Static conductivity of a quasi one-dimensional metal at finite temperatures

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The conductivity of a quasi one-dimensional metal is found with account of scattering of electrons from impurities and phonons at $T \gg \omega_D$ and $\omega_D\tau^* \ll 1$, where ω_D is the Debye frequency and $\tau^* = (\tau_i^{-1}+\tau_{2ph}^{-1})^{-1}$ is the effective collision time. The scaling hypothesis is applied. As a result, we get $\sigma = ae^2S^{-1}\omega_D v^{-1} l_{ph}^{-1} l^{*3}$, where $a \sim 1$, S is the area of the xy cross section of the cell, v is the Fermi velocity, and $l = v\tau$. It then follows that $\sigma \alpha T$ at $l_i \ll l_{ph}$ and $\sigma \alpha T^{-2}$ at $l_i \gg l_{ph}$. The experimental situation is discussed.

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1. INTRODUCTION

A method was developed in previous papers of the authors which made it possible to calculate the conductivity of quasi one-dimensional metallic systems at $T = 0$.[1–4] The only resistance mechanism in this case is the elastic scattering of electrons from impurities. At $T \neq 0$, inelastic scattering from phonons appears, to which the methods applied in Refs. 1–4 are no longer suitable.

The problem of the conductivity of a quasi one-dimensional metal with account taken of the scattering of electrons from phonons and impurities was considered earlier by Gogolin, Mel'nikov and Rashba.[5–7] They used a rather complicated method based on the Berezinskiĭ technique[8] and the Keldysh theory[9] for nonstationary problems at $T \neq 0$. In view of this the authors restricted themselves to the simpler case $\bar{\omega}\tau \approx \min(\bar{\omega}\tau_i, \omega\tau_{\rm ph}) \gg 1$ where $\bar{\omega}$ is the average frequency of the phonons, τ_i and $\tau_{\rm ph}$ are the impurity and phonon scattering times. This corresponds to low temperatures $T \ll \omega_D$ (or a small electron-phonon interaction constant) and very pure metals. At the same time, those well-known quasione-dimensional materials which preserve a metallic conductivity at $T \ll \omega_D$ possess large internal disorder, such that in them in all probability $\omega_D\tau_i \ll 1$.

We develop here a method which is a generalization of the method of Ref. 1, and which allows us in principle to consider the conductivity at finite temperatures and at any $\bar{\omega}\tau$. In order to avoid consideration of the quantum field of the phonons, we limit ourselves to temperatures $T \gg \omega_D$. The transition from the impurity-phonon mechanism of conductivity to the purely phonon mechanism is of most interest. Inasmuch as the case $\bar{\omega}\tau \gg 1$ was considered in Ref. 5, we limit ourselves to the case $\bar{\omega}\tau \ll 1$. The results recall qualitatively the behavior of $\sigma(T)$ obtained in Ref. 5, but the detailed temperature dependence turns out to be different.

2. METHOD OF CALCULATION OF THE CONDUCTIVITY AT $T \neq 0$

As has already been noted, at $\tau \gg \omega_D$, the phonon field becomes classical ($N_k \gg 1$) and it can be considered as a random external field acting on the electronic system. As in Ref. 1, we limit ourselves to the Born approximation, which corresponds to averaging with a Gaussian functional. The difference lies in the fact that this time the potential is time-dependent.

The method of finding the single-electron physical quantities in an arbitrary varying external field at $T \neq 0$ was developed by Gor'kov and Éliashberg.[10] The expression needed for the current has the form (the two spin projections are taken into account)

$$
\begin{aligned}
j_\omega(z) &= \frac{2evT}{S}\sum_\varepsilon \mathrm{Sp}\,\sigma_3 G_{\varepsilon,\varepsilon-\omega}(z,z) \\
&= \frac{2ev}{S}\mathrm{Sp}\,\sigma_3\Big\{\int\frac{d\varepsilon}{2\pi i}\left[G^A_{\varepsilon,\varepsilon-\omega}(z,z)n(\varepsilon) - G^R_{\varepsilon,\varepsilon-\omega}(z,z)n(\varepsilon-\omega)\right] \\
&\quad + \int\frac{d\varepsilon}{4\pi i}\int\frac{d\varepsilon_1}{2\pi}\int\frac{d\omega_1}{2\pi}\int dz_1 G^R_{\varepsilon,\varepsilon_1}(z,z_1)h_{\omega_1}(z_1)G^A_{\varepsilon_1-\omega_1,\varepsilon-\omega}(z_1,z) \\
&\quad \times\left(\mathrm{th}\frac{\varepsilon_1}{2T} - \mathrm{th}\frac{\varepsilon_1-\omega_1}{2T}\right)\Big\} - \frac{e^2}{mc}nA_\omega(z),
\end{aligned}
\tag{1}
$$

where S is the area of the cell section perpendicular to the filaments, G^A and G^R are the advanced and retarded Green's functions, and $h_{\omega_1}(z_1)$ is the total variable field, consisting of the external electromagnetic field and the phonon field:

$$
h_{\omega_1}(z_1) = -(ev/c)\sigma_3 A_{\omega_1}(z_1) + \Phi_{\omega_1}(z_1).
\tag{2}
$$

Since we are seeking the linear conductivity, this expression should be expanded in A with accuracy to first order. Here we obtain

$$
\begin{aligned}
j_\omega(z) &= \frac{-e^2}{mc}nA_\omega(z) + \frac{2e^2v^2}{cS}\mathrm{Sp}\Big\{\int\frac{d\varepsilon}{2\pi i}\int\frac{d\varepsilon_1}{2\pi}\int\frac{d\omega_1}{2\pi}\int dz_1 \cdot \\
&\quad \times\left[\sigma_3 G^R_{\varepsilon,\varepsilon_1}(z,z_1)\sigma_3 A_{\omega_1}(z_1)G^R_{\varepsilon_1-\omega_1,\varepsilon-\omega}(z_1,z)n(\varepsilon-\omega)\right. \\
&\quad \left.-\sigma_3 G^A_{\varepsilon,\varepsilon_1}(z,z_1)A_{\omega_1}(z_1)\sigma_3 G^A_{\varepsilon_1-\omega_1,\varepsilon-\omega}(z_1,z)n(\varepsilon)\right] \\
&\quad -\int\frac{d\varepsilon}{4\pi i}\int\frac{d\varepsilon_1}{2\pi}\int\frac{d\omega_1}{2\pi}\int dz_1\left[\sigma_3 G^R_{\varepsilon,\varepsilon_1}(z,z_1)A_{\omega_1}(z_1)\sigma_3 G^A_{\varepsilon_1-\omega_1,\varepsilon-\omega}(z_1,z)\right] \\
&\quad \times\left(\mathrm{th}\frac{\varepsilon_1}{2T} - \mathrm{th}\frac{\varepsilon_1-\omega_1}{2T}\right) - \int\frac{d\varepsilon}{4\pi i}\int\frac{d\varepsilon_1}{2\pi}\int\frac{d\omega_1}{2\pi}\int\frac{d\varepsilon_2}{2\pi}\int\frac{d\omega_2}{2\pi}\int dz_1\int dz_2 \\
&\quad \times\left[\sigma_3 G^R_{\varepsilon+\omega,\varepsilon_2+\omega_2}(z,z_2)A_{\omega_2}(z_2)\sigma_3 G^R_{\varepsilon_2,\varepsilon_1}(z_2,z_1)\Phi_{\omega_1}(z_1)G^A_{\varepsilon_1-\omega_1,\varepsilon}(z_1,z)\right. \\
&\quad \left.+\sigma_3 G^R_{\varepsilon,\varepsilon_1}(z,z_1)\Phi_{\omega_1}(z_1)G^A_{\varepsilon_1-\omega_1,\varepsilon_2}(z_1,z_2)A_{\omega_2}(z_2)\sigma_3 G^A_{\varepsilon_2-\omega_2,\varepsilon-\omega}(z_2,z)\right] \\
&\quad \times\left(\mathrm{th}\frac{\varepsilon_1}{2T} - \mathrm{th}\frac{\varepsilon_1-\omega_1}{2T}\right)\Big\}.
\end{aligned}
\tag{3}
$$

