Contribution to the theory of cyclotron resonance in a metal with almost specular boundary

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The cyclotron-resonance problem is solved for the case of almost specular reflection of the electrons from the metal boundary. The surface impedance Z is calculated and its dependences on the specularity parameter ρ and on the magnetic field H are analyzed. It is shown that in the principal approximation the impedance is independent of H and of the electron mean free path, but depends on the surface state of the metal (on ρ). Cyclotron resonance appears in the next approximation in the parameter $1-\rho$. The dependence of the character of the cyclotron resonance on the specularity parameter ρ is investigated qualitatively on the basis of Pippards "ineffectiveness concept."

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The reflection of electrons from the surface has a substantial influence on the character of the cyclotron resonance (CR) in a metal. It is known[1] that the resonance is due to "volume" electrons that do not collide with the surface of the sample and return to the skin layer after each revolution in the magnetic field **H**. An important role in the formation of the skin layer can be played, besides the volume electrons, also by "glancing" electrons. They collide with the metal boundary, and their entire orbits lie in the skin layer (see Fig. 1). The degree of specularity of the surface determines the relative role of the volume and glancing electrons in the skin effect, and by the same token influences the character of the CR—most of all the observed amplitude and shape of the resonance line. Consequently the amount and the quality of the information provided by the experiment turn out to be connected with the state of the sample boundary. This is precisely why the question of the effect of electron reflection on the CR is of fundamental significance and has been repeatedly considered by many workers (see, e.g.,[1–4]). This question is quite complicated and has not been completely solved theoretically to this day. Thus, for example, the result obtained by Meĭerovich[4] for the case of almost specular reflection is incorrect. It will be shown below that his mistake was not to take into account the δ-function character of the conduction operator of the glancing electrons. This result subsequently found its way into a number of other papers.[5,6] The δ-function term was also left out by Zherebchevskiĭ and one of us[3] in the calculation of the resonant increment; allowance for this term decreases the numerical coefficient by a factor 1.4.

We solve in the present paper the CR problem for the case of almost specular reflection of the electrons from the sample surface. The surface impedance of the metal is calculated and its dependences on the specularity parameter ρ and the magnetic field **H** are analyzed. In Sec. 1 we use straight forward physical considerations based on Pippard's "ineffectiveness concept" to analyze semiquantitatively the CR with account taken of the scattering of the electrons by the metal surface, in the entire range of variation of the specularity coefficient ρ, from zero to unity. We note that the ineffectiveness concept has not been applied previously to an investigation of the role of reflection in CR. The simplicity and lucidity of the analysis are attractive because they make it possible to interrelate all the various limiting cases. In Sec. 2 we obtain a rigorous solution of the problem of CR for almost specular and strictly specular reflection. The formulas of Sec. 1 turned out to be in quantitative agreement, accurate to real factors, with the result of the exact calculation. The expressions for surface impedance, calculated by solving the self-consistent system of Maxwell's equations and the kinetic equation, are asymptotically exact. Thus, we have obtained, in a certain respect, a complete solution of the problem of the influence of electron collisions with the surface on CR in metals.

I. PHYSICAL ANALYSIS OF THE PHENOMENON

Cyclotron resonance is observed in metals[1] under conditions of strong spatial dispersion

$$l, R \gg \delta, \tag{1.1}$$

when the radius of the orbit R in a constant magnetic field **H** and the effective mean free path l of the electron exceed the skin-layer depth δ. The conductivity of the volume electrons responsible for the CR can then be described qualitatively by the well-known formula

$$\sigma = N_{\text{eff}} e^2 \tau / m. \tag{1.2}$$

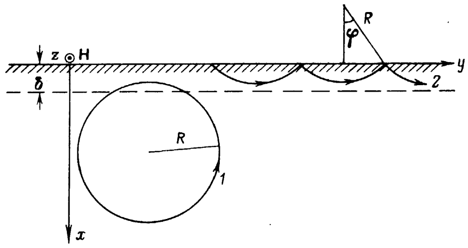

FIG. 1. Trajectories of the electrons that cause the conductivity of a metal: 1—volume electrons that make CR possible, 2—glancing electrons (glancing angles $\varphi \sim (\delta/R)^{1/2}$).

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Here N_{eff} is the number of volume electrons and τ is the time of their interaction with the electromagnetic field (e is the absolute value of the charge and m is the electron mass).

