during plastic deformation. At the present time the detailed mechanism of this process is being investigated.

1) The ZnS crystals were kindly provided by M. P. Kulakov, to whom the authors express their sincere gratitude.

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Helicoidal and sinusoidal spin ordering in quasi-one-dimensional magnets

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Using as an example a chain of spins interacting via the conduction electrons, the possibilities of helicoidal and sinusoidal ordering are considered. Expressions are obtained for the correlation functions and for the structure factor. Substances in which observation of this order is possible are indicated.

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1. INTRODUCTION

By now there are a large number of known substances that are quasi-one-dimensional in their magnetic properties (see the reviews[1,2]). The properties of most of them are satisfactorily described by a model with exchange interaction between the nearest spins that are arranged in a chain. The interaction between spins of different chains is assumed to be small. Depending on the type and magnitude of the single-ion anisotropy, the system is described by the Heisenberg model, by the XY model, or by the Ising model. If the interaction along a chain of spins that are not nearest neighbors becomes substantial, then more complicated states with helicoidal and sinusoidal magnetic structures are possible. We consider the properties of such systems using as a model spins interacting via conduction electron. The simplest example of the realization of such a model is a chain of atoms of transition elements. The s-shell electrons become collectivized, and they can be regarded as free, while the f or d electrons are localized on the sites. The localized spins interact with one another via direct exchange, and also indirectly via the conduction electrons. The last interaction is long-range and therefore, even it is small, can substantially alter the properties of a quasi-one-dimensional system.

Assume first that there are no fluctuations. We regard the functional of the free energy of the system as a function of the wave vector of the spin structure. The functional of the energy of the direct interaction of the spin is of the form

$$F_S[q]=2J(q)\mathbf{S}_q\mathbf{S}_{-q},$$

where

$$\mathbf{S}_q=N^{-1/2}\sum_i \mathbf{S}_i\exp(iqx_i),\qquad J(q)=\sum_i J_{ik}\exp[iq(x_i-x_k)].$$

If only the nearest spins interact, then the characteristic scale of the variation of the quantity $J(q)$ is of the order of the period Q_0 of the reciprocal lattice. The electrons are acted upon by a spin field with wave vector q and with an amplitude proportional to $\langle S\rangle$. If q is

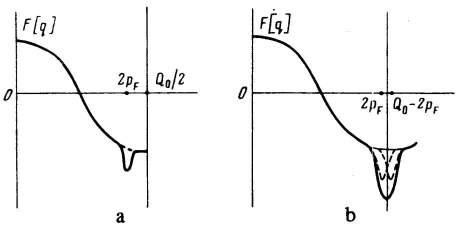

FIG. 1.

close to $2p_F$, then the energy of the interaction of the electrons and the spins has a logarithmic singularity:

$$F_{es}[q]\sim\frac{|VS_q|^2}{\varepsilon_F}\ln\frac{\varepsilon_F}{\{|2p_F-q|v_F,\ V\langle S\rangle,\ T\}},\tag{1}$$

where V is the matrix element of the electron-spin interaction.

If the logarithmic peak at $q=2p_F$ is near the minimum of $F_S[q]$, then even if it is small compared with F_S, it can determine the position of the absolute minimum and also the form of the free energy in the vicinity of this minimum (Fig. 1a). We shall henceforth assume throughout that the interaction is antiferromagnetic and that $F_S[q]$ has a minimum at $q=Q_0/2$. Then the condition that the absolute minimum be determined by the logarithmic peak at $2p_F$ can be written in the form

$$\left(\frac{F_{es}}{F_S}\right)^{1/2}>\frac{Q_0/2-2p_F}{Q_0}.$$

It is convenient to write the Hamiltonian of the bound system of electrons and spins in a form such that the electron momentum be reckoned from $\pm p_F$:

$$H=\int dx\,\psi^+(x)\left[-v_F\frac{\partial}{\partial x}\tilde\sigma^z+V\mathbf{S}_Q(x)\mathbf{s}\tilde\sigma^++V\mathbf{S}_{-Q}(x)\mathbf{s}\tilde\sigma^-\right]\psi(x)+\sum_{i\neq k}J_{ik}\mathbf{S}_i\mathbf{S}_k,\tag{2}$$

$$\psi(x)=\begin{pmatrix}\psi_+(x)\\ \psi_-(x)\end{pmatrix},$$

where $\psi_{\pm,k}=\psi_{\pm pF+k}$ is the operator for the annihilation of electrons with momentum close to $\pm p_F$, $\tilde\sigma^z$ and $\tilde\sigma^\pm=(\sigma^x\pm i\tilde\sigma^y)/2$ are operators acting on the column $\psi(x)$, $\mathbf{s}$ are Pauli matrices acting on the spin variables of the electron operators,

$$\mathbf{S}_{\pm Q}(x)=N^{-1/2}\sum_{|k|\ll Q}\mathbf{S}_{\pm Q+k}e^{ikx},\qquad Q=2p_F.$$

