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Interactions between electrons and a strong piezoacoustic wave

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The behavior of electrons trapped by the field of a strong piezoacoustic wave in nondegenerate semiconductors is studied. The resonant absorption of microwaves due to the quantization of one of the electron momentum components in the wave field and the effect of intense ultrasound on the galvanomagnetic properties of the crystal are considered.

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An ultrasonic wave (USW) of sufficiently high frequency and intensity can materially change the character of the motion of electrons in a crystal. A number of specific effects arise here, dependent in a nonlinear way on the intensity of the USW. Some of these effects have been investigated in a number of researches.[1–4] In particular, a quantum effect was considered in Ref. 1 and consists of the change in the energy spectrum of the electrons in the field of a strong hypersonic wave ($\nu > 10^9$ Hz); the effect of spatial redistribution of electrons in the field of a USW on sound propagation was studied in Refs. 2–4.

In the present paper, we study the behavior of electrons trapped by the field of a strong piezoacoustic wave in nondegenerate semiconductors. The resonant absorption of microwaves, connected with the quantization of one of the components of the momentum of the electron in the field of the wave, and the effect of a strong ultrasonic signal on the galvanomagnetic characteristics of the crystal, are investigated.

I. RESONANT ABSORPTION OF MICROWAVES

1. *Formulation of the problem. The energy spectrum of the electrons.* Consider the interaction of free electrons with a powerful ultrasonic wave in a nondegenerate piezo-semiconductor (n-type) with isotropic and quadratic dispersion law $E = p^2/2m^*$. In the effective-mass approximation, the action of the wave on the electron reduces to the classical non-stationary potential field

$$V(y, t) = V_0 \cos(ky - \omega t), \quad \omega = sk, \tag{1}$$

where s is the sound velocity (in the case of a cubic crystal, we deal with transverse sound), V_0 is expressed as a known function of the piezomoduli, of the dielectric tensor, and of the displacement vector in the USW. We shall be interested in the condition in which $V_0 \gg T$ (the temperature is in energy units). At a displacement amplitude $u_0 = 10^{-9}$ cm, wave frequency $\omega = 2\pi \cdot 10^9$ sec^{-1}, $V_0 = 176$ K for GaAs and $V_0 = 62$ K for InSb, the required power flux is 3.5 W/cm^2 and 1.7 W/cm^2, respectively.

After the carriers reach equilibrium with the wave, they will execute finite motion in y in small (by virtue of the condition $T \ll V_0$) vicinities of the minima of the potential (1). In a system of coordinates moving with the USW, such a motion corresponds to stationary states with quantum energy levels very close to the levels of the harmonic oscillator $E_n = \hbar\Omega(n + \frac{1}{2})$, $n = 0, 1, 2, \ldots$, where $\Omega = (V_0 k^2/m^*)^{1/2}$. The total energy of the electron is equal to $E_n + (p_x^2 + p_z^2)/2m^*$, and its spectrum is, of course, continuous. The effects of anharmonism are connected with the deviation of the actual potential $V_0 \cos ky$ from parabolic and become significant only at large numbers n. However, these levels are practically unpopulated at $T \ll V_0$. For the values of ω and u_0 used above, we obtain Ω(GaAs) $\approx 3.77 \times 10^{11}$ sec^{-1} ($\hbar\Omega \approx 3$ K) and Ω(InSb) $\approx 7.54 \times 10^{11}$ sec^{-1} ($\hbar\Omega \approx 6$ K).[1)] Strictly speaking, each of the found levels is spread out into a band because of the periodicity of the potential (1). However, it is easy to prove that the tunnel transparency of the barriers in the considered case is entirely negligible, i.e., the conditions of applicability of the strong-coupling approximation are satisfied by a wide margin, and we can neglect the widths of the bands.

The condition of good resolution of the discrete levels imposes a restriction on the momentum relaxation time τ of the electrons. That is, the inequality $\Omega\tau > 1$, or $kL(V_0/T)^{1/2} > 1$ must be satisfied, where L is the free path length of the electron. Finding τ from the expression for the mobility, we obtain the conditions μ (GaAs) $> 7.2 \times 10^4$ cm^2/V-sec, μ (InSb) $> 1.7 \times 10^5$ cm^2/V-sec.