If the inequality

$$(R\delta)^{1/2} \ll l, \tag{1.3}$$

is satisfied, then the time that the volume electron stays in the skin layer in one precession period is of the order of $\tau_0 = (R\delta)^{1/2}/v$ (v is the electron velocity). Such electrons have a momentum component normal to the metal surface $|p_x| \sim p_F(\delta/R)^{1/2}$, and N_{eff} turns out to be equal to $N(\delta/R)^{1/2}$ (N is the electron density). We must now take into account repeated return of the electrons to the skin layer. Since the "probability" of the next return in sequence, with allowance for the change of phase of the electromagnetic wave, is equal to $e^{-2\pi\gamma}$, the conductivity of the volume electrons takes the form

$$\sigma_{vol} = \frac{Ne^2}{m}\frac{(R\delta)^{1/2}}{v}\left(\frac{\delta}{R}\right)^{1/2}\left(1+2\sum_{n=1} e^{-2\pi n\gamma}\right) = \sigma_0\frac{\delta}{l}\operatorname{cth}\pi\gamma. \tag{1.4}$$

Here $\sigma_0 = Ne^2/m(\nu - i\omega)$, $\gamma = (\nu - i\omega)/\Omega$, $l = v/(\nu - i\omega)$, $\Omega = eH/mc$ is the cyclotron frequency, ν is the electron collision frequency, and ω is the frequency of the electromagnetic wave. The presence of the factor 2 in front of the summation sign in (1.4) is due to the fact that the time of the "first" sojourn in the skin layer turns out to be half as large as that of the succeeding ones. The appearance in (1.4) of the factor $\coth\pi\gamma$ leads to oscillations of the surface impedance of the metal in the magnetic field—to cyclotron resonance.

We proceed to estimate the conductivity of the glancing electrons. Their number is N_{eff} and the time between two successive collisions with the surface is determined by the glancing angle $\varphi \sim (\delta/R)^{1/2} \ll 1$; these values are of the same order as N_{eff} and τ_0 of the volume electrons. Following the universally accepted model used for the description of electron reflection from the boundary, we introduce the specularity parameter $\rho (0 \leqslant \rho \leqslant 1)$. The probability that an electron is not scattered in the volume of the metal in one cycle of its motion but is specularly reflected from the surface is then described by the quantity $\rho e^{-2\gamma\varphi}$. Taking into account the smallness of $|\gamma|\varphi \sim (R\delta)^{1/2}/l \ll 1$, we obtain with the aid of (1.2) the electric conductivity of the glancing electrons

$$\sigma_{sur} = \frac{Ne^2}{m}\left(\frac{\delta}{R}\right)^{1/2}\frac{(R\delta)^{1/2}}{v}\left(1+2\sum_{n=1}^{\infty}\rho^n e^{-2n\gamma\varphi}\right) = \sigma_0\frac{\delta}{l}\left[\frac{1-\rho}{1+\rho}+\gamma\left(\frac{\delta}{R}\right)^{1/2}\right]^{-1}. \tag{1.5}$$

To analyze the CR we can use the following estimate of the surface impedance of the metal:

$$Z = -4\pi i\omega\delta/c^2. \tag{1.6}$$

The complex quantity δ satisfies the equation

$$\delta^2 = ic^2/4\pi\omega\sigma,$$

where σ is the conductivity of the metal ($\sigma = \sigma_{vol} + \sigma_{sur}$).

According to (1.4) and (1.5) we have

$$\delta^3 = \frac{ic^2 l}{4\pi\omega\sigma_0}\left\{\operatorname{cth}\pi\gamma + \left[\frac{1-\rho}{1+\rho}+\gamma\left(\frac{\delta}{R}\right)^{1/2}\right]^{-1}\right\}^{-1}. \tag{1.7}$$

Equation (1.7) has simple solutions in two limiting cases. In one of them we can neglect the term $\gamma(\delta/R)^{1/2}$ compared with the remaining terms in the brackets, while in the other this term is the principal one.