Assuming that the electron subsystem is situated in a field of a fixed spin configuration, we obtain an expression for the electron Green's functions. We assume here that $S\gg1$, so that the spin operators in (2) can be replaced by their mean values. We shall subsequently calculate the contribution made to the spectrum by the long-wave excitations that determine the form of the correlation functions and of the structure factor.

We consider two possible types of structures with wave vector $Q\neq Q_0/2$: helicoidal and sinusoidal. For the helicoidal structure, in the absence of fluctuations and for a proper choice of the coordinate system we have $S^x=S\cos Qx$, $S^y=S\sin Qx$, $S^z=0$ or $S_Q^+(x)=S_{-Q}^-(x)=S$. The electron Green's function is

$$\mathscr{G}=-\frac{i\omega(\tilde\sigma^1s^1+\tilde\sigma^2s^2)+\xi(\tilde\sigma^1s^1-\tilde\sigma^2s^2)+\Delta(\tilde\sigma^+s^++\tilde\sigma^-s^-)}{\omega^2+\xi^2+\Delta^2}-\frac{i\omega(\tilde\sigma^1s^2+\tilde\sigma^2s^1)+\xi(\tilde\sigma^1s^2-\tilde\sigma^2s^1)}{\omega^2+\xi^2},\tag{3}$$

where $\Delta=VS$, $\tilde\sigma^1$, $\tilde\sigma^2$ and also s^1, s^2 are matrices in the form $\left(\begin{smallmatrix}1&0\\0&0\end{smallmatrix}\right)$ and $\left(\begin{smallmatrix}0&0\\0&1\end{smallmatrix}\right)$. For a sinusoidal structure we have $S^z=S\cos Qx$, $S^x=S^y=0$ or $S^z_{\pm Q(x)}=S/2$. The electron Green's function is

$$\mathscr{G}=-\frac{i\omega\tilde II+\xi\tilde\sigma^zI+\Delta\tilde\sigma^xs^z}{\omega^2+\xi^2+\Delta^2},\tag{4}$$

where $\Delta=VS/2$, and $\tilde I$ and I are unit matrices.

If the direct exchange exceeds the indirect one, then the helicoidal state is energywise favored. In the opposite case, when the indirect exchange is larger than or of the same order as the direct exchange, the situation is not clear. The configuration energies cannot be compared by using the molecular-field theory, since this theory is certainly not applicable for a sinusoidal structure. We do not know the ground state of this structure, and the term "sinusoidal" means only that there is a corresponding harmonic of the localized spin, which causes a restructuring of the electron spectrum (4).

We shall not consider effects connected with allowance for single-ion anisotropy. We point out only that the anisotropy, just as an external magnetic field, would decrease the number of Goldstone modes and would change correspondingly the low-temperature correlation functions.

2. HELICOIDAL STRUCTURE. CORRELATION FUNCTIONS

Let the direct exchange interaction J be much larger than the indirect one J_{eS}. Then, in the temperature interval $JS^2/3\gg T\gg J_{eS}S^2/3$, the system "does not feel" the true energy minimum. The correlation function of a one-dimensional antiferromagnet

$$K(x,\tau)=\langle\mathbf{S}(x,\tau)\mathbf{S}(0,0)\rangle=S^2\langle\mathbf{n}(x,\tau)\mathbf{n}(0,0)\rangle\cos(Q_0x/2)\tag{5}$$

can be calculated if we know the spin-wave spectrum

$$\omega^2=4S^2[J^2(0)-J^2(q)].$$

We write down the Lagrange function of the long-wave oscillations:

$$\mathscr{L}=\int dx\left[JS^2\left(\frac{\partial\mathbf{n}}{\partial x}\right)^2+\frac{1}{4J}\left(\frac{\partial\mathbf{n}}{\partial\tau}\right)^2\right].$$

Using Berezinskii's results[3] we obtain for $ux\ll\beta$ and $\tau\ll\beta$

$$K(x,\tau)=\left|\frac{x+i\tau u}{\xi_0}\right|^{-2u/v}\cos\frac{Q_0x}{2},$$

and for $ux\gg\beta$

$$K(x,\tau)=\exp(-4\pi T|x|/v)\cos(Q_0x/2),\tag{6}$$

where $u=2JS$, $v=8\pi JS^2$, $\beta=T^{-1}$, ξ_0 is of the order of the interatomic distance, which we shall hereafter assume equal to unity. We note that $u/v=1/4\pi S \ll 1$.

