Effective anharmonicity of the elastic subsystem of antiferromagnets

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The influence of antiferromagnetic exchange interaction on magnetoelastic coupling, as manifested in nonlinear acoustic phenomena, is investigated in the example of antiferromagnets with anisotropy of the "easy plane" type (AFEP). It is shown that under the simplest experimental conditions, the effective third-order elastic moduli $\hat{C}^{(3)}_{eff}$ are proportional to the square of the intersublattice exchange field and in real AFEP (for example, α-Fe_2O_3) may exceed by two orders of magnitude the usual values for solids. The values of the components of the tensor $\hat{C}^{(3)}_{eff}$ and the relations between them may vary over a wide range with change of the intensity and orientation of an external magnetic field, and also with change of constant mechanical stresses applied to the crystal. The possibility of observing various nonlinear acoustic effects is demonstrated with application to hematite.

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The elastic part of the potential-energy density of a solid, to terms of the third order in the components of the strain tensor $\hat{u}$, can be written in the form

$$F_e={}^1/_2\hat{C}^{(2)}\hat{u}\hat{u}+{}^1/_6\hat{C}^{(3)}\hat{u}\hat{u}\hat{u}.$$

The second-order elastic moduli ($\hat{C}^{(2)}\sim 10^{12}$ erg/cm^3) are responsible for linear acoustic effects, the third-order ($\hat{C}^{(3)}\sim 10^{13}$ to 10^{14} erg/cm^3) for nonlinear. With practically attainable strains, the values of $\hat{C}^{(3)}\hat{u}$ are so small in comparison with $\hat{C}^{(2)}$ that observation of the nonlinear effects in solids is at present difficult.[1]

In magnetic materials, the elastic subsystem interacts with the magnetic. Both the magnetic subsystems itself and the magnetoelastic (ME) coupling are by their very nature nonlinear and capable of introducing into the elastic subsystem an additional anharmonicity. In antiferromagnets (AF), observation of many manifestations of the ME coupling is appreciably facilitated by the fact that the antiferromagnetic exchange interaction weakens the coupling of the magnetic subsystem of the crystal with the external magnetic field[2]; this shows up especially clearly on comparison of AF with ferrites.[3] In particular, the ME coupling leads to the appearance of corrections $\Delta\hat{C}^{(2)}$ to the second-order elastic moduli. In AF with anisotropy of the "easy plane" type (AFEP), such as α-Fe_2O_3 ($T_M<T<T_N$) and $FeBO_3$, the experimental values of $\Delta C^{(2)}/C^{(2)}$ amount to tens of percent.[4,5]

It is natural to expect that the exchange interaction in AFEP will also promote the appearance of anharmonicity ($\Delta\hat{C}^{(3)}$) introduced by the magnetic subsystem into the elastic.[6]

In this paper, a calculation is made of the third-order effective elastic moduli of AFEP crystals, and an estimate is made of the possibility of observing a number of nonlinear accoustic effects. The ME coupling can be varied appreciably if the AFEP crystal is subjected both to an external magnetic field **H** and to compressive mechanical stresses P parallel to **H**.[7,8] Near a certain critical value $P_c(H)$, at which a spin reorientation (phase transition) occurs, the coupling is especially large: for certain acoustic waves, the coefficient of linear coupling with spin waves approaches its limiting value, close to unity.[8] Therefore the investigation of the nonlinear acoustic properties of AFEP is carried out with allowance for external static stresses.

The effective elastic moduli $\hat{C}^{(3)}_{eff}$ are determined by solution of a coupled system of Landau-Lifshitz equations and equations of elasticity, to terms of the second order in the amplitude of the alternating strains. In earlier work,[8,9] the linear ME coupling in AFEP was treated in the approximation of isotropy of the ME properties of the crystal in the basal plane. As measurements show, in actual AFEP of rhombohedral structure (for example, α-FE_2O_3) the ME anisotropy is large.[10,4] For this reason, some of the results presented here of the first approximation, relating to linear ME effects, contain appropriate changes as compared with those obtained in Refs. 8 and 9.

1. BASIC EQUATIONS

The energy density of an AFEP with two magnetic sublattices $\mathbf{M}_1$ and $\mathbf{M}_2$ can be represented in the form of a sum of the kinetic energy, due to the alternating elastic displacement **u**, the potential energy F, and the energy of interaction of the magnetic moments with the external magnetic field **H**:

$$w={}^1/_2\rho(\dot{\mathbf{u}})^2+F-2M_0(\mathbf{mH}).$$

The energy F consists of the magnetic energy F_m, the elastic F_e, and the magnetoelastic F_{me}: $F=F_m+F_e+F_{me}$. For crystals of rhombohedral structure (α-Fe_2O_3,

$FeBO_3$, $MnCO_3$; space group D_{3d}^6), each term can be expressed in the form[11]

$$F_m=2M_0\left[H_E m^2-H_D[\mathbf{ml}]_z+{}^1/{}_2 H_A l_z^2+{}^1/{}_2\alpha M_0\left(\frac{\partial \mathbf{l}}{\partial x}\right)^2\right], \tag{1}$$

$$\begin{gathered}F_e={}^1/{}_2\hat{C}^{(2)}\hat{u}\hat{u}={}^1/{}_2 C_{11}(u_{xx}^2+u_{yy}^2)+{}^1/{}_2 C_{33}u_{zz}^2\\ +C_{12}u_{xx}u_{yy}+C_{13}(u_{xx}+u_{yy})u_{zz}+(C_{11}-C_{12})u_{xy}^2\\ +2C_{44}(u_{xz}^2+u_{yz}^2)+2C_{14}[(u_{xx}-u_{yy})u_{yz}+2u_{xy}u_{xz}]-\hat{\tau}\hat{u},\end{gathered} \tag{2}$$

$$\begin{gathered}F_{me}=\mathbf{l}(\hat{B}\hat{u})\mathbf{l}=B_{11}(l_x^2u_{xx}+l_y^2u_{yy})+B_{12}(l_x^2u_{yy}+l_y^2u_{xx})\\ +2(B_{11}-B_{12})l_xl_yu_{xy}+B_{33}l_z^2u_{zz}+2B_{44}(l_yl_zu_{yz}\\ +l_xl_zu_{xz})+2B_{14}[(l_x^2-l_y^2)u_{yz}+2l_xl_yu_{xz}]+B_{41}[l_yl_z(u_{xx}-u_{yy})+2l_xl_zu_{xy}].\end{gathered} \tag{3}$$

