Resonance instability of small-scale plasma perturbations

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We consider the conditions for the excitation, in the reflection region of ordinary electromagnetic waves, of quasistationary small-scale plasma-concentration perturbations, elongated in the direction of the magnetic field. The stratification parameter, which characterizes the transverse dimension of the produced inhomogeneities, is found to be $\approx(D_e c/\omega_H)^{1/2}$ (D_e is the Debye radius and ω_H is the electron gyrofrequency). The stratification parameter is usually not large, less than the ion Larmor radius. A resonant amplification of the instability is observed at a perturbing-wave frequency equal to double the electron gyrofrequency.

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1. INTRODUCTION

An intensive linear transformation of a transverse electromagnetic wave by the concentration inhomogeneities, into natural longitudinal plasma oscillations, is possible in the region of plasma resonances. The electron heating resulting from absorption of the transverse-oscillation energy can cause the inhomogeneities to increase. This, in turn, leads to an enhancement of the transformation of the transverse wave, which again causes the inhomogeneities to increase, and so on. The resultant instability leads to a strong enhancement of the inhomogeneous structure of the plasma, i. e., to a destruction of the smooth plasma layer in the region of the resonances. This is called resonance instability. It develops only in the field of an ordinary electromagnetic wave in the region where this wave is reflected (see Fig. 1). An essential feature of the resonant instability is its nonlinear character—the effective growth of the concentration perturbations sets in only at a certain threshold $|\delta N| > \delta N_{thr}$.

Resonance instability was investigated earlier[1,2] under conditions when the decisive role is played by the thermal conductivity and the diffusion of the plasma along the force lines of the magnetic-field H. This is valid for sufficiently large-scale inhomogeneities. At the same time, as shown below (Sec. 3), the characteristic dimension of the inhomogeneities produced in the plasma across the field H is equal to

$$a \sim \left(D_e \frac{c}{\omega_H}\right)^{1/2}, \quad D_e=(T_e/4\pi e^2 N)^{1/2}, \quad \omega_H=eH/mc. \tag{1}$$

Here ω_H and D_e are the gyrofrequency and the Debye radius of the electrons. The dimension a usually turns out to be less than the ion Larmor radius ρ_{Hi} or the effective length $L_\perp$ of the transverse electronic thermal conductivity:

$$a^2 \ll \rho_{Hi}^2, \quad L_\perp^2 = \frac{\nu_e T_e}{m\omega_H^2}\tau_T, \tag{2}$$

where ν_e is the frequency of the collisions of the electrons with the heavy particles (ions), and τ_T is the characteristic time of the relaxation of their temperature T_e as a result of the indicated collisions.[3] In particular, the plasma stratification observed under conditions of experiments on the action of powerful radiowaves on the surface layers of the ionosphere[4-7] is small-scale in character (2). The development of these inhomogeneities is strongly influenced by plasma transport across the magnetic field. The present paper is devoted to the study of the conditions of excitation and development of resonance instability in the small-scale limit (2).

The fundamental equations are formulated in Sec. 2, and the optimal transverse dimension of the inhomogeneities (1) is determined in Sec. 3. In Sec. 4 we consider the conditions for the excitation of small-scale resonance instability. The region of double resonance at a transverse-wave frequency $\omega \approx 2\omega_H$ is investigated in Sec. 5, and a strong enhancement of the plasma perturbation in this region is observed. The results of the theory are compared with ionosphere experiments (Sec. 6).

2. FORMULATION OF PROBLEM. FUNDAMENTAL EQUATIONS

Let an ordinary electromagnetic wave

$$\tilde{\mathbf{E}} = \frac{1}{2}[\mathbf{E}e^{i\omega t} + \text{c.c.}] \tag{3}$$

with frequency

$$\omega > 2\omega_H \tag{3a}$$

be normally incident on a plane layer of a weakly-inhomogeneous magnetized plasma with infrequent collisions

$$\nu_e \ll \omega_H. \tag{4}$$

Assume that the region of wave reflection in the plas-

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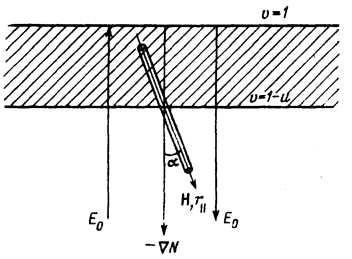

FIG. 1. Pattern of excitation of resonance instability in the region where an ordinary wave is reflected from a weakly inhomogeneous plasma ($v=4\pi e^2N/m\omega^2$, $u\omega_H^2/\omega^2$, ω is the plasma frequency. The region of the plasma resonance is shaded).

ma contains concentration inhomogeneities δN that are strongly elongated in the direction of the magnetic field **H**, but have a small dimension $a \ll c/\omega$ transverse to the field **H** (see Fig. 1). These inhomogeneities are produced in a strongly magnetized plasma (4), in which the diffusion is much faster along than across the magnetic field. We examine the time evolution of these inhomogeneities.

The point is that the polarization of the inhomogeneities in the field of the ordinary wave leads to an intense excitation of natural longitudinal oscillations of the plasma, which takes place in a small vicinity of the upper hybrid resonance[2,6]

$$v=1-u, \qquad v=\frac{\omega_0^2}{\omega^2}=\frac{4\pi e^2N}{m\omega^2}, \quad u=\frac{\omega_H^2}{\omega^2}, \tag{5}$$

where the group velocity of the longitudinal waves is small. As a result, an appreciable fraction of the energy of the longitudinal oscillations is absorbed (dissipated) directly in the inhomogeneity volume and heats the inhomogeneity. This heating changes the size and shape of the inhomogeneity. The strong increase of the concentration perturbations, i. e., the strong enhancement of the inhomogeneities, constitutes in fact the excitation of the resonance instability.