At temperatures $T \ll J_{eS}S^2/3$ the state of the system is determined by fluctuations within the logarithmic peak. If $2p_F$ is close to the boundary of the Brillouin zone, so that $|2p_F-Q_0/2| < VS/v_F$, then the energy minimum is as before at $Q=Q_0/2$ (Fig. 1b) and the correlation functions are described by formula (6), but with different u and v, which we shall calculate below. The ratio u/v will as before be much less than unity. If $|2p_F-Q_0/2| > VS/v_F$ (Fig. 1a), then the magnetic ordering is helicoidal. Such a state is characterized in the general case by a normalized complex vector $\mathbf{R}=\mathbf{n}_1+i\mathbf{n}_2$, whose real and imaginary parts are perpendicular and equal, so that

$$\mathbf{S}_i={}^1\!/_2 S(\mathbf{R}e^{iQx_i}+\mathbf{R}^*e^{-iQx_i}). \tag{7}$$

The orientation of $\mathbf{R}$ in space is determined by three angles. The Lagrange function in the absence of single-ion anisotropy depends only on the derivatives of these angles.

To determine this derivative we use the known expression for the spectrum of the spin waves in a helicoidal structure[4]:

$$\omega^2=4S^2[J(Q)-J(q)][J(Q)-{}^1\!/_2J(Q+q)-{}^1\!/_2J(Q-q)]. \tag{8}$$

The frequency becomes equal to zero at $q=0$ and $q=\pm Q$. The oscillations with $q=0$ correspond to rotation of all the spins through a certain angle in the $(\mathbf{n}_1, \mathbf{n}_2)$ plane, while the oscillations with $q=\pm Q$ corresponds to rotation of the plane itself. We denote by $\tilde{\Omega}_1$, $\tilde{\Omega}_2$, and $\tilde{\Omega}_3$ the angles of rotations relative to the axes of a coordinate system whose axes 1 and 2 lie along $\mathbf{n}_1$ and $\mathbf{n}_2$. From (8) we know the connection between the temporal and spatial derivatives of the angles $\tilde{\Omega}_1$, $\tilde{\Omega}_2$, and $\tilde{\Omega}_3$. In the approximation quadratic in these derivatives, we obtain for the Lagrange function

$$\begin{aligned}\mathscr{L}&=\int dx\left\{\left[J_3''S^2\left(\frac{\partial\tilde{\Omega}_3}{\partial x}\right)^2+\frac{1}{2[J(Q)-J(0)]}\left(\frac{\partial\tilde{\Omega}_3}{\partial\tau}\right)^2\right]\right.\\&\left.+\sum_{1,2}\left[\frac{1}{2}J_i''S^2\left(\frac{\partial\tilde{\Omega}_i}{\partial x}\right)^2+\frac{1}{4[J(Q)-J(0)]}\left(\frac{\partial\tilde{\Omega}_i}{\partial\tau}\right)^2\right]\right\}\\&\equiv\sum_i\int dx\frac{v_i}{8\pi}\left[\left(\frac{\partial\tilde{\Omega}_i}{\partial x}\right)^2+\frac{1}{u_i^2}\left(\frac{\partial\tilde{\Omega}_i}{\partial\tau}\right)^2\right].\end{aligned} \tag{9}$$

Expression (8) was obtained by starting from the assumption that the interaction between the spins is of the Heisenberg type:

$$H=\sum_{i\neq k}J_{ik}\mathbf{S}_i\mathbf{S}_k$$

and that $J(q)$ has a minimum at $q=Q$. The interaction via the conduction electrons, generally speaking, cannot be described with the aid of an effective Heisenberg Hamiltonian, since the effective exchange integral is itself a functional of S_i in this case. We therefore denote by J_i'' quantities that in the case of a pure Heisenberg Hamiltonian would be equal to

$$-\frac{\partial^2J(q)}{\partial q^2}\bigg|_{q=Q}.$$

The electron contribution to these quantities will be calculated separately. To this end we introduce new Green's functions