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2. *Absorption of microwaves.* An electromagnetic (EM) wave of frequency ω_0, propagating perpendicular to the USW and polarized along the y axis, will interact with the carriers according to the usual electric dipole mechanism. The corresponding oscillator strength for the transition $|n\rangle \to |n+1\rangle$ is obviously equal to simply n, since the electrons are described in the considered approximation by the wave functions of the harmonic oscillator. Thus, resonant absorption of the EM radiation at the frequency $\Omega = \omega_0$ should occur (in a system of coordinates connected with the USW, there is only the transverse Doppler effect, which we shall neglect relative to the parameter s^2/c^2). In absorption of an EM quantum, the momentum components p_x and p_z of the electron are conserved, so that we are dealing precisely with discrete, isolated absorption lines.

3. *Line shape in the traveling* USW *mode.* The USW damping leads to a decrease of the amplitude V_0 and consequently also of the transition frequency Ω, considered as a function of the number of the minimum of the potential energy (1). This means that the absorption line will be inhomogeneously broadened, since the different groups of electrons absorb at different frequencies. To this is added the radiation broadening (entirely negligible at the considered frequencies), Doppler broadening of the order of $\Omega (T/m^*c^2)^{1/2}$ due to thermal motion of the electrons along the direction of the EM wave, and the homogeneous collision broadening of the order of $\tau^{-1} \equiv \gamma$. It is easy to establish the fact that the principal contribution is made by the latter mechanism, and we can actually expect $\gamma/\Omega \sim 0.1-0.5$. As is well known, the collisions usually lead to a Lorentzian shape of the homogeneously broadened line:

$$J_L(\omega_0) \sim \frac{\gamma}{\gamma^2 + (\omega_0 - \Omega)^2}. \tag{2}$$

The real line shape is obtained as a superposition of the contours (2), in which Ω depends exponentially on the number of the potential well (the amplitude of the USW $\sim e^{-\alpha y}$). Assuming the attenuation coefficient α to be small in comparison with k, we can replace the summation by integration and find

$$J(\omega_0) \sim \frac{\gamma}{\gamma^2+\omega_0^2}\left[\frac{\omega_0}{\gamma}\left(\operatorname{arctg}\frac{\omega_0}{\gamma} - \operatorname{arctg}\frac{\omega_0-\Omega_0}{\gamma}\right) + \frac{1}{2}\ln\frac{\gamma^2+\omega_0^2}{\gamma^2+(\omega_0-\Omega_0)^2}\right]. \tag{3}$$

Here Ω_0 is the transition frequency in the first, deeper minimum of $V(y)$. (We write $\Omega = \Omega_0 \exp(-\alpha y)$ and integrate the difference $J_L(\omega_0)$ and this same expression with respect to y at $\Omega = 0$. This procedure leads to an integral that converges as $y \to \infty$, whence the formula (3) is obtained.) In a small region in the vicinity of $\omega_0 = \Omega_0$ the first derivative of the absorption signal is equal to

$$\frac{dJ}{d\omega_0} \sim -\frac{\gamma}{\Omega_0}\frac{1}{\gamma^2+(\Omega_0-\omega_0)^2}, \quad |\Omega_0-\omega_0| \sim \gamma \ll \Omega_0. \tag{4}$$

Thus, in the considered case, the derivative of a superposition of Lorentz contours with respect to frequency has itself a Lorentzian shape.

4. *Standing-wave regime.* In this situation, we are dealing with a frequency-modulated harmonic oscillator. The modulation frequency is equal to ω, i.e., it is not only smaller than the frequency Ω of the oscillator itself, but is also much smaller than γ. As is easily proven, in this case the line shape is close to Lorentzian (see (2)). The difference from (2) amounts to a small increment, equal in its principal order of magnitude to the expression

$$\frac{J(\omega_0)-J_L(\omega_0)}{J_L(\omega_0)} = \frac{\Omega\omega^2(\omega_0-\Omega)[(\omega_0-\Omega)^2-3\gamma^2]}{[\gamma^2+(\omega_0-\Omega)^2]^3}. \tag{5}$$

Here the condition $\Omega\omega^2 \ll \gamma^2$ is assumed, the correctness of which follows from the estimates given above. The expression (5) is odd relative to $\omega_0 - \Omega$, i.e., a small line asymmetry (in the parameter $\Omega\omega^2\gamma^{-3}$) arises. In a high-$Q$ resonator, the amplitude of the standing wave can increase (relative to the traveling wave) by several orders of magnitude. Consequently, the discussed effects can also occur in non-piezosemiconductors through interaction via the deformation potential.