We consider first the second case, which corresponds to specular reflection of the electrons from the sample boundary:

$$1-\rho \ll |\gamma|(\delta/R)^{1/2} \ll |\operatorname{th}\pi\gamma|. \tag{1.8}$$

In this situation the skin layer is produced by the glancing electrons and the surface impedance Z depends in first-order approximation monotonically on H. The $Z(H)$ oscillations due to the CR consistute a small fraction of the average value of the impedance

$$Z = \frac{4\pi\omega\delta_0}{c^2}e^{-3\pi i/10}\left[1-\frac{2}{5}\gamma\operatorname{cth}(\pi\gamma)\left(\frac{\delta_0}{R}\right)^{1/2}e^{\pi i/10}\right],$$
$$\delta_0 = (c^2R^{1/2}/4\pi\omega\sigma_0)^{2/5}. \tag{1.9}$$

Apart from real constant, these formulas agree with the expressions obtained in the previous papers.[2,3]

If the surface scattering of the glancing electrons prevails over the scattering in the volume of the metal ($1-\rho \gg |\gamma|(\delta/R)^{1/2}$, then the impedance Z takes the form

$$Z = \left(\frac{16\pi^2\omega^2 l}{c^4\sigma_0}\right)^{1/3}e^{-\pi i/3}\left[\operatorname{cth}\pi\gamma + \frac{1+\rho}{1-\rho}\right]^{-1/3}. \tag{1.10}$$

In the case of diffuse reflection ($\rho \ll 1$) formula (1.10) goes over into the known expression obtained by Azbel' and one of us[1]:

$$Z(H) = Z(0)[1-e^{-2\pi\gamma}]^{1/3}. \tag{1.11}$$

We note that the result (1.11) is valid when the resonant singularity is so strong that $|\coth\pi\gamma|$ greatly exceeds all the remaining terms in the curly brackets of (1.7). At so sharp a resonance, the surface impedance $Z(H)$ is not sensitive to the character of the interaction of the electrons with the boundary of the sample.

We consider the region of the parameters ρ and γ in which the following inequalities hold:

$$|\gamma|(\delta/R)^{1/2} \ll 1-\rho \ll |\operatorname{th}\pi\gamma|,\ 1. \tag{1.12}$$

Here, just as in the case of specular reflection (1.8), the surface impedance is determined mainly by the glancing electron. Cyclotron resonance appears in the next term of the expansion of Z in the parameter $(1-\rho)\times\coth\pi\gamma$. According to (1.10), we obtain

$$Z = \left[\frac{8\pi^2\omega^2 l(1-\rho)}{c^4\sigma_0}\right]^{1/3}e^{-\pi i/3}\left[1-\frac{1-\rho}{6}\operatorname{cth}\pi\gamma\right]. \tag{1.13}$$

Comparison of (1.3) with Meĭerovich's result[4] reveals substantial differences. It is impossible to reconcile the impedance given by (3.11) of[4] with the limiting cases of diffuse (1.11) and purely specular (1.9) reflection, the resonant term has a different character, and the phase factors in both terms of the impedance are different.

It will be shown in the next section that the foregoing physical analysis of the phenomenon is in full agreement with the exact solution obtained from Maxwell's equations.

II. SOLUTION OF MAXWELL'S EQUATIONS. SURFACE IMPEDANCE

Let the constant and homogeneous magnetic field **H** be parallel to the metal-vacuum interface. We choose a coordinate system with the y and z axes on the metal surface (the plane $x=0$), the z axis parallel to **H**, and the x axis directed into the interior of the sample (see Fig. 1). A plane monochromatic wave is incident on the interface $x=0$ in the direction of x. The electric field in the interior of the metal is $\mathbf{E}=\mathbf{E}(x)e^{-i\omega t}$. We introduce the Fourier transform

$$E_\alpha(x)=\frac{1}{\pi}\int_0^\infty dk\,\mathscr{E}_\alpha(k)\cos kx,\quad \mathscr{E}_\alpha(k)=2\int_0^\infty dx E_\alpha(x)\cos kx. \qquad (2.1)$$

Maxwell's equations for the Fourier component $\mathscr{E}_\alpha(k)$ of the field take the form

$$k^2\mathscr{E}_\alpha(k)+2E_\alpha'(0)=\frac{4\pi i\omega}{c^2}\left[K_\alpha(k)\mathscr{E}_\alpha(k)-\frac{1}{\pi}\int_0^\infty dk'\,Q_\alpha(k,k')\mathscr{E}_\alpha(k')\right], \qquad (2.2)$$
$$\alpha=y,\ z.$$

The Fourier component of the conductivity of an infinite metal $K_\alpha(k)$ is due to the volume electrons. The integral kernel of the conductivity operator $Q_\alpha(k,k')$ contains contributions from both the volume and the surface electrons, and depends on the parameter ρ. Exact expressions for the kernels $K_\alpha(k)$ and $Q_\alpha(k,k')$ for arbitrary reflection coefficients ρ are given in[2].