Notation: ρ is the density of the crystal; $\mathbf{m}\equiv(\mathbf{M}_1+\mathbf{M}_2)/2M_0$; $\mathbf{l}\equiv(\mathbf{M}_1-\mathbf{M}_2)/2M_0$; $|\mathbf{M}_1|=|\mathbf{M}_2|\equiv M_0$, whence $m^2+l^2=1$ and $(\mathbf{ml})=0$; H_E is the exchange field, H_D the Dzyaloshinskiĭ field, α the constant of nonuniform exchange interaction; the z axis is directed along the "hard" axis of the crystal (C_3), characterized by anisotropy field H_A; the x axis is directed along one of the second-order axes (U_2); $\hat{\tau}$ is the tensor of external mechanical stresses. For $H\ll 2H_E$, it is sufficient to take account in F_{me} only of the antiferromagnetic ME interaction (3), with the tensor $\hat{B}$ of ME constants. In the expression (2), the purely elastic anharmonicity is neglected because, as will be shown, it is small in comparison with the anharmonicity introduced into the elastic subsystem by the magnetic. With the same accuracy, it is possible to neglect the nonlinear dependence of $\hat{u}$ on the derivatives of the displacement vector.

Magnetoacoustic phenomena are described by a coupled system of Landau-Lifshitz equations and equations of elasticity[12]:

$$\dot{\mathbf{M}}^j=-\gamma[\mathbf{M}^j\times\mathbf{H}^j], \quad \rho\ddot{u}_i=\partial t_{ik}/\partial x_k, \tag{4}$$

where γ is the magnetomechanical ratio, $\mathbf{H}^j=-\delta W/\delta\mathbf{M}_j$ is the effective field acting on the j-th sublattice ($W=\int w dV$), and $\hat{t}$ is the stress tensor given by

$$\hat{t}=\frac{\partial F}{\partial\hat{u}}=\hat{C}^{(2)}\hat{u}+\hat{t}_{me}-\hat{\tau}, \quad \hat{t}_{me}=\frac{\partial F_{me}}{\partial\hat{u}}=\mathbf{l}\hat{B}\mathbf{l}. \tag{5}$$

The relation (5) describes the effect of the magnetic subsystem on the elastic. The inverse effect is described by the ME component of the effective fields,

$$\mathbf{H}_{me}=-\frac{1}{2M_0}\frac{\partial F_{me}}{\partial\mathbf{l}}=-\frac{1}{M_0}(\hat{B}\hat{u})\mathbf{l}.$$

As is well known, the spectrum of magnetic oscillations of an AFEP for $\mathbf{H}\perp\mathbf{z}$ consists of two branches: a quasiferromagnetic (QF) ω_{fk} and a quasiantiferromagnetic (QAF) ω_{ak}. For H, H_D, $H_A\ll H_E$, the frequencies of the branches (without allowance for ME interaction) are

$$\begin{gathered}(\tilde{\omega}_{fk}/\gamma)^2=H(H+H_D)+2H_EM_0\alpha k^2,\\ (\tilde{\omega}_{ak}/\gamma)^2=2H_EM_A+H_D(H+H_D)+2H_EM_0\alpha k^2.\end{gathered}$$

We shall hereafter be interested in oscillations with a not too high frequency $\omega\ll\omega_{ak}$; we shall furthermore assume $H_{me}\ll H_A$. These conditions allow us to neglect departure of the antiferromagnetic vector from the basal plane of the crystal ($l_z\approx 0$) and change of its modulus ($l^2\approx 1$). Then from equation (4) follows the relation

$$\gamma^{-2}[\mathbf{l}\ddot{\mathbf{l}}]_z=(\mathbf{Hl})\{[\mathbf{H}\times\mathbf{l}]_z+H_D\}+2H_EM_0\alpha[\mathbf{l}\times\nabla^2\mathbf{l}]_z+2H_E[\mathbf{l}\times\mathbf{H}_{me}]_z. \tag{6}$$

The presence of the factor $2H_E$ in the last term of this equation reflects the influence of exchange interaction on the manifestations of the ME coupling.

By expressing the quantities $\hat{u}$ and $\mathbf{l}$ in the form of a sum of constant (equilibrium) and variable components, $\hat{u}=\hat{u}_0+\hat{u}(\mathbf{r},t)$ and $\mathbf{l}=\mathbf{l}_0+\lambda(\mathbf{r},t)$, and linearizing the system of equations (4) with respect to $\hat{u}(\mathbf{r},t)$ and $\lambda(\mathbf{r},t)$, one can easily obtain all the known results relating to the linear manifestations of the ME coupling. In the calculation of the magnetic corrections to the third-order elastic moduli, the nonlinear equation (6) is solved to terms of the second order in $u(\mathbf{r},t)$.

2. EQUILIBRIUM STATE

The equilibrium state of an AFEP is described by the following system of equations:

$$\hat{t}=\hat{C}^{(2)}\hat{u}_0+\mathbf{l}_0\hat{B}\mathbf{l}_0-\hat{\tau}=0, \tag{7}$$

$$(\mathbf{Hl}_0)\{[\mathbf{H}\times\mathbf{l}_0]_z+H_D\}+\frac{2H_E}{M_0}\mathbf{l}_0(\hat{B}'\hat{u}_0)\mathbf{l}_0=0, \tag{8}$$

where the tensor $\hat{B}'$ is determined by the relation

$$[\mathbf{l}\times\mathbf{H}_{me}]_z=\frac{1}{M_0}\mathbf{l}(\hat{B}'u)\mathbf{l}. \tag{8a}$$

The second term in (7) is responsible for the spontaneous magnetostriction of the AFEP.[13–15] We shall hereafter assume that the tensor $\hat{\tau}$ has a single component $\tau_{\xi\xi}=-P$, where $\boldsymbol{\xi}\equiv\mathbf{H}/H$. This choice of the external stresses corresponds, for $P>0$, to compression of the crystal in the direction of a magnetic field oriented arbitrarily in the basal plane of the crystal (see Fig. 1). Substitution of the solution of equation (7) in equation (8) reduces the latter to the form

$$(\boldsymbol{\xi}\mathbf{l}_0)\{(H^2-2H_EH_p)[\boldsymbol{\xi}\times\mathbf{l}_0]_z+HH_D\}=0, \tag{9}$$

where the effective field H_p of the external pressure is

$$H_p=\frac{P}{M_0}\frac{[(B_{11}-B_{12})C_{44}-2B_{14}C_{14}]}{[C_{44}(C_{11}-C_{12})-2C_{14}^2]}. \tag{9a}$$

When $H_p<H_{pc}=H(H+H_D)/2H_E$, the antiferromagnetic vector is perpendicular to $\mathbf{H}$: that is, $(\boldsymbol{\xi}\cdot\mathbf{l})=0$. When $H_p>H_{pc}$, spin reorientation occurs,[7,9] and the vector $\mathbf{l}_0$ is determined by the second solution of equation (9):

$$[\boldsymbol{\xi}\mathbf{l}_0]_z=\frac{HH_D}{2H_EH_p-H^2}=\cos\varphi,$$

where φ is the angle between the directions of the field

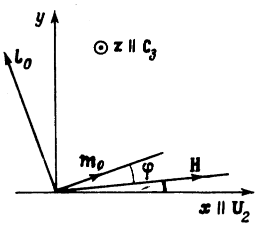

FIG. 1. Coordinate axes and equilibrium orientations of ferro- and antiferromagnetic moments ($\mathbf{m}_0\equiv\mathbf{m}_0|\mathbf{m}_0|\mu_0$; $\mathbf{H}\equiv|\mathbf{H}|\boldsymbol{\xi}$).