A quantitative estimate of the magnitude of the absorption of the microwaves can be obtained by calculating the absorption cross section σ. For a harmonic oscillator at exact resonance, $\sigma = 4\pi e^2/m^*\gamma$. At the values of the parameters used above, $\sigma \sim 10^{-10}$ cm^2.

5. *The role of screening.* At sufficiently high electron concentration N, the interaction (1) turns out to be screened and the condition $V_0 \gg T$ is violated. Even before this concentration is reached, the characteristic potential of the electrons will produce a distortion of the oscillator spectrum. The most stringent restriction on the applicability of the theory developed above follows from the requirement that these distortions be small relative to $\hbar\Omega$. For estimates, we assume that all the electrons are concentrated in the vicinities of the minima of the potential in a region with size of the order of the amplitude of the zero-point oscillations $(\hbar/m^*\Omega)^{1/2}$. The level shift is small if the potential produced by these oscillations on the boundary of the considered region is less than the bare potential (1) at this same point. This leads to the condition

$$\frac{8\pi^2e^2}{\varepsilon}N < \left(\frac{\hbar^2 V_0^3 k^{10}}{m^*}\right)^{1/4}. \tag{6}$$

Numerical estimates give a concentration limit $N_0 \sim 10^{12}$ cm^{-3}, beginning with which the screening has a significant effect on the spectrum of the electrons. This value of concentration corresponds to an absorption coefficient of the microwave radiation of the order of 10^2 cm^{-1}. Further, it is obvious that there exists a region of concentrations $N > N_0$ in which practically all the electrons still have a quantum spectrum, but the self-consistent potential already differs strongly from the oscillator potential. Resonant absorption takes place as before but is characterized by other frequencies.

II. GALVANOMAGNETIC PROPERTIES

In this section, we calculate the electrical conductivity tensor σ_{ij} of electrons captured by the USW in a constant homogeneous magnetic field perpendicular to the direction of propagation of the USW. We assume that the tem-

perature remains less than V_0 but is much greater than the energy level separation that arises through the joint action of the magnetic field and the potential $V(y) \approx m^*\Omega^2 y^2/2$ created by the USW. Under these conditions, the quasiclassical approximation is applicable and the method of the kinetic equation can be used. We shall solve the stationary kinetic equation, i.e., we shall neglect the time dependence of the USW field. The frequency of this field ($\omega \sim 10^9$ sec^{-1}) is much smaller than the frequency of the electron motions and the reciprocal relaxation time, which characterizes the scattering of the carriers. In the traveling-wave regime, after transition to the set of coordinates connected with the USW, the dependence of the external field on the time becomes even slower. It arises because of the decrease of the amplitude of the potential (and, consequently, Ω) as a result of the damping of the wave. The reciprocal characteristic time in this case is equal to $\alpha s \ll \omega$. If we are dealing with a moving system of coordinates, we must then take into account the Lorentz transformation for the electric (**F**) and magnetic (**H**) fields. The field $\mathbf{F}_L = \mathbf{s}\times\mathbf{H}/c$ can reach rather high values if the Larmor frequency becomes of the order of Ω. In this article we do not consider effects that are nonlinear in **F** and therefore we assume that an electric field transverse to **H** and to the direction of the USW and almost cancelling the field $\mathbf{F}_L$ is applied to the sample. It is the resulting (weak) field **F** which enters into the kinetic equation. The change in the magnetic field due to such cancellation is of the order of s^2/c^2 and need not be taken into account.

The kinetic equation is of the form

$$\frac{\partial f}{\partial y}\frac{p_y}{m^*}+\frac{\partial f}{\partial \mathbf{p}}\left(e\mathbf{F}+\frac{e}{m^*c}[\mathbf{p}\times\mathbf{H}]-\nabla V\right)-J(f)=0, \tag{7}$$

where $J(f)$ is the collision term. The equilibrium distribution function f_0 depends only on the combination $p^2/2m^* + V(y) \equiv E$ and in the linear approximation in **F** we obtain the following equation for the nonequilibrium contribution $g = f - f_0$:

$$\frac{\partial f_0}{\partial \mathbf{p}}e\mathbf{F}+\frac{\partial g}{\partial y}\frac{p_y}{m^*}+\frac{e}{m^*c}\frac{\partial g}{\partial \mathbf{p}}[\mathbf{p}\times\mathbf{H}]\frac{\partial g}{\partial p_y}m^*\Omega^2 y-J(f)=0. \tag{8}$$