An important feature of generation of longitudinal waves by inhomogeneites is its quasistationary character: the times of variation of the concentration perturbations δN are determined by diffusion processes that are slow in comparison with the frequency and lifetime of the plasma waves. The frequency of the excited longitudinal oscillations practically coincides therefore with the frequency ω of the transverse pump waves.

Because of the quasistationarity, the total problem of the development of resonance instability can be broken up into two parts:

1. Determination of the potential φ of the longitudinal waves generated in the field of the ordinary waves by a given inhomogeneity δN, and the calculation of the thermal-energy source $W(\delta N)$ resulting from the dissipation of these waves.

2. Calculation of the changes $\delta N(t)$ after finding the energy source $W(\delta N)$. The source W for collision absorption of longitudinal waves was investigated in[1,2,9]. In the region of the upper-hybrid resonance it is given by

$$W=\frac{\nu_e}{2\pi}\frac{1+u}{1-u}|\nabla\varphi|^2, \tag{6}$$

where the potential φ is obtained from the generalized Poisson equation

$$\left\{\frac{\partial}{\partial x_i}\hat{\varepsilon}_{ij}\frac{\partial}{\partial x_j}-k_0^2u\right\}\varphi=-4\pi\rho; \qquad k_0=\omega/c. \tag{7}$$

Here $\hat{\varepsilon}_{ij}(\omega)$ is the differential operator of the dielectric tensor of a collisionless plasma, in which the principal role is played by the cold-plasma tensor ε_{ij}.[1] In addition, in our case Eq. (7) takes into account the weak spatial dispersion of the longitudinal waves ($\hat{\varepsilon}_{ij}-\varepsilon_{ij}$) and the "transverse" correction k_0^2u.[9,10] The "extraneous" charge density ρ in the right-hand side of (7) is due to polarization of the perturbation δN in the field of the ordinary wave (3), and its value in the case of elongated inhomogeneities is

$$\rho=\frac{1}{8\pi(1-u)}\{\mathbf{E}_\perp+iu^{1/2}[\mathbf{E}_\perp\times\mathbf{h}]\}\nabla\delta v; \qquad \delta v=4\pi e^2\delta N/m\omega^2, \ \mathbf{E}_\perp\perp\mathbf{H}, \ \mathbf{h}=\mathbf{H}/H. \tag{7a}$$

The time evolution of minute small-scale concentration and electron-temperature perturbations δN and δT_e in a spatially homogeneous strongly ionized plasma (4) under the influence of the thermal-energy source W is described by the electronic-thermal conductivity and diffusion equations[11][1]:

$$\frac{3}{2}\frac{\partial}{\partial t}\frac{\delta T_e}{T_e}-D_\parallel^e\Delta_\parallel\left\{1.71\left(1.71\frac{\delta T_e}{T_e}+k_T^{-1}\frac{\delta N}{N}\right)+3.16\cdot0.5\frac{\delta T_e}{T_e}\right\}-D_\perp^e\Delta_\perp\left\{2.66\frac{\delta T_e}{T_e}-0.5k_T^{-1}\frac{\delta N}{N}\right\}=\frac{W}{NT_e},$$
$$\frac{\partial}{\partial t}\frac{\delta N}{N}-D_\parallel^e\Delta_\parallel\left\{1.71\frac{\delta T_e}{T_e}+k_T^{-1}\frac{\delta N}{N}\right\}-D_\perp^e\Delta_\perp\left\{-0.5\frac{\delta T_e}{T_e}+k_T^{-1}\frac{\delta N}{N}\right\}=0; \tag{8}$$
$$D_\parallel^e=\frac{T_e}{0.51m\nu_e}, \quad D_\perp^e=\frac{\nu_eT_e}{m\omega_H^2}, \quad k_T=\frac{T_e}{T_e+T_i}.$$

Here T_i is the ion temperature, and the symbols $\Delta_\parallel(\Delta_\perp)$ stand for Laplace operator with respect to the coordinates parallel (perpendicular) to **H**. Next, $D_\parallel^e$, $D_\perp$ are the electron diffusion coefficients along and across the magnetic field and k_T is the thermal-diffusion ratio. We note that we have omitted from (8) the relaxation terms, which are small in the considered case of strong transverse diffusion (2). We have also left out of the thermal-conduction equation (8) a nonlinear term of drift origin

$$T_i\mathbf{h}[\nabla\delta T_e, \nabla\delta N]/m\omega_HNT_e.$$

In the case of not too small perturbations $\delta N/N \sim \delta T_e/T_e > \nu_e/\omega_H$, this term, generally speaking, is appreciable in comparison with the linear dissipative terms and leads to symmetrization of the inhomogeneities relative to the magnetic field **H**.[2)] We therefore consider from now on axially-symmetrical (or planar) inhomogeneities with values of $\nabla_\perp\delta N\parallel\nabla_\perp\delta T_e$ for which this term vanishes.

We note that symmetrization is produced also by scattering of the excited longitudinal oscillations by the concentration inhomogeneities δN, a process considered in [12]. However, in the case of a symmetrical inhomogeneity strongly elongated along the magnetic field H, this interaction is also weakened to a negligibly small value (see conditions (20a) below).

3. HEATING OF CONCENTRATION INHOMOGENEITIES

The transformation of the transverse wave (3) into a longitudinal one by an inhomogeneity $\delta v = \delta v(r_\perp)$ elongated along the field H takes place at the point of the spatial (phase) synchronism

$$k_\parallel^2 = k_0^2 (n_{tr} \cos\alpha)^2, \tag{9}$$

where $\mathbf{k}$ is the wave vector of the excited longitudinal oscillations, n_{tr} is the refractive index of the ordinary wave, and α is the angle between the direction of its propatation and the magnetic field H. The subscripts $\parallel$, $(\perp)$ designate hereafter the components of the corresponding vectors parallel (perpendicular) to H. It follows from (9) that the region of excitation of the longitudinal oscillations with $k_1^2/k_0^2 \gg 1$ is located in the vicinity of the upper hybrid resonance $v = 1-u$, where $k_\parallel^2 \ll k_\perp^2$.

