High-frequency critical dynamics of ferromagnets above T_c

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The frequency dependence of the magnetic susceptibility of ferromagnets in the paramagnetic phase near the Curie point is considered. The asymptotic form of the susceptibility at high frequencies is obtained both for the case of pure exchange interaction and when account is taken of the dipole forces; in all the considered cases, the real part of the susceptibility was found to be negative and larger in absolute value than the imaginary one. The last section of the paper deals with the possible form of the frequency dependence at intermediate frequencies.

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1. INTRODUCTION

The critical dynamics of ferromagnets is presently investigated experimentally by two methods that complement each other: with the aid of inelastic neutron scattering and by studying the magnetic susceptibility in an alternating magnetic field. The main result obtained at the present time with the aid of neutrons is the confirmation of the dynamic-similarity hypothesis at temperatures not too close to the Curie point and at not too small momentum transfers i.e., in the region where the dipole forces and the associated demagnetization effects can be neglected (see, e.g., [1,2]). In addition, Lynn[3] recently observed, in the spectrum of neutrons scattered in iron above the Curie point, peaks whose position depended on the momentum transfer in almost the same manner as the position of the spin-wave peaks at room temperature, although their intensity was much lower. Lynn interpreted these peaks as spin waves above the Curie point. It should be noted that a conclusion that spin waves exist above T_c was deduced back in 1969 by Okorokov *et al.*[4] from data on small-angle scattering of neutrons in iron and in nickel; at the same time, no such waves were observed in EuO.[2]

Investigations of the frequency dependence of the magnetic susceptibility are devoted mainly to the relaxation of the homogeneous magnetization. The data obtained thereby determine therefore the role played in the critical dynamics by forces that violate the total-spin conservation law and are therefore responsible for this relaxation. These are primarily the dipole magnetic forces. They are evidently most significant in the temperature region where the static magnetic susceptibility χ is large ($4\pi\chi \gg 1$), and therefore magnetization-reversal process play an important role. At present it is still too early to draw final conclusions concerning the agreement between the experimental data[5–8] and the predictions of the various theories.[9–12] This is connected to a considerable degree with the fact that the experimental prerequisites without which no quantitative comparison of experiment with theory is possible have become clear only recently; the corresponding analysis can be found in the paper of Luzyanin and Khavronin.[6] It turns out, in particular, that owing to the critical slowing down of the relaxation the reciprocal time of the homogeneous relaxation Γ tends to zero and unless special measures are taken, the condition $\omega < \Gamma$ is violated, where ω is the frequency of the external field The experimental data can then no longer be interpreted with the aid of the simple Lorentz formula $\chi \sim \Gamma(-i\omega+\Gamma)^{-1}$ with a frequency-independent Γ. In the present paper, on the basis of the dynamic-similarity hypothesis proposed in[13], the behavior of the dynamic susceptibility is investigated in the high-frequency limit. In particular, asymptotic formulas are obtained for the homogeneous high-frequency susceptibilities; it appears that these formulas can be verified relatively simply in experiment. The last section of the paper is devoted to a discussion of the possibility of a resonant behavior of $\chi(\mathbf{k}, \omega)$, which might be interpreted in terms of relatively dampled spin waves above T_c.

The present paper is a direct continuation of earlier papers,[14,10,11] referred to hereafter as I, II, and III, and the same notation is used.

2. ASYMPTOTIC PROPERTIES OF THE SUSCEPTIBILITY

Consider a cubic ferromagnet, situated in a zero magnetic field, at temperatures higher than T_c. The magnetic susceptibility of such a system is described by a single scalar function $\chi(\mathbf{k}, \omega)$, where $\mathbf{k}$ is the momentum and ω is the frequency. The properties of χ which will be needed subsequently follow directly from the spectral representation for this quantity, and reduce to the following (see, e.g., Landau and Lifshitz[15]): 1) $\chi(\mathbf{k}, 0) > 0$. 2) $\chi(\mathbf{k}, \omega)$ is analytic in the upper ω half-plane; 3) $\chi(\mathbf{k}, \omega)$ assumes no real values anywhere in the upper half-plane, apart from the imaginary axis, where it is positive; 4) on the real axis the sign of $\mathrm{Im}\chi$ coincides with the sign of ω; in addition, $\chi(-\omega^*) = \chi^*(\omega)$ if ω is complex. It follows directly from these properties that χ decreases not faster than ω^{-2} as $\omega \to \infty$. We reckon the phase φ of the frequency ω in the usual manner from the positive real axis in a counterclockwise direction, and assume that $\chi(\omega)$ decreases in power-law fashion as $|\omega| \to \infty$. The requirement that χ be positive on the imaginary axis fixes the phase of this expression, and we have

$$\chi(\mathbf{k}, \omega) = \chi_\infty(\mathbf{k}) \left(\frac{i}{\omega}\right)^x = \chi_\infty(\mathbf{k}) \frac{1}{|\omega|^x} \left[\cos\left(\frac{\pi}{2} - \varphi\right)x + i\sin\left(\frac{\pi}{2} - \varphi\right)x\right], \quad (1)$$

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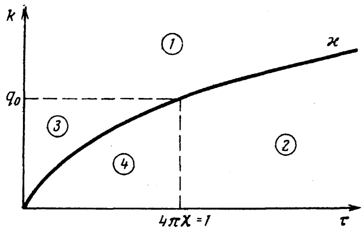

FIG. 1. Regions with different critical dynamics: 1—critical exchange region; 2—hydrodynamic exchange region; 3—critical dipole region; 4—hydrodynamic dipole region.

$$\mathrm{Re}\,\chi(\mathbf{k},\omega)=\chi_\infty(\mathbf{k})\frac{\cos(\pi x/2)}{|\omega|^x};\quad \mathrm{Im}\,\chi(\mathbf{k},\omega)=\chi_\infty(\mathbf{k})\frac{\varepsilon(\omega)}{|\omega|^x}\sin\frac{\pi x}{2},$$

where $\chi_\infty(\mathbf{k})>0$; $\mathrm{Re}\chi$ and $\mathrm{Im}\chi$ are written out on the real axis, and $\varepsilon(\omega)$ is the sign function.