We shall not describe the standard procedure of obtaining the asymptotic forms of $K_\alpha(k)$ and $Q_\alpha(k,k')$, and present the final result. For simplicity, we confine ourselves to isotropic electron dispersion (alkali metal).

Under the conditions of the anomalous skin effect, the function $K_\alpha(k)$ has an asymptotic form

$$K_\alpha(k)=\frac{3\pi Ne^2}{4mvk}\operatorname{cth}\pi\gamma. \qquad (2.3)$$

1. We consider first the case of almost specular reflection of the electrons from the sample boundary [Eq. (1.12)]. The asymptotic expression for the kernel $Q_\alpha\times(k,k')$ is a sum of three terms:

$$Q_\alpha(k,kx)=\frac{3Ne^2}{2mv(1-\rho)k^2}\left[\left(1+\frac{1-\rho}{2}\operatorname{cth}\pi\gamma\right)\frac{\ln x}{x^2-1}-\frac{\pi^2}{2\sqrt{x}}G_\alpha(x-1)\right]. \qquad (2.4)$$

The first term is the conductivity of the glancing electrons ($\varphi\sim(kR)^{-1/2}$; see the figure) and does not depend on the magnetic field H. It differs from the electric conductivity of the glancing electrons in specular reflection (cf. formula (3.7) of[2]). The differences are caused by the fact that in the specular case the glancing electrons are scattered in the interior of the metal, while under the conditions of (1.12) the scatterer is the sample boundary. The second term of (2.4) contains the CR and is due both to volume and to surface electrons with glancing angles $\varphi\sim\pi$. It is smaller by a factor $|\tanh\pi\gamma|/(1-\rho)\gg 1$ than the contribution of the glancing electrons. The third term is determined only by the surface electrons and is of the form

$$G_\alpha(t)=\frac{8\gamma(1-\rho)}{\pi^2}\int_0^{\pi/2}d\theta\,n_\alpha^2(\theta)\int_0^\pi\frac{d\varphi}{(e^{\gamma\varphi}-\rho e^{-\gamma\varphi})^2}\frac{\sin[kR\sin\theta(1-\cos\varphi)t]}{t}, \qquad (2.5)$$

where $n_y=\sin\theta$ and $n_z=\cos\theta$. It will be shown later that this term, alongside the others, plays an essential role in the solution of Maxwell's equations. If it is regarded then Eq. (2.2) has, generally speaking, no solution.

Let us analyze the behavior of the function $G_\alpha(t)$. It is easily seen that $G_\alpha(t)$ reaches a maximum of the order of $(1-\rho)kR\coth\pi\gamma$ at the point $t=0$. With increasing $|t|$ the function $G_\alpha(t)$ decreases, and in the region

$$(kR)^{-1}\ll|t|\ll|\gamma|^2/(1-\rho)^2kR$$

its behavior is characterized by the asymptotic form

$$G_\alpha(t)\approx\frac{2^{5/4}C_\alpha}{5\Gamma^2(1/4)}\frac{(1-\rho)(kR)^{1/4}}{\gamma}|t|^{-3/4},\quad C_y=3/2,\ C_z=1. \qquad (2.6)$$

Finally, far from the maximum at $|\gamma|^2/(1-\rho)^2kR\ll|t|$ we get

$$G_\alpha(t)\approx\frac{2^{1/2}\Gamma^2(1/4)}{3\pi^2}\frac{B_\alpha\gamma}{(1-\rho)(kR)^{1/4}}|t|^{-5/4}, \qquad (2.7)$$
$$B_y=1,\quad B_z=2.$$

It follows therefore that $G_\alpha(x-1)$ is a sharp function of x. The characteristic region of its variation is $|x-1|\sim|\gamma|^2/(1-\rho)^2kR$ and is small compared with unity. At the same time $\mathscr{E}_\alpha(kx)$ varies smoothly over an interval $\Delta x\sim 1$. We can therefore replace $G_\alpha(x-1)$ in (2.2) by the expression

$$G_\alpha(x-1)\to\delta(x-1)\int_{-\infty}^{\infty}dt\,G_\alpha(t)=\delta(x-1)\left(1-\frac{1-\rho}{2}\operatorname{cth}\pi\gamma\right).$$

