Particle-like solitons in superfluid ^{3}He phases

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All the types of particle-like solitons possible in the A and B phases of ^{3}He are considered with allowance for the spin-orbit interaction. These solitons do not have singularities anywhere in the field of the order parameter, and are characterized by a topological invariant and a finite dimension and energy. Asymptotic solutions of the Ginzburg-Landau equation at large distances from the soliton and at the soliton center are found. It is shown that there exists in the A phase a type of soliton possessing a momentum that stabilizes the soliton size. Like ring vortices, solitons of this type effect the transfer of momentum from the superfluid component of the liquid to the normal component.

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1. INTRODUCTION

The solitons in superfluid ^{3}He that have thus far been investigated are either moving domain walls, i.e., plane solitons (see, for example, Maki and Kumar's paper[1]), or coreless vortices, i.e., linear solitons (see Refs. 2–5). The possibility of the existence of particle-like solitons was pointed out in Ref. 6 by the present authors. These states are characterized by an integral topological invariant and a finite size, energy, and momentum. A well-known example of such states in superfluid He II is the ring vortex. However, in contrast to ring vortices, the particle-like solitons considered here do not possess singularities anywhere in the order-parameter field. The purpose of the present paper is to study in detail the structure of these states in both phases of superfluid ^{3}He with allowance for the spin-orbit interaction.

From the topological point of view there can exist in superfluid ^{3}He only two types of particle-like solitons, to each of which corresponds its own integral topological invariant. For solitons of the first type this number is the degree of the mapping of a three-dimensional sphere into a three-dimensional sphere ($S^3 \to S^3$). Solitons of this type are first considered, using as an example the B-phase (see Sec. 2), where it is easy to find an analytic solution for them. The energy of these solitons are proportional to their dimensions, and therefore they are unstable against a decrease in their size. Solitons of the first type in the A phase are studied in Sec. 3. It is shown that the superfluid-velocity field at large distances from the soliton obeys a dipole-type decay law, which indicates the possession of momentum by the soliton. The magnitude of the momentum of the soliton turns out to be proportional to the square of its dimension. Owing to the law of conservation of momentum, the soliton size gets stabilized. Such solitons also possess angular momentum directed along the momentum, and therefore they will be acted upon by the Magnus force as the superfluid stream flows around them. The appearance of a similar Magnus force acting on a solid immersed in the A phase is also considered in this section. Its appearance is connected with the fact that the stream of liquid around a body immersed in the liquid possesses nonzero angular momentum even when the body is stationary.

To solitons of the second type (see Sec. 4) corresponds the so-called Hopf invariant, which arises in the mapping of a three-dimensional sphere onto a two-dimensional sphere ($S^3 \to S^2$). Like solitons of the first type in the B phase, solitons of the second kind are unstable against a decrease of their dimensions, and can appear only in dynamical processes.

2. SOLITONS OF THE FIRST KIND. THE B PHASE

Let us consider solitons in the B phase with dimensions $R < R_D$, where R_D is the characteristic spin-orbit (dipole-dipole) interaction range (see the review article by Leggett[7]). In this case the order parameter A_{ik} is given by an arbitrary rotation matrix R_{ik}:

$$A_{ik}=\Delta(T)e^{i\Phi}R_{ik}. \qquad (2.1)$$

The components R_{ik} are functions of three parameters. These are, for example, the direction, $\boldsymbol{\omega}$, of the rotation axis and the angle, θ, of rotation. We choose as these parameters the components of the unit four-dimensional vector

$$n_\alpha=(\mathbf{n},\ n_4), \quad n_\alpha n_\alpha=\mathbf{n}^2+n_4^2=1, \qquad (2.2)$$

so that

$$R_{ik}=\delta_{ik}+2(n_in_k-\mathbf{n}^2\delta_{ik})-2e_{ikl}n_ln_4. \qquad (2.3)$$

(Here $\alpha=1, 2, 3, 4$; the Latin indices run through the values $1, 2, 3$.)