The Green's function in Eq. (3) contains in principle

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the terms of all orders in the interaction with impurities and phonons. They satisfy the equation

$$\left[\omega+iv\sigma_3\frac{\partial}{\partial z}-\Psi(z)\right]G_{\omega,\omega'}(z,z')-\int\frac{d\omega_1}{2\pi}\Phi_{\omega_1}(z)G_{\omega-\omega_1,\omega'}(z,z')$$
$$=2\pi\delta(\omega-\omega')\delta(z-z'), \tag{4}$$

where $\Psi(z)$ is the time-independent potential of the impurities, and $\Phi_{\omega_1}(z_1)$ is the potential of the phonon field. The boundary conditions in z depend on which function we determine, G^R or G^A. In principle, we can find also the "causal" function G, but it does not enter into the formula (3).

Equations of the type (4), with corresponding boundary conditions (of course, with a displacement in the momentum rather than frequency) were already solved in Refs. 2 and 3. We can write out the finished formulas:

$$G^R_{\omega\omega'\alpha\beta}(z>z')=-\frac{i}{v}\{S_{\alpha1}{}^{-1}(\infty,z)[S_{11}{}^{-1}(\infty,-\infty)]^{-1}[S^{-1}(z',-\infty)\sigma_3]_{1\beta}\}_{\omega\omega'},$$
$$G^R_{\omega\omega'\alpha\beta}(z<z')=\frac{i}{v}\{S_{\alpha2}(z,-\infty)[S_{22}(\infty,-\infty)]^{-1}[S(\infty,z')\sigma_3]_{2\beta}\}_{\omega\omega'},$$
$$\tag{5}$$
$$G^A_{\omega\omega'\alpha\beta}(z>z')=-\frac{i}{v}\{S_{\alpha2}{}^{-1}(\infty,z)[S_{22}{}^{-1}(\infty,-\infty)]^{-1}[S^{-1}(z',-\infty)\sigma_3]_{2\beta}\}_{\omega\omega'},$$
$$G^A_{\omega\omega'\alpha\beta}(z<z')=\frac{i}{v}\{S_{\alpha1}(z,-\infty)[S_{11}(\infty,-\infty)]^{-1}[S(\infty,z')\sigma_3]_{1\beta}\}_{\omega\omega'},$$

where

$$S_{\alpha\beta}(z,z')=T_z\exp\left\{\int_{z'}^{z}\left[i\omega\sigma_3-i\sigma_3\Psi(z_1)-i\sigma_3\int\frac{d\omega_1}{2\pi}\Phi_{\omega_1}(z_1)P_{\omega_1}\right]\frac{dz_1}{v}\right\}, \tag{6}$$

P_{ω_1} is the operator for the frequency shift by ω_1. The α and β components of $S_{\alpha\beta}$ remain operators acting on the variable ω. We must understand, for example, $[S_{22}(\infty,-\infty)]^{-1}$ in just this sense. The factors in the expressions (5) are non-commutative.

The processes of "forward" impurity scattering are unimportant in the longitudinal conductivity, as in the foregoing. In view of this, we can assume that

$$\Psi(z_1)=\zeta(z_1)\sigma_+/2+\zeta^*(z_1)\sigma_-/2. \tag{7}$$

As to the phonon field, we start from the usual model of Fröhlich (see, for example, Ref. 11) and therefore take the phonon operator in the form

$$\varphi(\mathbf{r})=i\left(\frac{2\pi^2\zeta}{Vp_0m}\right)^{1/2}\sum_{\mathbf{k}}\left(\frac{\omega_{\mathbf{k}}}{2}\right)^{1/2}\{c_{\mathbf{k}}\exp[i(\mathbf{kr}-\omega_{\mathbf{k}}t)]$$
$$-c_{\mathbf{k}}^+\exp[-i(\mathbf{kr}-\omega_{\mathbf{k}}t)]\}, \tag{8}$$

where

$$\langle c_{\mathbf{k}}^+c_{\mathbf{k}'}\rangle=N_{\mathbf{k}}\delta_{\mathbf{kk}'}=[\exp(\omega_{\mathbf{k}}/T)-1]^{-1}\delta_{\mathbf{kk}'}. \tag{9}$$

In the case considered, those processes are significant in which either $k_z\approx0$ or $k_z\approx\pm2p_0$. We shall assume that the corresponding vicinities are small, so that in all the factors of Eq. (8) we can set $k_z=0$ or $k_z=+2p_0$ with the exception of $e^{i\mathbf{k}\cdot\mathbf{r}}$.[1)] For example, we consider the case $k_z=0$. We denote

$$\sum_{k_z}c_{\mathbf{k}}e^{ik_zz}=c_{\mathbf{k}_\perp}(z).$$

In accord with (9), we have

$$\langle c^+_{\mathbf{k}_\perp}(z)c_{\mathbf{k}'_\perp}(z')\rangle=\delta_{\mathbf{k}_\perp\mathbf{k}'_\perp}N(\mathbf{k}_\perp,0)\sum_{k_z}\exp[ik_z(z-z')]$$
$$\to S(2\pi)^2\delta(\mathbf{k}_\perp-\mathbf{k}_\perp{}')\delta(z-z')N(\mathbf{k}_\perp,0), \tag{10}$$

where S is the area of the cell section perpendicular to the filaments. If $T\gg\omega_D$, then $N_{\mathbf{k}}\approx N_{\mathbf{k}}+1\approx T/\omega_{\mathbf{k}}\gg1$. In view of this, the phonon field operators become the classical components of the random potential.