The condition 3 leads directly to the restriction $x\leq 2$; if $1<x\leq 2$ we have $\mathrm{Re}\chi<0$.

When account is taken of the demagnetization effects considered in II, there are four temperature and momentum regions with different critical dynamics (Fig. 1). We determine x and χ_∞ for each of these region. If dynamic similarity obtains and there is no need to take into account additional weak interactions (regions 1 and 2 without dipole forces, and regions 3 and 4), this can be easily done with the aid of the usual similarity-theory assumption that if one of the variables is very large then only the dependence on this variable remains; but this should leave in regions 1 and 2 an additional factor k^2 that ensures conservation of the total spin. We, however, shall use a more detailed analysis with the aid of the Kubo formulas. As explained in II, it is more convenient to replace χ by a Green's function G (designated $G_\perp$ in II) connected with χ by the relation

$$\omega_0 G(\mathbf{k},\omega)=4\pi\chi(\mathbf{k},\omega),\quad \omega_0=4\pi(g\mu)^2 v_0^{-1}, \tag{2}$$

where ω_0 is the characteristic dipole energy. With the aid of the Kubo formulas we can rewrite G in the form (see I and II)

$$G(\mathbf{k},\omega)=G(\mathbf{k})\Gamma(\mathbf{k},\omega)[-i\omega+\Gamma(\mathbf{k},\omega)]^{-1}=G(\mathbf{k})F(\mathbf{k},\omega), \tag{3a}$$

$$\Gamma(\mathbf{k},\omega)=G^{-1}(\mathbf{k})(i\omega)^{-1}[\Phi(\mathbf{k},\omega)-\Phi(\mathbf{k},0)], \tag{3b}$$

$$\Phi(\mathbf{k},\omega)=\frac{i}{3}\int_0^\infty dt\, e^{i\omega t}\langle[\dot{S}_\mathbf{k}^\alpha(t),\dot{S}_{-\mathbf{k}}^\alpha(0)]\rangle, \tag{3c}$$

where $\mathbf{S}_\mathbf{k}$ is the Fourier transform of the spin density. If the exchange and dipole forces are taken into account, we have $\dot{\mathbf{S}}=(\dot{\mathbf{S}})_e+(\dot{\mathbf{S}})_d$, with

$$(\dot{S}_\mathbf{k}^\alpha)_e=N^{-1/2}\sum_{\mathbf{k}_1}(V_{\mathbf{k}_1}-V_{\mathbf{k}-\mathbf{k}_1})\varepsilon_{\beta\alpha\rho}S_{\mathbf{k}_1}^\beta S_{\mathbf{k}-\mathbf{k}_1}^\rho, \tag{4a}$$

$$(\dot{S}_\mathbf{k}^\alpha)_d=-N^{-1/2}\omega_0\sum_{\mathbf{k}_1}n_1^\beta\varepsilon_{\beta\alpha\rho}S_{\mathbf{k}-\mathbf{k}_1}^\rho(n_1S_{\mathbf{k}_1}),\quad \mathbf{n}=\mathbf{k}_1k_1^{-1}. \tag{4b}$$

Here $V_\mathbf{k}$ is the Fourier transform of the exchange integral.

We consider first the case of pure exchange interaction. It is known[13] that in this case we have dynamic similarity with an index $z_e=\frac{1}{2}(5-\eta)$. This means that the quantity Γ can be written in the form[1)]

$$\Gamma_e(\mathbf{k},\omega)=\varkappa^{z_e-2}k^2\gamma_e\left(\frac{\omega}{k^{z_e}},\frac{k}{\varkappa}\right), \tag{5}$$

where $\varkappa=a^{-1}\tau^\nu$ is the momentum and is equal to the reciprocal of the correlation radius, a is a length on the order of the lattice constant, and $\tau=(T-T_c)T_c^{-1}$. The factor k^2 is automatically separated in this expression by virtue of (4a); it is a consequence of the conservation of the total spin in exchange interaction. On the basis of (3c) we can represent Γ as a product of two factors, one of which is independent of ω, namely $\Gamma=[k^2G^{-1}(\mathbf{k})]\rho(\mathbf{k},\omega)$. The diagram series for Γ is shown in Fig. 2; it was investigated in detail in I for $k\ll\varkappa$ and $\omega\to 0$. We now want the result for large ω. To this end it suffices to note that in the limit as $\omega\to\infty$, after separating the factor k^2G^{-1}, all the diagrams cease to depend on $\mathbf{k}$ and $\varkappa$, i.e., ρ is a function of ω only. That this is indeed the case is easiest to trace with the first diagram of Fig. 2 as an example. As shown in I, the corresponding contribution to ρ is proportional to the integral

$$\frac{1}{(2\pi)^3 i}\int d\mathbf{q}\, q^{2-2\eta}G(\mathbf{k}+\mathbf{q})G(\mathbf{q})\frac{1}{\pi^2}\int\frac{dx_1\,dx_2\,\mathrm{Im}\,F(\mathbf{k}+\mathbf{q},x_1)\,\mathrm{Im}\,F(\mathbf{q},x_2)}{x_1x_2(x_1+x_2-\omega-i\delta)}. \tag{6}$$

In this expression we took into account the renormalization of the bare vertex with the aid of the Ward identity, and this led to the appearance of the additional factor $q^{-2\eta}$; this renormalization was discussed in detail in I. The Green's function satisfies the well-known sum rule

$$\frac{1}{\pi}\int\frac{dx}{x}\mathrm{Im}\,G(\mathbf{k},x)=G(\mathbf{k}), \tag{7}$$

by virtue of which the integral with respect to x_1 and x_2 converges well. But if we neglect x_1+x_2 in (6) in comparison with ω and take (7) into account, then the factor q^2 in the numerator gives rise to an integral that diverges in the region of large q. This means that as $\omega\to\infty$ the characteristic quantities of q under the integral sign are determined by the value of ω, and dynamic similarity gives for them the estimate $q\sim\omega^{1/x_e}$. As a result, the entire integral is proportional to ω^{-1+1/x_e}. Because of the dynamic similarity proposed by us, a similar estimate is obtained also for more complicated diagrams. As a result we obtain for Γ as $\omega\to\infty$:

$$\Gamma\sim k^2G^{-1}(\mathbf{k})\omega^{-1+1/x_e}=k^2G^{-1}(\mathbf{k})\omega^{1-x_e}, \tag{8}$$

$$x_e=\frac{4-\eta}{z_e}=\frac{2(4-\eta)}{5-\eta}\approx\frac{8}{5}$$

If this expression is written in the form (5), then we determine once more the dynamic exponent z_e. Thus, as $\omega\to\infty$ we have

$$\Gamma(\mathbf{k},\omega)=T_c(ka)^{z_e}\psi_e\left(\frac{k}{\varkappa}\right)\left(\frac{T_c(ka)^{z_e}i}{\omega}\right)^{(3-\eta)/(5-\eta)}. \tag{9}$$

