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Absorption of sound in helium II

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A theory is developed describing the propagation of high-frequency sound in helium II at low temperatures ($T < 0.6$ K) and high pressures ($P > 16$ atm), when the energy spectrum of the phonons becomes stable. The absorption coefficient and the sound dispersion are calculated under these conditions. The dependence of the velocity of second sound on the frequency is determined. The resonance properties of the obtained solution are discussed.

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INTRODUCTION

There has recently appeared a large number of works devoted to the study of kinetic phenomena in helium II at low ($T < 0.6$ K) temperatures (see the review of Ref. 1). Interest in this region has been raised by the assumption of Maris and Massey[2] that the phonon portion of the dispersion curve in He II can be unstable, i.e., the dependence of the photon energy ε on the momentum p has the form

$$\varepsilon = cp(1+\gamma p^2 - \delta p^4) \qquad (1)$$

(c is the velocity of sound, γ, $\delta > 0$).

Three-phonon processes become possible in such dispersion, and the effect of these processes on the kinetics and propagation of sound has been studied by Maris.[3] One of the results of this investigation has been the conclusion that, in addition to the ordinary second sound in He II at low temperatures, propagation of "new" sounds is possible. Also, it turned out that the velocity of ordinary second sound increases upon increase in the frequency, from the hydrodynamic value $c/\sqrt{3}$ to c. The qualitative explanation of this phenomenon has been given by Maris within the framework of the theory of Mandel'shtam-Leontovich for sound propagation in relaxing media.

The appearance of "new" sounds, i.e., weakly damped solutions of the dispersion equation, has been connected with the emergence of "quasiconserved" quantities, i.e., quantities which undergo almost no change within a time of the order of the period of the sound wave.

Such a situation was possible because $\omega t_{\parallel} \ll 1$ in the equilibrium regime (ω is the sound frequency, $t_{\parallel}$ is the time of establishment of equilibrium for phonons moving in a single direction) the transition from the region of hydrodynamics $\omega t_2 \ll 1$ (t_2 is the relaxation time of the second harmonic of the distribution function, expanded in Legendre polynomials) to the high frequency region $\omega t_2 \gg 1$ is described not by a single relaxation t_2, but by a whole series of times t_l, each of which corresponds to the relaxation of the harmonic $P_l(\cos\theta)$. For small l, a dependence $t_l^{-1} \sim l^4$ was obtained in Refs. 3–5. Therefore, with increase in frequency a situation becomes possible in which some of the harmonics are "conserved," and for them $\omega t_l \gg 1$, shile others are in the hydrodynamic regime $\omega t_l \ll 1$.

Kinetic phenomena in a gas of phonons for the case of four-phonon processes (under the assumption that $\gamma < 0$ in (1)) was considered by Khalatnikov and Chernikova.[6] They assumed that the transition from the hydrodynamic region to the high-frequency region is described by a single relaxation time τ_2.

In the present paper it will be shown that even in the

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case of four-phonon processes there are a number of times τ_l that describe the transition region. This leads to a decrease by a factor of two in the absorption coefficients and the sound dispersion in the region $(\omega\tau_2)^{1/2} \gg 1$ in comparison with the work of Ref. 6. Moreover, the dependence of the relaxation times on the number of the harmonic l allows us to extend the results of the investigations of Maris, Wehner and Benin[3,4,7] from three-phonon processes to four phonon processes.

According to the existing representations, the dispersion is anormalous ($\gamma > 0$) at low pressures and becomes normal ($\gamma < 0$) at high pressures ($P > 17$ atm).[8] Since the relaxation t_2 for three-phonon processes contains the parameter γ in second degree[3-5,9] ($t_2^{-1} \sim \gamma^2$), the time t_2 increases with increase in pressure. Therefore, the region of applicability of the present paper will be the entire region of normal dispersion and that part of the region of anomalous dispersion where $I_4 \gg I_3$; here I_4 and I_3 are respectively the four-phonon and the three-phonon collision integrals. By estimates, using the work of Jäckle and Kehr,[10] we find that this inequality is satisfied for pressures $p \geqslant 14$ atm.

