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Given-intensity approximation in the theory of nonlinear waves

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A large class of problems in the theory of linear waves can be solved only in the given-field (GF) approximation. In the present paper a new approximation, that of given intensity (GI), is developed, and takes into account the reaction to the phase of an intense wave. Physically this approximation is justified by the fact that the scales over which significant changes of the phase relation and transfer of the energy of the intense wave can take place can differ greatly in the presence of a mechanism that mismatches the phase relations between the interacting waves. It is shown that even in the absence of a phase mismatch the region in which the GI approximation is valid is larger than that of the GF approximation. The GI approximation is used to analyze the stationary interaction of waves in inhomogeneous nonlinear media and the nonstationary interaction of waves in homogeneous media. Expressions are obtained for the intensities and spectra of the excited or amplified waves. A number of effects that do not appear in the GF approximation are observed, particularly the influence on the parametric amplification of the intensity at the supplementary frequency and the dependence of the structure of the harmonic spectrum under nonstationary excitation conditions on the shape of the main beam.

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INTRODUCTION

The given-field (GF) approximation is widely used in the theory of nonlinear interaction of waves in dispersive media.[1–3] In this approximation, the complex amplitude of the intense initial wave is assumed to be given, as a result of which the nonlinear equations (which are partial differential equations in the general case) become simply coupled equations, and this facilitates greatly their solution for real wave beams and real nonlinear media. The GF approximation, however, describes correctly only the initial stage of the nonlinear wave interaction, so long as the reaction of the excited or amplified waves on the intense wave can be neglected. If the reaction is taken into account, however, the nonlinear wave equations can be solved exactly, even for homogeneous systems, only in a limited number of cases: for the interaction of plane waves or narrow wave packets (the so-called quasistatic approximation[4]) or for special cases of nonstationary wave interaction.[5–8] Recently, the method of the inverse scattering problem[9] has been applied to the analysis of nonlinear wave interactions, although this method imposes no limitation on the wave coupling coefficient, but its complexity is such that only asymptotic solutions are obtained. Numerical methods have recently been used to solve problems of nonlinear interaction of focused beams[10] and of the interaction of waves in inhomogeneous nonlinear media.[11] At the same time, of considerable interest in the theory of nonlinear waves is the development of analytic methods that make it possible to go beyond the framework of the GF approximation and at the same time produce results that can be easily interpreted.

In the present paper we develop, for the analysis of the interaction of waves in nonlinear dispersive media, the given-intensity (GI) approximation, in which, in contrast to the GF approximation, the reaction on the phase of the exciting wave is taken into account. The physical basis of the proposed approximation is the difference between the rates of change of the amplitudes and phases of the interacting waves. Therefore the GI approximation is effective in those cases where there is a mechanism of mismatching the phases of the interacting waves, such as wave detuning or group-velocity mismatch. However, even where there is no such mechanism, the accuracy of the solutions obtained by the GI

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approximation is higher than that of the GF approximation.

We consider in the approximation developed below a number of problems of nonstationary nonlinear processes and of stationary processes that occur in nonlinear media with inhomogeneities. It is shown that in the GI approximation the behavior of nonlinear processes can differ not only quantitatively but also qualitatively from the behavior predicted by the GF approximation.

It should be noted that the GI approximation has already been used by a number of workers.[12-14] They, however, have considered[12,13] stationary interaction of waves in homogeneous nonlinear media, and in[12] a problem having an exact solution was solved approximately.

1. PRINCIPLES OF THE METHOD AND CONDITIONS OF APPLICABILITY OF THE SOLUTIONS

The application of the GI approximation to certain problems of practical interest, but which have until recently solved in the GF approximation, will be illustrated by us in the subsequent sections. Here we consider the principle of finding solutions in the GI approximation, using as an example a problem for which an exact solution is known. This circumstance makes it possible to illustrate the effectiveness of the GI approximation and to determine the conditions under which the approximate solutions are valid.

Consider a degenerate three-frequency wave interaction described by the equation

$$\frac{\partial A_1}{\partial z}+\frac{1}{u_1}\frac{\partial A_1}{\partial t}+\delta_1 A_1=-i\beta_1 A A_1^* e^{i\Delta z}, \tag{1a}$$

$$\frac{\partial A}{\partial z}+\frac{1}{u_l}\frac{\partial A}{\partial t}+\delta A=-i\beta A_1^2 e^{-i\Delta z}, \tag{1b}$$

where A_1 and A are the complex amplitudes of the fundamental waves and of the second harmonic, respectively, β are the coefficients of the nonlinear coupling of the waves (real quantities), $\Delta=2k_1-k$ is the wave mismatch, u_n is the group velocity, and δ_l is the loss parameter.