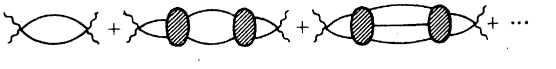

FIG. 2.

The phase was chosen in this expression to satisfy the requirement that Γ be positive on the upper part of the imaginary axis. This requirement can be easily obtained from the spectral representation that follows for Γ from the definition (3a). Formula (9) is valid at $k \ll \varkappa$ if $\omega \gg T_c(\varkappa a)^{z_e} \sim D\varkappa^2$, where D is the spin-diffusion coefficient introduced in[13] in the theory of dynamic similarity. On the other hand if $k \gg \varkappa$, then for (9) to be valid we must satisfy the condition $\omega \gg T_c(ka)^{z_e}$. In these two extreme cases we have

$$G(\mathbf{k}\omega) = \frac{\psi}{T_c}(ka)^2\left(\frac{iT_c}{\omega}\right)^{2(4-\eta)/(5-\eta)},$$
$$\operatorname{Re} G(\mathbf{k}, \omega) = -\frac{\psi}{T_c}(\mathbf{k}a)^2\left(\frac{T_c}{|\omega|}\right)^{2(4-\eta)/(5-\eta)}\cos\frac{\pi}{5-\eta}, \tag{10}$$
$$\operatorname{Im} G(\mathbf{k}, \omega) = -\operatorname{Re} G(\mathbf{k}, \omega)\,\varepsilon(\omega)\operatorname{tg}(\pi/(5-\eta)),$$

where $\psi \sim 1$ and we have used the well-known formula

$$G(\mathbf{k}) = Z[(k^2+\varkappa^2)a^2]^{-1+\eta/2}, \quad Z \sim T_c^{-1}.$$

As shown in II, in the temperature region $4\pi\chi(0, 0) \gg 1$ and at momenta $k \ll q_0 = a^{-1}(\omega_0 T_c^{-1})^{1/2}$ (regions (3, 4) the critical dynamics is determined by the dipole forces and a new dipole dynamic-similarity regime sets in, with an exponent $z_d = z_e - 1/\nu$, where ν is the index of the correlation radius. If $k > q_0$, then the dipole forces have little effect on the dynamics, and exchange dynamic similarity takes place as before, while at $k \sim q_0$ the two regimes are joined together. The characteristic energy in the region of dipole similarity is given by

$$\Gamma_d(\mathbf{k}) = T_c(q_0 a)^{1/\nu}(ka)^{z_e-1/\nu} g\left(\frac{k}{\varkappa}\right) \approx \omega_0\left(\frac{T_c}{\omega_0}\right)^{1/4} g\left(\frac{k}{\varkappa}\right)(ka)^{z_e-1/\nu}. \tag{11}$$

In the right-hand side of this expression, use was made of the approximate equation $\nu \approx \frac{2}{3}$. At nonzero ω we have in lieu of (5)

$$\Gamma_d(\mathbf{k}, \omega) = T_c q_0^{1/\nu}\varkappa^{z_e-1/\nu}\gamma_d\left(\frac{\omega}{T_c q_0^{1/\nu}\varkappa^{z_e-1/\nu}}, \frac{k}{\varkappa}\right). \tag{12}$$

Now, by virtue of (4b), the bare vertices of the diagram series of Fig. 2 no longer depend on the momentum, but only on its direction. This leads, as shown in II, to suppression of the fluctuations polarized parallel to the momentum. It turns out as a result that the dynamics is determined by diagrams with rescattering (the second and more complicated diagrams of Fig. 2), and the specifics of the rescattering are such that (see II) the integrals of type (6) contain in place of $q^{2-2\eta}$ the factor $[q^{2-1/\nu-\eta}\varphi(q/\varkappa)]^2$. If $3 - 2/\nu > 0$, then the corresponding expression at large ω and at zero k and $\varkappa$, just as in the exchange case, remains finite and we obtain instead of (8)

$$\Gamma \sim G^{-1}(\mathbf{k})\,\omega^{-1+(3\nu-2)(\nu z_d)^{-1}} = G^{-1}(\mathbf{k})\,\omega^{1-x_d}, \quad x_d = \frac{2(2-\eta)}{2-\eta+3-2/\nu} \approx 2 + \frac{\alpha}{\nu}, \tag{13}$$

where $\alpha = 2 - 3\nu$ is the heat-capacity exponent. Writing down the expression for Γ in the form (8), we can determine z_d once more. We see that $x_d < 2$ only if $\alpha < 0$, as was already assumed in the derivation of (13). According to calculations by Fisher and Aharony,[16] we have in the dipole region $\alpha \approx -\frac{1}{34}$, i.e., actually $x_d < 2$, although this inequality is satisfied "at the limit." It should be noted in this connection that in II, in the derivation of the dipole similarity, we used in fact a correlation-coalescence rule that is valid precisely if $\alpha < 0$. The point is that, for example, for the four-particle vertex $\Gamma(\mathbf{p}, \mathbf{q})$ at $p \gg q \gg \varkappa$ this rule must be written in the form

$$\Gamma(\mathbf{p}, q) = p^{1/\nu-1-\eta}[a_1 q^{2-1/\nu-\eta} + a_2 q^{1/\nu-1-\eta}\varkappa^{3-2/\nu}], \tag{14}$$

where $a_{1,2} \sim 1$, and for $\alpha > 0$ the principal term is the second one, while at $\alpha < 0$ it is the first, which we used in fact in II. If we use the second term, then at $k < \varkappa$ the characteristic dipole-similarity energy will be the same as in II, and at $k \gg \varkappa$ we have

$$\Gamma_d(k) = T_c g(q_0 a)^{1/\nu}(\varkappa a)^{3-2/\nu}(ka)^{z_e-3+1/\nu}, \tag{15}$$

where $g \sim 1$; we then have in place of (13)

$$x_d = \frac{2(2-\eta)}{2-\eta-3+2/\nu} \approx 2 - \frac{\alpha}{\nu} < 2. \tag{16}$$