On the basis of everything given above, we can take as the phonon operator in (6)

$$\Phi_\omega(z)=\Phi_{1\omega}(z)+\Phi_{2\omega}(z), \tag{11}$$

where

$$\Phi_{1\omega}(z)=\frac{2\pi}{\sqrt{2}}\int\frac{d\mathbf{k}_\perp}{(2\pi)^2}\{c_{\mathbf{k}}(z)\delta(\omega+\omega_{\mathbf{k}})+c_{\mathbf{k}}^*(z)\delta(\omega-\omega_{\mathbf{k}})\}, \tag{12}$$

$\mathbf{k}=(0,\mathbf{k}_\perp)$ and

$$\langle c_{\mathbf{k}}(z)c_{\mathbf{k}'}^*(z')\rangle=vS\tau_{ph1}^{-1}\delta(z-z')(2\pi)^2\delta(\mathbf{k}_\perp-\mathbf{k}_\perp{}') \tag{13}$$

($\tau_{\mathrm{ph1}}^{-1}=\lambda_1T$, where λ_1 is the dimensionless constant of the electron-phonon interaction);

$$\Phi_{2\omega}(z)=\frac{2\pi}{\sqrt{2}}\int\frac{d\mathbf{k}_\perp}{(2\pi)^2}\left\{[a_{\mathbf{k}}(z)\delta(\omega+\omega_{\mathbf{k}})+b_{\mathbf{k}}^*(z)\delta(\omega-\omega_{\mathbf{k}})]\frac{\sigma_+}{2}\right.$$
$$\left.+[a_{\mathbf{k}}^*(z)\delta(\omega-\omega_{\mathbf{k}})+b_{\mathbf{k}}(z)\delta(\omega+\omega_{\mathbf{k}})]\frac{\sigma_-}{2}\right\}, \tag{14}$$

and $\mathbf{k}=(2p_0,\mathbf{k}_\perp)$,

$$\langle a_{\mathbf{k}}(z)a_{\mathbf{k}'}^*(z')\rangle=\langle b_{\mathbf{k}}(z)b_{\mathbf{k}'}^*(z')\rangle=vS\tau_{ph2}^{-1}\delta(z-z')(2\pi)^2\delta(\mathbf{k}_\perp-\mathbf{k}_\perp{}') \tag{15}$$

($\tau_{\mathrm{ph2}}^{-1}=\lambda_2T$, where λ_2 is a dimensionless constant).

We shall first show that certain terms in the formula (3) are unimportant. If, as in Ref. 1, we add and subtract an expression corresponding to the integral term in j without impurities and phonons, we can then substitute in the difference the expressions (5), which are suitable for the vicinity of the Fermi boundary. In addition, the "free" expression with the integrals taken in correct order, vanishes, while the free expression in the difference cancels the first term in (3). Thus, the integral term (3) remains with the G functions in the form (5). Upon substitution of these expressions in the first integral term in (3), which does not contain the products G^RG^A, we obtain terms with the unequal number ζ and ζ^* (or a and a^*, and b^*) (see Ref. 1). Therefore, zero is obtained upon averaging.

Further, we can obtain the following relation from Eq. (5):

$$G^R(z,z_1)\sigma_3G^R(z_1,z)=-\left(\frac{i}{v}\right)[\theta(z-z_1)\theta(z_1-z')$$
$$-\theta(z'-z_1)\theta(z_1-z)]G^R(z,z'). \tag{16}$$

The integral over z_1 gives the following in this case:

$$\int G^R(z,z_1)\sigma_3G^R(z_1,z)dz_1=-(i/v)(z-z')G^R(z,z'). \tag{16'}$$

The situation is similar for G^A.

We now consider the last term in (3). If the vector potential A did not depend on the time, then $\omega_2 = 0$ and, taking the integral over z_2, we would have obtained zero. This is natural, because the current should be proportional to $E = (i\omega/c)A_\omega$. Further, transforming to the representation of the interaction in ω and recognizing that in the averaging over the impurity and the phonon fields only factors of the type $\exp[iz_1(\omega_1 - \omega_2)/v]$ can always appear, we see that the point from which the frequencies are reckoned is unimportant, i.e., we can always set one of the ε equal to zero in the products of G functions. If we set $\varepsilon_1 = 0$ in the last term in (3), there is left

$$\int_{-\infty}^{\infty} \frac{d\varepsilon_1}{2\pi}\left(\operatorname{th}\frac{\varepsilon_1}{2T} - \operatorname{th}\frac{\varepsilon_1-\omega_1}{2T}\right) = \frac{\omega_1}{\pi}, \tag{17}$$

where ω_1 corresponds to the field Φ_{ω_1}.

Since the last term in (3) vanishes in the case of a time-independent A, we can then expand in ω_2 and keep the term of first order. Then, applying the relation (16), we immediately obtain the contribution to the conductivity

$$\left(\frac{ie^2v}{\pi S}\right) \mathrm{Sp} \int \frac{d\varepsilon}{2\pi} \int \frac{d\varepsilon_1}{2\pi} \int \frac{d\omega_1}{2\pi} \int dz_1\, dz_2 [\sigma_3 G^R_{\varepsilon,\varepsilon_2}(z, z_2)\, G^R_{\varepsilon_2,\varepsilon_1}(z_2, z_1)$$
$$\times (z_2 - z_1)\, \Phi_{\omega_1}(z_1)\, \omega_1 G^A_{\varepsilon_1-\omega_1,\varepsilon}(z_1, z) - \sigma_3 G^R_{\varepsilon,\varepsilon_1}(z, z_1)\, \Phi_{\omega_1}(z_1)\, \omega_1 G^A_{\varepsilon_1-\omega_1,\varepsilon_2}(z_1, z_2)$$
$$\times (z_1 - z_2)\, G^A_{\varepsilon_2,\varepsilon}(z_2, z)]. \tag{18}$$

It was shown in Refs. 1 and 2 that differences in the coordinates of the order of l^* are significant in the integrals. Here l^* is the total path length: $l^* = (l_{2\mathrm{ph}}^{-1} + l_i^{-1})^{-1}$. We can therefore make the following substitution:

$$G^R_{\varepsilon_2,\varepsilon_1}(z_2, z_1)\,(z_2 - z_1) \to l^{*2}\delta(z_2 - z_1)\,[G^R_{\varepsilon_2,\varepsilon_1}(z_1 + 0, z_1) - G^R_{\varepsilon_2,\varepsilon_1}(z_1 - 0, z_1)]$$
$$\to -(i/v)\, l^{*2}\delta(z_2 - z_1)\,\sigma_3 \cdot 2\pi\delta(\varepsilon_2 - \varepsilon_1). \tag{19}$$

Substituting the results in (12), we get zero as a result.[2]

Thus, only the second term in (3) remains, corresponding completely to the expression in Ref. 1 for $T = 0$. We can again set $\varepsilon_1 = 0$ in the product G and obtain the integral (17).