H and of the ferromagnetic moment $\mathbf{m}_0$ (see Fig. 1). We shall hereafter define the direction of $\mathbf{m}_0$ by the unit vector μ_0 (when $H_p < H_{pc}$, $\mu_0 = \xi$).

3. DYNAMIC SECOND-ORDER ELASTIC MODULI

We shall treat dynamic effects principally with application to the crystal hematite (α-Fe_2O_3), an antiferromagnet ($H_E \sim 10^7$ Oe) with anisotropy of the "easy plane" type at room temperature, for which the magnetic, elastic, and magnetoelastic parameters are known[4,10,16] (see Appendix A). The velocity of spin waves in hematite is higher than the velocity of sound; therefore the interaction of normal volume sound waves $\tilde{\omega}_{s\mathbf{k}}$ with spin waves $\tilde{\omega}_{f\mathbf{k}}$ is greatest in the range of small wave vectors $\mathbf{k}$, where under the usual experimental conditions $\tilde{\omega}_{s\mathbf{k}} \ll \tilde{\omega}_{f\mathbf{k}} \approx \tilde{\omega}_{f0}$. This gives us a basis, in the analysis of the effect of the magnetic subsystem on the elastic, for restricting ourselves to the quasistatic approximation ($\omega \ll \tilde{\omega}_{f\mathbf{k}}$), which corresponds to neglect of the left side of equation (6).

Dispersion of spin waves plays an important role only in the immediate neighborhood of critical pressures ($H_p \approx H_{pc}$). Without allowance for this dispersion, the solution of equation (6) linearized with respect to the variables $\hat{u}(\mathbf{r}, t)$ and $\boldsymbol{\lambda}(\mathbf{r}, t)$ has the form

$$\lambda^{(1)} = -\mu_0 \left(\frac{2H_E}{M_0}\right) \frac{1}{(\omega_{f0}/\gamma)^2} [\mathbf{l}_0 (\hat{B}'\hat{u}) \times \mathbf{l}_0], \tag{10}$$

where for $H_p < H_{pc}$

$$(\omega_{f0}/\gamma)^2 = 2H_E H_{ms} + H(H+H_D) - 2H_E H_p, \tag{10a}$$

whereas for $H_p > H_{pc}$

$$(\omega_{f0}/\gamma)^2 = 2H_E H_{ms} + \frac{(2H_E H_p - H^2)^2 - H^2 H_D^2}{2H_E H_p - H^2}. \tag{10b}$$

The term $2H_E H_m$ is the magnetostrictive gap in the AFMR spectrum.[13,14] The effective field H_m of the spontaneous magnetostriction is[4,14]

$$H_{ms} = \frac{1}{M_0} \frac{C_{44}(B_{11}-B_{12})^2 - 4C_{14}(B_{11}-B_{12})B_{14} + 2B_{14}^2(C_{11}-C_{12})}{[C_{44}(C_{11}-C_{12}) - 2C_{14}^2]}. \tag{10c}$$

The relations (10a), (10b), and (9a) with $B_{14} = C_{14} = 0$ agree with those obtained in Ref. 8. The tensor of ME stresses in equations (5), to terms linear in the deflection of the vector $\mathbf{l}$, is

$$\hat{t}_{me}^{L} = 2\mathbf{l}_0 \hat{B} \lambda^{(1)}.$$

With use of the relation (10) and of the equality $\mathbf{l}_0 \cdot \hat{B}\mu_0 = \mathbf{l}_0 \cdot \hat{B} \cdot \mathbf{l}_0$, we have

$$\hat{t}_{me}^{L} = -\left(\frac{4H_E}{M_0}\right) \frac{1}{(\omega_{f0}/\gamma)^2} [\mathbf{l}_0 \times (\hat{B}'\hat{u})\mathbf{l}_0]^2, \tag{11}$$

where

$$[\mathbf{l}_0(\hat{B}'\hat{u})\mathbf{l}_0] = [(B_{11}-B_{12})(u_{xx}-u_{yy}) + 4B_{14}u_{yz}] l_{0x} l_{0y} - [(B_{11}-B_{12})u_{xy} + 2B_{14}u_{xz}](l_{0x}^2 - l_{0y}^2). \tag{11a}$$

The relations (11) and (5) allow us to interpret the effect of the linear ME coupling on the elastic subsystem as a change of the second-order dynamic moduli of elasticity, whose effective values are[1)]

$$\hat{C}^{(2)}_{\text{eff}} = \hat{C}^{(2)} - \Delta\hat{C}^{(2)}; \quad \Delta\hat{C}^{(2)} = \left(\frac{4H_E}{M_0}\right) \frac{1}{(\omega_{f0}/\gamma)^2} (\mathbf{l}_0 \hat{B}' \mathbf{l}_0)^2. \tag{12}$$

Dispersion of spin waves, which is important when $H_p \approx H_{pc}$, leads to a nonlocal coupling of the stresses with the strains; this is taken into account in the characteristic equation given in Appendix B, which determines the dispersion law $\omega_{s\mathbf{k}}$ and the polarization $\mathbf{e}_{s\mathbf{k}}$ of normal sound waves.[17]

It was shown in Ref. 8 that in uniaxial AFEP (that is, ones elastically and magnetically isotropic in the basal plane), for $\mathbf{H} \parallel \mathbf{x}$ and $H_p \to H_{pc}$ the velocity of transverse sound waves with $\mathbf{k} \parallel \mathbf{x}$ and $\mathbf{e} \parallel \mathbf{y}$ approaches zero as $|\mathbf{k}| \to 0$.[2)] By use of equations (B1) and B2), one can easily show that in AFEP of rhombohedral structure, this property is possessed by a wave with $\mathbf{e} \parallel \mathbf{x}$ and with $\mathbf{k}$ oriented in the (z, y) plane of the crystal at angle θ to the y axis, where

$$\operatorname{tg}\theta = -\left[C_{14} - \frac{B_{14}(B_{11}-B_{12})}{M_0 H_{ms}}\right] \Big/ \left(C_{44} - \frac{2B_{14}^2}{M_0 H_{ms}}\right). \tag{13}$$

For the crystal α-Fe_2O_3, it follows from (13) and the experimental data (see Appendix A) that $\theta \approx 52°$.