It should be noted that in the variant with $\alpha > 0$, owing to the factor $\varkappa^{3-2/\nu}$ in (15), there is no natural matching of the dipole and exchange energies at $k \sim q_0$; moreover, at these values of k the dipole energy depends on the temperature and turns out to be much higher than the exchange energy. It appears that this circumstance is an indirect confirmation of the correctness of the sign of α obtained in (16). We assume below that $\alpha < 0$; then

$$\Gamma(\mathbf{k}, \omega) = T_c(\varkappa a)^{2-\eta}(q_0 a)^{1/\nu}\left(\frac{T_c(q_0 a)^{1/\nu} i}{\omega}\right)^{\xi}\gamma_d\left(\frac{k}{\varkappa}\right), \quad \xi = \frac{2-\eta-|\alpha|/\nu}{2-\eta+|\alpha|/\nu}. \tag{17}$$

It is convenient to write down formulas analogous to (10) directly in approximate form, by setting η equal to zero, putting $\nu = \frac{2}{3}$ where possible, and expanding in terms of α. As a result we have at $k < \varkappa$ and $\omega_0(T_c/\omega_0)^{1/4}(\varkappa a) \ll \omega \ll T_c(q_0 a)^{z_e} \approx \omega_0(\omega_0/T_c)^{1/4}$ and at $k \gg \varkappa$ and $\omega_0(T_c/\omega_0)^{1/4} ka \ll \omega \ll \omega_0(\omega_0/T_c)^{1/4}$

$$G(\mathbf{k}, \omega) = (\omega_0 T_c)^{-1/2}\gamma\left(\frac{i\omega_0}{\omega}\right)^{2-3|\alpha|/2}, \quad \gamma \sim 1,$$
$$\operatorname{Re} G(\mathbf{k}, \omega) = -(\omega_0 T_c)^{-1/2}\gamma\left(\frac{\omega_0}{|\omega|}\right)^{2-3|\alpha|/2}\cos\frac{3\pi|\alpha|}{4}, \tag{18}$$
$$\operatorname{Im} G(\mathbf{k}, \omega) = -\operatorname{Re} G(\mathbf{k}, \omega)\,\varepsilon(\omega)\operatorname{tg}(3\pi|\alpha|/4).$$

It is easy to verify that at $k \sim q_0$ and $\omega \sim T_c(q_0 a)^{z_e} \approx \omega_0(\omega_0/T_c)^{1/4}$ formulas (18) become joined to (10).

It is of interest to note that both in the exchange and in the dipole cases the susceptibility is independent of temperature in the limit as $\omega \to \infty$. In both cases, its real part is negative and is larger than the imaginary part in absolute magnitude.

It remains to explain the role of the dipole forces in the exchange regions 1 and 2, where these forces can be taken into account by perturbation theory (cf.[9] and II). Principal interest attaches in fact to the homogeneous high-frequency susceptibility ($k = 0$), since it is determined fully by the dipole forces. In the limit of small ω, the corresponding result was obtained by Huber.[9]

As shown in II, to this end it suffices to estimate the first diagram on Fig. 2 with the dipole vertices, in which the internal lines correspond to the Green's functions of the exchange similarity theory.

We obtain here the same estimate in the high-frequency limit:

$$\omega \gg T_c(\varkappa a)^{z_e} \sim D\varkappa^2.$$

As follows from I and II, at finite ω and at $\mathbf{k}=0$ the analytic expression for the first diagram of Fig. 2 with the dipole vertices is of the form

$$\Gamma_H = \frac{2}{3}\omega_0^2 G^{-1}(0)\frac{v_0}{(2\pi)^3 i}\int d\mathbf{q}\, G^2(\mathbf{q})\frac{T_c}{\pi^2}\int\frac{dx_1\,dx_2\,\mathrm{Im}\,F(q,x_1)\,\mathrm{Im}\,F(q,x_2)}{x_1x_2(x_1+x_2-\omega-i\delta)} \tag{19}$$

This expression differs from (6) in the absence of any power of q in the numerator, as a result of which it behaves like ω^{-1} at large ω. This can be easily verified if we neglect the sum x_1+x_2 in comparison with ω and use the sum rule (7). As a result we get for Γ_H

$$\Gamma_H^{(1)} = A\omega_0\frac{i\omega_0}{\omega}(\varkappa a)^{1+\eta}, \tag{20}$$

where $A\sim 1$. $G(0,\omega)$ decreases here like ω^{-2}, and its imaginary part is equal to zero, so that we must determine in the expansion of Γ_H the next term, which decreases faster than ω^{-1}. To this end we represent (14) in the form

$$\Gamma_H = \Gamma_H^{(1)} + \frac{2}{3}\omega_0^2\frac{1}{G(0)}\frac{v_0}{(2\pi)^3 i\omega}\int d\mathbf{q}\, G^2(\mathbf{q})\frac{T_c}{\pi^2}\cdot \times\int\frac{dx_1\,dx_2\,(x_1+x_2)}{x_1x_2(x_1+x_2-\omega-i\delta)}\mathrm{Im}\,F(q,x_1)\,\mathrm{Im}\,F(q,x_2). \tag{21}$$