The integral equation (2.2) can then be written in the form

$$(k)+2E'(0)=\frac{i}{\delta^3}\left[\frac{\mathscr{E}(k)}{k}-\frac{2}{\pi^2}\int_0^\infty dk'\frac{\ln(k/k')}{k^2-k'^2}\mathscr{E}(k')\right]. \qquad (2.8)$$

We have left out the subscript of the sought function, $\mathscr{E}_\alpha(k)\equiv\mathscr{E}(k)$, since Eq. (2.8) contains no quantities that depend on the polarization. The independence of the field distribution of the wave polarization is a result of the assumption that the electron dispersion is isotropic. The quantity δ is the thickness of the skin layer in the considered case of almost specular reflection of the electrons

$$\delta=\delta_0\left(1+\frac{1-\rho}{2}\operatorname{cth}\pi\gamma\right)^{-1/3},\quad \delta_0=\left[\frac{mvc^2(1-\rho)}{3\pi^2Ne^2\omega}\right]^{1/3}. \qquad (2.9)$$

Equation (2.8) coincides in form with the equation for the Fourier component of the electric field in the case of the anomalous skin effect at $H=0$ and diffuse scattering of the electrons.[7] This agreement is not accidental, since in both cases the skin layer is formed by the effective electrons moving along the metal surface. Consequently we can use the results of the theory of the anomalous skin effect and write down directly the expression for the surface impedance with δ given by (2.9):

$$Z=\frac{2\pi 3^{1/2}\omega\delta_0}{c^2}e^{-\pi i/3}\left[1-\frac{1-\rho}{6}\operatorname{cth}\pi\gamma\right]. \qquad (2.10)$$

The principal term in (2.10) does not depend on the magnetic field and on the mean free path (on ν), but depends substantially on the state of the sample surface. The CR oscillations are small compared with the dc component of the impedance. It is of interest to note that the exact formula (2.1) differs from (1.13) only by a factor $(3/\pi^2)^{1/6}\approx 0.82$.

2. To complete the picture, we present in conclusion the results of the exact solution of Maxwell's equation and of the calculation of the surface impedance in the case of specular reflection of the electrons from the metal boundary [Eq. (1.8)]. In this case the asymptotic form of the integral kernel $Q_\alpha(k, kx)$ differs from (2.4) only in the form of the first term, namely:

$$\frac{3Ne^2}{2mv(1-\rho)k^2}\frac{\ln x}{x^2-1}$$

is replaced by

$$\frac{c^2k_\alpha^{5/2}}{4\omega k^{3/2}}[x(x+1)]^{-1/2}.$$

The quantity $|k_\alpha|$ is the reciprocal of the depth of the skin layer formed by the glancing electrons:

$$k_\alpha=\left(\frac{12\pi^2 2^{1/2}C_\alpha}{5\Gamma^2(1/4)}\frac{\omega\sigma_0}{c^2R^{1/2}}\right)^{2/5}.$$

The resultant integral equation is solved with the aid of the method of Hartmann and Luttinger.[8] The result is the following expression for the surface impedance:

$$Z_{\rm spec}=a\frac{\omega}{c^2k_\alpha}e^{-3\pi i/10}\left[1-A\frac{\gamma\operatorname{cth}\pi\gamma}{C_\alpha(k_\alpha R)^{1/2}}e^{\pi i/10}\right], \qquad (2.11)$$

where the real constants a and A are given by

$$a=\frac{4\pi^2\Gamma^{-2}(2/5)}{(4\pi/5)^{1/5}\sin(2\pi/5)}\approx 6.98,$$

$$A=\frac{\Gamma^2(1/4)\sin(2\pi/5)}{2(2\pi)^3(50\sqrt{\pi})^{1/5}}\int_0^\infty dt\, t\operatorname{cth}(\pi t)\left|\Gamma\left(\frac{1+2it}{5}\right)\Gamma\left(\frac{3+2it}{5}\right)\right|^2\approx 0.12.$$

The results of this section thus corroborate the validity of the physical description proposed in the preceding section for the phenomenon.

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