We change over from the stationary first-order equations (1) to second-order equations, neglecting the losses ($\partial/\partial t\equiv 0, \delta_1=\delta=0$),

$$\frac{d^2A_1}{dz^2}-i\Delta\frac{dA_1}{dz}-\beta_1[\beta_1 I(z)-\beta I_1(z)]A_1=0, \tag{2}$$

$$\frac{d^2A}{dz^2}+i\Delta\frac{dA}{dz}+2\beta\beta_1 I_1(z)A=0, \tag{3}$$

where $I(z)=AA$ is the intensity. This yields GF approximations: 1) for the parametric gain from (2) at $\beta=0$ and $I(z)=I(0)$ and 2) for second-harmonic generation from (3) at $\beta_1=0$. To solve (2) and (3) rigorously it would also be necessary to write down, for example for the intensity I_1, an equation whose behavior is determined by terms of the type[1)] $A_1^2 A$, which in turn depend on I_1^2 and II_1 etc. In the developed approximate theory we can confine ourselves to Eqs. (2) and (3).

By way of example we consider in detail the second-harmonic generation process; it can be assumed here that the initial amplitude of the harmonic is $A(0)=0$, and $A_1(z=0)=A_{1,0}$. If we put in (3) for the intensity $I_1(z)=I_1(z=0)=I_{1,0}$, then solution of the equation obtained in this manner will correspond to the GI approximation, namely, no limitations whatever are imposed in the nonlinear medium on the phase φ_1 of the exciting wave.

In the GI approximation, Eq. (3) has as its solution

$$A(z)=-i\beta A_{1,0}^2 z e^{-i\Delta z/2}\operatorname{sinc}(\varkappa z) \tag{4}$$

or we have for the real amplitudes a and the phase

$$a=\beta z\operatorname{sinc}(\varkappa z)a_{1,0}^2, \tag{5}$$

$$\varphi=2\varphi_{1,0}-\pi/2-\Delta z/2. \tag{6}$$

In (4)–(6) we put

$$\varkappa=(\Delta/2)[1+8(l_{nl}\Delta)^{-2}]^{1/2},\quad l_{nl}=(\beta\beta_1 I_{1,0})^{-1/2},\ \operatorname{sinc}x=\sin x/x. \tag{7}$$

In the GF approximation the characteristic nonlinear length is $l_{nl}\to\infty$.

1.1. *Inexact matching of the phase velocities*, $\Delta\neq 0$. A rigorous solution of equations (1) in this case ($\partial/\partial t=0, \delta=\delta_1=0$) yields for the amplitude of the harmonic ($\beta=\beta_1$)[15]

$$a=v_b\operatorname{sn}(v_c z/l_{nl},\gamma)a_{1,0}, \tag{8}$$

where $\operatorname{sn}(\zeta,\gamma)$ is the elliptic sine and

$$v_c=v_b^{-1}=|l_{nl}\Delta|/4+[1+(l_{nl}\Delta/4)^2]^{1/2},\quad \gamma=v_b/v_c. \tag{9}$$

At $\Delta=0$ we have $v_b=v_c=1$ and we get

$$a=a_{1,0}\operatorname{th}(\beta a_{1,0}z). \tag{10}$$

In the case $\Delta>4/l_{nl}$ we have

$$v_b=v_c^{-1}=[1+8(l_{nl}\Delta)^{-2}]^{1/2}(l_{nl}\Delta/2). \tag{10a}$$

Under this condition $\gamma<0.2$, and using the expansion[16] of the sn function, we arrive at the approximate result (5).

On the other hand if $\Delta\gg 4/l_{nl}$ we obtain from (5) and (8) the result of the GF approximation:

$$a=\beta z a_{1,0}^2\operatorname{sinc}(\Delta z/2). \tag{11}$$

Thus, the condition for the applicability of the solution in the GI approximation is

$$\Delta>4/l_{nl},\ \text{or}\ l_{nl}>(4/\pi)l_\Delta \tag{12}$$

($l_\Delta=\pi/\Delta$ is the "coherent" length[11]), and interaction length z can in this case be arbitrary.