Thus, the vector $n_\alpha(\mathbf{r})$ which defines the mapping of the coordinate space of the vector $\mathbf{r}$ onto the three-dimensional sphere $n_\alpha n_\alpha=1$, is specified at each point of the vessel with the ^{3}He-B. At large distances from the soliton the order-parameter field is unperturbed: $R_{ik}(\infty)=R^0_{ik}$. Let us, for definiteness, choose $R^0_{ik}=\delta_{ik}$. This implies that to all infinitely remote points of the coordinate space corresponds the vector $n^0_\alpha=(0, 0, 0, 1)$. A three-dimensional space whose points at infinity are all equivalent is, from the standpoint of its topological structure, a three-dimensional sphere S^3 in a four-dimensional space, in the same way as a plane with identical points at infinity is equivalent to a two-dimen-

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sional sphere S^2. Consequently, we have a mapping of the three-dimensional sphere corresponding to the coordinate space onto the three-dimensional sphere $n_\alpha n_\alpha = 1$. Each such continuous mapping is characterized by the integral invariant

$$N = \frac{1}{2\pi^2} \int \frac{d^3 r}{8} e_{\alpha\beta\gamma\delta} e_{ijk} n_\alpha \frac{\partial n_\beta}{\partial x_i} \frac{\partial n_\gamma}{\partial x_j} \frac{\partial n_\delta}{\partial x_k}, \tag{2.4}$$

which is called the degree of the $S^3 \to S^3$ mapping, where $e_{\alpha\beta\gamma\delta}$ and e_{ijk} are completely antisymmetric tensors. The integral in (2.4) is equal to the area of the unit sphere $n_\alpha n_\alpha = 1$ multiplied by the number of times the vector n_α traces this sphere as the vector $\mathbf{r}$ runs through the entire coordinate space.

Let us now find for R_{ik} an expression corresponding to the degree of mapping $N = 1$. For this purpose we must solve the Ginzburg-Landau equations obtained by minimizing the gradient-energy functional:

$$E \sim \frac{\rho^s \hbar^2}{m^2 \Delta^2(T)} \int d^3 r \left\{ \frac{1}{2} \frac{\partial A_{ik}}{\partial x_l} \frac{\partial A_{ik}^*}{\partial x_l} + \frac{\partial A_{ik}}{\partial x_k} \frac{\partial A_{il}^*}{\partial x_l} \right\}. \tag{2.5}$$

On account of the spherical symmetry, let us seek the solution to the corresponding equation in the form

$$\begin{aligned} n_\alpha &= (\hat{\mathbf{r}} \sin \chi(r), \cos \chi(r)) \\ &= (\sin \chi(r) \sin \theta \cos \varphi, \sin \chi(r) \sin \theta \sin \varphi, \sin \chi(r) \cos \theta, \cos \chi(r)). \end{aligned} \tag{2.6}$$

Here r, θ, φ are spherical coordinates. Notice that the angles χ, θ, and φ, where $0 \leqslant \chi \leqslant \pi$, $0 \leqslant \theta \leqslant \pi$, and $0 \leqslant \varphi \leqslant 2\pi$, specify the position of the vector n_α on the sphere $n_\alpha n_\alpha = 1$. Thus, if $\chi(r)$ in (2.6) varies from zero to π as r varies from infinity to zero, then the vector n_α runs over the entire sphere $n_\alpha n_\alpha = 1$ once as $\mathbf{r}$ runs through the entire space. This can also easily be verified by substituting (2.6) into (2.4) and obtaining $N = 1$.

As a result of the minimization of the functional (2.5) we obtain for the function $\chi(r)$ the equation

$$\chi'' + \frac{2}{r} \chi' - \frac{1}{r^2} (3 - 2 \cos 2\chi) \sin 2\chi = 0 \tag{2.7}$$

with the boundary conditions $\chi(0) = \pi$ and $\chi(\infty) = 0$.

Equation (2.7) is a self-similar equation, i.e., its solution depends on r/R, where R is an arbitrary parameter determining the dimension of the soliton. This solution has the following asymptotic forms:

$$\chi(r) \to \begin{cases} \pi - c_1 r/R, & r \ll R \\ c_2 (R/r)^2, & r \gg R \end{cases} \tag{2.8}$$

where $c_{1,2}$ are numbers of the order of unity.

An investigation of the phase trajectories of Eq. (2.7) shows that this equation does not possess a solution with a continuous derivative and the asymptotic forms (2.8).[1)] It is, however, easy to construct a continuous solution whose derivative is discontinuous at $r \sim R$. This discontinuity can actually be neglected, since we must take into consideration at distances $\sim \xi$ from the discontinuity surface the terms of fourth order in the gradients in the Hamiltonian (2.5), which allows us to construct a solution with a continuous derivative in the entire region $0 < r < \infty$.