Since the changes in the frequency are cancelled upon averaging over the phonons, it follows that $\omega_1 = \omega$. This makes it possible to transform immediately to the static conductivity:

$$\sigma = \frac{e^2v^2}{\pi S} \mathrm{Sp} \int_{-\infty}^{\infty} \frac{d\varepsilon}{2\pi} \int_{-\infty}^{\infty} dz_1 [G^A_{0,\varepsilon}(z_1, z)\, \sigma_3 G^R_{\varepsilon,0}(z, z_1)\, \sigma_3]. \tag{20}$$

We could have written any ε_1 in place of the frequency argument zero and consequently,

$$\lim \frac{1}{2\Omega} \int_{-\Omega}^{\Omega} d\varepsilon_1 \quad \text{as} \quad \Omega \to \infty.$$

But at large Ω, the integral $\int d\varepsilon\, d\varepsilon_1$ gives Sp_ε. If we now substitute Eq. (5) there, we obtain

$$\mathrm{Sp}_\varepsilon\{[S_{11}^{-1}(\infty, -\infty)]^{-1}[S_{11}(\infty, -\infty)]^{-1}\}.$$

Finally, taking it into account that even this expression does not depend on ε_1, we can cross out the result

$$\lim \frac{1}{2\Omega} \int_{-\Omega}^{\Omega} d\varepsilon_1 \quad \text{as} \quad \Omega \to \infty.$$

Thus, we obtain, in complete analogy with Ref. 2,

$$\sigma = \frac{e^2L}{\pi S} \int \frac{d\varepsilon}{2\pi} [S_{11}^{-1}(L, 0)]^{-1}_{0,\varepsilon} [S_{11}(L, 0)]^{-1}_{\varepsilon,0}. \tag{21}$$

We transform here to the finite interval $(L - 0)$ in place of $(\infty, -\infty)$.

3. INTERSECTING AND NONINTERSECTING DIAGRAMS

Equations (20) and (21) can be described by means of diagrams. Naturally, just as in Refs. 2–4, the problem arises as to the role of "nonintersecting" diagrams. We find the phase factors of the series of intersecting diagrams shown in Fig. 1, where the impurity averages $\langle|\zeta|^2\rangle$ are indicated by the dashed lines and the corresponding phonon averages by the wavy lines. The results are

$$\text{a) } \exp[2i\omega_{\mathbf{k}}(z_l - z_i)/v], \quad \text{b) } \exp[2i\omega_{\mathbf{k}}(z_l - z_m)/v],$$
$$\text{c) } \exp[2i\omega_{\mathbf{k}}(z_l - z_m)/v], \quad \text{d) } 1, \quad \text{e) } \exp[2i\omega_{\mathbf{k}}(z_l - z_i)/v],$$
$$\text{f) } \exp\{2i[\omega_{\mathbf{k}1}(z_i - z_l) - \omega_{\mathbf{k}2}(z_m - z_l)]/v\},$$
$$\text{g) } \exp\{2i[\omega_{\mathbf{k}1}(z_l - z_m) + \omega_{\mathbf{k}2}(z_i - z_l) + \omega_{\mathbf{k}3}(z_m - z_i)]/v\}. \tag{22}$$

Thus, it is seen that any interaction, with the exception of the intersection of the c-phonon lines with one another (and, of course, of the impurity lines with one another), give factors of the type $\exp[2i\omega_{\mathbf{k}}(z_i - z_l)]$. Such

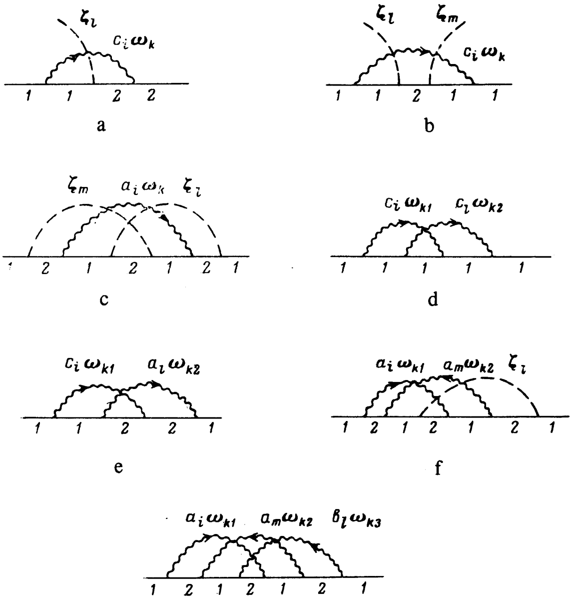

FIG. 1.

a result is quite natural. If we transform in the S matrix to the representation of the interaction in ω, then the c-operators transform to $c_i \exp[-i\omega_k\sigma_3 z_i/v]$, i.e., to expressions that do not contain ω and therefore commute with one another. All the remaining operators will contain factors of the type $\exp[\pm 2i\omega z_i/v]$, which change under the action of the shifts $P_{\omega k}$.

The significant distances are $|z_i - z_l| \sim l^*$. Therefore, two limiting cases are possible.

1) If $\bar{\omega} l^*/v \gg 1$, then averaging over the small interval $\Delta k_\perp$ leads to vanishing of phase factors of the type (22). Consequently, in this case, we do not need to take into account any intersections other than the intersection of the impurity lines with one another.[3] The limit $\bar{\omega} l^*/v \gg 1$ was considered in Ref. 5, but from the viewpoint of real objects this case is evidently not of interest. Actually, taking into account the equality $l^* = (l_i^{-1} + l_{2\mathrm{ph}}^{-1})^{-1}$, we obtain the result that the condition $\bar{\omega} l^*/v \gg 1$ means that $(\bar{\omega}/v)\min(l_i, l_{\mathrm{ph}} \gg 1$. Consequently, at any rate, it is necessary to have $\omega_D l_i/v \gg 1$, i.e., a sufficiently pure material. Since the free path length $l_{\mathrm{ph}} \sim (v/\lambda T)(\omega_D/T)^2$ and $\bar{\omega} \sim T$ at $T \ll \omega_D$, and at $T \gg \omega_D$ we have $l_{\mathrm{ph}} \sim v/\lambda T$, $\bar{\omega} \approx \omega_D$, where λ is the dimensionless constant of electron-phonon interaction, and usually $\lambda \sim 1$, it follows that the condition $(\bar{\omega}/v)\, l_{\mathrm{ph}} \gg 1$, is feasible only at $T \ll \omega_D$. This means that the material should not experience a Perierls transition to a dielectric at $T_c \sim \omega_D$. But in experiment, such a transition is absent only in materials with internal disorder, for which the condition $\omega_D l_i/v \gg 1$ is most readily violated.

There is one more important point. At $T \ll \omega_D$ the phonons are essentially quantum. This leads to the appearance of diagrams of the type of Fig. 2, which, in contrast to the three-dimensional situation, are not small; however, they are not considered in Ref. 5. It is difficult to state to what consequences these diagrams lead; therefore, the calculation of Gogolin, Mel'nikov and Rashba is justified only at $T \gg \omega_D$. In principle, the simultaneous satisfaction of the conditions $(\bar{\omega}/v) l_{\mathrm{ph}} \gg 1$ and $T \gg \omega_D$ is possible, but for this case it is necessary to assume, without special justification, that the dimensionless constant $\lambda \ll 1$. Such a calculation would furthermore be valid only in the region of temperatures $\omega_D/\lambda \gg T \gg \omega_D$. Thus, strictly speaking, the calculation of Ref. 5 refers only to the exotic situation in which it is also necessary that $\omega_D l_i/v \ll 1$.

2) If $T \gg \omega_D$ (and $\lambda \sim 1$), then $\bar{\omega} l^*/v \sim \omega_D l^*/v \ll 1$ always, and the relation between l_i and l_{ph} can be arbitrary. We shall consider just this case. The phase factors in this case differ little from unity.