Figure 1 shows the dependence of the amplitude A of the harmonic on the phase mismatch at a constant length z, from which it is seen that at $\Delta<2/l_{nl}$ there is a con-

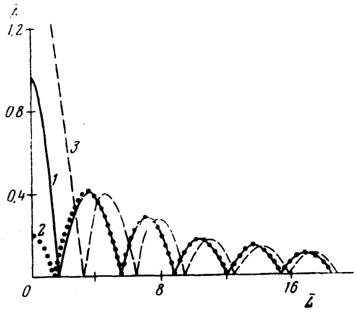

FIG. 1. Dependence of the relative amplitude of the second harmonic $\tilde{A}=|A/A_{1,0}|$ on the parameter $\tilde{\Delta}=\Delta z/2$ for $z/l_{nl}=2$, as calculated: 1—exactly (the curve is taken from[17]); 2—in the GI approximation (point); 3—in the GF approximation.

siderable discrepancy between the exact calculation by formula (3) and the calculation in the GI approximation. In the opposite case ($\Delta z \gtrsim 2z/l_{nl}$) these calculations give practically identical values, which, however, differ strongly from the result of the GF approximation.

It follows from (5) and (7) that the period $\Delta^{(per)}z=2[n^2 \times \pi^2-2(z/l_{nl})^2]^{1/2}$ of the spatial beats of the amplitude of the harmonic agrees with the GF approximation only at large values of n ($n=1, 2, \ldots$ is the number of the period) (see also Fig. 1).

The change of the phases of the fundamental radiation is determined according to (1a) in the GI approximation by the expression

$$\varphi_1(z)=\varphi_{1,0}-(1-\mathrm{sinc}\,2\varkappa z)\Delta z[8+(l_{nl}\Delta)^2]^{-1}=\varphi_{1,0}-\psi. \tag{13}$$

We see therefore that at $\Delta\neq 0$ the phase velocity of the fundamental wave depends only on its intensity ($l_{nl}^{-1}\neq 0$); in other words, self-action takes place.[2)] Taking into account the transverse distribution, the divergence of the fundamental radiation in a nonlinear medium decreases at $\Delta>0$ (self-focusing) and increases at $\Delta<0$ (self-defocusing). From the point of view of the efficiency of the harmonic generation, what is important is not the change of the phase φ_1 of the fundamental radiation itself, but the change of the phase relation[1]:

$$\Phi=2\varphi_1-\varphi_2-\Delta z=(\pi-\Delta z-4\psi)/2.$$

The relative increment to the phase in the GI approximation, compared with the GF approximation,

$$\tilde{\psi}(\Delta\neq 0)=\psi/(\Delta z/2)=2[8+(l_{nl}\Delta)^2]^{-1}$$

while small ($\tilde{\psi}<1$), exerts a substantial influence on the conversion efficiency. The absolute change of ψ, on the other hand, can be large.

1.2. *Exact matching of the phase velocities*, $\Delta=0$. Let us compare the rigorous solution (10) with the approximate one (5) at $\Delta=0$. Expanding the functions in these expressions, we readily find that the intensity of the harmonic is described, accurate to $(z/l_{nl})^6$, by the same expression

$$I_2=(z/l_{nl})^2[1-{}^2/_3(z/l_{nl})^2]I_{1,0}. \tag{14}$$

The result of the GF approximation is given only by the first term of the expansion (14). Consequently, even in the absence of the phase mismatch mechanism ($\Delta=0$) the range of applicability of the GI approximation is larger than that of the GF approximation.

Figure 2 shows the dependence of the effectiveness of harmonic generation on the length t of the nonlinear medium; the calculations were performed in accord with the exact formula (10) and in the discussed approximations. Over a length $z=l_{nl}$, the difference between the conversion coefficient calculated in the GI approximation in the exact value reaches only 9%. Thus, at $\Delta=0$ the GI approximation can be used in practice all the way to $z\lesssim l_{nl}$. This circumstance can be used to solve nonlinear problems with allowance for nonequilibrium losses in the interacting waves. Thus, stationary solution of Eq. (1) at $\delta_1\neq\delta$ in the GI approximation yields

$$A=-i\beta z A_{1,0}^2\,\mathrm{sinc}\{z[2l_{nl}^{-2}-(\delta_1-\delta/2)^2]^{1/2}\}\exp\{-(2\delta_1+\delta)z/2\}. \tag{15}$$

In (15) we have taken into account the damping of the intensity I_1 of the fundamental radiation as a result of the absorption ($\delta_1\neq 0$), while the decrease of the intensity I_1, due to the transfer to the harmonic, is disregarded. We note that a somewhat different approach to the derivation of the results in the GI approximation in the presence of losses in the nonlinear medium was considered by Emel'yanov and Klimontovich.[13]

2. WAVE INTERACTION IN LINEARLY INHOMOGENEOUS MEDIA

We turn now to wave interaction in nonlinear media with nonlinear inhomogeneities. This problem has recently attracted particular interest in connection with the problem of plasma heating by laser radiation (see, e.g., [19,20] and the literature cited therein).