$$\tilde{\mathscr{G}}(\bar{x},\bar{x}')=\exp[-{}^1\!/_2 i\mathbf{\Omega}(\bar{x})\mathbf{s}]\mathscr{G}(\bar{x},\bar{x}')\exp[{}^1\!/_2 i\mathbf{\Omega}(\bar{x}')\mathbf{s}], \quad \bar{x}=(x,\tau),$$

where $\mathbf{\Omega}(\bar{x})=\{\Omega_1(\bar{x}), \Omega_2(\bar{x}), \Omega_3(\bar{x})\}$ is the aggregate of the rotation angles that make $\mathbf{R}(0)$ congruent with $\mathbf{R}(\bar{x})$. This corresponds to a changeover to a coordinate system that rotates with $\mathbf{R}(x)$, and in which the order parameter is constant. From the equations of motion for $\mathscr{G}(\bar{x}, \bar{x})$ it is easily seen that the effective electron Hamiltonian takes in the rotating coordinate system the form

$$\begin{aligned}\tilde{H}=\int dx\,\tilde{\psi}^+(\bar{x})\Big\{&-iv_F\frac{\partial}{\partial x}\tilde{\sigma}^zI+V\langle S_Q^+\rangle s^-\tilde{\sigma}^++V\langle S_{-Q}^-\rangle s^+\tilde{\sigma}^-\\&-iv_F\exp[-{}^1\!/_2 i\mathbf{\Omega}(\bar{x})\mathbf{s}]\frac{\partial}{\partial x}\exp[{}^1\!/_2 i\mathbf{\Omega}(\bar{x})\mathbf{s}]\tilde{\sigma}^z\\&+\exp[-{}^1\!/_2 i\mathbf{\Omega}(\bar{x})\mathbf{s}]\frac{\partial}{\partial\tau}\exp[{}^1\!/_2 i\mathbf{\Omega}(\bar{x})\mathbf{s}]I\Big\}\tilde{\psi}(\bar{x}).\end{aligned} \tag{10}$$

The last two terms, expressed in terms of the local rotation angles $\tilde{\Omega}_i$, are equal to

$$\tilde{\psi}^+(\bar{x})\left[\frac{1}{2}v_F\frac{\partial\tilde{\Omega}_i}{\partial x}s^i\tilde{\sigma}^z+\frac{i}{2}\frac{\partial\tilde{\Omega}_i}{\partial\tau}s^iI\right]\tilde{\psi}(\bar{x}).$$

Regarding them as a perturbation, we calculate the correction, quadratic in the derivatives of the angles $\tilde{\Omega}_i$ (Fig. 2—the solid lines are the Green's functions (3)). Integrating with respect to the coupling constant or, equivalently, with respect to the gap in the electron spectrum:

$$\int_0^\Delta\frac{d\Delta'}{\Delta'}\langle H_{int}^{(2)}\rangle,$$

we obtain for the electronic contribution to the value J_i'' of the Lagrangian (9):

$$(J_3''S^2)^e=v_F/16\pi, \quad (J_{1,2}''S^2)^e=0. \tag{11}$$

The electron contribution to the terms with the time derivatives in (9) is small, like J/v_F, in comparison with the exchange contribution. We note that for an antiferromagnet (Fig. 1b) the electron contribution to $J''S^2$ is described by the diagram of Fig. 2 with a scattering potential $\Delta\tilde{\sigma}^xs^x$, where the solid lines are the Green's functions (4), and is equal to $v_F/8\pi$. Thus, at $T \ll J_{eS}S^2/3$ we have in formulas (6) $v=v_F$ and $u=(Jv_F/2\pi)^{1/2}$.

The correlation function of the spins in a helicoidal structure is expressed in terms of the order-parameter correlator:

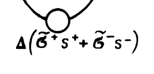

FIG. 2.

$$K(\bar{x}) \equiv \langle \mathrm{S}(\bar{x})\mathrm{S}(0)\rangle = {}^1/_2 S^2 \cos Qx \langle \mathbf{R}^*(\bar{x})\mathbf{R}(0)\rangle = {}^1/_2 S^2 \cos Qx \langle \cos\Omega_1(\bar{x}) \cos\Omega_3(\bar{x})\rangle. \qquad (12)$$

Assume first that the Lagrangian (9) is isotropic relative to the axes 1, 2, 3, i.e., $u_i = u$ and $v_i = v$. Then the fluctuations about each angle can be regarded as independent and the problem reduces to the calculation of the independent mean values $\langle\cos\Omega(\bar{x})\rangle$ with the Lagrangian

$$\mathscr{L} = \frac{v}{8\pi}\int dx \left[\left(\frac{\partial\Omega}{\partial x}\right)^2 + \frac{1}{u^2}\left(\frac{\partial\Omega}{\partial\tau}\right)^2\right].$$