The total flux of the energy transformed by a given inhomogeneity into the energy of longitudinal oscillations is determined by solving Eq. (7). It was determined earlier in[1,2,8], and its value in the case of quasilongitudinal propagation of the ordinary wave in the region $v = 1-u$ (i.e., at $\sin^2\alpha < 2u^{1/2}\cos\alpha$) is

$$P = \frac{|E_0|^2}{n_{tr}} \frac{1-u^{1/2}}{1+u^{1/2}} \frac{\omega}{8\cos\alpha\mu_1} \int [\delta v(r_\perp)]^2 d^2 r_\perp; \qquad \mu_1 = |\nabla v|, \; n_{tr} \approx u^{1/2}. \tag{10}$$

Here μ_1 is the gradient of the relative concentration in the vicinity of the upper hybrid resonance, and E_0 is the amplitude of the plane wave (3) incident on the plasma or the wave reflected from it.

We now obtain the fraction of the total energy (10) absorbed in an inhomogeneity region with transverse dimension a. This value is determined by the time of group departure of the longitudinal oscillations and the rate of their dissipation. To determine this fraction we consider the dispersion properties of the excited longitudinal waves. The dispersion equation corresponding to (7) takes in the region of the upper hybrid resonance the form

$$G = -(v+u-1) + u(1-u)\frac{k_\parallel^2}{k_\perp^2} + u(1-u)\frac{k_0^2}{k_\perp^2} - 3(k_\perp D_e)^2 \frac{(1-u)^2}{1-4u} - i\frac{\nu_e}{\omega}(1+u) = 0;$$

$$k_0 = \omega/c, \quad D_e^2 = T_e/4\pi e^2 N = T_e/m\omega^2(1-u). \tag{11}$$

The first two terms in the left-hand side of (11) correspond to cold-plasma longitudinal oscillations. The third describes the transverse corrections $\sim k_0^2/k^2 \ll 1$, and the fourth term

$$3(k_\perp D_e)^2 \frac{(1-u)^2}{1-4u} \ll u(1-u) \tag{11a}$$

takes into account the corrections necessitated by the thermal motion of the electrons. The last two corrections, which are generally speaking small, are significant in the process considered below, since it is precisely they which determine the group velocity of the longitudinal waves in the region of the upper hybrid resonance.[10] Equation (11) includes also a small dissipative term corresponding to weak collisional absorption of longitudinal oscillations in a cold plasma with $\nu_e \ll \omega$.[13]

With the aid of (11) we can easily obtain the group velocity ($\mathbf{V}_{gr} = \partial\omega/\partial\mathbf{k}$):

$$\mathbf{V}_{gr\perp} = \frac{\omega \mathbf{k}_\perp}{k_\perp^2} u(1-u)\left\{\frac{k_\parallel^2 + k_0^2}{k_\perp^2} + 3(k_\perp D_e)^2 \frac{1-u}{u(1-4u)}\right\} D^{-1}(k_\perp); \tag{12}$$

$$\mathbf{V}_{gr\parallel} = -\frac{\omega \mathbf{k}_\parallel}{k_\perp^2} u(1-u) D^{-1}(k_\perp);$$

$$D(k_\perp) = 1 + 3(k_\perp D_e)^2 \left(\frac{1-u}{1-4u}\right)^2 \tag{12a}$$

and the collision decrement γ_{coll} of the total damping of the longitudinal waves $\gamma = \mathrm{Im}\,\omega = \gamma_{coll} + \Delta\gamma$:

$$\gamma_{coll} = \frac{1}{2}\nu_e(1+u) D^{-1}(k_\perp). \tag{13}$$

The additional decrement $\Delta\gamma$, which is connected with cyclotron absorption and with allowance for collisions in the thermal correction (11) is significant only in a small vicinity of the double resonance $\omega \approx 2\omega_H$. It will be considered separately in Sec. 5.

We recognize furthermore that the total number $k_\perp$ of the excited longitudinal oscillations is of the order of the reciprocal of the transverse dimension a of the inhomogeneity: $k_\perp \approx a^{-1}$. Using this estimate and expressions (12) and (13), we easily find the time of group departure of the longitudinal waves from the volume of an inhomogeneity that is strongly elongated along the field H:

$$t_{gr} = a/V_{gr\perp} \quad (k_\perp = a^{-1}) \tag{14}$$

and the relative value of the ensuing absorption:

$$F(a) = 2\gamma t_{gr} = \frac{\nu_e}{\omega} F_1(a); \tag{15}$$

$$F_1(a) = \frac{1+u}{1-u}\left(\frac{c}{\omega_H a}\right)^2 \left[u^{1/2}\cos^2\alpha + 1 + \left(\frac{a_m}{a}\right)^4\right]^{-1}. \tag{15a}$$

We have taken into account here the relation (9) $k_\parallel^2/k_0^2 = u^{1/2}\cos^2\alpha$ and have introduced the characteristic dimension

$$a_m = \left(D_e \frac{c}{\omega_H}\right)^{1/2} \left\{3\frac{1-u}{|1-4u|}\right\}^{1/4} = \left\{\frac{3T_e}{mc^2}\frac{u}{|1-4u|}\right\}^{1/4} \frac{c}{\omega_H}. \tag{16}$$

With the aid of (10) and (15) we easily obtain a simple expression for the thermal-energy $\tilde{W}$ averaged along the field H over a distance $r_\perp < a$ from the axis of the elongated inhomogeneity[3]:

$$\tilde{W}(\delta v) = \int W\, dr_\parallel = \frac{P}{\pi a^2} F\left(\frac{a}{2}\right) = \frac{|E_0|^2 \nu_e}{u^{1/2}\mu_1} \frac{1-u^{1/2}}{1+u^{1/2}} \frac{(\delta v_1)^2}{16\cos\alpha} F_1\left(\frac{a}{2}\right), \tag{17}$$

where we have used for the energy flux P an expression corresponding to a Gaussian distribution of the concentration

$$\delta v(r_\perp)=\delta v_1 \exp\{-r_\perp^2/a^2\}, \qquad (17a)$$

and the appearance of the factor 1/2 in the argument of $F(a/2)$ is due to the concrete form of the perturbation (17a) at which the transverse wave number of the effective longitudinal oscillations is $k_\perp \approx 2/a$.