We write down the equations of the three-frequency wave interactions ($\omega_1+\omega_2=\omega_3$), which in the case of decay processes take the form[1,15]

$$\begin{aligned}\frac{dA_{1,2}}{dz}+\frac{1}{u_{1,2}}\frac{\partial A_{1,2}}{\partial t}&=-i\beta_{1,2}A_3A_{2,1}^{*}e^{i\Phi(z)},\\ \frac{\partial A_3}{\partial z}+\frac{1}{u_3}\frac{\partial A_3}{\partial t}&=-i\beta_3A_1A_2e^{-i\Phi(z)},\end{aligned} \tag{16}$$

where the phase shift is

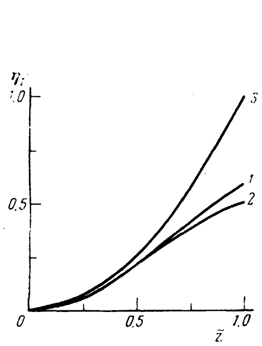

FIG. 2. Coefficient of conversion of the second harmonic $\eta_I=I/I_{1,0}$ as a function of the relative length $\tilde{z}=z/l_{nl}$ for the case $\Delta=0$: 1—exact calculation; 2—GI approximation; 3—GF approximation.

$$\Phi(z)=\Delta z+\int_0^z \Delta(z')dz',\quad \Delta+\Delta(z)=k_3-k_2-k_1,$$

Δ and $\Delta(z)$ are the constants and alternating parts of the wave mismatch. The remaining quantities in the equation are conventional and analogous to those in (1).

In the case of tearing interactions it is necessary to make in (16) the substitution $\beta_n \to i\sigma_n$. It is the last process which we shall consider here for the sake of argument.

In the GI approximation, the stationary behavior of the complex amplitude A_1 is described by the equation

$$\frac{d^2V_1}{dz^2}+i[\Delta+\Delta(z)]\frac{dV_1}{dz}-\left(\Gamma_1^2-i\frac{d\Delta(z)}{dz}\right)V_1=0, \tag{17}$$

where $V_1=A_1e^{-i\Phi}$, $\Gamma_1^2=\sigma_1(\sigma_2 I_{3,0}+\sigma_3 I_{2,0})$, and $I_{n,0}$ is the intensity at the entrance into the nonlinear medium. Equation (17) is valid for the case of equal group velocities of the interacting waves ($u_1=u_2=u_3=u$), and makes it possible to analyze the waveform of the pulse within the framework of the quasistatic approximation, but we shall not dwell on this question here.

We consider first the case of a homogeneous medium ($\Delta(z)=0$) and obtain the condition for stabilization of the tearing instability, without solving Eq. (17). The character of the solution of Eq. (17) depends on the parameter $g^2=4\Gamma_1^2-\Delta^2$. At $g^2>0$ Eq. (17) has a growing solution that leads to pairing instability. If $g^2<0$, then the solution is oscillatory, in accord with the condition for the elimination of the tearing instability. From the stabilization condition

$$4\sigma_1(\sigma_2 I_{3,0}+\sigma_3 I_{2,0})<\Delta^2 \tag{18}$$

it follows that the threshold pumping density $I_{3,0}^{(\mathrm{thr})}$ depends not only on the wave mismatch Δ, but also on the intensity $I_{2,0}$ at the supplementary frequency ω_2 relative to the frequency ω_1. Putting $\omega_2=0$ in (18), we obtain the result of [21].

For an inhomogeneous medium, the solution of Eq. (17) at $\Delta=0$ and $\Delta(z)=\alpha z$ takes the form

$$A_1(z)=\{A_{1,0}F(\mu_1,{}^1/_2;\zeta)+\sigma_1 A_{3,0}A_{2,0}^*F(\mu_2,{}^3/_2;\zeta)\}\exp(-i\alpha z^2/4), \tag{19}$$

where $F(\mu,m;\zeta)$ is a confluent hypergeometric function,

$$\mu_1=i\Gamma_1^2/2\alpha,\quad \mu_2=\mu_1+{}^1/_2,\quad \zeta=i\alpha z^2/4.$$

At short distances $z(\alpha z^2<1, gz<1)$, using the expansion of the function $F(\mu,m;\zeta)$, we obtain for the intensity

$$I_1(z)=\{1+(\Gamma_1 z)^2[1-(\alpha z^2)^2/45]\}I_{1,0}. \tag{20}$$

It is important that the inhomogeneities of the medium during the initial stage decrease the growth of the parametrically amplified signal. At large distances ($\alpha z^2>1$), on the other hand, the amplification is practically saturated; at $\Gamma_1^2/2\alpha<1$ the intensity takes the form

$$\begin{aligned}I_1=\{&1+\pi(\Gamma_1^2/2\alpha)^2+(\pi/2)^{1/2}\sigma_1 A_{3,0}A_{2,0}^*(2A_{1,0})^{-1}\\&+(2\pi)^{1/2}(\Gamma_1^2/2\alpha)\cos\psi\}\exp(\pi\Gamma_1^2/\alpha)I_{1,0},\\ \psi=&{}^1/_4\{3\pi+2\alpha z^2-2(\Gamma_1^2/\alpha)\ln[\alpha z^2/2]\}.\end{aligned} \tag{21}$$

Thus, the inhomogeneities of the medium lead to elimination of the tearing instability. However, both the instability threshold and the saturation level depend not only on the intensity of the pump wave but also on the intensity of the wave at the supplementary frequency.