We call attention to the great similarity of the case considered here to that treated in Ref. 2. There the violation of localization takes place because of the non-one-dimensionality, and we could isolate two limiting cases. In the case of "large non-one-dimensionality" we could neglect diagrams with intersection and the kinetic equation turns out to be applicable. In the case of "small non-one-dimensionality," the effects of localization were significant, and we used the similarity hypothesis ("scaling"). In the considered problem, the role of non-one-dimensionality is played by inelasticity and there are also two limiting cases.

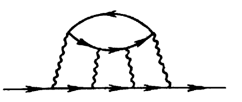
FIG. 2.

4. CONDUCTIVITY AT $T \gg \omega_D$, $\omega_D l^*/v \ll 1$.

Thus the case considered is similar to the situation studied in Refs. 2–4. Consequently, we must expect that the conductivity will be nonanalytic in the delocalization parameter in the vicinity of zero. In order to get around this difficulty, we again apply the similarity hypothesis, but in a different variant. A finite sample along the z direction was considered in Refs. 2–4 and it was assumed that $\sigma = qf(\gamma/q^\nu)$, where $k = \exp(-L/4l_2)$, and γ is the delocalization parameter. But it is clear physically that the transition from a finite sample to an infinite one is not a neutral procedure and it is desirable to avoid it. Also suspicious is the fact that the expansion of σ in γ in Refs. 2–4 does not have a "scaling" form.[4] In view of this, we consider the infinite sample, but we shall find the static conductivity as the limit of $\sigma(\omega_0)$ as $\omega_0 \to 0$. This leads to the transition $G^R_{\varepsilon 0} \to G^R_{\varepsilon+\omega 0\omega 0}$ in formula (20); formula (21) is inapplicable in this case.

We now make the assumption that scaling holds in the theory and σ has the self-similar form

$$\sigma = \sigma_0 f\left[\gamma\left(\frac{iv}{\omega_0 l^*}\right)^\nu\right]. \tag{23}$$

Here σ_0 is the value of σ at $\gamma = 0$, i.e., the result for a purely one-dimensional case (see, for example, Ref. 1). Taking the two spin projections into account, we have

$$\sigma_0 = \frac{8\zeta(3)e^2 l^*}{\pi S}\left(\frac{\omega_0 l^*}{iv}\right). \tag{24}$$

The function f possesses the following properties: $f(0) = 1$, and as $\omega_0 \to 0$ the conductivity σ should not depend on ω_0. For this, it is necessary that $f(x \to \infty) \propto x^{1/2}$, and then $\sigma(0) \sim (e^2 l^*/\pi S)^{1/\nu}$.

We must now find the delocalization parameter γ and determine the exponent ν. Here we must distinguish two cases.

a) If $l_i \gg l_{\mathrm{ph}}$, then $l^* \approx l_{2\mathrm{ph}}$.[5] Since the reason for the delocalization is the inelasticity of the scattering, the only quantity which can serve for γ is $\gamma = \omega_D l^*/v$.

We determine the exponent ν from the expansion of σ in a series in γ (at $\omega_0 \neq 0$) σ is an analytic function of γ as $\gamma \to 0$). However, there is no need of completing this complicated calculation. The exponent ν can be found with the help of a qualitative discussion. For example, we consider the first term of the expansion. For this purpose, we isolate a single phonon line corresponding to some z. The Green's functions inside this line have a frequency greater by ω_k than those which are located outside this line. In the expansion in ω_k, we must take terms of even order, because $\pm\omega_k$ enters symmetrically. At $T \gg \omega_D$, we can assume $\omega_k \sim \omega_D$.

Just as in Ref. 1, the result is expressed in final analysis in terms of the functions B_n (see Ref. 1, formula (52)). These functions satisfy the set of equations (57) of Ref. 1 (we need only the part of B_n which does not depend on z):

$$n^2(B_{n-1}+B_{n+1}-2B_n)+i\beta n B_n=0,$$

where $\beta=2\omega_0 l^*/v$. The boundary condition is $B_0=1$. At $\beta \ll 1$, the large n will be significant; we can transform to the continuous variable n and from the difference equations to the differential equation

$$n^2 B_n''+i\beta n B_n=0. \tag{25}$$

A solution is

$$B_n=uK_1(u), \quad u=2\sqrt{-i\beta n}, \tag{26}$$

where K_1 is the Hankel function of imaginary argument. Thus, we can assume that the $n\sim\beta^{-1}$ are significant.

In the expansion over ω_k we obtain one extra Green's function for each degree of ω_k. These functions are expressed by the formulas (5). Consequently, each of the degrees of ω_k corresponds to an increase in the degree of A_{11} or S_{22} in the denominator. In the reduction of the general expression to the functions B_n, we make use of the formula

$$S_{11}^{-k-1}=(S_{11}^{(1)}S_{11}^{(2)}+S_{12}^{(1)}S_{21}^{(2)})^{-k-1}=\sum_{n=0}^{\infty}(-1)^n\frac{(n+k)!}{n!k!}\frac{(S_{12}^{(1)}S_{21}^{(2)})^n}{(S_{11}^{(1)}S_{11}^{(2)})^{n+k+1}} \tag{27}$$

where

$$S_{11}=S_{11}(\infty,-\infty), \quad S_{11}^{(1)}=S_{11}(\infty,z), \quad S_{11}^{(2)}=S_{11}(z,-\infty),$$

and a similar formula for S_{22}. Evidently, the increase in the degree k by unity leads to the appearance of an extra degree of the number n under the summation sign, and this latter leads to the appearance of the factor β^{-1}. We can then conclude that to each degree of ω_k in the denominator there corresponds a single degree of ω_0; consequently, in the formula (23), $\nu=1$.

The following question can arise here: is not the increase in the degree of β in the denominator due to the isolation of the phonon line? In fact, for example, when a single phonon operator is isolated

$$G_A(zz')\to\int G_A(zz_i)\Phi(z_i)G_A(z_iz')\,dz_i,$$

i.e., the number of the Green's functions increases. We can immediately answer this question in the negative. Actually, imagine that we have somewhat renormalized the phonon operators. This leads to a small change in l^* and, in accord with formula (24), this cannot lead to the appearance in σ of terms with another degree of ω_0. On the other hand, in first order, the result of renormalization is obtained by the isolation of the phonon line for z_i, by multiplication by the renormalization, and by summation over all z_i. Consequently, the isolation of the phonon line does not lead to a change in the degree of β. The validity of the given discussion can be demonstrated directly by calculation of the lowest order in ω_D/ω_0, i.e., $(\omega_D/\omega_0)^2$.

It follows from formula (23) with $\nu=1$ that the static conductivity at $l_i \gg l_{ph}$ has the order[6]

$$\sigma\sim(e^2l^*/\pi S)(\omega_D l^*/v). \tag{28}$$

b) The situation is more complicated at $l_i \ll l_{ph}$. Since the inelasticity is connected with the phonons, it follows that in spite of the fact that $l_{ph}^{-1} \gg l_i^{-1}$ and $l^*\approx l_i$, all the l_{ph} are significant and two small parameters l_i/l_{ph} and $\omega_D l^*/v$ enter into the theory. In this case it is no longer so simple to write down the similarity relation.