We seek a solution of Eq. (8) in the form

$$g=f_0'(Ay+\mathbf{Bp}), \quad f_0' \equiv \frac{\partial f_0}{\partial E}. \tag{9}$$

We shall assume that the electrons experience only elastic collisions. Then, upon substitution of (9) in (8), the component with Ay drops out of the integral because of the locality of the elementary scattering act. Then the collision term takes on the standard form

$$J(f)=\frac{1}{\tau}f_0'\mathbf{Bp},$$

where τ is the transport relaxation time. Equation (8) is satisfied at the following values of the constants entering into (9):

$$A=e\tau\omega_H F_x - eF_y, \quad B_x=-\frac{e\tau}{m^*}F_x, \quad B_y=0, \quad B_z=-\frac{e\tau}{m^*}F_z, \tag{10}$$

where $\omega_H = eH/m^*c$. It follows from (10) that the only nonvanishing components of σ_{ij} are σ_{xx} and σ_{zz}, and (as at **H** = 0)

$$\sigma_{xx}=\sigma_{zz}=\frac{2}{3}\frac{Ne^2\langle\varepsilon\tau\rangle}{m^*T}, \tag{11}$$

the angular brackets denote averaging over the kinetic energy of the electron ε. Thus, in the given situation, nondiagonal (Hall) coefficients σ_{ij} are absent and the magnetoresistance is zero even if τ depends on the kinetic energy of the carrier. In order to make clear the origin and physical meaning of the results, which at first glance are unexpected, it is useful to consider a simple model in which the scattering of the particles is described by "frictional force" $\mathbf{F}_{fr} = -m^*\mathbf{u}/\tau$ (**u** is the drift velocity of the particle). The equations of motion

$$m^*(\dot{\mathbf{u}}+\mathbf{u}/\tau)=\frac{e}{c}[\mathbf{u}\times\mathbf{H}]+e\mathbf{F}-\nabla V(y) \tag{12}$$

have the following solution, corresponding to the steady state (as $t \to +\infty$):

$$u_{x,z}=\frac{e\tau}{m^*}F_{x,z}, \quad u_y=0, \quad y=y_0\equiv\frac{e}{m^*\Omega^2}(F_y-\omega_H\tau F_x). \tag{13}$$

It is easy to see that this solution corresponds in accuracy to the form of the tensor σ_{ij} given above (see (11)). The relaxation processes are described by the solution of the system (12) at **F** = 0:

$$u_z\sim\exp(-t/\tau), \quad u_{x,y}\sim\exp(-\beta t), \tag{14}$$

where

$$\beta^3+\frac{2}{\tau}\beta^2+\beta\left(\tilde{\omega}^2+\frac{1}{\tau^2}\right)+\frac{\Omega^2}{\tau}=0, \quad \tilde{\omega}^2\equiv\Omega^2+\omega_H^2.$$

The roots of the characteristic equation in the limiting case $\tilde{\omega}\tau \gg 1$ are approximately equal to

$$\beta_{1,2}=\pm i\tilde{\omega}-\frac{1}{\tau}\left(1-\frac{\Omega^2}{2\tilde{\omega}^2}\right), \quad \beta_3=-\frac{\Omega^2}{\tilde{\omega}^2\tau}. \tag{15}$$

We see that the presence of a potential which limits the motion of the electrons along a single coordinate leads to a change in the relaxation times of the system and to the appearance of a new relaxation parameter $\tau_0 \equiv \tau\tilde{\omega}\Omega^{-2}$.