Because of the rapid convergence of the energy integral

$$E \sim \frac{\rho^s \hbar^2}{m^2} \int d^3 r \left\{ \left(\frac{d\chi}{dr} \right)^2 + \frac{(1 - \cos 2\chi)^2 + (1 - \cos 2\chi)}{r^2} \right\}$$

the characteristic dimension of the integration domain is of the order of R, and, consequently,

$$E \sim \rho^s R (\hbar/m)^2. \tag{2.9}$$

Since the energy is proportional to the soliton dimension, such solitons are unstable, since they can continuously reduce their radius, conserving the invariant (2.4) in the process. As soon as R becomes $\sim \xi$, the coherence length, the order parameter ceases to be described by the rotation matrix R_{ik}, and the topological invariant (2.4) ceases to have meaning. Therefore, a soliton at these distances can vanish, although the possibility of its being stable at distances $R \sim \xi$ is not to be excluded. This question, as well as the question of the possibility of the stabilization of solitons with dimensions $R \gg \xi$, e.g., by a spin current, remains open.

Let us write out the expression for the spin current j_{ik} (a current of spin S_i in the direction x_k) at large distances from the soliton:

$$j_{ik} \sim \rho^s \frac{\hbar}{m} \frac{R^2}{r^3} \left(\delta_{ik} - 3 \frac{x_i x_k}{r^2} \right). \tag{2.10}$$

3. SOLITONS OF THE FIRST KIND. THE *A* PHASE

The order parameter of the A phase has the form

$$A_{ik} = \Delta(T) V_i (\Delta_k' + i \Delta_k''), \tag{3.1}$$

where $\mathbf{V}$ is a unit vector characterizing the spin motion, while $\mathbf{\Delta}'$, $\mathbf{\Delta}''$, and $\mathbf{l} = \mathbf{\Delta}' \times \mathbf{\Delta}''$ are unit vectors characterizing the orbital motion. We shall consider the solitons in orbital motion. With that end in view, let us write the orbital part of the order parameter in the form

$$\Delta_i' + i \Delta_i'' = R_{ik}(\mathbf{r}) (\hat{x}_k + i \hat{y}_k), \quad l_i = R_{ik}(\mathbf{r}) \hat{z}_k, \tag{3.2}$$

where R_{ik} is a three-dimensional rotation matrix; $\hat{\mathbf{x}}$, $\hat{\mathbf{y}}$, $\hat{\mathbf{z}}$ are the basis vectors of the Cartesian coordinate system. Thus, there exists at each point of the coordinate space, as in the B phase, a rotation matrix that can be parametrized with the aid of the formula (2.3) in terms of the four-dimensional unit vector n_α. Consequently, as in the B phase, there exist particle-like solitons characterized by the invariant (2.4).

Using the expression for the superfluid velocity

$$\mathbf{v}^s = \frac{\hbar}{2m} \Delta_i' \nabla \Delta_i'', \tag{3.3}$$

from which follows the Mermin-Ho relation[2]:

$$(\operatorname{rot} \mathbf{v}^s)_i = \frac{\hbar}{4m} e_{ikl} \mathbf{l} \left[\frac{\partial \mathbf{l}}{\partial x_j} \frac{\partial \mathbf{l}}{\partial x_k} \right], \tag{3.4}$$

we can rewrite the invariant (2.4) in another form

$$N = \left(\frac{m}{2\pi\hbar} \right)^2 \int d^3 r \, \mathbf{v}^s \operatorname{rot} \mathbf{v}^s. \tag{3.5}$$

Let us construct the corresponding solution for $N=1$. In contrast to the B phase, the solution for the vector n_α will not be spherically symmetric, since there is a preferred direction of the vector $\mathbf{l}$ for $r\to\infty$, namely, $\mathbf{l}(\infty)=\hat{\mathbf{z}}$. Therefore, generally speaking, the solution to the Ginzburg-Landau equation should be sought in the form

$$n_\alpha=((\hat{\mathbf{z}}\cos\Theta(\theta)+\hat{\boldsymbol{\rho}}\sin\Theta(\theta))\sin\chi(r,\theta),\ \cos\chi(r,\theta)). \tag{3.6}$$

However, the nonlinear equations for $\Theta(\theta)$ and $\chi(r,\theta)$ cannot be solved analytically. Therefore, we shall consider the asymptotic behavior of the solution for $r\to\infty$ and $r\to0$.