We again consider the lowest order. Isolating the phonon line, we do not obtain an increase in the degree of ω_0 in the denominator, but then performing the expansion in ω_k, we establish the fact, as before, that this is in fact an expansion in $(\omega_D/\omega_0)^2$. Thus, in first order in l_{ph}^{-1}, we get

$$\sigma_1=\sigma_0\frac{l^*}{l_{ph}}f\left(\frac{i\omega_D}{\omega_0}\right). \tag{29}$$

At first glance, it appears that this is not the same function f as in formula (23) for the previous case, because there we had the entire sum of diagrams with many phonon lines and in the given case there is only a single phonon line. However, we can have recourse to the following reasoning. If we carry out a small renormalization of the phonon operators in the previous case, then this does not affect the function f, which depends only on ω_D/ω_0, and, on the other hand, this reduces to the isolation of a single phonon line. This means that the function f in (29) must be the same as in the previous case. It then follows that at $\omega_0 \ll \omega_D$ we have $f\sim i(\omega_D/\omega_0)$, and consequently,

$$\sigma_1\sim\frac{e^2}{\pi S}\frac{l^{*2}}{l_{ph}}\frac{\omega_D l^*}{v}. \tag{30}$$

We can consider the obtained $\sigma_0(23)$ and $\sigma_1(30)$ as terms of the expansion of σ in the quantity $q=(l^*/l_{ph})\times(i\omega_D/\omega_0)$. This expansion holds for $\omega_D \gg \omega_0 \gg (l^*/l_{ph})\omega_D$. Since, by assumption, $l^*\approx l_i \ll l_{ph}$, such a region does exist. An now we again make an assumption of the type (23). Comparing (30) and (24), we find $\nu=1$ and

$$\gamma=\frac{l^*}{l_{ph}}\frac{\omega_D l^*}{v}. \tag{31}$$

The expression for σ at $\omega_0=0$ turns out to be of the order of σ_1:

$$\sigma(0)\sim\frac{e^2}{\pi S}\frac{l^{*2}}{l_{ph}}\frac{\omega_D l^*}{v}. \tag{32}$$

This formula for the static conductivity is a general one and is suitable at arbitrary relation of l_i and l_{ph}.

As has already been noted, only l_{2ph} enters into l^*. So far as l_{ph}^{-1} is concerned in (32), this quantity is $al_{ph1}^{-1}+bl_{ph2}^{-1}$, where $a, b\sim 1$, but $a\neq b$. This can be seen by calculating Eq. (29) in lowest order in ω_D/ω_0, i.e., $(\omega_D/$

$\omega_0)^2$. However, at $T \gg \omega_D$ we can assume $l_{1\text{ph}} \sim l_{2\text{ph}}$.

Equation (32) can be given a "diffusion" interpretation. In the localized state, the electron executes a finite motion with frequency of the order of v/l^* and amplitude l^*. The decay probability of this state, or the level width, is proportional to the probability of capture of the phonon during the period of the oscillations, i.e., $\tau^*/\tau_{\text{ph}} = l^*/l_{\text{ph}}$ and the smearing out of the line due to inelasticity, i.e., ω_D. Consequently, the mean lifetime of the state is

$$t \sim [(\tau^*/\tau_{ph})\omega_D]^{-1}. \tag{33}$$

Applying the diffusion formula, we obtain

$$\sigma \sim e^2\nu(\mu)D \sim e^2\nu(\mu)\overline{l^2}/t \sim e^2(\pi v S)^{-1}l^{*2}\omega_D(l^*/l_{ph}),$$

i.e., Eq. (32) ($\nu(\mu)$ is the density of states on the Fermi boundary). We also note that at $\omega_D l^*/v \sim 1$, Eq. (32) joins together with the formulas of Ref. 5, obtained under the assumption $\omega_D l^*/v \gg 1$.

Substituting in (32)

$$l^{*-1} = l_i^{-1} + l_{2ph}^{-1}, \quad l_{ph1,2}^{-1} = \lambda_{1,2}T/v,$$

we obtain

$$\sigma(0) \sim \frac{e^2}{\pi S}\frac{\omega_D}{v}\frac{\lambda T}{v}(l_i^{-1} + \lambda_2 T/v)^{-3}. \tag{34}$$

In particular,$^{7)}$ $\sigma(0) \propto T$ at $l_i \ll l_{\text{ph}}$ and $\sigma(0) \propto T^{-2}$ at $l_i \gg l_{\text{ph}}$.

It is seen from Eq. (34) that the phonons play a double role. At $l_i \gg l_{\text{ph}}$ the basic role of the phonons is the delocalization of the electrons as a consequence of the inelasticity of the collisions. Here σ increases proportionally with T. Upon further increase in the temperature, when $l_{\text{ph}}^{-1} \sim \lambda T/v \gg l_i^{-1}$ the phonons form an additional random potential, strengthening the localization. Here σ falls off as T^{-2}. This situation is also similar at $\omega_D l^*/$ $\gg 1$ (see Ref. 5), although the detailed laws for this case are different ($\sigma \propto T$ at $l_i \ll l_{\text{ph}}$ and $\sigma \propto T^{-1}$ at $l_i \gg l_{\text{ph}}$).

We note that if the material is very pure, so that $\omega_D l_i/v \gg 1$, or undergoes a phase transition into a dielectric at not too low temperatures, then our theory gives a good description of the high-temperature region, where $\sigma \propto T^{-2}$.

The formulas obtained also make it possible to trace the dependence of σ on the frequency ω_0. We have found that in the "richest" case $l_i \ll l_{\text{ph}}$ there are three regions.

a) If $\omega_0 \gg \omega_D$, then $\sigma = \sigma_0$ and is given by Eq. (24). The real part of σ in this case is the same as for the purely one-dimensional metal:

$$\operatorname{Re}\sigma \sim (e^2/\pi S)l^*(\omega_0 l^*/v)^2 \ln^2(v/\omega_0 l^*).$$

Actually, this region corresponds to infrared light and at such frequencies there are many other mechanisms of interaction of radiation with matter.

b) If $\omega_D l^*/l_{\text{ph}} \ll \omega_0 \ll \omega_D$, then $\sigma = \sigma_0 + \sigma_1$ where σ_1 is given by Eq. (30).

c) Finally, if $\omega_0 \ll \omega_D l^*/l_{\text{ph}}$, then

$$\sigma = \sigma_0(c_1 q + c_2 + \ldots) \approx \sigma(0) + c_2\sigma_0, \tag{35}$$

where

$$q = (l^*/l_{ph})(i\omega_D/\omega_0), \quad c_1, c_2 \sim 1.$$

Consequently, in this region, $\operatorname{Re}\sigma$ and $\operatorname{Im}\sigma$ are the same type as in the previous case; however, the numerical coefficients are different.

At $l_i \gg l_{\text{ph}}$, the region b) is missing. We note that at $\omega_D l^*/v \gg 1$, $l_{\text{ph}} \gg l_i$ and at low frequencies ($\omega_0 l^*/v \ll 1$) σ is simply σ_0 with the replacement $\omega_0 \to \omega_0 + iv/l_{\text{ph}}$ (see Ref. 5), i.e., it corresponds qualitatively to our result for the regions (b) and (c) at $\omega_D l^*/v \sim 1$.