In closing this section, we note that the symmetry properties of the tensor $\hat{C}^{(2)}_{\text{eff}}$ differ from the symmetry properties of the tensor $\hat{C}^{(2)}$. By means of the relations (B1) and (B2) one can show, in particular, that the ME interaction lifts the degeneracy of the frequencies of transverse sound waves with $\mathbf{k} \parallel \mathbf{z}$. The waves with polarization $\mathbf{e} \parallel \mathbf{y}$ have no linear interaction with the magnetic subsystem and are propagated with phase velocity $[\rho^{-1} C_{44}]^{1/2}$. The waves with $\mathbf{e} \parallel \mathbf{x}$ have phase velocity $[\rho^{-1}(C_{44} - \Delta C_2)]^{1/2}$, where ΔC_2 is given in (B. 3). In an α-Fe_2O_3 crystal located in a field $H \approx 1$ kOe, with $H_p = 0$, the relative correction to the elastic modulus is $\Delta C_2 / C_{44} \approx 25\%$.[4,5]

4. THIRD-ORDER DYNAMIC MODULI OF ELASTICITY

We shall first consider relatively longwave oscillations with wave vectors satisfying the condition $2H_E M_0 \alpha k^2 \ll (\omega_{f0}/\gamma)^2$. In this case the effect of dispersion of the magnetic subsystem on the nonlinear acoustic properties of the AFEP can be neglected. On representing the solution of equation (6), to terms of the second order of smallness with respect to $\hat{u}(\mathbf{r}, t)$, in the form $\mathbf{l} = \mathbf{l}_0 + \boldsymbol{\lambda}^{(1)}(\mathbf{r}, t) + \boldsymbol{\lambda}^{(2)}(\mathbf{r}, t)$, we get

$$\lambda^{(2)} = \mu_0 \left(\frac{2H_E}{M_0}\right)^2 \frac{1}{(\omega_{f0}/\gamma)^4} \Big\{ 2[\mathbf{l}_0(\hat{B}'\hat{u}) \times \mu_0] [\mathbf{l}_0(\hat{B}'\hat{u}) \times \mathbf{l}_0] - \frac{1}{2} N(H, H_p) [\mathbf{l}_0(\hat{B}'\hat{u}) \times \mathbf{l}_0]^2 \Big\} - \mathbf{l}_0 \left(\frac{2H_E}{M_0}\right)^2 \frac{1}{(\omega_{f0}/\gamma)^4} \frac{1}{2} [\mathbf{l}_0(\hat{B}'\hat{u}) \times \mathbf{l}_0], \tag{14}$$

where for $H_p > H_{pc}$

$$N(H, H_p) = 3 \frac{H H_D [(2H_E H_p - H^2)^2 - H^2 H_D^2]^{1/2}}{(\omega_{f0}/\gamma)^2 (2H_E H_p - H^2)}, \tag{14a}$$

whereas for $H_p < H_{pc}$

$$N(H, H_p) = 0; \tag{14b}$$

$$[\mathbf{l}_0(\hat{B}'\hat{u})\boldsymbol{\mu}_0] = [(B_{11}-B_{12})(u_{xx}-u_{yy})+4B_{14}u_{yz}]^{1}/_{2}(l_{0x}\mu_{0y}+l_{0y}\mu_{0x}) - [(B_{11}-B_{12})u_{xy}+2B_{14}u_{xz}](l_{0x}\mu_{0x}-l_{0y}\mu_{0y}). \tag{14c}$$

The value of $\boldsymbol{\lambda}^{(1)}$ is determined by the equality (10).

The nonlinear part $\hat{t}_{me}^{NL}$ of the tensor of magnetoelastic stresses can be represented in the form

$$\hat{t}_{me}{}^{NL} = 2(\mathbf{l}_0\hat{B}\boldsymbol{\lambda}^{(2)}) + (\boldsymbol{\lambda}^{(1)}\hat{B}\boldsymbol{\lambda}^{(1)}) \equiv {}^{1}/_{2}\Delta\hat{C}^{(3)}\hat{u}\hat{u}. \tag{15}$$

With the aid of relations (14), (10), and (15) and of the equality

$$\boldsymbol{\mu}_0(\hat{B}\hat{u})\boldsymbol{\mu}_0 - \mathbf{l}_0(\hat{B}\hat{u})\mathbf{l}_0 = 2\mathbf{l}_0(\hat{B}'\hat{u})\boldsymbol{\mu}_0,$$

we find the third-order effective dynamic elastic moduli

$$\Delta\hat{C}^{(3)}_{3\Phi\Phi} = \left(\frac{4H_E}{\mu_0}\right)^2 \frac{1}{(\omega_{f0}/\gamma)^4}[3(\mathbf{l}_0\hat{B}'\boldsymbol{\mu}_0)(\mathbf{l}_0\hat{B}'\mathbf{l}_0)^2 - N(\mathbf{l}_0\hat{B}'\mathbf{l}_0)^3]. \tag{16}$$

For hematite ($B \sim 10^7$ erg/cm^3, $H_E \sim 10^7$ Oe, $M_0 \sim 10^3$ G) with $H_p = 0$ and $H \sim 1$ kOe, we have $(\omega_{f0}/\gamma)^2 \sim 30$ kOe^2, whence $\Delta\hat{C}^{(3)} \sim 10^{15}$ to 10^{16} erg/cm^3; that is, much larger than the ordinary third-order elastic moduli ($\hat{C}^{(3)}$).[1] Because of the factor $(\omega_{f0}/\gamma)^4$ in the denominator of $\Delta\hat{C}^{(3)}$ and because of the function $N(H, H_p)$ the effective moduli $\hat{C}^{(3)}_{eff} = \hat{C}^{(3)} \approx \Delta\hat{C}^{(3)} \approx \Delta\hat{C}^{(3)}$ are sensitive to the intensity of the external magnetic field H and to the external pressures and become especially large in the vicinity of the spin reorientation[3)] $H_p \to H_{pc}$.