We note that the characteristic dimension over which the source W is averaged in (17) is as a rule small

$$\Delta r_\parallel \approx a\frac{V_{gr\parallel}}{V_{gr\perp}}(k_\perp=a^{-1})=u^{1/2}\cos\alpha\frac{c}{\omega_H}\left[u^{1/2}\cos^2\alpha+1+\left(\frac{a_m}{a}\right)^4\right]^{-1}$$
$$<\frac{u(1-u)k_\parallel^2}{\mu_1\cos\alpha k_\perp^2}=\left(\frac{\omega_H a}{c}\right)^2 u^{1/2}(1-u)\cos\alpha/\mu_1. \qquad (18)$$

Therefore in the derivation of (17) we could neglect the weak inhomogeneity of the medium.[4)]

It follows from (15)–(17) that the most intense heating of the inhomogeneities takes place at $a\approx a_m$. The reason is that the group velocity $V_{gr\perp}(k_\perp=a^{-1})$ increases with increasing $a>a_m$, on account of thermal effects, and increases with decreasing a in the case $a<a_m$ on account of transverse effects. As a result, the heating intensity in both cases decreases. The parameter a_m is therefore the optimal transverse dimension of the inhomogeneities. It is determined by the dispersion properties of the natural longitudinal oscillations of the plasma in the region of the upper hybrid resonance. At $k_\perp^{-1}=a\approx a_m$ the thermal and the transverse corrections in (11) and (12) become equal. The value of the parameter a_m (16), naturally, is of the order $(D_e c/\omega_H)^{1/2}$, since the thermal corrections are proportional to D_e/a^2, and the transverse ones are $\propto \omega_H^2 a^2/c^2$.

We note that the simple expressions (15)–(17) for the heating intensity agree well with the results of a rigorous calculation of the source $\tilde{W}$, obtained in[9] by numerical integration of Eqs. (6) and (7) for a Gaussian inhomogeneity (17a). If the problem is rigorously solved, the function F_1 in the right-hand side of (17) describes the spatial distribution of the source $\tilde{W}(r_\perp)$, and depends, naturally, on the distance $r_\perp$ to the inhomogeneity axis: $F_1=F_1(r_\perp)$. In Fig. 2 this function is represented by two terms of the expansion in the region $(r_\perp/a)^2<1$:

$$F_1(r_\perp)\approx F_1^{(1)}\left[1-\beta^{-2}\frac{r_\perp^2}{a^2}\right] \qquad (19)$$

depending on the value of $a/2a_m$. The coefficient $F_1^{(1)}(a)$ in (19) characterizes the intensity of the heat source at the center of the Gaussian inhomogeneity, while the parameter $\beta^2(a)$ characterizes the transverse dimension of the heated region ($\Delta r_\perp \sim \beta a$). For comparison, Fig. 2 shows the function $F_1(a/2)$ corresponding to the approximation (15a) and (17) for the same plasma parameters. The transverse dimension of the heated region in the approximation (17) is of the order of a, i.e., $\beta\sim 1$. The agreement between the approximate (17) and rigorous (19) calculations of the heat source is quite good.

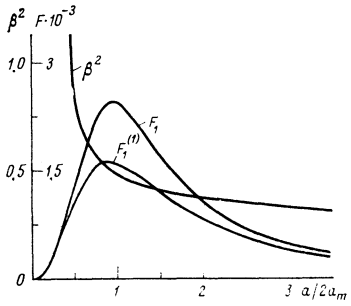

FIG. 2. Comparison of the approximate (F_1) and of the rigorous ($F_1^{(1)}$) calculations of the source of the heat energy at the center of a Gaussian inhomogeneity with transverse dimension a. Plot of the parameter β^2 against a. The plasma parameters are $u=0.04$, $T_e=1700\,^\circ$C, $\omega_H=8.5\times10^6$ sec^{-1}, $\mu_1=1/100$ km, and $\cos\alpha=1$. The optimal transverse dimension of the inhomogeneity is $a_m\approx 50$ cm.

We note that in these calculations we have neglected the influence of the inhomogeneity on the propagation of the longitudinal waves. Actually the scattering of the longitudinal oscillations by the concentration perturbations δv leads to a diffusion redistribution of their energy in wave-vector space.[12] By using the method of[12] we can obtain the characteristic time of this process. In the considered case of axially symmetrical (or planar) inhomogeneities $\delta v=\delta v(r_\parallel, r_\perp^2)$ this time is

$$t_k\approx\left(\frac{l_1}{\delta v_1}\right)^2\frac{1-u}{\omega u^{1/2}\cos^2\alpha}\left(\frac{\omega_H a}{c}\right)^4\left[u^{1/2}\cos^2\alpha+1+\left(\frac{a_m}{a}\right)^4\right]^3, \qquad (20)$$

where l_1 is the parameter of elongation of the inhomogeneity (i.e., the ratio of its longitudinal and transverse dimensions) and δv_1 is the value of the perturbation at the maximum of the inhomogeneity. A comparison of the characteristic times (14) and (20) shows that in the case of strongly elongated inhomogeneities, when

$$\frac{t_k}{t_{gr}}\sim\left(\frac{l_1}{\delta v_1}\right)^2\left(\frac{\omega_H a}{c}\right)^6\frac{(1-u)^2D^{-1}(k_\perp=a^{-1})}{u^{1/2}\cos^2\alpha}\left[u^{1/2}\cos^2\alpha+1+\left(\frac{a_m}{a}\right)^4\right]^4\gg 1, \qquad (20a)$$

the scattering of the longitudinal waves by the concentration perturbation δv can actually be neglected in comparison with the stronger group departure of their energy from the inhomogeneity region.