The corresponding Gauss integral was calculated a number of times (see, e.g., [5]):

$$G(\bar{x}) \equiv \langle\cos\Omega(\bar{x})\rangle = \left|\left(1-\exp\left[-2\pi T\left(\frac{|x|}{u}+i\tau\right)\right]\right)\xi_0\frac{2\pi T}{u}\right|^{-u/v}\exp\left[-\frac{2\pi T|x|}{v}\right]. \qquad (13)$$

Let the "anisotropy" of the Lagrangian (9) be small. Since $u/v \ll 1$, we shall assume the correlation function to depend only on x. We separate the isotropic and anisotropic parts of the Lagrangian, expressing the velocities in the form

$$v_{1,2} = v - {}^1/_3 v'(\cos^2\Omega_1(x)\cos^2\Omega_2(x) - {}^1/_4),$$
$$v_3 = v + v'(\cos^2\Omega_1(x)\cos^2\Omega_2(x) - {}^1/_4). \qquad (14)$$

Substituting in (14) the mean values $\cos^2\Omega_1(x)$ and $\cos^2\Omega_2 \times (x)$, calculated with the isotropic Lagrangian, and calculating the correlation function with x-dependent velocities (14), we obtain

$$G_3(x) \equiv \langle\cos\Omega_3(x)\rangle \equiv \exp\left\{-\frac{2\pi T|x|}{v}+\frac{5}{32}\alpha - \frac{\alpha}{32}\left[4\exp\left(-\frac{8\pi T|x|}{v}\right)+\exp\left(-\frac{16\pi T|x|}{v}\right)\right]\right\}, \quad \alpha = \frac{v'}{v}; \qquad (15)$$

at $x \ll v/2\pi T$ we have

$$G_3(x) = \exp\left\{-\frac{2\pi T|x|}{v}\left(1-\frac{3}{4}\alpha\right)\right\}$$

and at $x \gg v/2\pi T$

$$G_3(x) = \exp\left\{-\frac{2\pi T|x|}{v}+\frac{5}{32}\alpha\right\}.$$

The expression for $G_{1,2}(x)$ is obtained by making the substitution $\alpha \to -\alpha/3$. We note that the effective shift of the origin over large distances in (15) does not depend on the temperature.

We consider now the general case of arbitrary anisotropy. We can calculate the correlation function at short distances, when the angles Ω_i are small and can be regarded as independent. This approach is correct if $G_i(x) \approx 1$. If this condition is violated, i.e., $\Omega_i \sim 1$ (since the rotations do not commute, all the angles become of the order of unity simultaneously even in the case of strong anisotropy), we can take into account only the isotropic part of the Lagrangian (9). To calculate this part it is necessary to average the velocity ellipsoid $v_1 = v_2 \neq v_3$ over the rotation angles Ω_1, Ω_2, and Ω_3 in the interval $[0, 2\pi]$. The result is

$$v = \frac{1}{(2\pi)^2}\int_0^{2\pi}\int_0^{2\pi}\frac{v_1 v_3 d\Omega_1\, d\Omega_2}{[v_3^2-(v_3^2-v_1^2)\cos^2\Omega_1\cos^2\Omega_2]^{1/2}}. \qquad (16)$$

The substitution $v_i \to v_i/u_i^2$ in (16) yields an expression for the isotropic term with the time derivatives in (9). The correlator (12) thus breaks up into a product of correlators (13) with isotropic u and v:

$$\langle\mathbf{R}^*(\bar{x})\mathbf{R}(0)\rangle = e^{-\alpha}[G(\bar{x})]^2,$$

where $\alpha \sim 1$ and does not depend on the temperature (see (15)).

3. THE STRUCTURE FACTOR

Just as the correlation functions, the dynamic structure factors can be calculated in the limit of small or large frequencies compared with $T(u/v_{\max}$. We consider first the case of short times and distances, when $\Omega_i \ll 1$ and the correlation function is equal to

$$K(x,\tau) = {}^1/_2 S^2 \cos Qx \exp\{D_1(x,\tau)-D_1(0,0)+D_3(x,\tau)-D_3(0,0)\}, \qquad (17)$$

where D_i is the Green's function of the angles Ω_i;

$$D_i(\omega_n, q) = -\frac{4\pi u_i^2/v_i}{\omega_n^2+u_i^2q^2}. \qquad (18)$$

By analytically continuing $K(x, \tau)$ to real times $t > 0$, by the method developed in the book of Akhiezer and Berestetskiĭ[6] (see also[7]), we obtain for the retarded correlation function