1. DERIVATION OF THE KINETIC EQUATION

We consider the linearized four-phonon collision integral

$$I_4 = \int d\sigma(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3, \mathbf{p}_4)\,(\delta n_1 + \delta n_2 - \delta n_3 - \delta n_4)\,d\mathbf{p}_2, \tag{2}$$

where $d\sigma$ is the scattering cross section and diverges if no account is taken of dispersion. As in Ref. 11, we have

$$d\sigma \sim 1/(1-\cos\theta)^2 \tag{3}$$

(θ is the angle between the colliding phonons p_1 and p_2). The deviation of the distribution function from equilibrium is sought in the form[12]

$$\delta n = -n_0(n_0+1)\left[\varepsilon\sum_l \nu_l P_l(\cos\theta) + pc\sum_k \beta_k P_k(\cos\theta) + \frac{\partial\varepsilon}{\partial\rho}\rho'\right]. \tag{4}$$

Here ρ is the density while ν_l and β_k are the expansion coefficients of the local temperature and the relative velocity in Legendre polynomials.[1)]

As has been shown in Ref. 11 for the second harmonic, and as we shall show here for all harmonics, the four-phonon collision integral diverges if no account is taken of the dispersion parameter γ. For this reason, we can assume that ε depends linearly on p and we can combine the fuctions $\beta(\theta)$ and $\nu(\theta)$ in (4), writing δn in the form

$$\delta n = -n_0(n_0+1)\left[\varepsilon\sum_l \nu_l P_l(\cos\theta) + \frac{\partial\varepsilon}{\partial\rho}\rho'\right], \tag{5}$$

i.e., we actually consider four-phonon scattering for linear phonons. We shall verify below that such an approximation is always valid in the equilibrium regime, $\omega t_{\parallel} \ll 1$.

We shall be interested in the behavior of I_4 at $l \gg 1$. Since the $P_l(\cos\theta)$ has different asymptotic forms at small and large θ, we represent the integration over the angles of the colliding phonons in the form

$$I_4 = \int_0^{\theta_0} f(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3, \mathbf{p}_4)\,d(\cos\theta) + \int_{\theta_0}^{\pi} f(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3, \mathbf{p}_4)\,d(\cos\theta). \tag{6}$$

Here θ_0 is a small angle, such that the transition from one asymptotic form to the other takes place at $l\theta_0 \sim 1$. We have[13]

$$0 \leqslant \theta \leqslant \theta_0 \qquad P_l(\cos\theta) \underset{l\to\infty}{\to} J_0(l\theta) \tag{7}$$

(J_0 is a Bessel function of order zero),

$$\theta_0 \leqslant \theta \leqslant \pi \qquad P_l(\cos\theta) \to \frac{1}{l^{1/2}}\frac{\cos(l\theta - \pi/4)}{(\sin\theta)^{1/2}}. \tag{8}$$

The second integral in (6) oscillates rapidly because of (8), and at $\theta > \theta_0$, we can neglect it. Substituting (7) in (6), we see that the zeroth term in the expansion of $J_0(l\theta)$ in powers of $l\theta$ is cancelled out because of the law of energy conservation. The term that is quadratic in $l\theta$ is eliminated by subtracting from the expansion $J_0(l\theta)$ the quantity $l^2(1-\cos\theta)$ obtained from the difference, multiplied by l^2, between the law of conservation of energy and momentum

$$l^2[p_2(1-\cos\theta) - p_3(1-\cos\theta_3) - p_4(1-\cos\theta_4)] = 0 \tag{9}$$

Evidently the equality (9) is valid only for linear phonons, since we neglect terms $\sim l^2p^2$, i.e., the derivation is valid at $l^2\gamma p^2 \ll 1$. It becomes clear from the foregoing consideration that the relaxation rate of the l-th harmonic depends on l like

$$\tau_l^{-1} \sim \tau^{-1}l^2. \tag{10}$$

Recognizing that for the equilibrium regime, we should have

$$\tau^{-1}l^2 \ll t_{\parallel}^{-1}, \tag{11}$$

and that for "strong" (see Sec. 3) normal dispersion

$$\tau^{-1} \sim t_{\parallel}^{-1}\gamma p^2, \tag{12}$$

we obtain the required inequality $l^2\gamma p^2 \ll 1$.