At $t \ll \tau_0$ the general solution of the system (12) transforms into a solution corresponding to the collisionless regime:

$$\bar{u}_x=\frac{eF_x}{m^*}\left(\frac{\Omega^2}{\tilde{\omega}^2}\right), \quad \bar{u}_y=-\frac{cF_x}{H}\left(\frac{\omega_H^2}{\tilde{\omega}^2}\right), \quad \bar{u}_z=\frac{eF_z}{m^*}, \tag{16}$$

where $\bar{u}_{x,y,z}$ are the components of the velocity averaged over the oscillations of frequency $\tilde{\omega}$. The formulas (16) also follow from a quantum mechanical consideration of the problem (see Ref. 5) if we set $\tau = \infty$ from the beginning, i.e., we neglect the scattering entirely. They describe a two-dimensional (in the x, z plane) quasi-particle with an anisotropic effective mass

$$m_{xx}=m^*(\tilde{\omega}^2/\Omega^2), \quad m_{zz}=m^*, \quad m_{xz}=m_{zx}=0.$$

The Hall components σ_{ij} in this case are different from zero and equal to

$$\sigma_{yx}=-\sigma_{xy}=\frac{Nec}{H}\left(\frac{\omega_H^2}{\tilde{\omega}^2}\right). \tag{17}$$

On the other hand, in the spatially homogeneous case ($\Omega=0$, $\tilde{\omega}=\omega_H$, $\tau_0\to\infty$) the solution of Eqs. (12) lead to the usual form of σ_{ij}, from which follow the well-known formulas of the Hall effect and the magnetoresistance. In particular,

$$\sigma_{yx}=-\sigma_{xy}=\frac{Ne^2\omega_H}{m}\langle\tau^2/(1+\omega_H^2\tau^2)\rangle. \tag{18}$$

Thus the Hall components σ_{ij} differ from zero only in the absence of scattering, or in a spatially homogeneous problem, since $\tau_0=\infty$ in both cases. We see, however, that the solution (16) and (17) is unstable relative to allowance for collisions, and (18) is unstable relative to the introduction of the external potential $V(y)$. The quantities $u(t)$ and $y(t)$ found from (12) relax at $t\gg\tau_0$ to the values (13) with the decrements $\mathrm{Re}\,\beta$, $\mathrm{Re}\,\beta_2$ and β_3. The Hall components σ_{ij} are damped with a time constant $\tau_0=\tau\tilde{\omega}^2\Omega^{-2}$.

Naturally, that the kinetic equation (8) which describes the behavior of the system as $t\to+\infty$ leads immediately to values of σ_{ij} corresponding to the formulas (13). The well-known statement that in a strong magnetic field ($\omega_H\tau\gg1$) the Hall coefficients can be obtained in the collisionless approximation turns out to be valid only for spatially homogeneous systems.

If τ in (12) depends on the kinetic energy of the electron ε, then the formulas (13) give the steady-state solution for carriers with given ε. The physical meaning of this solution is very simple: each carrier undergoes only elastic collisions and is therefore characterized by its own values of τ: for a given τ the electron drifts along the y axis until it reaches the point y_0 at which the electric force eF_y, the Lorentz force $e\omega_H\tau F_x$, and the force produced by the USW field $m^*\Omega^2y_0$ are balanced.

With the help of the solution of the kinetic equation (9), we can find the particle coordinate distribution function $W(y)$:

$$W(y)=\int(f_0+g)\,d^3\mathbf{p}$$

and then calculate the mean value of the coordinate y. As it should, it turns out to be equal to y_0 from (13) in which τ is replaced by $\langle\tau\rangle$. Thus, the nonequilibrium contribution g corresponds to a shift in the equilibrium distribution in the momentum and in the coordinate spaces. For σ_{ij} we again obtain the result (11), i.e., in the sense of a drift motion, the electrons behave as a gas of two-dimensional ($y=\mathrm{const}$) particles with isotropic effective mass. In this sense, in the orientation of the magnetic field that we have chosen, both longitudinal and transverse magnetoresistance is lacking (**H** is parallel to the plane of motion of the particles).

Thus, in the field of a strong USW (upon complete capture of the carriers) the transverse magnetoresistance vanishes. In a real experiment, the fraction of the captured electrons depends on the intensity of the wave and can be determined from measurements of the transverse magnetoresistance. The number of captured electrons can be reduced also because of the heating of the wave. The quantitative consideration of these effects goes beyond the framework of this paper. We only note that in the situation in which the USW plays the role of the external wave, the transfer of energy to the captured carriers is greatly reduced, since the drift velocity in the direction of the wave is identical with the sound velocity (or is generally equal to zero in the standing wave regime).

[1)] Thus, $\hbar\Omega\ll V_0$, i.e., the number of levels in the well is large.

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