5. COMPARISON WITH EXPERIMENT

In a comparison of the obtained formulas with experimental data, we must first look out for the satisfaction of the criteria of applicability of our calculation:

$$T \gg \omega_D, \quad \omega_D\tau^* \ll 1.$$

If we speak of a real substance with large number of branches of the vibrational spectrum, then the condition $T \gg \omega_D$ means, strictly speaking that the temperature should be greater than all the limiting frequencies, some among which can be rather high. Taking it into account that real substances decompose at high temperatures and possibly, therefore, all the experimental data refer to $T < 300$ K, we see that the criterion $T \gg \omega_D$ may not be satisfied.

At $T < \omega_D$ the quantum nature of the photons begins to appear and diagrams of type of Fig. 2 begin to be significant. But, even if we make the assumption that these diagrams are unimportant, then at not too low an impurity path length the second condition $\omega_D\tau^* \ll 1$ can be violated.

In view of the fact that it is very difficult to tell ahead of time with what case one is dealing in any given experiment, we shall attempt to understand what follows from the experimental data themselves.

The clearest statement is that in the range $100\text{ K} < T < 300\text{ K}$ we have

$$\rho(T)/\rho(300\text{ K}) = a + bT^{\xi}, \tag{36}$$

where the constant a depends on the sample, but the second term is identical in both samples of the given material. This fact was established in the materials TTF–TCNQ ($\xi = 2.33$),[14] HMTSF–TCNQ ($\xi = 2.39$)[15] and TTT_2I_3 ($\xi = 2$).[16]

This result can be interpreted as follows. We have seen that, in accord with formula (34), the phonon and impurity resistances add up in a far from simple manner. It is then clear that the first term in (36) is due not with the impurities but to some three-dimensional resistances, connected in series with the sample. Whether there are some breaks in the principal chains of the sample or the effect of the measurement contacts—only the authors of the experiment can decide. But, one way or the other, the effect is apparently constant and it is necessary to consider only the second term in (36). The independence of this term of the sample indicates that under the experimental conditions, the impurity scattering is small in comparison with the phonon scattering.[8] But in this case, our theory predicts $\xi = 2$ while experimentally, this quantity is sometimes larger, although not by much.

It remains to assume that the effective Debye temperature lies somewhere inside the investigated range or near it. The question arises: is it impossible to "touch up" the theory, so as to keep $T \sim \omega_D$. If we assume that diagrams of the type of Fig. 2 are unimportant, we could take the value

$$l_{ph}^{-1} = (\lambda\omega_D/v)[\exp(\omega_D/T)-1]^{-1} \tag{37}$$

for l_{ph}^{-1} instead of the value $l_{\text{ph}}^{-1} = \lambda T/v$ used earlier. Such a form for l_{ph}^{-1} corresponds to the assumption that processes with $p_0 \to -p_0$ are essential in phonon scattering and that there is no substantial transverse dispersion (i.e., $\omega(2p_0, \mathbf{k}_\perp) \approx \omega_D$).

However, the condition $\bar{\omega}\tau \ll 1$ is violated at $T < \omega_D$, as has already been noted. But processes with $p_0 \to p_0$ and with $p_0 \to -p_0$ are insufficiently sharply distinguished there. A more precise treatment of the derivation given in Ref. 5 shows that if $l_{\text{ph}} \ll l_i$, then $\sigma = e^2 l_{\text{ph}2}(\pi S)^{-1}$. Consequently, ρ is proportional to the number of phonons with $k_z = 2p_0$, i.e., at $T \ll \omega_D$ it depends exponentially on the temperature. It appears that at $T \ll \omega_D$ and without account of the diagrams of Fig. 2, the assumptions $\omega_D \ll 1$ and $\omega_D\tau \gg 1$ give qualitatively the same result: in the first case, $\rho \propto \exp(-2\omega_D/T)$, in the second, $p \propto \exp(-\omega_D/T)$. Since, on the one hand, the quantity ω_D used in our model does not acquire a very definite character, in comparison with experiment, while, on the other hand, under real conditions of experiment, we have more readily $\omega_D \sim T$ rather than any of the limiting cases, it is reasonable to write down the simplest interpolation, which gives the dependences $\omega_D \sim T^2$ at $T \gg \omega_D$ and $\rho \propto \exp(-a\omega_D/T)$ at $T \ll \omega_D$ (where $a \sim 1$). The formula (34) obtained by us with $l_i \gg l_{\text{ph}}$ and l_{ph} in the form (37) can be used as such an interpolation.

For comparison with experiment, we can write

$$\rho = \rho_0 + \rho_1(T), \quad \frac{\rho_1(T)}{\rho_1(300\,\text{K})} = \left(\frac{e^{\alpha}-1}{e^{\alpha/\tau}-1}\right)^2, \tag{38}$$

where $\alpha = \omega_D/300$ K, $\tau = T/300$ K. Rewriting Eq. (36) in this form, we obtain

$$\rho_1(T)/\rho_1(300\,\text{K}) = \tau^{\xi}. \tag{36'}$$

In the comparison of the curves obtained from both formulas, we must keep it in mind that the breakup of ρ by ρ_0 and ρ_1 is not single-valued, because ρ_1 can include an arbitrary constant (identical, naturally, for both samples). Hence formula (36′) should be compared not with (38) but with the formula

$$\frac{\rho_1(T)}{\rho_1(300\,\text{K})} = \frac{(e^{\alpha/\tau}-1)^{-2}+c}{(e^{\alpha}-1)^{-2}+c}. \tag{38'}$$

By choice of the two constants α and c, we can make the curves (38′) and (36′) congruent at $\xi = 2-2.4$ and $\tau \approx 0.25-1$ with accuracy to within 2–3%, which corresponds to the accuracy of the congruence of the curves for different samples and their description by Eq. (36′). We note that the constant c was necessary for us only in the comparison of Eqs. (38) and (36′). Actually, for treatment of the experiment, we set up the dependence $\ln[1-\rho(T)/\rho(300\,\text{K})]$. Here the curves with different ρ_0 turn out to be identical, but shifted along the vertical. They can then be made coincident by means of a shift. Consequently, the experimental data can be compared with (38).

It seems to us that we can then draw the following conclusions. First, one must not attach to Eq. (36) the meaning of an accurately established physical law, because the completely different formula (38), which depends, as does (36′) on a single adjusted parameter, describes the high-temperature data with the same accuracy; a formula of the type (36′) can serve only as a convenient empirical formula. It seems that the formula (38) has two advantages in this sense. First, it is exact at $T \gg \omega_D$; second, the determination of α from comparison with experimental data makes it possible to find ω_D, i.e., some measure of the limiting vibrational frequency.

Comparison shows that $\xi \leqslant 2.4$, which corresponds to $\omega_D \leqslant 150$ K; this is quite reasonable.

Of course, it would be far more interesting to compare formula (34) with the data for such objects for which the criteria of its applicability are satisfied and, in particular, to trace the role of the impurity scattering. Without speaking of searches for new materials, one should advance in the already existing materials, so far as this is possible, towards higher temperatures and, in addition, to try to introduce additional "impurities" for example, by the method of disturbing the stoichiometry.