3. NONSTATIONARY WAVE INTERACTIONS. EFFECTS OF PHASE MODULATION OF THE PULSES

Nonstationary wave interactions are of considerable interest in nonlinearly optical processes that occur in the fields of ultrashort laser pulses. They will be studied below for homogeneous media ($\Delta(z)=0$).

3.1. *Frequency multiplication.* According to Eq. (1), nonstationary frequency doubling is described in the GI approximation by the equation (without allowance for the losses, $\delta_1=\delta=0$)

$$\frac{\partial^2 V}{\partial\xi\,\partial\eta}+\frac{\Gamma^2}{4}V=0, \tag{22}$$

where

$$V=A\exp(-i\nu^{-1}\Delta\eta),\quad \Gamma^2=-8\beta\beta_1\nu^{-2}I_{1,0}(r,\xi),$$

$$\xi=t-\frac{z}{u_1},\quad \eta=t-\frac{z}{u},\quad \nu=u^{-1}-u_1^{-1}.$$

The coordinate r has been introduced in the intensity $I_{1,0}(r,\xi)$ of the fundamental radiation to take into account the transverse distribution.

The boundary conditions for (22) take the form

$$V(r,\xi,\eta)|_{\xi=\eta}=0,\quad \frac{dV}{d\xi}\bigg|_{\xi=\eta}=-i\beta\nu^{-1}A_{1,0}^2(r,\eta)\exp(-i\nu^{-1}\Delta\eta). \tag{23}$$

The solution of (22) by the Reimann method assumes, after a number of mathematical transformations, the form

$$A(r,\eta,z)=-i\beta\int_0^z A_{1,0}^2(r,\eta-\nu z_1)e^{-i\Delta z_1}J_0[g(r,\eta,z_1)]dz_1, \tag{24}$$

where $J_0(g)$ is a Bessel function of real argument.

$$g^2(r,\eta,z_1)=8\beta\beta_1(z-z_1)\int_0^{z_1} I_{1,0}(r,\eta-\nu x)dx.$$

The complex amplitudes of ultrashort laser pulses can be represented in the form

$$A_{1,0}(r,\eta)=G(r)F(\eta)A_0. \tag{25}$$

The function $G(r)$ characterize the shape of the beam ($G(0)=1$), while the function

$$F(t)=\exp\{-(\tau^{-2}+i\varepsilon)t^2\} \tag{25a}$$

characterizes the shape of the pulse and the modulation of the phase in time. Usually the width of the spectrum of ultrashort pulses $\Delta\omega=4\tau^{-1}(1+\varepsilon^2\tau^4)^{1/2}$ exceeds the

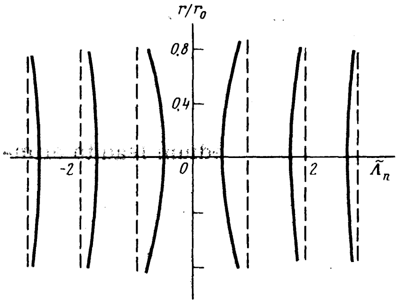

FIG. 3. Spatial arrangement of the minima in the spectrum of the harmonic ($\tilde{\Lambda}_n = c\nu z \lambda_2^{-2} \Lambda_{n,\min}$). The solid curves were calculated in the GI approximation at $z/l_{nl} = 2^{-1/2}\pi$; the dashed curves were calculated in the given-field approximation ($l_{nl} \to \infty$, $\beta_1 = 0$).

width due to the duration τ, i.e., $\varepsilon\tau^2 > 1$. We can therefore neglect the group delay of the interacting pulses in comparison with the duration τ.