$$K^R(x,t) = \frac{S^2\cos Qx}{2}\left\{\left[\left(\frac{T_1}{T}\right)^2 \mathrm{sh}(\pi T t_1^+)\,\mathrm{sh}(\pi T t_1^-)\right]^{-u_1/v_1} \times\left[\left(\frac{T_3}{T}\right)^2 \mathrm{sh}(\pi T t_3^+)\,\mathrm{sh}(\pi T t_3^-)\right]^{-u_3/v_3}\right\} \times \sin\left(\pi\left[\frac{u_1}{v_1}\theta\left(t-\frac{|x|}{u_1}\right)+\frac{u_3}{v_3}\theta\left(t-\frac{|x|}{u_3}\right)\right]\right), \qquad (19)$$

where

$$t_i^\pm = t \pm x/u_i, \quad T_3 = u_3 VS/v_3, \quad T_1 = JS.$$

The condition for (19) to be valid is that the expression in the curly brackets differ little from unity, i.e., $t_i^\pm \ll (u_1 T/v_1)^{-1}$, or that the corresponding frequencies be equal to

$$\omega_i^\pm = (\omega \pm u_i k)/2 \gg u_1 T/v_1.$$

The system structure factor that determines the neutron scattering

$$S(q,\omega) \sim d\sigma/d\omega d\Omega,$$

is equal to the imaginary part of $K^R(k, \omega)(k = q - Q)$. The resultant expressions are cumbersome, and we present only the formulas for the quantum region $\omega_i^\pm \gg T$. We can then make in (19) the substitution $\sinh(\pi T t_i^\pm) \to \pi T t_i^\pm$. By the same token, we neglect in the expressions given below the terms $\sim e^{-\omega/T}$, which would cause some smearing of the step functions.

Strong anisotropy, $u_1/v_1 \gg u_3/v_3$:

1) at $\omega_1^+ \sim \omega_1^-$

$$S(k,\omega)=C\sin\left(\pi\frac{u_1}{v_1}\right)\Gamma^2\left(1-\frac{u_1}{v_1}\right)|\omega_1^+\omega_1^-|^{-(1-u_1/v_1)} \tag{20a}$$

$$\times\begin{cases}\sin\pi\dfrac{u_1}{v_1}, & \omega^2-u_1^2k^2>0\\ \sin\pi\dfrac{u_3}{v_3}, & \omega^2-u_1^2k^2<0\end{cases};$$

2) at $\omega_1^+ \gg \omega_1^-$

$$S(k,\omega)=C\sin\left[\pi\left(\frac{u_1}{v_1}+\frac{u_3}{v_3}\right)\right]\Gamma\left(1-\frac{u_1}{v_1}\right)\Gamma\left(1-\frac{u_1}{v_1}-2\frac{u_3}{v_3}\right)$$

$$\times|\omega_1^+|^{-(1-u_1/v_1)}|\omega_1^-|^{-(1-u_1/v_1-2u_3/v_3)}\left(\frac{u_3}{u_1}\right)^{2u_3/v_3}\sin\left[\pi\left(\frac{u_1}{v_1}\theta(\omega-u_1k)+\frac{u_3}{v_3}\right)\right]; \tag{20b}$$

3) at $\omega_3^+ \gg \omega_3^-$

$$S(k,\omega)=C\sin\left(\pi\frac{u_3}{v_3}\right)\Gamma\left(1-\frac{u_3}{v_3}\right)\Gamma\left(1-\frac{u_3}{v_3}-2\frac{u_1}{v_1}\right)$$

$$\times|\omega_3^+|^{-(1-u_3/v_3)}|\omega_3^-|^{-(1-u_3/v_3-2u_1/v_1)}\sin\left[\pi\left(\frac{u_1}{v_1}+\frac{u_3}{v_3}\right)\theta(\omega-u_3k)\right]. \tag{20c}$$

Weak anisotropy, $u_{1,3}=u\mp\Delta u$, $\Delta u \ll u$:

1) at $\omega^+ \sim \omega^-$

$$S(k,\omega)=C\sin^2\left[\pi\left(\frac{u_1}{v_1}+\frac{u_3}{v_3}\right)\right]\Gamma^2\left(1-\frac{u_1}{v_1}-\frac{u_3}{v_3}\right)$$

$$\times|\omega^+\omega^-|^{-(1-u_1/v_1-u_3/v_3)}\theta(\omega-uk); \tag{21a}$$

2) at $\omega_1^+ \gg \omega_1^- u/\Delta u$

$$S(k,\omega)=C\sin\left[\pi\left(\frac{u_1}{v_1}+\frac{u_3}{v_3}\right)\right]\Gamma\left(1-\frac{u_1}{v_1}\right)\Gamma\left(1-\frac{u_1}{v_1}-2\frac{u_3}{v_3}\right)$$

$$\times|\omega_1^+|^{-(1-u_1/v_1)}|\omega_1^-|^{-(1-u_1/v_1-2u_3/v_3)}\left(\frac{\Delta u}{u}\right)^{-u_3/v_3}\sin\left[\pi\left(\frac{u_1}{v_1}\theta(\omega-u_1k)+\frac{u_3}{v}\right)\right]; \tag{21b}$$

3) at $\omega_3^+ \gg \omega_3^- u/\Delta u$

$$S(k,\omega)=C\sin\left(\pi\frac{u_3}{v_3}\right)\Gamma\left(1-\frac{u_3}{v_3}\right)\left(\frac{\Delta u}{u}\right)^{-u_1/v_1}\Gamma\left(1-\frac{u_3}{v_3}-2\frac{u_1}{v_1}\right)$$

$$\times|\omega_3^+|^{-(1-u_3/v_3)}|\omega_3^-|^{-(1-u_3/v_3-2u_1/v_1)}\sin\left[\pi\left(\frac{u_1}{v_1}+\frac{u_3}{v_3}\right)\theta(\omega-u_3k)\right], \tag{21c}$$

In formulas (20) and (21)

$$C={}^1/_2S^2(\pi T_1)^{-u_1/v_1}(\pi T_3)^{-u_3/v_3}.$$

In the classical region $\omega^{\pm} \ll (u/v)_{\max}T$ the structure factor is determined by analytically continuing the correlation function (12) calculated with an isotropic Lagrangian. Substituting $u_1=u_3=u$ and $v_1=v_3=v$ in (21), we get

$$S(k,\omega)=\frac{S^2e^{-\alpha}\sin^2(\pi u/v)\pi T^2u/v}{[(2\pi Tu/v)^2+\omega_+^2][(2\pi Tu/v)^2+\omega_-^2]}. \tag{22}$$

4. SINUSOIDAL STRUCTURE

The average spin density in a sinusoidal structure is described by expression (7) with $\mathbf{R}=\mathbf{n}e^{i\varphi}$, where $\mathbf{n}$ is a unit real vector and φ is the phase. The correlation function breaks up thus into a product of the correlation functions of the vectors and of the phases:

$$\langle\mathbf{R}^*(\bar{x})\mathbf{R}(0)\rangle=\langle\mathbf{n}(\bar{x})\mathbf{n}(0)\rangle\langle e^{i\varphi(\bar{x})-i\varphi(0)}\rangle. \tag{23}$$

The expressions for the two correlators are already known [Eqs. (6) and (13)], but in contrast to all preceding cases the velocities u_φ and v_φ in the phase correlator are of the order of the Fermi velocity. In the direction correlator, the velocities $\mathbf{u}_n$ and $\mathbf{v}_n$ are of the same order as the velocities calculated with the aid of the diagram of Fig. 2 for the case of an antiferromagnet. The structure factor can be calculated only in the region $\omega^{\pm} \gg T$, where it is described by formulas (20) and (21) with

$$u_1,v_1 \to u_\varphi,v_\varphi,\quad u_3,v_3 \to u_n,v_n/2.$$

An interesting analogy exists between a Peierls dielectric with order parameter $\Delta e^{i\varphi}$, on the one hand, and a sinusoidal structure with order parameter $\mathbf{n}e^{i\varphi}$, on the other. The substitution $\varphi \to \varphi+\varphi_0$ transforms both system into a state that can be made coincident with the initial one only via a spatial translation. In our case there is therefore a conduction mechanism analogous to the Froehlich mechanism. This means only that although the electron spectrum does have a gap (see (4)) no real dielectric transition can occur in a one-dimensional system. Whereas in a Peierls dielectric the motion of the charge-density wave is accompanied by oscillations of the heavy lattice, which leads to renormalization of the mass of the elementary excitations, in a magnet the motion of the spin-density wave is accompanied by a change in the mean values of the spins at the lattice sites. Since the spin-configuration energy in a sinusoidal structure is lower than the electron energy, the effective mass of the elementary excitations is of the order of the electron mass.