APPENDIX

The relations (32)–(34), (36) and (37) connecting the components of S with the components of S^{-1} and $G(z, z')$ with $G(z'z)$, and also the transformations of S and G in the substitution $\xi \rightleftharpoons \xi^*$, were all established in Ref. 1. All these referred to the impurity potential, which is time-independent. We can generalize these relations for the phonon potential (11) of the present work (see also (12) and (14)).

First of all, we connect the components of the matrix S^{-1} with the components of S. In the presence of the impurity potential only, using expressions of the type (23)–(27) from Ref. 1, we can obtain the relation (32). In the case considered in the present work, the situation is somewhat different. There is no direct connection between the components of S^{-1} and S. However we can do the following. In the function with which the averaging is carried out, there appear $|c|^2$, $|a|^2$ and $|b|^2$, in which the latter enter symmetrically. If we make the substitutions $c \rightleftarrows c^*$, $a \rightleftarrows b^*$ in all the averaged expressions, then the result does not change. On the other hand, taking the first terms of formulas of the type (23)–(26) from Ref. 1, we establish the fact that

$$\begin{aligned} &S^{-1}_{11\omega,\omega'}(z,z') \to S_{22\omega',\omega}(z,z'), \quad S^{-1}_{12\omega,\omega'}(z,z') \to -S_{12\omega',\omega}(z,z'), \\ &S^{-1}_{22\omega,\omega'}(z,z') \to S_{11\omega',\omega}(z,z'), \quad S^{-1}_{21\omega,\omega'}(z,z') \to -S_{21\omega',\omega}(z,z'). \end{aligned} \tag{A.1}$$

Further, transforming the left and right sides in the relation $(z > z_1 > z')$

$$S^{-1}(z,z') = S^{-1}(z_1,z')S^{-1}(z,z_1) \tag{A.2}$$

according to Eqs. (A.1), we confirm the validity of the formulas (A.1). And now, making use of Eqs. (5) of the present work, we obtain transformation formulas for G^R and G^A (they are identical):

$$\begin{aligned} &G_{11\omega,\omega'}(z,z') \to G_{22\omega',\omega}(z',z), \quad G_{12\omega,\omega'}(z,z') \to G_{12\omega',\omega}(z',z), \\ &G_{22\omega,\omega'}(z,z') \to G_{11\omega',\omega}(z',z), \quad G_{21\omega,\omega'}(z,z') \to G_{21\omega',\omega}(z',z). \end{aligned} \tag{A.3}$$

In order to obtain the transformation formulas similar to (36) and (37) in Ref. 1, we write down the equation for S;

$$\left(\omega + iv\sigma_3\frac{\partial}{\partial z}\right)S_{\omega,\omega'}(z,z') - \int\frac{d\omega_1}{2\pi}\Phi_{\omega_1}(z_1)S_{\omega-\omega_1,\omega'}(z,z') = 0.$$

We multiply on the left and right by σ_1, and obtain

$$\left(-\omega + iv\sigma_3\frac{\partial}{\partial z}\right)\tilde{S}_{\omega,\omega'}(z,z') + \int\frac{d\omega_1}{2\pi}\tilde{\Phi}_{\omega_1}(z)\tilde{S}_{\omega-\omega_1,\omega'}(z,z') = 0,$$

where $\tilde{S} = \sigma_2 S \sigma_1$ and similarly for Φ_{ω_1}.

It follows from Eqs. (11), (12) and (14) that $\tilde{S}_{\omega,\omega'}(z,z')$ satisfies the same equation as $S_{-\omega,-\omega'}(z,z')$ if we change the sign of ω and transform $a \to -a^*$, $b \to -b^*$, $c \to -c^*$. The same result is obtained if we write down the equation for $S(z,z')$ relative to z'. Thus, this transformation, yields the relations

$$S_{11,\omega,\omega'} \rightleftarrows S_{22,-\omega,-\omega'}, \quad S_{12,\omega,\omega'} \rightleftarrows S_{21,-\omega,-\omega'}. \tag{A.4}$$

From Eqs. (5) we get for G

$$G^R_{\omega,\omega'} \rightleftarrows -\sigma_1 G^A_{-\omega,-\omega'}\sigma_1. \tag{A.5}$$

---

1) The corresponding momenta are

$$\Delta k_z \sim \frac{1}{l^*}, \quad \frac{\partial\omega_0}{\partial k_z}t\Delta k_z \sim \frac{sz\Delta k_z}{v} \ll z\Delta k_z$$

(s is the speed of sound).

2) In the approximation (19), two terms in (12) are cancelled. Actually, using the G-function properties found in the Appendix, we can show that the two terms in (12) are equal to one another and both vanish in the approximation (19).

3) So far as the c-lines are concerned, it appears at first glance that their intersections with one another are admissible even at $\bar{\omega}l^*/v \gg 1$. However, in actuality, this is not the case. As has already been made clear, intersections of c-lines with a-, b- and ζ-lines are not admissible. Consequently, all the paired c-operators pertaining to intersecting lines always belong to Green's functions with a single pseudospinor index, for example, 1. Taking it into account that the null and averaged Green's functions are proportional to $\theta(z)$ or $\theta(-z)$, it is easy to see that the intersections of c-lines are impossible.

4) Taking the above into account, there is every reason for assuming that the results obtained in Refs. 2–4 are in error. The correct formulas for the conductivity in the models considered in Refs. 2–4, obtained with use of the method developed below, are given in Ref. 12.

5) The effective path length l^* is determined by scattering from impurities and phonons in the approximation $\omega_k l^*/v = 0$. Here the a- and b-phonons are similar to the ζ-field, while the c-phonons are similar to the η-field, which, as is well known, (see Ref. 1) does not take part in the scattering. Consequently, $l^{*-1} = l_i^{-1} + l_{2\text{ph}}^{-1}$.

6) We note that the conductivity at $\omega_0 \to 0$ is real, while all the terms of the expansion of σ in ω_D have the form $i\omega_0(i\omega_D/\omega_0)^{2k}$, i.e., they are purely imaginary. The appearance of the first degree of ω_D in formula (28) at $\omega_0 \to 0$ and the change of the imaginary conductivity to real can take place if the function f contains a term of the type $[1-(\omega_D/\omega_0)^2]^{1/2}$.

7) The case $l_i \gg l_{ph}$, $T \gg \omega_D$, $\omega_D l_{ph}/v \ll 1$ was considered recently in the work of Madhukar and Cohen[13] where the result $\sigma \propto T^{-2}$ was obtained. Since the note[13] is short, it is difficult to assess the accuracy of this calculation. Doubt is raised also by the fact that the authors throw away part of the diagrams while at $\omega_D l^*/v \ll 1$ all the diagrams are essential. The final formula (15) in Ref. 13 contains a nontrivial coefficient of T^{-2} and even has an incorrect dimensionality.

8) Strictly speaking, this is not obligatory, since the "impurities" frequently correspond to innate disorder in the crystal, and are therefore the same in different samples. Nevertheless, in the experiments under discussion, the form of the temperature dependence $\rho(T)$ show more readily that the impurity scattering is unimportant.

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