We note one more peculiarity of the acoustic nonlinearity of AFEP. The components of the tensor $\hat{B}$ are determined by the properties of the crystal and are related to the crystallographic axes. But the components of the vectors $\mathbf{l}_0$ and μ_0 are determined by the orientation of the magnetic field in the basal plane (and, in general, by the value of the external pressure). This makes it possible by change of the direction of **H** to change the relation between the components of the tensor $\hat{C}^{(3)}_{eff}$; that is, to control the interaction between sound waves.

In the high-frequency (hypersonic) range, especially when $H_p \approx H_{pc}$, it is necessary to allow for dispersion of spin waves. Here, as in the linear case, the relation of the tensor $\hat{t}_{me}^{NL}$ to the strains becomes nonlocal; this, however, is unimportant in the problems considered below.

5. INTERACTION OF SOUND WAVES

We shall treat the interaction of sound waves, caused by the effective elastic anharmonicity, by the method of slowly varying amplitudes.[18] We represent the elastic displacement vector in the form of an expansion in normal quasiacoustic waves:

$$u(\mathbf{r}, t) = \frac{1}{2(2\pi)^3}\sum_s \int d\mathbf{k}\, \mathbf{e}_{s\mathbf{k}} u_{s\mathbf{k}}(\mu\mathbf{r}, \mu t)\exp(-i\omega_{s\mathbf{k}}t + i\mathbf{k}\mathbf{r}) + \text{c.c.},$$

where μ is a smallness parameter, characterizing the slowness of variation of the amplitudes $u_{s\mathbf{k}}$ of the normal waves. The frequencies $\omega_{s\mathbf{k}}$ and the polarizations $\mathbf{e}_{s\mathbf{k}}$ are determined with allowance for the linear ME coupling (12); in particular, for $\mathbf{H} \parallel \mathbf{x}$ and $H_p \lesssim H_{pc}$, by the relations (B1) and (B2). In the first approximation with respect to the parameter μ, one can obtain from equations (4), by use of the expressions (12) and (15), the following system of equations for the slowly varying amplitudes of the normal quasiacoustic waves:

$$\left[\frac{\partial}{\partial t} + (\mathbf{V}_{s\mathbf{k}}\nabla) + \eta_{s\mathbf{k}}\right]u_{s\mathbf{k}} + \frac{1}{(2\pi)^3}\sum_{s's''}\int d\mathbf{k}'\, \Phi_{ss's''}(\mathbf{k}, \mathbf{k}', \mathbf{k}'-\mathbf{k})u_{s'\mathbf{k}'}\begin{Bmatrix} u_{s''(\mathbf{k}-\mathbf{k}')} \\ \overset{*}{u}_{s''(\mathbf{k}'-\mathbf{k})} \end{Bmatrix} = 0, \tag{17}$$

where the upper multiplier corresponds to processes of fusion of waves, with conservation-of-energy law $\omega_{s'\mathbf{k}'} + \omega_{s''(\mathbf{k}'-\mathbf{k})} = \omega_{s\mathbf{k}}$, and the lower to processes of splitting, with conservation-of-energy law $\omega_{s'\mathbf{k}'} = \omega_{s''(\mathbf{k}'-\mathbf{k})} + \omega_{s\mathbf{k}}$; $\mathbf{V}_{s\mathbf{k}}$ and $\eta_{s\mathbf{k}}$ are, respectively, the group velocities and the relaxation frequencies of the normal waves; $\Phi_{ss's''}$ is the interaction amplitude, equal to

$$\Phi_{ss's''}(\mathbf{k}, \mathbf{k}', \mathbf{k}'-\mathbf{k}) = \frac{1}{4\rho\omega_{s\mathbf{k}}}\hat{C}^{(3)}_{\text{eff}}\hat{a}_{s\mathbf{k}}\hat{a}_{s'\mathbf{k}'}\hat{a}_{s''(\mathbf{k}-\mathbf{k}')}, \qquad \hat{a}_{s\mathbf{k}} \equiv {}^{1}/_{2}[e_{si}(\mathbf{k})k_j + e_{sj}(\mathbf{k})k_i]. \tag{18}$$

With the aid of equation (17) we shall estimate the possibility of observing several nonlinear effects, with application to a hematite crystal.

Parametric generation of sound by sound. Assuming $\mathbf{H} \parallel \mathbf{x}$ and $H_p \lesssim H_{pc}$, we shall consider the interaction of transverse sound waves with wave vectors $\mathbf{k}_0$, $\mathbf{k}_1$, and $\mathbf{k}_2$ parallel to the z axis. In Sec. 3 we mentioned that without allowance for magnetoelastic interaction, the spectrum of these waves is degenerate. The ME interaction lifts the degeneracy in such a way that the velocity of a wave with polarization $\mathbf{e} \parallel \mathbf{y}$ continues to be described by the elastic modulus C_{44}, whereas the velocity of a wave with $\mathbf{e} \parallel \mathbf{x}$ is determined by the effective modulus $C_{44}^{eff} = C_{44}(1-\varkappa)$, where

$$\varkappa \equiv \frac{\Delta C_2}{C_{44}} = \frac{H_E}{M_0}\frac{1}{C_{44}}\left(\frac{2B_{14}}{\omega_{f0}/\gamma}\right)^2. \tag{19}$$

(The effect of dispersion of the magnetic subsystem on the velocity of a wave with $\mathbf{e} \parallel \mathbf{x}$ in this case can be neglected over the whole H_p range, all the way to $H_p \to H_{pc}$, because of the considerable difference of the direction of propagation from that prescribed by equation (13).)

If the pumping wave, with amplitude u_0 and frequency ω_0, has polarization $\mathbf{e}_0 \parallel \mathbf{y}$ and $\mathbf{k}_0 \parallel \mathbf{z}$, then there is a possibility of a synchronous process of interaction of it with waves u_1 and u_2, whose frequencies ω_1 and ω_2 satisfy the condition $\omega_1 + \omega_2 = \omega_0$, if the wave vectors $\mathbf{k}_1$ and $\mathbf{k}_2$ are antiparallel ($\mathbf{k}_1 \parallel -\mathbf{k}_2 \parallel \mathbf{k}_0$), $\mathbf{k}_1 + \mathbf{k}_2 = \mathbf{k}_0$, and the polarizations $\mathbf{e}_{1,2} \parallel \mathbf{x}$ (see Fig. 2).

It is to be expected that for the pumping wave considered, there will be no linear coupling with the magnetic subsystem; nevertheless it will participate effectively in the nonlinear ME interaction of the waves.