Since $\sin\chi\to0$ for $r\to\infty$ and $r\to0$, the expression (3.6) can be rewritten in these limiting cases in the form

$$n_\alpha=(\mathbf{n},1),\ |\mathbf{n}|\ll1. \tag{3.7}$$

Retaining in the Hamiltonian (2.5) only the terms quadratic in the gradients $\mathbf{n}$, we obtain for $R<R_D$ ($\mathbf{V}=\text{const}$)

$$E\sim\frac{\rho^s\hbar^2}{m^2}\int d^3r\left\{\frac{1}{2}\left(\frac{\partial n_i}{\partial x_k}\right)^2+\frac{1}{2}\left(\hat{\mathbf{z}}\frac{\partial\mathbf{n}}{\partial x_k}\right)^2+(\text{rot}\,\mathbf{n})^2-(\hat{\mathbf{z}}\,\text{rot}\,\mathbf{n})^2\right\}. \tag{3.8}$$

Varying this functional, we obtain the equation

$$\Delta\mathbf{n}+\hat{\mathbf{z}}(\hat{\mathbf{z}}\Delta\mathbf{n})-2\,\text{rot}\,\text{rot}\,\mathbf{n}-2[\hat{\mathbf{z}}\times\nabla](\hat{\mathbf{z}}\,\text{rot}\,\mathbf{n})=0. \tag{3.9}$$

$[\hat{\mathbf{z}}\times\Delta]$

The solution has the form

$$\mathbf{n}=\hat{\mathbf{r}}\begin{cases}c_1r/R, & r\ll R,\quad c_1\sim1\\ c_2R^2/r^2, & r\gg R,\quad c_2\sim1\end{cases}, \tag{3.10}$$

i.e., the asymptotic behavior of the solution is exactly the same as in the B phase.

The superfluid velocity $\mathbf{v}^s$ for $r\to0$ and $r\to\infty$ is determined from the formula

$$m\mathbf{v}^s/\hbar=n_z\nabla n_i-n_i\nabla n_z+n_x\nabla n_y-n_y\nabla n_x$$
$$\approx-\nabla n_z=\begin{cases}-c_1\hat{\mathbf{z}}/R, & r\ll R\\ -R^2c_2[\hat{\mathbf{z}}-3\hat{\mathbf{r}}(\hat{\mathbf{z}}\hat{\mathbf{r}})]/r^3, & r\gg R\end{cases}. \tag{3.11}$$

The current density for $\mathbf{v}^n=0$, $j_i=\rho^s_{ik}v^s_k+C_{ik}(\text{curl}\,\mathbf{l})_k$ has the following form:

$$\mathbf{j}\sim\rho^s\frac{\hbar}{mR}\hat{\mathbf{z}},\quad r\ll R;\quad \mathbf{j}\sim\rho^s\frac{\hbar R^2}{mr^3}(\hat{\mathbf{z}}-3\hat{\mathbf{r}}(\hat{\mathbf{r}}\hat{\mathbf{z}})),\quad r\gg R. \tag{3.12}$$

The same velocity ($\mathbf{v}^s$) and current ($\mathbf{j}$) field distributions arise in the motion of a solid in an ideal liquid. This motion is characterized by a momentum $\mathbf{P}$ and a soliton-drift velocity $\mathbf{u}$:

$$\mathbf{P}\sim\rho^s\frac{\hbar}{m}R^2\hat{\mathbf{z}},\quad \mathbf{u}\sim\frac{\hbar}{mR}\hat{\mathbf{z}}. \tag{3.13}$$

These expressions resemble the corresponding expressions for quantized vortex rings in a superfluid liquid, except for the absence of the factor $\ln(R/\xi)$ in the expression for $\mathbf{u}$, since solitons do not possess singular cores.

As in the B phase, the soliton energy

$$E\sim\rho^s(\hbar/m)^2R. \tag{3.14}$$

Solitons in the A phase are stable owing to the coupling of the dimension R to the momentum, which, in the absence of a normal component, is conserved during the motion.

Notice in this connection that, in solving the present problem, it would have been more rigorous to have varied the functional $E-\mathbf{P}\cdot\mathbf{u}$, i.e., the functional E for a given momentum $\mathbf{P}$. Such a treatment alters the asymptotic behavior of $\mathbf{n}(\mathbf{r})$, but not the dipole character of $\mathbf{v}^s$ and $\mathbf{j}$. In this case there appears a characteristic soliton dimension $R\sim\hbar/mu$, as a result of which the scaling invariance of the Ginzburg-Landau equation is destroyed. Consequently, the solution to this equation with a continuous derivative can exist in the entire space, and, in contrast to the B phase, it is not necessary to take terms of higher order in the gradients into account.

Such solitons can arise in the presence of a flow of the superfluid component relative to the normal component, effecting, just as vortex rings do in He II, the transfer of momentum from the superfluid component to the normal component.

The influence of the soliton states on the superfluid properties of ^{3}He has been discussed by Anderson and Toulouse.[4] Although these authors considered vortices without singularities, i.e., linear solitons, the arguments they adduce in their paper are applicable to the case of particle-like solitons, especially as the above-considered soliton with $N=1$ is a vortex without a singularity bent into a ring.

