner that the symmetry axis of the ellipsoid of the hyperfine interaction would coincide with the direction z'. The muonium can occupy different positions in the crystal cell. Even if muonium occupies, for example, only the tetrapore position there are still several such positions in a cell. We assume that the muonium does not move from one position to another (there is no diffusion) and we consider only some one position. In a real situation the results obtained will have to be applied to all possible positions of the muonium. Thus, we begin our investigation with the hyperfine interaction of the form

$$\mathscr{H}_{\text{hf}} = \hbar a_{xy}(\sigma_{x'\mu}\sigma_{x'e} + \sigma_{y'\mu}\sigma_{y'e}) + \hbar a_z \sigma_{z'\mu}\sigma_{z'e} + |\mu_e|(\boldsymbol{\sigma}_e\mathbf{H}) - \mu_\mu(\boldsymbol{\sigma}_\mu\mathbf{H}). \tag{4}$$

Here a_{xy}, a_z are hyperfine interaction constants, $\mathbf{H}$ is the magnetic field intensity vector.

But the laboratory coordinate system is determined by direction not associated with the direction of the crystal axes: by the direction of the initial polarization of the muons in beam $\mathbf{P}_0$, by the magnetic field H and the vector orthogonal to these two directions. In the case of the interaction (4) the directions x' and y' are physically equivalent and therefore a rotation about the z' axis is not significant. The x' axis can be brought by such a rotation into the x, y plane. Then the position of the ellipsoid in space is specified by two angles: $\theta = \mathbf{P}_0 \cdot \mathbf{z}'$ and φ—the angle between the projection of the z' axis on the plane perpendicular to $\mathbf{P}_0$ (the z axis) and the direction of the transverse magnetic field H (or, in the case of an inclined field, with the direction $[[\mathbf{P}_0 \times \mathbf{H}] \times \mathbf{P}_0]$—the y axis. Going over in (4) to the laboratory coordirate system we obtain

$$\begin{aligned}\mathscr{H}_{\text{hf}} = {}& {}^1\!/_4\hbar\omega_0(\boldsymbol{\sigma}_\mu\boldsymbol{\sigma}_e) + \hbar b\{\sin^2\theta(\sigma_{xe}\sigma_{x\mu}\sin^2\varphi + \sigma_{ye}\sigma_{y\mu}\cos^2\varphi) + \cos^2\theta\,\sigma_{z\mu}\sigma_{ze} \\ & + \sin\theta[\sin\varphi\cos\varphi\sin\theta(\sigma_{x\mu}\sigma_{ye} + \sigma_{xe}\sigma_{y\mu}) + \sin\varphi\cos\theta(\sigma_{ze}\sigma_{x\mu} + \sigma_{z\mu}\sigma_{xe}) \\ & + \cos\varphi\cos\theta(\sigma_{z\mu}\sigma_{ye} + \sigma_{ze}\sigma_{y\mu})]\} + |\mu_e|(\boldsymbol{\sigma}_e\mathbf{H}) - \mu_\mu(\boldsymbol{\sigma}_\mu\mathbf{H}).\end{aligned} \tag{5}$$

Here we have changed the notation to $\omega_0 = 4a_{xy}$, $b = a_z - a_{xy}$. Following the work of Nosov and Yakovleva[2] we represent the density matrix in the form

$$\rho = \rho_{i0}\sigma_{i\mu} + \rho_{0i}\sigma_{ie} + \rho_{ik}\sigma_{i\mu}\sigma_{ke}. \tag{6}$$

Then the equation for the density matrix operator is obtained by using its commutator with the Hamiltonian

$$i\hbar\partial\rho/\partial t = [\mathscr{H}, \rho]. \tag{7}$$

Substituting (5) into (7) we obtain the right-hand side in which in addition to the terms appearing in the equations of Nosov and Yakovleva there are also additional ones corresponding to the violation of the spherical symmetry of the hyperfine interaction. It can be easily seen that the whole difference between the "nonspherically-symmetric" Hamiltonian and the "spherically-symmetric" one can be represented in the form

$$\mathscr{H}_{\text{nonsph}} - \mathscr{H}_{\text{sph}} = \tfrac{1}{4}\hbar A_{ik}(\sigma_{i\mu}\sigma_{ke} + \sigma_{ie}\sigma_{k\mu}), \tag{8}$$

where

$$\begin{gathered} A_{ik} = A_{ki} \quad \text{and} \\ A_{11} = 2b\sin^2\theta\sin^2\varphi, \quad A_{33} = 2b\cos^2\theta, \quad A_{12} = 2b\sin^2\theta\sin\varphi\cos\varphi, \\ A_{22} = 2b\sin^2\theta\cos^2\varphi, \quad A_{13} = 2b\sin\theta\cos\theta\sin\varphi, \\ A_{23} = 2b\sin\theta\cos\theta\cos\varphi. \end{gathered} \tag{9}$$

But if we do not introduce specific expressions for A_{ik} then expression (8) is the most general expression for a deformed hyperfine interaction.