We note in conclusion that in the frequency region $\omega_H<\omega<2\omega_H$ converse to (3a) the thermal correction in expression (12) for the group velocity $V_{gr\perp}$ reverses sign. The sign of the analogous terms $(a_m/a)^4$ in formulas (15a), (20), and others is simultaneously reversed. The intensity of the longitudinal waves increases (the corresponding characteristic time (20) decreases sharply at $a\approx a_m$). As a result, the local heating of the inhomogeneities can greatly weaken in the case $\omega<2\omega_H$ and $a\approx a_m$, while the transverse dimension of the heated region ($\Delta r_\perp\sim\beta a$) can increase noticeably.

4. EXCITATION OF RESONANCE INSTABILITY

We proceed now to the second part of the problem, the investigation of the character of the development of the plasma inhomogeneity with account taken of its additional heating by dissipation of longitudinal plasma

oscillations. This process is described by Eqs. (8). Their solution for elongated inhomogeneities is conveniently represented in the form

$$\delta v(r_\perp,t)=(1-u)\int_{-\infty}^{t} G(t-t',|\mathbf{r}_\perp-\mathbf{r}_\perp'|^2)\frac{\tilde{W}(r_\perp',t')}{NT_e}d^2r_\perp'\,dt'+\delta v_0(r_\perp,t);$$
$$\tilde{W}(r_\perp,t)=\int W\,dr_\parallel, \tag{21}$$

where $\tilde{W}$ is the averaged source of thermal energy, localized in accordance with (9) and (18) in a small vicinity of the upper-hybrid resonance $v=1=u$, $\delta v = 4\pi e^2\delta N/m\omega^2$ is the dimensionless perturbation of the concentration in the same region, and δv_0 is the initial "bare" perturbation, which is not connected with the inhomogeneity-heating mechanism considered here.

A general investigation of (21) shows [14] that the small-scale concentration perturbations are generated only by heat sources $\tilde{W}(t)$ that increases rapidly enough, so that processes of plasma transport across the magnetic field can be neglected. [In the opposite, case the concentration perturbations die out rapidly over a region with the transverse dimension on the order of the effective length $L_\perp$ of the electronic thermal conductivity (see Eq. (2)).]

Omitting therefore the dissipative terms proportional to $D_\perp^e$ in the thermal-conductivity and diffusion equations (8), we easily obtain an explicit expression for their Green's function

$$G(t,r_\perp^2)=-\frac{\mathcal{H}}{(D_\parallel^e t)^{1/2}}\delta(\mathbf{r}_\perp);$$
$$\mathcal{H}\approx 0.3k_T\{[0.966+k_T^{1/2}]^2+1.82k_T\}^{-1/2}, \tag{22}$$
$$k_T=T_e/(T_e+T_i),\quad D_\parallel^e=T_e/0.51m\nu_e.$$

Here $\delta(\mathbf{r}_\perp)$ is the Dirac delta function and T_i is the ion temperature.

We recognize also that in accordance with the results of the numerical calculations of Sec. 3, the transverse dimension of the heat-energy source $\tilde{W}(r_\perp,t)$ at $a\sim a_m$ is of the order of the dimension of the inhomogeneity. This gives grounds for neglecting in the near-axis region $r_\perp \lesssim a$ the changes of the transverse dimension a. As a result, at $r_\perp<a$ Eq. (21) can be written in a form similar to that given in [1,2]:

$$\delta v_1(t)=-\mathcal{H}_1(a)\int_0^t\frac{dt'}{(t-t')^{1/2}}[\delta v_1(t')]^2+\delta v_{01}(t), \tag{23}$$

where $\delta v_1(t)$ is the maximum concentration perturbation at the temperature of the inhomogeneity ($r_\perp=0$), $t=0$ is the instant when the field E_0 is turned on, and the coefficient $k_1(a)$ is equal, according to (17) and (22),

$$\mathcal{H}_1(a)=\frac{|E_0|^2\nu_e}{NT_e u^{1/2}\mu_1}\frac{1-u^2}{(1+u^{1/2})^2}\frac{\mathcal{H}(k_T)}{16\cos\alpha(D_\parallel^e)^{1/2}}\left(\frac{2c}{\omega_H a}\right)^2\left[1+\left(\frac{2a_m}{a}\right)^4\right]^{-1}. \tag{23a}$$

We have neglected here small terms $u^{1/2}\cos^2\alpha<1$ in the denominator of $F_1(a)$ (15a). It is easy to verify that Eq. (23) has a growing solution that corresponds to the resonance instability considered here. In the asymptotic limit $|\delta v_1|>|\delta v_{01}|$ the solution takes the form

$$-\delta v_1(t)\mathcal{H}_1(a)=\frac{1}{\pi}(t_0-t)^{-1/2};\quad t_0=\text{const}. \tag{24}$$

According to (24), the resonance instability has an "explosive" character and causes a rapid increase of the negative concentration perturbations.