3.2. *Second-harmonic spectrum.* The spectral density of the harmonic at $\nu z < \tau$ is given by

$$S(\omega;\, r,\, z) = G^4(r) I_0^2 \operatorname{sinc}^2(zd/2) S^{qu}(\omega, z), \tag{26}$$

$$d^2 = d^2(\omega) = (\Delta + \nu\omega)^2 + 8\beta\beta_1 I_0 G^2(r), \tag{27}$$

where $S^{qu}(\omega, z)$ is the spectrum of the harmonic in the given-field approximation in the quasistatic doubling regime ($\nu = 0$):

$$S^{qu}(\omega, z) = (\beta z)^2 \left| \int_{-\infty}^{+\infty} F^2(t) e^{i\omega t} dt \right|^2 .$$

It follows from (26) that the structure of the spectrum of the harmonic depends on the dispersion properties of the medium, on the intensity I_0 of the fundamental radiation, and on the shape of the beam $G(r)$. The spatial picture of the distribution of the minima in the spectrum of the harmonic is determined by the expression ($\Delta = 0$)

$$\Lambda_{n,\,min} = (n\lambda_2^2/c\nu z)\{1 - 2(z/nl_{nl})^2 G^2(r)\}^{1/2}, \tag{28}$$

where λ_2 is the wavelength of the second harmonic ($\lambda_2 = \lambda_1/2$), Λ is the deviation from λ_2, $l_{nl} = (\beta\beta_1 I_0)^{-1/2}$, and n is the order of the minimum.

Figure 3 shows the form of the function (28) for a Gaussian main beam: $G(r) = \exp\{-(r/r_0)^2\}$. It is seen that the structure of the spectrum of the harmonic in intense fields $z \gtrsim l_{nl}$ differs strongly from that calculated

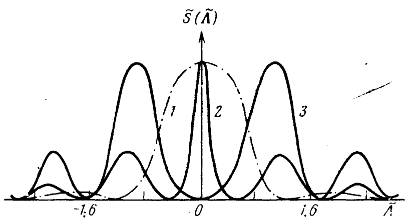

FIG. 4. Normalized spectrum of second harmonic $\tilde{S}(\Lambda)$ for the center of the beam ($r = 0$) at different ratios z/l_{nl}: 1—0($l_{nl} \to \infty$); 1—1.36; 3—$2^{-1/2}\pi = 2.23$ ($\tilde{\Lambda} = c\nu z \lambda_1^{-1} \Lambda$).

in the given-field approximation. These distinguishing features are: the bending of the bands in opposite directions away from the center of the spectrum, the narrowing of the central band, and the broadening of the lateral bands. Figure 4 illustrates the change of the spectrum of the harmonic for the center of the beam with changing ratio z/l_{nl}.

3.3. *Energy of second harmonic.* We calculate now the energy density of the harmonic

$$W(z) = \int_{-\infty}^{+\infty} S(\omega;\, r,\, z)\, d\omega. \tag{29}$$

The difficulty of the exact calculation of (29) is due to the presence of the function $\operatorname{sinc}^2[zd(\omega)/2]$ (see (26)), which, however, can be replaced approximately by $\exp(-0.09 z^2 d^2)$. The energy density in the harmonic, for an initial pulse in the form (25a) is then

$$W = 2^{1/2}\pi (z/l_{nl})^2 [1 + (z/l_{qu})^2]^{-1/2} \exp\{-0.72(z/l_{nl})^2\} W_1, \tag{30}$$

where $W_1 = 2\pi^{1/2} I_{1,0}(r)\tau$ is the energy density of the fundamental radiation, and $l_{qu} = \tau\{\nu^2(1 + \varepsilon^2\tau^4)\}^{1/2}$ is the quasistatic length. At constant lengths z and l_{qu}, the energy conversion coefficient $\eta_e = W/W_1$ reaches a maximum value when $l_{nl} \approx 0.85z$. With increasing phase-modulation coefficient ε (with decreasing length l_{qu}), the effectiveness of conversion into the harmonic decreases.

3.4. *Frequency mixing.* We consider the generation of the summary frequency, assuming the group velocities of the mixed waves to be equal ($u_1 = u_2$). In terms of new coordinates $\xi = t - z/u_1$ and $\theta = t - z/u_3$, the function V_3, which is connected with the amplitude of the excited wave by the relation

$$V_3 = A_3 \exp(i\nu_{31}^{-1}\Delta\theta),$$

is defined in the GI approximation by the equation (cf. (22))

$$\frac{\partial^2 V_3}{\partial\xi\,\partial\theta} - \beta_3 \nu_{31}^{-2} [\beta_1 I_{2,0}(\theta) + \beta_2 I_{1,0}(\theta)] V_3 = 0. \tag{31}$$

Its solution

$$A_3(r, \theta, z) = -i\beta_3 \int_0^z A_{1,0}(r, \theta - \nu_{31}\xi_1) A_{2,0}(r, \theta - \nu_{31}\xi_1) e^{-i\Delta\xi_1} J_0(g_1)\, d\xi_1,$$

$$g_1^2(r, \theta, \xi_1) = 4\beta_3 (z - \xi_1) \int_0^{\xi_1} [\beta_1 I_{2,0}(r, \theta - \nu_{31}z') + \beta_2 I_{1,0}(r, \theta - \nu_{31}z')]\, dz' \tag{32}$$

is of the same form as expression (24). Consequently, all the singularities of nonstationary second-harmonic generation by phase-modulated pulses are obtained in the GI approximation in the case of nonstationary mixing of such pulses. The result of the given-field approximation is obtained from (32) if $\beta_1 = \beta_2 = 0$.