The modification of the Nosov and Yakovleva equations for the case of scattering of magnetized electrons by muonium was obtained in Ref. 3. Using these equations and substituting (8) into (7) we obtain the following system of equations:

$$\begin{aligned} d\rho_{\varkappa 0}/dt &= -{}^1/_2\omega_0\varepsilon_{\varkappa\lambda l}\rho_{\lambda l} - \zeta\varepsilon_{\varkappa\mu\lambda}\omega_\mu'\rho_{\lambda 0} \\ &\quad -{}^1/_2 A_{ik}[\varepsilon_{i\varkappa p}\rho_{pk} + \varepsilon_{k\varkappa p}\rho_{pi}], \\ d\rho_{0k}/dt &= {}^1/_2\omega_0\varepsilon_{k\lambda l}\rho_{\lambda l} + \varepsilon_{kml}\omega_m'\rho_{0l} - 2\nu\rho_{0k} + 2\nu\mathscr{P}_k \\ &\quad -{}^1/_2 A_{qr}[\varepsilon_{rkp}\rho_{qp} + \varepsilon_{qkp}\rho_{rp}], \\ d\rho_{\varkappa k}/dt &= {}^1/_2\omega_0[\varepsilon_{\varkappa k\lambda}\rho_{\lambda 0} - \varepsilon_{\varkappa k\lambda}\rho_{0l}] - \zeta\varepsilon_{\varkappa\mu\lambda}\omega_\mu'\rho_{\lambda k} \\ &\quad + \varepsilon_{kml}\omega_m'\rho_{\varkappa l} - 2\nu\rho_{\varkappa k} + 2\nu\rho_{\varkappa 0}\mathscr{P}_k - A_{\varkappa s}\varepsilon_{skp}\rho_{0p} \\ &\quad - A_{qs}\varepsilon_{q\varkappa p}\rho_{p0}. \end{aligned} \tag{10}$$

Here we have utilized the notation generally accepted in muon physics: the first subscript in the density matrix refers to muonium, the second to the electron, ω_0 is the frequency of hyperfine splitting, related to the spherically symmetric part of the hyperfine interaction, $\omega' - e\mathbf{H}/m_e c$ is the vector representing the Larmor precession frequency of the electron spin, ζ is the ratio of the absolute values of the magnetic moments of muonium and of the electron, $\mathscr{P}_k$ is the polarization vector for the electrons which exchange their spins with the muonium electron or, what is the same: the polarization of the spins of the electrons in muonium. In the problem under consideration $\omega_p' = eH\delta_{2p}/m_e c$, $\mathscr{P}_k = \mathscr{P}\delta_{2k}$. The initial polarization of the muonium is given by $P_k^0 = P_0\delta_{k3}$.

We consider the situation in which spin exchanges with the medium occur so frequently that even during short times, much smaller than those which can be observed experimentally, all the processes associated with the relaxation of electron spin can be regarded to be established. But then some of the components of the polarization density matrix can be taken to be equal to zero:

$$\rho_{31} = \rho_{13} = \rho_{33} = \rho_{23} = \rho_{01} = \rho_{03} = \rho_{21} = \rho_{11} = 0. \tag{11}$$

The components ρ_{02}—the polarization of the electron spin in muonium along the y axis becomes equal to the stationary polarization:

$$\rho_{02} = \mathscr{P} = \frac{\exp(\mu_e H/kT) - \exp(-\mu_e H/kT)}{\exp(\mu_e H/kT) + \exp(-\mu_e H/kT)} \approx -|\mu_e|\frac{H}{kT}. \tag{12}$$

Because of this the components of the matrix corresponding to the correlations of the spin of the muon with the y-component of polarization of the spin of the electron in muonium become equal to

$$\rho_{12} = \rho_{10}\mathscr{P},\ \rho_{22} = \rho_{20}\mathscr{P},\ \rho_{32} = \rho_{30}\mathscr{P}. \tag{13}$$