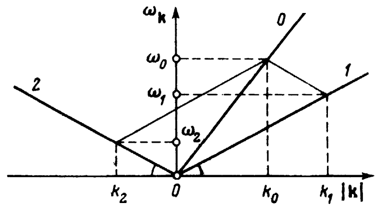

FIG. 2. Condition for parametric generation of waves with frequencies ω_1 and ω_2 by a pumping wave with frequency ω_0. Straight line 0: $\mathbf{k} \parallel \mathbf{z}$; $\mathbf{e} \parallel \mathbf{y}$; $\omega(\mathbf{k}) = (C_{44}/\rho)^{1/2} |\mathbf{k}|$. Straight lines 1 and 2: $\mathbf{k} \parallel \mathbf{z}$; $\mathbf{e} \parallel \mathbf{x}$; $\omega(\mathbf{k}) = [(1-\varkappa)C_{44}/\rho]^{1/2} |\mathbf{k}|$.

On setting $u_{\mathbf{s}'\mathbf{k}'} = u_0(z)\delta(\mathbf{k}' - \mathbf{k})$ in equation (17) and calculating the amplitudes of interaction (18), we get in the prescribed pumping-field approximation a system of equations for the statjonary amplitudes u_1 and u_2:

$$\frac{\partial u_1}{\partial z} + R_1 u_1 + \Psi_2 u_0 e^{-R_0 z} u_2^* = 0,$$
$$\frac{\partial u_2^*}{\partial z} - R_2 u_2 + \Psi_1 u_0^* e^{-R_0 z} u_1 = 0, \tag{20}$$

where

$$\Psi_n = (-1)^{n-1} \left(\frac{2H_E}{M_0}\right)^2 \frac{B_{14}^3}{(\omega_{f0}/\gamma)^4} \frac{k_0 k_n}{C_{44}^{\text{eff}}}. \tag{20a}$$

$R_n = \eta_n / V_n \equiv Q_n^{-1} k_n / 2$ is the attenuation coefficient, Q_n is the quality factor of the wave with wave vector k_n, $u_0 \equiv u_0(z=0)$, and $n = 1, 2$. We shall solve equations (20) with the boundary conditions $u_1|_{z=0} = u_1(0)$, $u_2|_{z=L} = 0$. If we neglect the attenuation of sound waves, the solution has the form (L is the length of the specimen along z)

$$u_1 = u_1(0) \frac{\cos \Gamma(L-z)}{\cos \Gamma L}, \qquad u_2 = u_1^*(0) \frac{\Gamma}{\Psi_2 u_0^*} \frac{\sin \Gamma(z-L)}{\cos \Gamma L}, \tag{21}$$

where $\tau = |u_0| \, |\Psi_1 \Psi_2|^{1/2}$ is the growth increment. The condition for parametric generation, $2\Gamma L = \pi$, [19] determines the value of the critical deformation ε_{cr} in the pumping wave,

$$\varepsilon_{cr} \equiv \frac{1}{2} k_0 |u_0|_{cr} = \pi \frac{2|B_{14}|}{C_{44}} \frac{1-\varkappa}{(k_0 L) \varkappa^{5/2}}. \tag{22}$$

With $\omega_0/2\pi \sim 10^8$ Hz, $L \sim 1$ cm, $H \sim 1$ kOe, and $H_p = 0$, we get for α-Fe_2O_3 the estimate $\varepsilon_{cr} \sim 2 \cdot 10^{-6}$. Such ultrasonic deformations are experimentally quite attainable. In Appendix C it is shown that allowance for a finite quality factor $Q \gtrsim 10^3$ of the sound waves does not change the order of magnitude of ε_{cr} as estimated by formula (22).

Doubling of the sound frequency. Analysis of the interaction amplitudes (18), with use of the expression (16), shows that a synchronous process of frequency doubling is possible, for example, for a normal wave with polarization $\mathbf{e} \parallel \mathbf{z}$ and with projections of the wave vector determined by the relations (see (B.1))

$$\frac{k_x^2}{k_y^2} = \frac{C_{44}\Delta C_2 + (C_{13}+C_{44})(3C_{14}+\Delta C_3/2)}{(C_{13}+C_{44}+\Delta C_2)(C_{14}+\Delta C_3/2)}$$
$$\frac{k_z}{k_y} = -\frac{2C_{14}+\Delta C_3/2}{C_{13}+C_{44}+\Delta C_2}. \tag{23}$$

The frequency of the wave under consideration satisfies the equation

$$\rho \omega_k^2 = C_{33} k_z^2 + (C_{44} - \Delta C_2) k_x^2 + C_{44} k_y^2. \tag{24}$$

A pumping wave with amplitude u_0 and frequency ω_0 can generate a second-harmonic wave ($\omega_2 = 2\omega_0$) whose amplitude u_2 is described, in the approximation of prescribed pumping field, by the equation

$$\frac{\partial u_2}{\partial \zeta} + R_2 u_2 - \Psi u_0^2 e^{-2R_0\zeta} = 0, \tag{25}$$

where ζ is the coordinate measured in the direction of $\mathbf{k}$ (conditions are assumed to be uniform in directions perpendicular to $\mathbf{k}$); R_0 and R_2 are the attenuation coefficients of the pumping and second-harmonic waves; $u_0 \equiv u_0(\zeta = 0) = 0$; and

$$\Psi = \frac{3}{2} \left(\frac{2H_E}{M_0}\right)^2 \frac{B_{14}^3}{\rho \omega_{k_0}^2 (\omega_{f0}/\gamma)^4} k_0 k_{0x}^2 k_{0y}^2. \tag{25a}$$

From the solution of equation (25) with the boundary condition $u_2(\zeta = 0) = 0$, it follows it follows that the spatial distribution of the amplitude of the second harmonic has a form similar to that given in Ref. 1:

$$u_2 = u_0^2 \frac{\Psi}{R_2 - 2R_0} (e^{-2R_0\zeta} - e^{-R_2\zeta}). \tag{26}$$

The maximum value $u_{2\max}$ is reached at a distance $\zeta_0 = (R_2 - 2R_0)^{-1} \ln(R_2/2R_0)$. If we suppose, following Ref. 20, that the quality factor of the wave $Q \sim \omega^{-1}$, then $R_2 = 4R_0$ and $\zeta_0 = (2R_0)^{-1} \ln 2$. Then the maximum second-harmonic output is determined by the relation

$$\left|\frac{u_{2\max}}{u_0}\right| = 3\varepsilon_0 \left(\frac{H_E}{M_0}\right)^2 \frac{|B_{14}|^3 Q_0}{\rho v^2 (\omega_{f0}/\gamma)^4} \left|\frac{k_{0x}^2 k_{0y}}{k_0^3}\right| \tag{27}$$

where $\varepsilon_0 = k_0 |u_0| / 2$ is the deformation amplitude in the pumping wave at the input to the crystal ($\zeta = 0$); $v = \omega/k$ is the phase velocity of the waves. On taking $H \sim 1$ kOe, $\omega_0/2\pi \sim 10^8$ Hz, and $Q_0 \sim 10^3$ and using the data on the magnetoelastic parameters of hematite given in Appendix A, we get the following estimates:

$$k_x^2/k_y^2 \approx 2.5; \quad k_z/k_y \approx 0.5; \quad \zeta_0 \sim 0.5 \text{ cm}; \quad |u_{2\max}/u_0| \sim 3 \cdot 10^5 \varepsilon_0.$$