Thus, the angular part of the function f in (6) is of the form

$$f(\theta) \sim \frac{1}{(1-\cos\theta)^2}[J_0(l\theta) - 1 + l^2(1-\cos\theta)] \sim l^4 + l^2, \quad \theta \to 0. \tag{13}$$

Integrating up to θ_0, we obtain

$$I_4 \sim \sum_l \nu_l J_0(l\theta)\,\frac{C_1l^2 + C_2}{\tau} \tag{14}$$

(C_1, C_2 are constants). This derivation is valid for $l \gg 1$ and for $\theta \ll 1$, respectively. We can write I_4 in a form that is valid for all angles if we recognize that the function $J_0(l\theta)$ is an eigenfunction of I_4, i.e., I_4 should have the form of a differential Bessel operator

$$\hat{l}^2 = \frac{1}{\theta}\frac{\partial}{\partial\theta}\left(\theta\frac{\partial}{\partial\theta}\right). \tag{15}$$

At angles $\theta \sim 1$, this operator is replaced by the angular part of the Laplace operator

$$\hat{l}^2 = \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right). \tag{16}$$

We assume the constant C_2 to be equal to 1, and choose C_1 from the condition that at $l=2$ the time τ_2 is identical with that calculated in Ref. 12. Finally, we have the following kinetic equation (see Ref. 12):

$$\left[\bar{z} - \cos\theta + \frac{\bar{\omega}}{7i\omega\tau_2}\hat{l}^2\right]\nu(\theta) = -\frac{\bar{\omega}}{7i\omega\tau_2}(\nu_0 + 3\nu_1\cos\theta) + (u+\cos\theta)\cos\theta\frac{\rho'}{\rho}. \tag{17}$$

Here $\bar{z} = \bar{\omega}(1 - 1/7i\omega\tau_2)$ and $\bar{\omega} = \omega/kc$. We note the following: the time τ_3 is determined in this fashion with an accuracy to within a coefficient ~ 1, i.e., we have an inconsequential indeterminacy in the boundary of the region $(\omega\tau_2)^{1/2} \gg 1$. The choice of $\omega\tau_2$ has practically no effect on the absorption coefficient and the sound dispersion.

2. PROPAGATION OF FIRST AND SECOND SOUNDS

We first investigate how the considered process affects the propagation of first sound. For high frequencies and small angles, where the quantity $\hat{l}^2/\omega\tau_2$ is important, we obtain from (17)

$$\left(\frac{\theta^2}{2} + \frac{1}{7i\omega\tau_2}\hat{l}^2\right)\nu(\theta) = \frac{\rho'}{\rho}(u+1). \tag{18}$$

This is an inhomogeneous Bessel equation and its solution is described by means of the Lommel function. Being interested in the asymptotic form at small angles, we write down approximately

$$\left(\frac{\theta^2}{2} - \frac{1}{7i\omega\tau_2\theta^2}\right)\nu(\theta) = \frac{\rho'}{\rho}(u+1). \tag{19}$$

Making use of the connection between the sound absorption coefficient and the function $\nu(\theta)$,[3,14] we have

$$\alpha = -\frac{\pi^2}{30}\frac{(u+1)^2\omega T^4}{\hbar^3c^6\rho}\int_0^\infty \mathrm{Im}\frac{d(\theta^2/2)}{\theta^2/2 + i/7\omega\tau_2\theta^2} = \frac{\pi^3}{120}\frac{(u+1)^2\omega T^4}{\hbar^3c^6\rho}, \tag{20}$$

i.e., smaller than in Ref. 6 by a factor of 2.

For the anomalous dispersion, Gurevich and Laĭkhtman[14] obtained a result that was smaller than in Ref. 6 by a factor of 3. What has been said above pertains also to the sound dispersion. It is seen from (19) and (20) that the result (20) is valid under the condition $(\omega\tau_2)^{1/2} \gg 1$: the characteristic angles in this case are $\theta^2 \sim (\omega\tau_2)^{-1/2}$. If the frequencies are not high enough, i.e., $\omega\tau_2 \gg 1$, but $(\omega\tau_2)^{1/2} \sim 1$, then the result will be practically no different from that obtained in Ref. 6.