In the second term, the integrals with respect to x_1 and x_2 have a much worse convergence than in (19), and the sum x_1+x_2 cannot be neglected in comparison with ω. Since $F(q,x)\sim q^2$, it follows that the integral with respect to x_1 and x_2 vanishes[2)] as $q\to 0$, and the region $q\sim\varkappa$ is not singled out in the integral. By virtue of the dynamic singularity, the characteristic momenta under the integral sign turn out to be of the order of ω^{1/z_e}, and the second term in (21) is proportional to $\omega^{-1-(1-2\eta)/z_e}$. The phase is chosen on the basis of the condition $\mathrm{Re}\,\Gamma>0$, which is a consequence of the spectral representation for Γ (it is satisfied by formulas (19) and (21)); as a result we get

$$\Gamma_H(\omega) = \omega_0\left(\frac{i\omega_0}{\omega}\right)(\varkappa a)^{1+\eta}\left[A - B\left(\frac{D\varkappa^2 i}{\omega}\right)^{(1-2\eta)/z_e}\right], \tag{22a}$$

$$G(\omega) = \frac{i}{\omega}G(0)\Gamma_H(\omega);$$

$$\mathrm{Re}\,G(\omega) = Z\left(\frac{\omega_0}{\omega}\right)^2\left[-\frac{A}{\varkappa a} + B\left(\frac{T_c}{|\omega|}\right)^{2/5}\cos\frac{\pi}{5}\right],$$
$$\mathrm{Im}\,G(\omega) = ZB\left(\frac{\omega_0}{\omega}\right)^2\left(\frac{T_c}{|\omega|}\right)^{2/5}\varepsilon(\omega)\sin\frac{\pi}{5}. \tag{22b}$$

Here $B\sim 1$, and in the formulas for $\mathrm{Re}\,G$ and $\mathrm{Im}\,G$ we have put $\eta=0$ and $Z\sim T_e^{-4}$. It is easy to verify that at finite k the dipole corrections to Γ as $\omega\to\infty$ decrease in the same manner as (22a), and are therefore negligibly small compared with the corresponding expressions in (9). The rapid decrease of Γ_H (like ω^{-1}) is due to the fact that the integrand in (19) is very "rigid" at small q, i.e., it contains no additional powers of q. In the cases considered above, the analogous expressions contain the "softening" factors $q^{2-2\eta}$ in the exchange case and $(q^{2-\eta-1/\nu})^2$ in the dipole case.

Thus, to estimate Γ_H we have considered the simplest diagram of Fig. 2. As shown in II, all the remaining diagrams make a contribution of the same order and do not change the estimate. This, however, is correct if there is no cancellation similar to that occurring in the case of the Ward identity (see, e.g., the book of Patashinskiĭ and Pokrovskiĭ[17]). To this end it would be necessary to cancel a contribution on the order of unity not in the scalar vertex, but in the pseudotensor expression, which takes in the static limit the form

$$F_{\alpha\rho\sigma} = \varepsilon_{\alpha\beta\rho}m_\sigma m_\beta + \frac{2}{5(2\pi)^3}\int d\mathbf{q}\, G^2(\mathbf{q})\,\Gamma^{(2)}_{\mu\nu\rho\sigma}(\mathbf{q},\mathbf{p})\,\varepsilon_{\alpha\beta\mu}m_\beta m_\nu, \tag{23}$$

where $\Gamma^{(2)}$ is the coefficient of $P_2(\cos\vartheta)$ in the expansion of the four-particle vertex in Legendre polynomials, ϑ is the angle between the vectors $\mathbf{p}$ and $\mathbf{q}$, and $\mathbf{m}=\mathbf{p}p^{-1}$. In the experimental study of Γ, at $4\pi\chi<1$[6] in the τ region where the usual behavior $\chi\sim\tau^{-1.3}$ takes place, Γ is practically independent of τ, in contradiction to Huber's theory. It is therefore reasonable to analyze the question of the homogeneous susceptibility in the "cancellation" region, i.e., in the case when the estimate $F\sim\partial G^{-1}/\partial\tau$ holds for the vertex part of (23). The corresponding calculations are perfectly analogous to those made above, and we present only the final results. For the reciprocal time of the homogeneous relaxation we have

$$\Gamma_H = C\frac{\omega_0^2}{T_c(\varkappa a)^{1/2(1+\eta)+\alpha/\nu}} \approx \frac{C\omega_0^2}{T_c\tau^{1/3}}, \tag{24}$$

where $C\sim 1$; this is smaller by a factor $\tau^{2/3}$ than Huber's result, which becomes, as shown in II, joined to the homogeneous damping in the dipole region at temperatures $4\pi\chi\sim 1$. Now there is no such joining. In general this is not surprising, for when τ is decreased the restructuring of the critical dynamics is due to the suppression of the critical fluctuations that are polarized along the momentum, and this mechanism is turned on quite rapidly on passing through the temperature region where $4\pi\chi\sim 1$. In the region of high frequencies, the result depends on the sign of α, inasmuch as for $\partial G^{-1}/\partial\tau$ we have at $k\gg\varkappa$ the formula

$$\frac{\partial G^{-1}}{\partial\tau} = g_1 k^{2-\eta-1/\nu} + g_2 k^{1/\nu-\eta-1}\varkappa^{-\alpha/\nu}, \tag{25}$$

and it is necessary to use the second term at $\alpha>0$ and the first at $\alpha<0$. The final result can then be represented in the form

$$\Gamma_H(\omega) = \omega_0[C_1 + C_2(\varkappa a)^{-2\alpha/\nu}](\varkappa a)^{2-\eta}\left(\frac{\omega_0}{T_c}\right)^{|\alpha|/\nu}\left(\frac{i\omega_0}{\omega}\right)^{1-|\alpha|/\nu},$$
$$G(0,\omega) = G(0)\cdot i\Gamma_H(\omega)/\omega, \quad \omega\gg D\varkappa^2, \tag{26}$$

where $C_1\sim C_2\sim 1$. It is of interest to note that in this case, regardless of the sign of α, we have $x=2-|\alpha|/\nu$

<2, i.e., the situation here is very similar to the one obtained in the dipole region.