The solution (24) has been obtained under the condition that the transport processes across the magnetic field (electronic thermal conductivity and diffusion) are negligible:

$$\Delta t=\frac{a^2}{4D_\perp^e}>(t_0-t);\quad D_\perp^e=\frac{\nu_e T_e}{m\omega_H^2}. \tag{25}$$

Eliminating (t_0-t) from (24) and (25), we rewrite this condition in the form

$$|\delta v_1|=\frac{4\pi e^2|\delta N(r_\perp=0)|}{m\omega^2}>\frac{\delta v_1^*}{2}\frac{a}{2a_m}\left[1+\left(\frac{2a_m}{a}\right)^4\right]$$
$$\delta v_1^*=\frac{16\cos\alpha}{\pi\mathcal{H}(k_T)}\frac{(1+u^{1/2})^2}{1-u^2}\mu_1 a_m\frac{\omega_H^2}{c^2\nu_e}(4D_\parallel^e D_\perp^e)^{1/2}\frac{NT_e u^{1/2}}{|E_0|^2}$$
$$\approx 91\mu_1 a_m\frac{\omega_H^2}{c^2}\left(\frac{2T_e}{m\nu_e^2}\frac{2T_e}{m\omega_H^2}\right)^{1/2}\frac{NT_e u^{1/2}}{|E_0|^2}, \tag{26}$$

where δv_1^* is determined by the plasma parameters in the region of the upper hybrid resonance $v=1-u$. The numerical coefficient in the last expression for δv_1^* corresponds to values typical of the ionosphere, $k(k_T=0.5)=0.078$, $\cos\alpha=1$, and $u\ll 1$. The condition (26) restricts the value of the perturbation, $\delta N(r_\perp=0)$ is the function of the transverse dimension a of the inhomogeneity. It is seen that the limiting value $|\delta v_1|=\delta v_1^*$ is minimal when a is approximately equal to a_m from (16).

In the case opposite to (26), the processes of transverse transport play the decisive role, and Eq. (21) has no growing solutions—the initial inhomogeneities spread out and attenuate in time. The condition (26) determines therefore the region of the existence of the resonance instability, while the parameter δv_1^* determines the threshold concentration perturbation δv_{thr}, starting with which the resonance instability develops. The quantity δv_1^* is proportional to the gradient of the relative concentration in the layer $\mu_1=|\nabla v|$ and decreases with increasing intensity $|E_0|^2$ of the ordinary pump wave incident on the plasma. It follows from (26) that the parameter δv_1^* increases sharply with increasing electron temperature T_e. Thus, in a strongly ionized plasma with $\nu_e\sim T_e^{-3/2}$ we have $\delta v_1^*\sim T_e^{15/4}$.

According to (24)–(26), negative perturbations of the concentration with $|\delta v_1|>\delta v_1^*$ and $a\approx a_m$ increase rapidly as a result of the development of resonance instability. Suppression of this instability is due apparently to the anomalous absorption of the transverse wave as a result of the transformation into longitudinal oscillations on the developed structure of the small-scale inhomogeneities. It was shown earlier [2,8] that to obtain an appreciable anomalous absorption the mean squared perturbation of the concentration $\overline{\delta v^2}$ in the plasma should exceed the characteristic value δv_c^2:

$$\overline{\delta v^2}\geqslant\delta v_c^2=\frac{c\mu_1}{\omega}\frac{u^{1/2}}{\pi}\frac{1+u^{1/2}}{1-u^{1/2}}. \tag{27}$$

Inhomogeneities with concentration perturbations $|\delta v_1| > \delta v_c$ are broadened and then fade out. Thus, excitation of resonance instability produces in the plasma a quasi-stationary structure of inhomogeneities with transverse dimension $a \approx a_m$ (16) and with a concentration perturbation $\delta v_1 \approx -\delta v_c$. The growth time of these homogenities $(t_0 - t)$ (24) decreases with increasing initial perturbation δv_1, and the characteristic relaxation time Δt (25) is determined by the rate of the transverse diffusion.

A more detailed analysis of the dynamics of the development of the inhomogeneities shows that in the case $\beta^2 < 1$ (i. e., in the region $a \gtrsim a_m$, see Fig. 2) the inhomogeneities not only grow but are also compressed (collapse). This process has some analogy with the collapse of Langmuir waves.[15] However, the collapse noted here does not have the character of self-compression. It occurs only in an external magnetic field and is of thermal rather than of striction origin. The mechanism causing the compression is connected not with a change in pressure, but with the inhomogeneity of the heating. Only the transverse inhomogeneity dimension a is contracted. We note that the compression of the inhomogeneities in the case $a > a_m$ and the analogous broadening process at $a < a_m$ ($\beta^2 > 1$) contribute to stabilization of their transverse dimension near the value $a \approx a_m$.

5. RESONANCE ENHANCEMENT OF THE EFFECT AT $\omega \approx 2\omega_H$

We have considered above only the cold-plasma approximation for the collision absorption of longitudinal waves, since the collisionless dissociation and the additional absorption, which are connected with allowance for collisions in the thermal corrections to the dispersion equation (11), are small under the conditions $(k_\perp D_e)^2 \ll 1$. However, in the case when the wave frequency ω is close to double the gyrofrequency of the electrons $2\omega_H$ (or $n\omega_H$), the omitted dissipative processes can become substantial and even decisive. This greatly increases the heating of the inhomogeneities and leads to a more intense excitation of the resonance instability.

Just as in Sec. 3, we consider here the case of weak absorption:

$$\nu_e < |k_\parallel| v_{Te} < |\omega - 2\omega_H|; \quad v_{Te} = (2T_e/m)^{1/2}, \tag{28}$$

where v_{Te} is the thermal velocity of the electrons. In this case (see also the condition (11a)) the group velocity of the longitudinal oscillations is described as before by Eqs. (12) and (12a). It is necessary, however, to add to the left-hand side of the dispersion equation (11) an additional dissipative term[13]

$$\Delta G = -i2\left(\frac{3}{4}\right)^3 (k_\perp D_e)^2 \left[\frac{\nu_e}{\omega}(1-2u^{1/2})^{-2} + \pi^{1/2}\frac{\omega}{|k_\parallel| v_{Te}} \exp\left\{-\left(\frac{\omega - 2\omega_H}{k_\parallel v_{Te}}\right)^2\right\}\right]; \quad D_e^2 = T_e/3m\omega_H^2, \tag{29}$$

which turns out to be important at small values of the detuning $|\omega - 2\omega_H| \ll 2\omega|$. The first term in (29) corresponds to allowance for collisions to the thermal corrections of Eq. (11), while the second corresponds to collisionless cyclotron absorption of the longitudinal waves: it plays an essential role in the narrow frequency region $|\omega - 2\omega_H| \sim |k_\parallel| v_{Te}$.