We now take up the problem of the propagation of second sound. We consider the range of frequencies satisfying the condition

$$1 \ll \omega\tau_l \ll (\rho_n/\rho)^{-1},\ \rho_n/\rho \ll \omega\tau_l \ll 1, \tag{21}$$

where ρ_n/ρ is the density of the normal component. This allows us to avoid consideration of the entanglement of first sound with second. We then obtain from Eq. (17)

$$\left(\bar{z} - \cos\theta + \frac{\bar{\omega}}{7i\omega\tau_2}\hat{l}^2\right)\nu(\theta) = -\frac{\bar{\omega}}{7i\omega\tau_2}(\nu_0 + 3\nu_1\cos\theta). \tag{22}$$

For a qualitative test of the idea of Maris on the connection of the quasi-conserved quantities with the appearance of new sounds, we consider a model in which the times t_l change so rapidly with the number l (i.e., $t_l^{-1} \sim l^n$ and n is large) that the following intervals are possible:

$$\omega\tau_2 \ll 1, \tag{I}$$

$$1 \ll \omega\tau_2,\ \omega\tau_3 \ll 1, \tag{II}$$

$$1 \ll \omega\tau_2,\ 1 \ll \omega\tau_3,\ \omega\tau_4 \ll 1. \tag{III}$$

Let all the harmonics beginning with the fourth relax with the same time τ. The kinetic equation (22) in this case can be written in the form [2)]

$$(\bar{\omega} - \cos\theta)\nu(\theta) = \frac{\bar{\omega}}{i\omega\tau}(\nu(\theta) - \nu_0 - \nu_1\cos\theta) + \frac{\bar{\omega}}{i\omega\tau}\left(\frac{\tau}{\tau_2} - 1\right)\nu_2 P_2(\theta) + \frac{\bar{\omega}}{i\omega\tau}\left(\frac{\tau}{\tau_3} - 1\right)\nu_3 P_3(\theta). \tag{23}$$

In region I, it then takes the form

$$(\bar{\omega} - \cos\theta)\nu(\theta) = \frac{\bar{\omega}}{i\omega\tau_2}(\nu(\theta) - \nu_0 - \nu_1\cos\theta) \tag{24}$$

and at $\omega\tau_2 \ll 1$ we obtain the well known solution

$$\bar{\omega}^2 = \frac{1}{3} - \frac{4}{15}i\omega\tau_2. \tag{25}$$

In region II,

$$(\bar{\omega} - \cos\theta)\nu(\theta) = \frac{\bar{\omega}}{i\omega\tau}(\nu(\theta) - \nu_0 - \nu_1\cos\theta - \nu_2 P_2(\cos\theta)). \tag{26}$$

In region III

$$(\bar{\omega} - \cos\theta)\nu(\theta) = \frac{\bar{\omega}}{i\omega\tau}(\nu(\theta) - \nu_0 - \nu_1\cos\theta - \nu_2 P_2(\cos\theta) - \nu_3 P_3(\cos\theta)). \tag{27}$$

Equations (26) and (27) have the form of kinetic equations for three and four conserved quantities, respectively. It is possible to solve them by the method used in Ref. 12.

We obtain the first three equations by equating the coefficients in the case of identical $P_l(\theta)$ in (27). We obtain the fourth equation by solving (27) relative to $\nu(\theta)$ and removing the zeroth harmonic from the solution. As a result, we have

$$\bar{\omega}\nu_0 - {}^1/_3\nu_1 = 0,\ -\nu_0 + \bar{\omega}\nu_1 - {}^2/_5\nu_2 = 0,\ -{}^2/_3\nu_1 + \bar{\omega}\nu_2 - {}^3/_7\nu_3 = 0,$$
$$2\nu_0 = -\frac{\bar{\omega}}{i\omega\tau}[\nu_0 2Q_0(\bar{z}) + \nu_1 2Q_1(\bar{z}) + \nu_2 2Q_2(\bar{z}) + \nu_3 2Q_3(\bar{z})] \tag{28}$$

($Q_l(\bar{z})$ are the Legendre polynomials of the second kind). Expanding the fourth equation in the case $\omega\tau \ll 1$ in terms of $1/\bar{z}$ with accuracy to $(1/\bar{z})^4$, we obtain for region III

$$\mathrm{Re}\,\bar{\omega}_1^2 \approx \frac{26}{35},\quad \mathrm{Re}\,\bar{\omega}_2^2 \approx \frac{4}{35},\quad \mathrm{Im}\,\bar{\omega}^2 \sim \omega\tau. \tag{28'}$$