Thus at strain amplitudes $\varepsilon_0 \sim 10^{-6}$, the second-harmonic output in hematite may already amount to tens of percent, whereas at the same amplitudes the second-harmonic output in, for example, a magnesium-aluminum alloy amounts to fractions of a percent. [1]

Acoustic detection. If there is incident on the crystal a sound wave with carrier frequency ω, amplitude-modulated at frequency Ω, then on the basis of the acoustic nonlinearity of the crystal, detection may occur, with selection of a signal u_Ω at the modulation frequency Ω. This phenomenon has been treated earlier[21] on the assumption that the signal at the modulation frequency occurs under conditions of volume resonance with the specimen. The large value of the effective elastic nonlinearity in AFEP allows us to count on obtaining an appreciable detection effect with traveling waves.

A quantitative estimate of the efficiency of acoustic detection in hematite, for example for a wave with polarization $\mathbf{e} \| \mathbf{z}$, propagated in the direction ζ, and having at the crystal boundary a displacement $u(\zeta=0)=u_0(1+a\cos\Omega t)\cos\omega t$, with $\omega/2\pi \sim 10^8$ Hz, $Q_\omega \sim 10^3$, $H \sim 1$ kOe, and $L \sim 1$ cm, is $|u_\Omega/u_0| \sim 2\cdot 10^5\, \varepsilon_0 a$.

Thus in AFEP, all the nonlinear effects considered can be observed on traveling sound waves with actually attainable strains $\varepsilon_0 \sim 10^{-6}$; this is difficult or impossible in ordinary solids.

APPENDIX A:

MAGNETOELASTIC PARAMETERS OF HERMATITE (α-F_2O_3)

Elastic moduli C, 10^{11} erg/cm^3 [16]:

$$C_{11}=24.2,\quad C_{33}=22.6,\quad C_{12}=5.5,$$

$$C_{44}=8.5,\quad C_{13}=1.6,\quad C_{14}=-1.3.$$

Density $\rho=5.29\ g/\mathrm{cm}^3$. [4]

Magnetoelastic interaction constants B, 10^6 erg/cm^3 [10]:

$$B_{11}-B_{12}=8\pm 2,\quad 2B_{14}=27\pm 3,$$

$$B_{11}+B_{12}=-(32\pm 4),$$

$$2B_{44}=\mp(53\pm 24),\quad B_{33}=-(3.9\pm 0.9),$$

$$B_{41}=\mp(47\pm 13).$$

Magnetic parameters:

$$M_0=870\ G^{4)},\quad H_E=9.2\cdot 10^6\ \mathrm{Oe},$$

$$H_D=2.2\cdot 10^4\ \mathrm{Oe}.$$

APPENDIX B:

CHARACTERISTIC EQUATION FOR NORMAL QUASIACOUSTIC WAVES IN A CRYSTAL OF RHOMBOHEDRAL STRUCTURE

The characteristic equation for normal waves, when $H_p \quad H_{pc}$ and $\mathbf{H} \| \mathbf{x}$ (or $\mathbf{H} \| \mathbf{y}$), has the form

$$\mathbf{e}_{sk}\hat{T}=0; \tag{B.1}$$

$$T_{11}=C_{11}k_x^2+[{}^1/_2(C_{11}-C_{12})-\Delta C_1]k_y^2+(C_{44}-\Delta C_2)k_z^2+(2C_{14}-\Delta C_3)k_yk_z-\rho\omega_{sk}^2,$$

$$T_{22}=[{}^1/_2(C_{11}-C_{12})-\Delta C_1]k_x^2+C_{11}k_y^2+C_{44}k_z^2-2C_{14}k_yk_z-\rho\omega_{sk}^2,$$

$$T_{33}=C_{33}k_z^2+C_{44}(k_x^2+k_y^2)-\Delta C_2k_x^2-\rho\omega_{sk}^2,$$

$$T_{12}=[{}^1/_2(C_{11}+C_{12})-\Delta C_1]k_xk_y+(2C_{14}-{}^1/_2\Delta C_3)k_xk_z,$$

$$T_{13}=[(C_{13}+C_{44})-\Delta C_2]k_xk_z+(2C_{14}-{}^1/_2\Delta C_3)k_xk_y,$$

$$T_{23}=(C_{13}+C_{44})k_yk_z+C_{14}(k_x^2-k_y^2)-{}^1/_2\Delta C_3k_x^2;$$

$$\Delta C_1=\frac{H_E}{M_0}\left(\frac{B_{11}-B_{12}}{\omega_{fk}/\gamma}\right)^2,\quad \Delta C_2=\frac{H_E}{M_0}\left(\frac{2B_{14}}{\omega_{fk}/\gamma}\right)^2,\quad \Delta C_3=\frac{H_E}{M_0}\frac{4B_{14}(B_{11}-B_{12})}{(\omega_{fk}/\gamma)^2}. \tag{B.2}$$

APPENDIX C:

EFFECT OF ATTENUATION OF THE CONDITION FOR PARAMETRIC GENERATION OF SOUND

If the attenuation coefficients R_n of the waves are nonzero, one can derive from the system of equations (20) the following equation:

$$\frac{\partial^2 u_1}{\partial z^2}+(R_0+R_1-R_2)\frac{\partial u_1}{\partial z}+[\Gamma^2e^{-2R_0z}+R_1(R_0-R_2)]u_1=0. \tag{C.1}$$