It should be noted that if cancellation does take place, a change takes place in the previously obtained results (see[16] and III) for anisotorpic magnets. It is then necessary in III to replace $\varkappa^{3/2}$ and τ by $\varkappa^{1/2}$ and $\tau^{1/3}$ in formulas (31), replace $\varkappa_0^{7/2}$ by $\varkappa_0^{5/2}$ in the second term of (32a), and replace $\varkappa_0^{3/2}$ and $\varkappa_0^{1/2}$ in the second term of (32c). The formulas for the characteristic energies at large anisotropy take in the case of the easy plane the form[3)]

$$\Gamma_{k\perp}=T_c(\varkappa^2+k^2)a^2[(ka)^2(\varkappa_0 a)^{-3/2}h_\perp(k/\varkappa_0)+\psi_\perp\varkappa_0^{3/2}],$$
$$\Gamma_{k\parallel}=T_c(k^2+\varkappa_0^2)a^2\left[(ka)^2\frac{\varkappa_0}{k}h_\parallel\left(\frac{k}{\varkappa_0}\right)(\varkappa_0 a)^{-3/2}+d\frac{\omega_0^2}{T_c}(\varkappa a)^{-2}(\varkappa_0 a)^{-3/2}\right]. \quad (27)$$

In concluding this section, we make one more remark. The critical absorption of EuS was investigated in[8] in a magnetic field and it was observed that the critical damping Γ, regarded not as a function of the temperature but as a function of the susceptibility $\chi(H)$, is independent of the field. From the point of view of similarity theory this is natural, since all the critical-dynamics formulas are expressed in terms of the quantity $\varkappa\sim\chi^{-1/2}(H)$. A special situation can arise only at the resonant frequencies due to the fact that a magnetic moment that can precess around the field exists in a nonzero field.

3. DYNAMIC SUSCEPTIBILITY IN THE INTERMEDIATE REGION

We shall examine the possible behavior of $G(\mathbf{k},\omega)$ at intermediate frequencies. In all the cases considered we had $\operatorname{Re}G<0$ as $\omega\to\infty$, and consequently this quantity reverses sign somewhere at $\omega\sim\Gamma(\mathbf{k})$. This means, in particular, that if we investigate $\operatorname{Re}G(0,\omega)$ at a fixed frequency as a function of the temperature, then this quantity first increases as τ decreases, owing to the increase of $G(0)$, reaches a maximum at $\Gamma(0)\sim\omega$, and then decreases and becomes negative as $\tau\to 0$. Next, the asymptotic value of Γ was obtained above in the form $\gamma(k)(i/\omega)^{x-1}$. If this expression is substituted in (3a) and, in contrast to the procedure above, Γ is not neglected in the denominator, then we get

$$G(\mathbf{k},\omega)=G(\mathbf{k})\frac{\gamma(i/\omega)^{x-1}}{-i\omega+\gamma(i/\omega)^{x-1}},$$
$$\operatorname{Re}G(\mathbf{k},\omega)=G(\mathbf{k})\frac{\gamma(\gamma-|\omega|^{2-\delta}\cos(\pi\delta/2))}{(|\omega|^{2-\delta}-\gamma\cos(\pi\delta/2))^2+\gamma^2\sin^2(\pi\delta/2)}, \quad (28)$$
$$\operatorname{Im}G(\mathbf{k},\omega)=G(\mathbf{k})\frac{\gamma|\omega|^{2-\delta}\varepsilon(\omega)\sin(\pi\delta/2)}{(|\omega|^{2-\delta}-\gamma\cos(\pi\delta/2))^2+\gamma^2\sin^2(\pi\delta/2)}$$

where $\delta=2-x$ is the deviation of x from the limiting value. This Green's function has all the necessary analytic properties, is correctly normalized by the condition $G(\mathbf{k},0)=G(\mathbf{k})$, and can be easily shown to satisfy the sum rule (7); it can therefore be regarded as a possible variant of an interpolation formula. At $|\omega|^{2-\delta}\sim\gamma$ the function $\operatorname{Im}G$ takes a resonant form because the Green's function has two symmetrical poles $\omega_\pm$ in the lower ω half-plane (Fig. 3). The expressions for them and for the corresponding residues $r_\pm$ are of the form

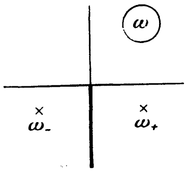

FIG. 3.

$$\omega_\pm=\mp e^{\mp i\Phi}\gamma^{1/x}=\gamma^{1/x}(\pm\cos\Phi-i\sin\Phi),$$
$$r_\pm=-\omega_\pm/x, \qquad \Phi=\pi\delta/2x. \quad (29)$$

In addition to these poles, the function G has along the negative imaginary axis a cut with a discontinuity

$$\Delta G(\omega)=\frac{1}{2i}[G(\omega-\varepsilon)-G(\omega+\varepsilon)]=\frac{\gamma|\omega|^{2-\delta}\sin\pi\delta}{\gamma^2+2\gamma|\omega|^{2-\delta}\cos\pi\delta+|\omega|^{4-2\delta}}. \quad (30)$$

It is interesting to note that as $\delta\to 0$ the poles approach the real axis and the discontinuity vanishes. In the pure exchange case at $k\ll\varkappa$ (region 2) we have $\gamma^{1/x}\sim D\varkappa^2(k/\varkappa)^{5/4}\ll D\varkappa^2$, and formula (28) can certainly not be used at $\omega\sim\gamma^{1/x}$. At these values ordinary diffusion takes place[12] subject to the small corrections considered in I. In exactly the same manner, in the case of homogeneous damping, the resonance frequencies in region 2 are small compared with the characteristic value $D\varkappa^2$, whereas the dispersion should begin with $\omega\sim D\varkappa^2$ (this is easily seen from (19)). Therefore the frequency dependence at $\omega<D\varkappa^2$ is determined by the usual Lorentz formula, with the width taken from Huber's paper,[9] or else with the modified width (24), and at $\omega\gg D\varkappa^2$ the asymptotic behavior discussed above sets in. These two cases are special in that the frequencies at which the damping dispersion begins are high in comparison with $\Gamma(0)$. In all the remaining cases (regions 1, 3 and 4 of Fig. 1) the characteristic dispersion frequencies coincide with $\Gamma(\mathbf{k})$. Of course, formulas (28), (39) and (30) cannot be used literally, if only because they are not obtained following substitution in the diagram series of Fig. 2. Thus, if (28) is substituted in the integral (6) at $\varkappa=0$, the resultant expression has in the lower ω half-plane several branch points at $|\omega|\sim k^{z_e}$, a detailed analysis of which is hardly of interest, while formula (8) holds only in the limit $|\omega|\gg k^{z_e}$.