Similarly, for region II,

$$\mathrm{Re}\,\bar{\omega}_1^2=\frac{3}{5},\quad \mathrm{Re}\,\bar{\omega}_2=0,\quad \mathrm{Im}\,\bar{\omega}^2\sim\omega\tau_3. \tag{29}$$

It is seen that for such a model, the velocity of second sound increases with increase in the frequency, in agreement with the numerical calculations.[3,15] Moreover, a new sound appears in region III. [3)]

A model with a single relaxation time, as is well known, does not give solutions in the region $\omega\tau_2\gg 1$.[12]

3. CONCLUSION. DISCUSSION OF EXPERIMENTAL PERSPECTIVES

An important quantitity in the theory of kinetic phenomena in helium II is $t_{\parallel}$—the time of establishment of energy equilibrium for parallel phonons. In the calculation of $t_{\parallel}$, the decisive role is played by two factors: 1) the sign of γ in the expressions (1); 2) the participation of multiphonon processes, i.e., corrections to the expression calculated in the lowest order perturbation theory. The parameter determining these corrections is the quantity[5,17]

$$\alpha=\frac{(u+1)^2}{\gamma\rho c\hbar^3}\left(\frac{kT}{c}\right)^2$$

for the dispersion law (1). This is none other than the ratio of the line widths, due to three-phonon processes, to some energy "deficit" $\Delta\varepsilon\sim cp\gamma p^2$. If the dispersion is "strong", i.e., $\alpha\ll 1$, then the perturbation theory series diverges and the time $t_{\parallel}$ as a function of the sign of γ is governed either by parallel three-phonon processes,[3] or by parallel four-phonon processes.[11,12]

If $\alpha\gg 1$ (the dispersion is "weak"), then the time $t_{\parallel}$ is calculated from the renormalized perturbation theory.[18] [4)] The sign of γ here is unimportant.

In the case of "strong" normal dispersion, the establishment of equilibrium in the energy and in the number of phonons is controlled respectively by four-phonon and and five-phonon parallel processes,[11,12] in which

$$t_5^{-1}\ll t_4^{-1}.$$

Therefore, for very high frequencies $(\omega\tau_2)^{1/2}\gg 1$, no equilibrium in the number of phonons can be established. However, substituting all the values of the quantities in the parameter α (the $\gamma(P)$ dependence is taken from Ref. 8, the rest from Ref. 19), we find that the condition $\alpha\ll 1$ is satisfied at $P>17$ atm for temperatures $T<0.5$–0.7 K (the corresponding pressures are ~ 20–23 atm). Using the expression for $t_{\parallel}^{-1}$ from Ref. 12,

$$t_{\parallel}^{-1}=\frac{4{,}15}{192\pi^3}\frac{(u+1)^4}{\rho^2 c}\left(\frac{kT}{\hbar c}\right)^7\frac{x^2}{\gamma}\left(1+\frac{14{,}7}{x}\right),\quad x\gg 1,$$

we find that for equilibrium we should have

$$\omega\ll 10^5.$$

This corresponds to very long wavelengths ($\lambda\gg 10$ cm) and is beyond the limits of experimental possibilities.

In conclusion, we find the range of temperatures, pressures and frequencies, where observation of the effects discussed in the present paper is possible. Since the quantities $t_{\parallel}$ increase with increase in pressure and with decreasing temperature, it is more convenient to work at very low pressures and high temperatures. But at low pressures ($P<10$ atm) three-phonon processes are more effective, and at much higher pressures the parameter α becomes ~1; this does not allow the possibility of exactly determining the quantity $\omega t_{\parallel}\ll 1$. At high temperatures ($T>0.6$ K) the effect of rotons is important. Therefore, it appears that the only possible point where observation of the effects given in Sec. 2 is possible is the transition point $P_c=17$ atm, where $\gamma=0$. Here $\alpha\gg 1$, equilibrium is established simultaneously in energy and the number of phonons by renormalized three-phonon processes.