This shows once more that in this case the mean-field approximation cannot be used and expression (4) can be used only for estimates.

5. CONCLUSION

One of the substances in which a one-dimensional structure of the helicoidal or sinusoidal type can be observed, is the intermetallic compound $Al_{11}Mn_4$. The exchange interaction between the Mn atoms that are arranged in a chain is effected via the conduction electrons, while the interaction between the atoms of different chains is apparently small.[8] At $T=100$ K the susceptibility of the samples has a maximum and then decreases. The low-temperature behavior of the susceptibility is unclear because of the paramagnetic impurities. Dunlop and Gruner[8] attribute the behavior of the susceptibility to the formation of single magnetic systems out of Mn atom pairs. The point is that the Mn atoms can occupy in the structure two possible positions, *A* and *B*. The relative differences between the distances *AA*, *AB*, and *BB* is less than 5%. For agreement with experiment, however, it is necessary to introduce a very large gap, $\Delta \sim 150$ K. Since the interaction via the electrons is long-range, there is little likelihood that the small difference between the distances can lead to a substantial difference between the *AA*, *AB*, and *BB* exchange integrals. The converse is more likely, namely, at temperatures such that the impurities play no role the susceptibility has a behavior typical of one-dimensional magnets in which all the spins are equivalent (see, e.g., [1]).

Some anomalies in the susceptibility are observed also in the compound VF_2.[1] The quantity $\chi_{\max}T(\chi_{\max})$ is somewhat smaller than that calculated from the model with nearest-neighbor interaction. The three dimensional transition in this substance is to a state with helicoidal structure. It is therefore perfectly possible that in the temperature interval in which the system is quasi-one-dimensional the ordering of the system is close to helicoidal. There are no conduction electrons in the system and the interaction of the non-nearest neighbors is probably via direct exchange, on account of the weak localization of the *d* electrons. Of course, the most convincing confirmation of the existence of the structures described above would be a direct neutron-diffraction observation of a two-velocity structure factor.

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Thermally stimulated emission of surface polaritons

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The angular dependences of the emission spectra of heated zinc selenide films were experimentally obtained in *p*-polarized light in the frequency band of the surface polaritons in the IR region of the spectrum. The dispersion curves of the surface polaritons are reconstructed from their emission spectra. The influence of a metallic substrate on the dispersion curves of the surface polaritons of a dielectric film is demonstrated theoretically. The developed theory is in good agreement with the experimental results. The possibilities of using this method to determine the optical characteristics of metal films in an insulator-metal-insulator sandwich are discussed.

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INTRODUCTION

The thermal vibrations of atoms in condensed media can give rise to alternating dipole moments at the characteristic frequencies of the medium. These alternating dipole moments emit electromagnetic waves under certain conditions. Subject to satisfaction of the energy and momentum conservation laws, these electromagnetic waves can leave the medium and be recorded as thermal radiation. The thermal vibrations of the atoms near the free surface of a crystal lattice also produce alternating dipole moment. The magnetic field produced by them, however, is "tied" to the interface and can not "break away" from it, since the wave vector of the surface polariton on the crystal–vacuum interface is always larger than the wave vector of light in vacuum.[1] The use of prisms with anomalous total internal reflection (ATIR), as is known, makes it possible to equalize the wave vector of the light in the ATIR prism with the wave vector of the surface polariton. When this condition is satisfied, the surface polariton absorbs the light wave. If some system has absorbed light, then, in accordance with Kirchoff's law, it must emit it, i.e., the picture can be reversed. Thermally stimulated emission of surface polaritons has been investigated many times (see, e.g., [2]), but the thermal emission of surface polaritons of single crystals in the regime of inverted ATIR has so far been observed apparently only in[3].

The purpose of the present study was to check on the feasibility of observing the emission of surface polaritons of dielectric film in the inverted ATIR regime, as well as to investigate the interaction of the surface polaritons of the film with a metallic substrate (in particular, to investigate the "metallic" quenching of surface polaritons[1]).

1. EXPERIMENT

The tests were made on ZnSe films sputtered on an aluminum film, obtained in turn by sputtering in vacuum on a hot substrate. The thickness of the aluminum layers prior to the annealing was ≈ 0.1 and ≈ 1 μm, while the ZnSe film, after crystallizing annealing in an argon atmosphere, was ≈ 1 μm thick. The ATIR prism was a half-cylinder of single-crystal silican, and the size of the air gap between the prism and the sample was set with the aid of a frame of lavsan polyster film 25 to 6 μm thick, depending on the emission angle. The "sam-