At the same time, the character of the behavior of $G(\mathbf{k},\omega)$ at intermediate ω is obviously determined by those of its singularities that are closest to the real axis and lie in the lower half-plane of the variable ω. By virtue of the property $G(-\omega^*)=G^*(\omega)$ mentioned at the beginning of the preceding section, these singularities either lie on the imaginary axis are symmetrically disposed about it. Nothing is known at present concerning the character of these singularities.

Let us discuss the simplest variant, when the nearest singularities are the poles of the function $G(\mathbf{k},\omega)$. It is obvious here that the pole lying on the imaginary axis corresponds to that contribution to $\omega^{-1}\operatorname{Im}G$ which has a maximum at $\omega=0$, while the poles $\omega_\pm^{(i)}(\mathbf{k})=\pm\omega_1^{(i)}(\mathbf{k})-i\omega_2^{(i)}(\mathbf{k})$ lying outside the imaginary axis correspond to contributions made to $\operatorname{Im}G$ and having maxima at $|\omega|\approx\omega_1^{(i)}$. The general picture of the behavior of the $\omega^{-1}\operatorname{Im}G$ is then determined by which of the poles are closer to the real axis, and the maximum due to the nearest pole can completely "smear out" the maxima due to the other poles. If $\omega_2 < \omega_1$ for some pair of poles, then these poles should be perceived in the experiment as resonances corresponding to relatively weakly damped excitations. Thus if we consider by way of examples formulas (28), then the peak of ImG is well pronounced even in the exchange case, when $\delta = \frac{2}{5}$, while in the dipole case, when $\delta = |\alpha|/\nu \sim 0.1$, one can speak of a rather narrow resonance at $\omega \approx \omega_0(T_c/\omega_0)^{1/4}\varphi(k/\varkappa)(\varkappa a)$, where $\varphi(0) \sim 1$, $\varphi(x)|_{x\to\infty} = \varphi_\infty x^{-1}$ and $\varphi_\infty \sim 1$. It is not excluded that Lynn's "spin-wave" peaks[3] are due to the existence of such poles. It is of interest to note that Lynn was unable to describe his data on large-momentum spin wave, both below and above T_c, with the aid of the dispersion law $Ak^2(1-\beta k^2)$; at the same time it is easy to verify that they fit well the $A'k^{5/2}$ curve. Here, however, the condition $k \gg \varkappa$ was not satisfied; the data were obtained at $k > \varkappa$. We note also that the problem of spin waves above T_c was discussed many times theoretically (see, e.g., Hubbard's paper), but no convincing results have been obtained so far.

The presence of poles in the lower half-plane leads to the appearance of more complicated singularities connected with the "pole-interaction" phenomenon. Thus, it was shown in I that the diffusion pole generates a branch point that is closer to the real axis than this pole; the influence of this branch point on $G(\mathbf{k}, \omega)$, however, turned out to be weak.

Let us discuss the unusual situation that results from the "pole interaction" in the case of homogeneous relaxation and leads to a very complicated nonmonotonic behavior of the function $G(0, \omega)$. Assume that in the dipole region the function $G(\mathbf{k}, \omega)$ has poles at the points $\omega_\pm = \pm\Omega(\mathbf{k})\exp(\mp i\Phi(\mathbf{k}))$, and let $\Phi(\mathbf{k}) \ll 1$ (expression (29) yields in the dipole region $\Phi(\mathbf{k}) \approx 3\pi|\alpha|/8 \sim 0.1$). In this case the function $G(0, \omega)$ has a resonant peak at $\omega \approx \Omega(0)$ with a width $\Omega(0)\sin\Phi(0) \approx \Omega(0)\Phi(0)$. At $k \ll \varkappa$, the quantities Ω and Φ can be expanded in powers of k^2:

$$\Omega(\mathbf{k}) = \Omega_0 + \Omega_1 k^2, \quad \Phi(\mathbf{k}) = \Phi_0 + \Phi_1 k^2,$$

where $\Omega_1 \sim \Omega_0\varkappa^{-2}$ and $\Phi_1 \sim \varkappa^{-2}$. We assume that $\Omega_1 > 0$ and neglect for simplicity the dependence of Φ on k. If we now substitute the corresponding pole expression in the diagram series of Fig. 2, then we immediately obtain a series of singular points of the function $\Gamma(\omega)$ at $\omega_n^{(\pm)} = \pm n\Omega_0 e^{\mp i\Phi}$.

Let us analyze this question in greater detail, using as an example a two-particle intermediate state. Recalling the statements made in the preceding section concerning the structure of this state in the dipole region, we obtain for the contribution from the two poles ω_+ to $\Gamma(\omega)$

$$\Gamma_2^{(+)} \sim \frac{1}{i}\int d\mathbf{q}\, \varkappa^{4-2\eta-2/\nu} f\left(\frac{q}{\varkappa}\right) G^2(\mathbf{q}) \frac{r_+^2(\mathbf{q})}{\omega_+^2(\mathbf{q})\,[2\omega_+(\mathbf{q})-\omega-i\delta]}, \tag{31}$$

where $f(0) \sim 1$ and $r_+(\mathbf{q})$ is the residue of $G(\mathbf{q}, \omega)$ at the pole ω_+, it being assumed that $r_+(0) \neq 0$. This expression has at $\omega = 2\Omega_0 e^{-i\Phi}$ a singularity near which, as is easy to verify, we can represent Γ in the form

$$\Gamma(\omega) = \Gamma_1 + g e^{i\Phi}[\Omega_0(\omega - 2\Omega_0 e^{-i\Phi})e^{i\Phi}]^{1/2} = \Gamma_1 + \gamma(\omega). \tag{32}$$

Here Γ_1 is that part of Γ which is regular near $\omega = 2\Omega_0$ and is a complex quantity, while $g \sim 1$. Near this singularity we have

$$G(0, \omega) = G(0)\,[F_1(\omega) - 2i\Gamma_1^{-2}F_1^2(2\Omega_0)\gamma(\omega)], \quad F_1 = \Gamma_1(-i\omega + \Gamma_1)^{-1}. \tag{33}$$