When process (29) is taken into account, the total decrement of the longitudinal oscillations that are effectively excited at $\omega \approx 2\omega_H$ by an elongated inhomogeneity with a transverse dimension $a (k_\perp \approx a^{-1}$, $k_\parallel \approx k_0 u^{1/4} \cos\alpha)$ takes the form

$$\gamma = \gamma_{coll} + \Delta\gamma; \quad \gamma_{coll} = \frac{5}{8}\nu_e D^{-1}(k_\perp = a^{-1}),$$
$$\Delta\gamma = \left(\frac{3}{4}\right)^3 \left(\frac{D_e}{a}\right)^2 \left[\frac{\nu_e}{(1-2u^{1/2})^2} + \pi^{1/2}\omega\left(\frac{mc^2}{T_e \cos^2\alpha}\right)^{1/2} \times \exp\left\{-\frac{mc^2}{T_e\cos^2\alpha}(1-2u^{1/2})^2\right\}\right] D^{-1}(k_\perp = a^{-1}). \tag{30}$$

The first term, γ_{coll}, coincides with (13) and describes the collision absorption of the longitudinal waves in the cold-plasma approximation, while the second term $\Delta\gamma$ corresponds to the additional absorption, which is important only in the region of the double resonance $|\omega - 2\omega_H| \ll 2\omega_H$; the coefficient $D(k_\perp)$ is defined in (12a).

Substituting the total decrement (30) in the general relations (15) and (17), we easily obtain an expression for the heated source $\tilde{W}$ at the center of the inhomogeneity with allowance for the additional dissipation $\Delta\gamma$. It follows from (30) that for the case of small detunings

$$|\omega - 2\omega_H|/2\omega_H < \left(\frac{27}{40}\right)^{1/3} \frac{D_e}{a} \tag{31}$$

the additional absorption is $\Delta\gamma > \gamma_{coll}$, i. e., the heating of the inhomogeneities increases substantially. It is important that at $\omega \approx 2\omega_H$ the maximum heating intensity is reached not in the narrow region $a \approx a_m$, as in Sec. 3, but in the entire region of small transverse scales

$$a \lesssim a_m = \left(D_e \frac{c}{\omega_H}\right)^{1/3} \left(\frac{9}{8}\frac{1}{|1-2u^{1/2}|}\right)^{1/4}. \tag{32}$$

If the conditions (31) and (32) are satisfied, the heat source does not depend on the transverse dimension a of the inhomogeneity (cf. (15a), (17)):

$$\tilde{W}(\delta v) = \frac{|E_0|^2 2^{1/3}}{\mu_1} \frac{(\delta v_1)^2}{48\cos\alpha}\left[\frac{\nu_e}{|1-2u^{1/2}|} + 2\omega_H \pi^{1/2} |z_2| e^{-z_2^2}\right]; \quad z_2 = (1-2u^{1/2})\left(\frac{mc^2}{T_e\cos^2\alpha}\right)^{1/2}. \tag{33}$$

According to (33) $\tilde{W}$ increases with decreasing detunings $|\omega - 2\omega_H|$, first like $|1 - 2u^{1/2}|^{-1}$, and then exponentially in the region $|1 - 2u^{1/2}| \sim (T_e/mc^2)^{1/2}$. This strong increase of the intensity of the heating of the small-scale inhomogeneities should lead to an improvement of the conditions for the excitation of the resonance instability, i. e., to an enhancement of the stratification of the plasma in the double-resonance region $\omega \approx 2\omega_H$.

We note in conclusion that an analogous effect of resonance amplification of inhomogeneities as a result of an increase of their heating can be observed also at ω

$\sim 3\omega_H$. However, the additional absorption of the longitudinal waves in the region $\omega \approx 3\omega_H$ turns out to be weaker by a factor $k_1^2 T_e/m\omega_H^2$ than in the double-resonance case $\omega \approx 2\omega_H$ considered here.

6. COMPARISON WITH EXPERIMENTS IN THE IONOSPHERE

Small-scale stratification of plasma was observed following the action of high-power radioemission on the F layer of the ionosphere.[4-7] The stratification takes place only under the influence of the ordinary waves in the region of its reflection, i.e., in the resonance region (see Fig. 1). The relative perturbation of the concentration in the produced inhomogeneities $\delta N/N \sim 10^{-2}$, their characteristic scale across the magnetic field is $a \approx 0.5$ m, and along the field $H - l_\parallel > 200$ m. The condition (2) is well satisfied for these inhomogeneities. They can therefore result from the development of small-scale resonance instability. Indeed, under the conditions of the experiments[4-7] $|E_0|^2/NT_e \sim 10^{-3}$, frequency $\nu_e \approx 3\times 10^2$ Hz, $\omega \approx 3\times 10^7$ Hz (night time) and $\nu_e \approx 10^3$ Hz, $\omega \approx 5\times 10^7$ Hz (daytime), the relative gradient is $\mu_1 = |\nabla_v| = (1-2)\times 10^{-7}$ cm^{-1}. Consequently, the threshold values $\delta N/N$ of the initial concentration perturbations, needed to excite resonance instability, were in accordance with (26) $|\delta N/N|_{thr} \approx \delta_{\nu_1}^* \sim 10^{-3}$. These are relatively small values. Initial perturbations of this order can exist in the ionosphere under natural conditions. In addition, they can be generated on account of other nonlinear processes (self-focusing and drift instability[10,11], or else dissipative parametric instabilities).[17,18]

The characteristic transverse and longitudinal dimensions of the plasma stratification in resonance instability are, according to (16) and (25) and under the condition of the F-layer of the ionosphere, respectively $a \approx a_m \approx 0.5$ m and $l_\parallel \approx (4D_\parallel^e(t_0 - t))^{1/2} \approx 2$ km, i.e., they agree sufficiently well with the experimental results.