Substituting expressions (11)–(13) into the system of equations (10) we obtain the following system of three equations:

$$\begin{aligned} d\rho_{30}/dt &= [\zeta\omega_2' - ({}^1/_2\omega_0 + A_{22})\mathscr{P}]\rho_{10} + A_{12}\mathscr{P}\rho_{20}, \\ d\rho_{20}/dt &= -A_{12}\mathscr{P}\rho_{30} + A_{23}\mathscr{P}\rho_{10}, \\ d\rho_{10}/dt &= -[\zeta\omega_2' - ({}^1/_2\omega_0 + A_{22})\mathscr{P}]\rho_{30} - A_{23}\mathscr{P}\rho_{20} + A_{22}\mathscr{P}\rho_{30}. \end{aligned} \tag{14}$$

The system (14) must be solved with the initial conditions:

$$\rho_{20}(0) = \rho_{10}(0) = 0,\ \rho_{30} = P_0. \tag{15}$$

Here P_0 is the initial polarization of the muon directed along the z axis. Solving the system (14), (15) we obtain

$$P = P_0\frac{A_{23}^2\mathscr{P}^2 + (\tilde{\omega}_\mu + A_{12}^2\mathscr{P}^2)\cos[\tilde{\omega}_\mu^2 + (A_{23}^2 + A_{12}^2)\mathscr{P}^2]^{1/2}t}{\tilde{\omega}_\mu^2 + (A_{23}^2 + A_{12}^2)\mathscr{P}^2}. \tag{16}$$

Here

$$\tilde{\omega}_\mu = \zeta\omega_2' + ({}^1/_2\omega_0 + A_{22})|\mathscr{P}|. \tag{17}$$

Under the conditions of relatively high temperatures, at which it is possible to neglect terms quadratic in the polarization $\mathscr{P}$, the quantity $\tilde{\omega}_\mu$ becomes equal to the whole shifted frequency of muon precession.

From (16) it can be seen that violation of the spherical nature of the hyperfine interaction leads to two effects, firstly, to the appearance of a nonprecessing part of the polarization, secondly, to a quite complicated temperature dependence of the muon precession frequency. In this case the situation is such as if we were observing the muon precession in an inclined field the magnitude and the direction of which vary with temperature, as is represented in formulas (1) and (2). The precession frequency as can be seen from (16) varies with temperature in accordance with

$$\Omega \sim [(A + B/T)^2 + C^2/T^2]^{1/2}. \tag{18}$$

Thus, we have shown that a rigorous calculation carried out under the assumption of rapid relaxation leads to the same formula (2) as the qualitative arguments. It is clear that one can extract from experiment the values of all the quantities A_{2k}. In such a situation these quantities must vary as the crystal is rotated.

It must be emphasized that formula (16) describes the polarization of the muon situated at a certain given position in the crystal. If there are several positions nonequivalent with respect to the laboratory system of coordinates (momentum of the beam, magnetic field), then the polarization will be described by a sum of several terms analogous to (16) with coefficient proportional to the problability of the muonium being situated at definite sites. In such a case the complexity of the Fourier picture will increase. All this makes it very promising and interesting to conduct experimental searches for such a region of concentration of doping by donor impurities, at which muonium still exists, but the probability of scattering by the electrons of the medium becomes very large.

The application of the density matrix apparatus enables us to draw conclusions already from the very form of Eqs. (9) and (10) that each Fourier-component of the polarization will have its own width related to the fact that the precession frequencies corresponding to different positions of muonium in the crystalline lattice have different damping rates the magnitudes of which will also depend on the rotation of the crystal.

[1]V. G. Grebennik, I. I. Gurevich, V. A. Zhukov, I. G. Ivanter, A. N. Manych, B. A. Nikol'skiĭ, V. I. Selivanov, and V. A. Suetin, Pis'ma Zh. Eksp. Teor. Fiz. **22**, 36 (1975) [JETP Lett. **22**, 16 (1973)].

[2]N. G. Nosov and I. V. Yakovleva, Zh. Eksp. Teor. Fiz. **43**, 1750 (1962) [Sov. Phys. JETP **16**, 1236 (1963)].

[3]I. G. Ivanter, Zh. Eksp. Teor. Fiz. **63**, 1873 (1972) [Sov. Phys. JETP **36**, 990 (1973)].

Translated by G. Volkoff