If $\omega = 2\Omega_0$, then $\gamma(\omega) \approx 0$ and measurement of G at this frequency makes it possible to obtain $F_1(2\Omega_0)$ and Γ_1, after which it is possible to determine the complex coefficient of $\gamma(\omega)$. As a result we can express the singular part of $G(0, \omega)$ at $\omega \sim 2\Omega_0$ in the form

$$\begin{gathered} \tilde G(0, \omega) = G(0)\,(A + iB)\,\gamma(\omega), \\ \operatorname{Re}\tilde G(0,\omega) = G(0)\,g[\Omega_0|\omega - 2\Omega_0|]^{1/2}[A\cos\psi - B\sin\psi], \\ \operatorname{Im}\tilde G(0,\omega) = G(0)\,g[\Omega_0|\omega - 2\Omega_0|]^{1/2}[B\cos\psi + A\sin\psi], \\ \psi = \frac{1}{2}\operatorname{arctg}\frac{2\Omega_0\Phi}{\omega - 2\Omega_0}. \end{gathered} \tag{34}$$

It is clear from these formulas that $\tilde G$ depends in a rather complicated manner on ω and on the magnitudes and signs of the constants A and B.

The interaction of the two poles ω_+ and ω_- gives rise to a singular point $\omega = -2i\Omega_0\sin\Phi$ on the imaginary axis. Arguments perfectly analogous to those just presented yield

$$\Gamma_{+-}^{(2)}(\omega) = \Gamma_0^{(2)} - \frac{g_0\Omega_0}{\sin\Phi} + \frac{g_1}{(\sin\Phi)^{1/2}}[\Omega_0(2\Omega_0\sin\Phi - i\omega)]^{1/2}, \tag{35}$$

where $\Gamma_0^{(2)}$ is the regular part of Γ, which should be real near the imaginary axis, and $g_{0,1} \sim 1$. The complicated dependence of this expression on Φ is due to the fact that $\omega_+ + \omega_- \sim \sin\Phi$. The value of Ω_0 should be of the order of the static value $\Gamma(0) = \Gamma_0$. It follows therefore from (35) that the dispersion of $\Gamma(\omega)$ begins very early, at $\omega \sim \Omega_0\Phi$. The reason is that the sum $\omega_+ + \omega_-$ lies very close to the point $\omega = 0$, whereas $\omega_\pm$ are not small.[4)] It is obvious that $\Gamma_{+-}^{(2)}(0) = \Gamma_0 \sim \Omega_0$; this means that $g_1\sqrt{2} = g_0$ and $\Gamma_0^{(2)} = \Gamma_0$. As a result we have at $\omega \ll 2\Omega_0\Phi$:

$$\begin{gathered} \operatorname{Re} G(0, \omega) = G(0)\left\{1 + \frac{(\omega/2\Omega_0\Phi)^2(g_0\Omega_0/\Gamma_0)^2}{1 + (g_0\Omega_0/2\Gamma_0\Phi)^2(\omega/2\Omega_0\Phi)^2}\right\}, \\ \operatorname{Im} G(0, \omega) = G(0)\frac{\omega}{\Gamma_0}\left[1 + \left(\frac{g_0\Omega_0}{2\Gamma_0\Phi}\right)^2\left(\frac{\omega}{2\Omega_0\Phi}\right)^2\right]^{-1}. \end{gathered} \tag{36}$$

It is interesting to note that the second term in the denominator is not small, so that the frequency dependence of ω^{-}ImG sets in very early. The remaining singular points, which are connected with the interaction of a large number of poles, lead to weaker singularities in Γ. Thus, when three poles ω_i are taken into account, a factor $[\omega_+(\mathbf{q}_1) + \omega_+(\mathbf{q}_2) + \omega_+(q_1 + q_2) - \omega]^{-1}$ appears under the integral sign and must be integrated with respect to q_1 and q_2. As a result, the corresponding singular term in Γ behaves like $(\omega - 3\Omega_0 e^{-i\Phi})^2\ln(\omega - 3\Omega_0 e^{-i\Phi})$.

So far we have dealt only with singularities of $G(0, \omega)$ as a function of the frequency, and the positions of the singularities were determined by the condition $\omega_n \approx n\Omega_0 \sim n\omega_0(T_c/\omega_0)^{1/4}\tau^\nu$. Obviously, if we fix the frequency and vary the temperature, then as a result of these singularities the temperature dependences of ImG and ReG will exhibit anomalies at

$$\tau_n \sim \left[\frac{\omega}{n\omega_0}\left(\frac{\omega_0}{T_c}\right)^{1/4}\right]^{1/\nu} \approx \left(\frac{\omega}{n\omega_0}\right)^{3/2}\left(\frac{\omega_0}{T_c}\right)^{3/8}. \quad (37)$$

It should be noted that Luzyanin and Khavronin[6] have observed a complicated temperature and frequency dependence of $\chi(0, \omega)$ in the temperature region $4\pi\chi > 1$, and it turned out that the dispersion of χ begins with very low frequencies. It is not excluded that these phenomena are connected with the mechanism discussed by us, but of[6] are insufficient for a detailed comparison of the theory with experiment. In particular, this calls for measurements at much higher frequencies than used in[6].

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1) Formulas such as (5), in which the functional dependence on the argument is important, will be written out where possible, without separating the corresponding dimensional factors. The correct physical dimensionality will be restored only in the final expressions.

2) This can be easily verified in examples by substituting for F the diffusion formula from I or else the expression discussed in the next section.

3) I take the opportunity to note that formulas (33a) and (33c) of III are incorrect. The correct results are those of Kruger and Huber.[18,19] I am grateful to Professor Huber for pointing out this circumstance.

4) It must be noted that the dispersion of Γ at $\omega \sim \Omega_0\Phi$ begins also if the singularities of $G(0, \omega)$ at the points $\omega_\pm$ are more complicated that mere poles; when functions $G(\omega)$ having such singularities are substituted in the corresponding integral, a singular point of $\Gamma(\omega)$ appears at $\omega \approx -2i\Omega_0\Phi$.

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