3.5. *Parametric amplification.* Another important case of nonstationary wave interaction, when the group velocities at the frequency ω_2 and, say, the frequency ω_1 can be regarded as equal. This case applies, besides processes of amplification and generation of the difference frequency, also to processes of stimulated scattering (Raman scattering, Mandel'shtam–Brillouin scattering, etc.).

The behavior of the complex amplitude A_2 of frequency ω_2 is given, in accordance with Eqs. (2) in the GI approximation, by the equation

$$\frac{\partial^2 V_2}{\partial\xi\,\partial\eta}+\beta_2\nu_{32}^{-2}[\beta_1 I_{3,0}(\eta)-\beta_3 I_{1,0}(\eta)]V_2=0 \tag{33}$$

with boundary conditions

$$V_2|_{\eta=\xi}=0,\quad \left.\frac{\partial V_2}{\partial\eta}\right|_{\eta=\xi}=-i\beta_2\nu_{32}^{-1}A_{3,0}(\xi)A_{1,0}^*(\xi), \tag{34}$$

where

$$V_2=A_2\exp(i\nu_{32}^{-1}\Delta\xi),\ \xi=t-z/u_2,\ \eta=t-z/u_3. \tag{35}$$

For the analysis of the considered process it is convenient to change from the time-dependent form of the solution of (33)

$$A_2(\eta,z)=-i\beta_2 e^{-i\Delta z}\int_0^z A_{3,0}(\eta-\nu_{32}\xi)A_{1,0}^*(\eta-\nu_{32}\xi)e^{-i\Delta\xi}I_0[g]d\xi,$$

$$g^2=(z-\xi)\int_0^\xi\Gamma(\eta-\nu_{32}x)\,dx,\qquad \Gamma^2(\eta)=4\beta_2[\beta_1 I_{3,0}(\eta)-\beta_3 I_{1,0}(\eta)] \tag{36}$$

($I_0[g]$ is a Bessel function of imaginary argument) to the Fourier spectrum

$$A_2(\omega,z)=-i\beta_2\exp\{-i(\Delta-\nu_{32}\omega)z/2\}$$
$$\times\int_{-\infty}^{+\infty}A_{3,0}(t)A_{1,0}^*(t)\frac{\operatorname{sh}(z/2)[\Gamma^2(t)-(\Delta+\nu_{32}\omega)^2]^{1/2}}{[\Gamma^2(t)-(\Delta+\nu_{32}\omega)^2]^{1/2}}e^{i\omega t}\,dt. \tag{37}$$

In the derivation of (37), just as in Sec. 3.1, it was assumed that the time lag is $\nu_{32}z<\min(\tau_1,\tau_3)$ (τ_n are the durations of the initial pulses), and the nonstationary character of the process is due to phase modulation.

In the case of large gains $\Gamma(0)z=\Gamma_0 z>1$ and identical exciting pulses, such that

$$A_{3,0}(t)A_{1,0}^*(t)=A_{3,0}(0)A_{1,0}(0)\exp\{-(2\tau^{-2}+i\varepsilon)t^2\}, \tag{38}$$

the spectral density at the frequency ω_2 is equal to

$$I_2(\omega,z)=\frac{\pi^2\beta_2^2 I_{1,0}(0)I_{3,0}(0)}{4(\Gamma_0^2-\nu_{32}^2\omega^2)(D^2+\varepsilon^2)^{1/2}}\exp\left\{\left(\Gamma_0-\frac{\nu_{32}^2\omega^2}{2\Gamma_0}\right)z-\frac{D\omega^2}{D^2+\varepsilon^2}\right\}, \tag{39}$$

where

$$D=\Gamma_0 z[1+\nu_{32}^2\omega^2(2\Gamma_0^2)^{-1}](2\tau_3^2)^{-1}.$$

It follows from (39) that the width of the spectrum at the excited frequency is equal to

$$\Delta\omega_2=2[1+(\nu_{32}z/\tau_3)^2]^{-1/2}\{\Gamma_0 z[1+4\varepsilon^2\tau_3^4(\Gamma_0 z)^{-2}]\}^{1/2}\tau_3^{-1}. \tag{40}$$

The growth of the gain increment $\Gamma_0 z$ and of the frequency-modulation coefficient ε make the spectrum narrower, while the group detuning ν_{32} causes a decrease of $\Delta\omega_2$. At $\Gamma_0 z=2\varepsilon\tau_3^2$ the spectrum width at the frequency ω_2 is minimal:

$$\Delta\omega_{2,min}=2[2\varepsilon\tau_3^2/(1+(\nu_{32}z/\tau_3)^2)]^{1/2}\tau_3^{-1}. \tag{41}$$

Thus, in the presence of phase modulation an increase in the gain does not lead in all cases to an increase in the width of the spectrum (cf. [22]).