Its solution is represented in the form

$$u_1=\exp\left(-\frac{R_0+R_1-R_2}{2}z\right)\left[c_1H_\nu^{(1)}\left(\frac{\Gamma}{R_0}e^{-R_0z}\right)+c_2H_\nu^{(2)}\left(\frac{\Gamma}{R_0}e^{-R_0z}\right)\right] \tag{C.2}$$

where $H_\nu^{(1,2)}$ are the Hankel functions of order ν, $\nu=|R_0-R_1-R_2|/R_0$, and $c_{1,2}$ are constants of integration. By use of equation (C.2) we get the solution of the second of equations (20): when $R_0 \leqslant R_1+R_2$,

$$u_2^*=-\frac{\Gamma}{\Psi_2u_0}\exp\left(-\frac{R_0+R_1-R_2}{2}z\right)\left[c_1H_{1+\nu}^{(1)}\left(\frac{\Gamma}{R_0}e^{-R_0z}\right)+c_2H_{1+\nu}^{(2)}\left(\frac{\Gamma}{R_0}e^{-R_0z}\right)\right] \tag{C.3}$$

when $R_0 \geqslant R_1+R_2$,

$$u_2^*=-\frac{\Gamma}{\Psi_2u_0}\exp\left(-\frac{R_0+R_1-R_2}{2}z\right)\left[c_1e^{i\pi(1-\nu)}H_{1-\nu}^{(1)}\left(\frac{\Gamma}{R_0}e^{-R_0z}\right)+c_2e^{-i\pi(1-\nu)}H_{1-\nu}^{(2)}\left(\frac{\Gamma}{R_0}e^{-R_0z}\right)\right]. \tag{C.4}$$

The generation condition, given in the form $u_2|_{z=L}=0$, $u_1|_{z=0}=u_1(0)\to 0$, gives the following equations for determination of the critical value of the growth increment Γ (J_ν and N_ν are the Bessel and Neumann functions): when $R_0 \leqslant R_1+R_2$,

$$J_\nu(x)N_{1+\nu}(px)=J_{1+\nu}(px)N_\nu(x), \tag{C.5}$$

when $R_0 \geqslant R_1+R_2$,

$$\mathrm{tg}\,\nu\pi=\frac{J_\nu(x)N_{1-\nu}(px)-N_\nu(x)J_{1-\nu}(px)}{J_\nu(x)J_{1-\nu}(px)+N_\nu(x)N_{1-\nu}(px)}, \tag{C.6}$$

here $x=\Gamma/R_0$, $p=e^{-R_0L}$. If we use the facts that $\omega_2=[1-(1-\varkappa)^{1/2}]\omega_0/2$ and that for α-Fe_2O_3 at $H\sim 1$ kOe, $\varkappa\approx 0.25$, then $\omega_2\approx 6\cdot 10^{-2}\ \omega_0 \ll \omega_0$. Supposing that $Q^{-1}\sim\omega$, i.e., $R\sim\omega^2$, we may consider that $R_2 \ll R_0 \approx R_1$, i.e., $\nu\to 0$ (we neglect the dependence of the attenuation on the polarization).

Then from (C.5) or (C.6), the generation condition takes the form

$$\Gamma L=\frac{\pi}{2}\frac{R_0L}{1-e^{-R_0L}}\beta(R_0L), \tag{C.7}$$

where the function β varies over the range $1\leqslant\beta<1.6$ as its argument varies over the range $0\leqslant R_0L<\infty$. [22] It follows from the expression (C.7) that the critical value of the strain in the pumping wave is

$$\varepsilon_{cr}=\varepsilon_{cr}(R\to 0)\frac{R_0L}{1-e^{-R_0L}}\beta(R_0L), \tag{C.8}$$

where $\varepsilon_{cr}(R\to 0)$ is determined by the expression (22). For $\omega_0/2\pi\sim 10^8$ Hz, $L\sim 1$ cm, $H\sim 1$ kOe, $H_p=0$, and $Q_0\sim 10^3$, we get for α-Fe_2O_3 $\varepsilon_{cr}\approx 2\varepsilon_{cr}(R\to 0)$.

CONCLUSION

1. In antiferromagnets, in contrast to ferromagnets, the exchange interaction plays a part in the formation of a magnetoelastic coupling between the magnetic and elastic subsystems of the crystal. This leads to a sub-

stantial renormalization not only of the second-order, but also of the third-order elastic moduli.

2. For values of the field and of the external mechanical stresses not too close to those that cause an orientational phase transition from the "Zemman" phase to the angular, the magnetic contribution to specific components of the tensor of third-order elastic moduli $\Delta\hat{C}^{(3)}$ is proportional to the square of the exchange field (H_E) and inversely proportional to the fourth power of the AFMR frequency (ω_{f0}); therefore for high-temperature (with high T_N and therefore large H_E), weakly anisotropic (with small ω_{f0}) antiferromagnets of the type of α-Fe_2O_3, the effective moduli $\hat{C}^{(3)}_{eff}=\hat{C}^{(3)}+\Delta\hat{C}^{(3)}$ may attain unusually large values, exceeding by two orders of magnitude the value of the purely elastic anharmonic moduli $\hat{C}^{(3)}$.

3. The renormalizable components $\hat{C}^{(3)}_{eff}$ depend strongly on the values of a magnetic (H) and mechanical stresses (P) oriented in a definite manner; they increase to limiting values at $P\to P_c(H)$ (P_c is the critical uniaxial pressure corresponding to spin reorientation; $P_c(0)=0$).

4. In AF of the type of α-Fe_2O_3, the large values of $\hat{C}^{(3)}_{eff}$ that are realized under quite simple experimental conditions may appreciably facilitate the observation of such nonlinear ultrasonic and hypersonic dynamic effects as parametric excitation of sound by sound, doubling of the sound frequency, acoustic detection, and a number of others.

5. The significant magnetic renormalization of the third-order moduli and the possibility of resultant control through it by means of external fields and external stresses make practical the theoretical and experimental investigation of the relaxational parameters of quasi-acoustic magnetoelastic waves in high-temperature, weakly anisotropic antiferromagnetic monocrystals.

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[1)] An analysis of the expression for $\Delta C^{(2)}$ and, see below, for $\Delta C^{(3)}$ should be carried out, obviously, by use of the expressions (10a) and (10b) for $(\omega_{f0}/\gamma)^2$. In particular, the dependence of $\Delta C^{(2)}/C^{(2)}$ on H_E is more complicated than a linear dependence, which occurs only for the "Zeeman" phase (when $\mathbf{m}_0 \parallel \mathbf{H}$) and when $\Delta C^{(2)}/C^{(2)}\ll 1$.

[2)] We emphasize that not all transverse waves with $\mathbf{e}\parallel y$ are strongly coupled with spin waves. Thus for waves with $\mathbf{k}\parallel z$ and $\mathbf{e}\parallel y$ there is no linear ME coupling.

[3)] For $H_p<H_{pc}$, the condition $H_p\to H_{pc}$ is formally equivalent to the condition $H\to 0$ when $P=0$. Under these conditions, however, the possibility of an expansion of the vector $\mathbf{l}$ and of the strains $\hat{u}$ as series in small departures from the equilibrium values becomes problematical.

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