We should have

$$\omega t_{\parallel}\ll 1,\quad (\omega\tau_2)^{1/2}\gg 1,\quad \omega t_{rr}\gg 1$$

(the latter is the condition of absence of rotons). The wavelengths are bounded from above by the size of the vessel, for which we take $d=10$ cm, which leads to $\omega\gg\omega_c=2.3\times 10^4$.

Taking $t_{\parallel}$ from Ref. 18, we find

$$\omega\sim 2\cdot 10^5-3\cdot 10^5 \text{ and } T=0.4-0.5\text{ K},\quad P=17\text{ atm}.$$

At $T>0.7$ K, all the effects are determined by rotons. In this same region of frequencies and temperatures, observation of the new sounds is possible. The parameters $u=2.32$, $c=3.35\times 10^4$, $\rho=0.166$[19] and $\Delta=7.5$ K were used as estimates.

We note also that the inclusion of parallel processes in the kinetic equation (17) leads to the result that in the region $\omega t_{\parallel}>1$, the second sound will also propagated also in the case of four-phonon processes (see note 2). This follows from the fact that the mathematical form of Eq. (17) and the equation used in Refs. 3, 7, and 15 are identical. Thus, in the region $\omega t_{\parallel}>1$, resonant behavior of first sound is possible.[3] Summing up, we can say that the unexpected effects associated with the propagation of first and second sound more readily are due to the weak departure of the phonon spectrum from the linearity than to the sign of this departure.

I thank I. M. Khalatnikov who read through the manuscript and made a number of useful remarks.

[1)] Equilibrium with respect to the number of phonons is established (see Sec. 3).

[2)] The kinetic equation written in this form differs from the equation used by Benin[14] and Wehner[7] for three-phonon processes only in the absence of a term describing "parallel" processes. In the equilibrium regime, $\omega t_{\parallel}\ll 1$, this term is equal to zero after integration over all energies of the phonons moving in a single direction.

[3)] The results of (25), (28) and (29) can be obtained more formally by using the method of continued fractions.[16] To be

precise, expanding the function $\nu(\theta)$ in Legendre polynomials, we equate the coefficients for the same polynomials. Limiting ourselves to two, three and four harmonics, we obtain the results (25), (28) and (29), respectively. But the role of the conserved quantities would not be revealed in such a method of solutions.

[4)]The times $t_{\parallel}$ calculated for "strong" anomalous dispersion dispersion[3] and weak dispersion[18] differ only by a numerical factor ~ 2.

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Thermal stability of condensed systems that contain trapped atoms

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A consistent analysis of the thermal stability of atom-containing condensed systems, based on recently obtained experimental data, shows that, notwithstanding the previous opinion expressed in the literature, that when the temperature of the experiment is lowered one should expect an appreciable increase of the concentration of the trapped atoms. The analysis shows that to trap atoms in a solid matrix by means of a magnetic field the field intensity required is quite large and exceeds by several orders the fields at which $\mu_e B/k_B T \simeq 1$. In addition, a magnetic field can be effectively used for trapping only if there is no contact between the atoms prior to the establishment of equilibrium between them and the magnetic field. The use of beams polarized in advance does not facilitate the problem to any extent.

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Recent progress in cryogenics and in the technique of obtaining strong magnetic fields have increased in the stabilization of appreciable concentrations of atoms in solid matrices at low temperatures. The most attractive in this respect are metstable (with high energy content) mixtures, both from the point of view of their use as rocket fuel[1,2] and in connection with the possiblity of producing metallic superconducting phases (metallic hydrogen and nitrogen[3]) and superfluid phases (triplet hydrogen[4]). In his review[5] devoted to prospects of progress in science, Ginzburg included the problem of obtaining these phases among the most interesting and important tasks of modern physics.

The necessary condition for the existence of metastable systems containing atoms whose stable states are molecules is that recombination of the atoms be kinetically hindered. The cause of the kinetic hindrances may be either the existence of an energy barrier to the motion of the atoms over the matrix (activated diffusion), or the presence of an activation energy in the recombination act itself, because of the interaction of the atoms with the matrix and with one another, or because of alignment of their electron spins in strong magnetic fields.[1)]

However, in view of the large energy reserves in systems that contain appreciable numbers of "quasi-free" atoms, slowness of the isothermal decay of the system in comparison with the observation time is far from a sufficient condition of the system stability. The reason lies in the thermal explosion of chemically ac-