The development of resonance instability is accompanied by excitations of plasma waves as a result of linear transformation of the transverse pump waves by the quasistationary inhomogeneities of the plasma. The frequency of these waves coincides with the frequency of the perturbing transmitter. These waves were observed in experiment in the investigation of the spectrum of the scattered high-frequency signal in the form of plasma lines.[4,5] It is important that the plasma lines are very narrow, (1-10) Hz, thus fully confirming the linear mechanism of the transformation: nonlinear mechanisms would be characterized by an appreciable broadening of the plasma lines (~ 10 kHz).[19] The ratio of the intensities of the signals scattered in the plasma and in the principal lines, in accordance with the theory[2], increases rapidly with decreasing transverse dimension of the inhomogeneities.[5]

In the region $v \approx v_r = (1-u)/(1-u\cos^2\alpha)$ the wave number of the excited longitudinal oscillations increases resonantly. Therefore near $v = v_r$ collisionless absorption of the plasma waves takes place[1,2] and leads to the appearance of the experimentally observed accelerated electrons.[20]

The experimentally observed perturbations of the concentration $\delta N/N \sim 10^{-2}$ exceed the characteristic value δv_c (27). (Under the conditions of the F layer of the ionosphere we have $\delta v_c \approx 5\times 10^{-3}$.) These perturbations ensure absorption of not only the perturbing wave, but also of other "sounding" waves propagating in the perturbed zone. This "anomalous" absorption increases when the sounding-wave frequency approaches the critical frequency of the F layer. The experimentally observed dependence of the anomalous absorption on the frequency of the sounding waves[21,22] is in good agreement with the theory.[2,8]

The small-scale inhomogeneities that occur when the ionosphere is perturbed ensure an intense aspect scattering of the radio waves with frequencies $f < 500$ MHz. This phenomenon has been investigated experimentally in detail. The corresponding measurements have indeed made it possible to determine the characteristic dimension of the inhomogeneities, the perturbations of the concentration in them, etc.,.[4-7]

The nonlinear character of the instability that leads to the small-scale stratification is attested to by the nonstationary processes. If the ionosphere is unperturbed, the small-scale inhomogeneities take an appreciable time, $t = 2-8$ min after the turning on of the field, to develop. On the other hand, if the inhomogeneities have already developed, then the characteristic time of their variation with changing power of the perturbing station is much shorter, on the order of 0.1-5 sec.[4] This decrease of the time corresponds to the growth of the increment of the resonance instability at large initial perturbations of the concentration (24). The time of relaxation of the inhomogeneities after turning on the perturbing field is $t = 0.1-5$ sec, and increases with increasing transverse dimension of the inhomogeneities in proportion to a^2.[7] This points to a shrinking of the inhomogeneities and is in full agreement with the theoretical estimate (25).

The theory agrees also with the strong acceleration of the growth of the inhomogeneities when the ionosphere is perturbed by double the gyrofrequency, an enhancement observed in[4]. It must be emphasized that, as shown in Sec. 5 at $\omega \approx 2\omega_H$ there should occur not only an enhancement of the growth of the inhomogeneities, but also an appreciable change of their spectrum, manifest in the appearance of exceedingly small scales $a \ll a_m$. This should lead to an appreciable increase of the intensity of the aspect scattering for radiowaves of high frequency $f \approx 10^2-10^3$ MHz.

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[1)] We have neglected here the striction effects in comparison with thermal effects. The corresponding smallness condition takes the form $2T_e la/m\omega_H \nu_e \ll 1$, where λ and a are the characteristic dimensions of the source W[Eq. (6)] along and across the magnetic field.

[2)] If large-scale inhomogeneities are present in the plasma, the drift instability leads to the development of the small-scale drift-dissipative instability considered in[11].

[3)]We note that expression (17), which was derived here for an axially symmetrical inhomogeneity, remains valid, apart for a factor of the order of unity, also in the case of a planar inhomogeneity.

[4)]In the case opposite to (18), the heat intensity was calculated in [9].

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Neutron-diffraction investigation of the magnetic state of $Ni_{1-x}S$, $N_{1-x}S_{1-y}O_y$, and $N_{1-x}S_{1-z}Se_z$ in the metal-semiconductor phase transition region

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A neutron-diffraction investigation of the effect of oxygen and selenium on the variation of the magnetic state of hexagonal $Ni_{1-x}S$ has been carried out in the temperature range from 4.2 to 293 K, which includes the critical temperature of the phase transition from the antiferromagnetic to the paramagnetic state. It is found that the doping elements do not change the magnetic structure and the magnitude of the magnetic moment of the nickel ions. From the independence of the diffuse scattering of temperature it is concluded that, in the metallic state, all the solid solutions are Pauli paramagnets.

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The existence of correlation between the nature of the variation of the binding forces in a number of 3d-transition metal sulfides, selenides, and tellurides and the type of conduction of these compounds, which was pointed out in Refs. 1 and 2, has led the authors of these papers to infer the possibility of a directional variation of the electrophysical properties through the variation of the relation between the ionic and homopolar components of the binding forces. The latter is realized by doping these compounds with anions with electronegativity different from the electronegativity of the anions of the original compound. It was assumed that the variation of the difference between the mean electronegativities of the anions and cations can significantly change the degree of ionicity of the phases, on which quantity clearly depends the degree of localization of the valence electrons.

Experiments with pure and doped $Ni_{1-x}S$, in which the intermediate character of the electrophysical properties manifests itself most clearly,[1,2] confirmed the correctness of the stated hypothesis. It was shown

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