In the GI approximation, Γ_0 is determined by the intensities $I_{3,0}$ and $I_{1,0}$ respectively at the pump frequencies ω_3 and ω_1 (see (36)). An increase of $I_{1,0}$, however, produces the opposite of the effect due to the increase of $I_{3,0}$.

CONCLUSION

We have developed a theory of nonlinear waves in the GI approximation. The application of this approximation to media with linear and nonlinear inhomogeneities and nonstationary processes is physically more justified than the hitherto used GF approximation. One of the linear coefficients in the abbreviated equations (16) of the nonlinear wave interaction in the GF approximation, β_n, is in effect assumed to be equal to zero. Actually, however, the coefficients β_n are as a rule of the same order, and it is the rates of change of the amplitudes and phases of the interacting waves which are different. This circumstance is the basis of the developed GI approximation, in which a number of new effects have been obtained. We mention among them the influence of the effect exerted on the threshold by parametric amplification, and the threshold of intensity stabilization at the additional frequency. In the case of multiplication and mixing of frequencies in the nonstationary regime or when these processes take place in inhomogeneous media, the conversion coefficient may experience saturation at a level that differs appreciably from complete conversion in the case of stationary processes that occur in homogeneous media (see also [23]). An analysis of nonstationary multiplication and mixing of frequencies in the GI approximation yields a qualitatively different picture of the spectrum than the GF approximation. This result agrees with experiments on frequency multiplication of intense ultrashort pulses.[14]

The condition for the applicability of the GI approximation in the presence of loss of phase coherence between the interacting waves is determined by inequality (12), which is valid for arbitrary lengths, z, and the coherent length l_Δ, depending on the character of the interaction, must be replaced by the quasistatic length l_{qu} or by the inhomogeneous length $l_{inhom}\approx\pi/\Delta(z)$. At distances $z<l_{nl}$ satisfaction of condition (12) is not obligatory.

We have confined ourselves here to an illustration of the use of the developed GI approximation to cases of three-particle interactions. This approximation, of course, can be used also for the analysis of four-frequency interactions. We note finally that, in accordance with the space-time analogy in the theory of nonlinear waves[5] the GI approximation can be applied to certain problems of nonlinear interaction of wave beams.

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[1)]We note that in the presence of a mismatch ($\Delta \neq 0$) it is impossible to obtain equations in closed form for the intensities I and I_1 (cf.[13]).

[2)]This phenomenon was apparently first pointed out by Ostrovskiĭ,[18] who analyzed it qualitatively.

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Resonance excitation of light and dynamic electro-optical effects

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An analysis is reported of the propagation of a weak plane light wave in a gas placed in strong constant and uniform alternating external electric fields. The frequency of the latter is in the radio band and the frequency of the incident light wave is close to the frequency of one of the allowed transitions in the gas molecules. The time modulation of the molecular transition frequency due to the Stark effect in the external electric field is taken into account. The degree of modulation and the mean intensity of the light wave transmitted through the medium under consideration are investigated as functions of the amplitude of the external alternating field, the constant external field, and the frequency of the incident light wave. A number of features of this functional dependence is noted. The possibility of observing these effects in a gas of molecules of the symmetric spinning-top type is discussed. Possible applications of these effects are examined.

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1. INTRODUCTION

Modulation of electromagnetic waves can be produced in media in which the refractive index is a function of the electric field. This type of modulation of light waves by an external electric radio-frequency field has been observed in dielectrics[1–4] and has subsequently found application in lasers where it is used for mode locking.[5–7] The modulation is also possible in the electron plasma of semiconductors and in gas plasma in an external magnetic field with an rf component modulating the cyclotron frequency.[8–13] The modulation of waves by an external electric low-frequency rf field in gas plasma, due to the modulation by this field of the electron mean free time, has been discussed by Kumar *et al.*,[14] and that due to the hydrodynamic modulation of the plasma density by this field has been discussed by Kumar *et al.*,[14] Aliev and Silin,[15] and Ostrovskiĭ and

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