Thus far we have considered solitons with $R<R_D$ in the A phase. It can be shown that the asymptotic forms of the solutions for solitons with dimension $R>R_D$ differ from those obtained for $R<R_D$ only by a scale transformation along the z axis. To wit, for $R>R_D$ it is necessary to set $\mathbf{V}\parallel\mathbf{l}$ in the order parameter (3.1) (see Ref. 7). In the process the coefficients of the free-energy expansion (3.8) change and new terms appear, with the result that Eq. (3.9) will have a more complex form. Nevertheless, the $\mathbf{v}^s$ and $\mathbf{j}$ fields at large distances retain their dipole character, with the only difference that the vector $\mathbf{r}$ in the formulas (3.11) and (3.12) should be replaced by $\mathbf{r}'=p\mathbf{z}+q\boldsymbol{\rho}$, where the constants p and q depend on the coefficients in the energy expansion.

The asymptotic expression (3.11) does not allow a judgment to be made about the magnitude of the angular momentum, $\mathbf{L}$, of the particle-like soliton:

$$\mathbf{L}=\int d^3r[\mathbf{r}\times\mathbf{j}].$$

To estimate $\mathbf{L}$, let us consider the following term of the expansion of $\mathbf{v}^s$ in powers of $\mathbf{n}$ for $r\to\infty$:

$$\mathbf{v}^s+\frac{\hbar}{m}\nabla n_z\approx\frac{\hbar}{m}e_{ijk}n_i\nabla n_j\hat{z}_k$$
$$=\frac{\hbar}{m}\hat{\boldsymbol{\varphi}}\frac{R^4}{r^5}\sin^2\theta, \tag{3.15}$$
$$[\mathbf{r}\times\mathbf{j}]_z\approx\frac{\hbar}{m}\frac{R^4}{r^4}\sin^3\theta.$$

It can be seen from (3.15) that the angular momentum has the order of magnitude

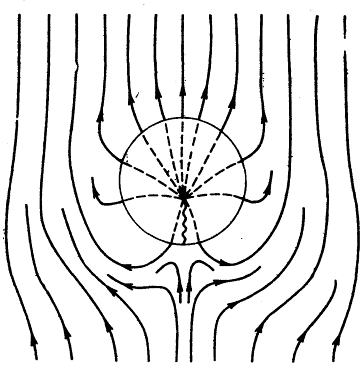

FIG. 1. Qualitative spatial distribution of the l field near a solid (a sphere). The dashed lines represent the fictitious l field inside the sphere, while the wavy line represents the vortex line.

$$\mathbf{L}\sim\rho^s\frac{\hbar}{m}R^3\hat{\mathbf{z}}. \tag{3.16}$$

This should lead to the appearance of the Magnus force when a superfluid current perpendicular to $\mathbf{l}(\infty)$ flows around the soliton. And in this respect a particle-like soliton is similar to a normal solid with dimension $R>\xi$ immersed in ^{3}He-A. Even if the body is at rest relative to the liquid, there arise near it $\mathbf{l}$ and $\mathbf{v}^s$ fields possessing a nonzero angular momentum $L\sim\rho^s\hbar R^3/m$. This is connected with the fact that we should have fulfilled on the surface of the solid the boundary condition $\mathbf{l}\parallel\boldsymbol{\nu}$, where $\boldsymbol{\nu}$ is a vector normal to the surface (see Ref. 8). This boundary condition can be satisfied with the aid of a single singular point (a simple hedgehog) in the l-vector field inside the solid. Since the l field at infinity is uniform, this hedgehog should be compensated by an anti-hedgehog on the surface of the body: an "island" of zero dimension, in Mermin's terminology[9] (see Fig. 1). In Ref. 10 the present authors showed that a hedgehog and an anti-hedgehog in the field of the vector l are connected by a vortex with two circulation quanta. This vortex is located inside the solid. Therefore, around it exists a nonzero circulation of the velocity $\mathbf{v}^s$. Consequently, if a superfluid current $\mathbf{j}^s$ flows around a solid in the A phase, then the solid will experience a Magnus force

$$F\sim\frac{\hbar}{m}Rj^s. \tag{3.17}$$

4. SOLITONS OF THE SECOND KIND

It remains for us to consider the spin solitons with dimensions $R<R_D$ in the field of the vector $\mathbf{V}$ in the A phase and the solitons with dimensions $R>R_D$ in the B phase. In both cases the order parameter is determined by a unit three-dimensional vector: $\mathbf{V}$ in the A phase and $\boldsymbol{\omega}$ in the B phase, respectively. The angle, θ, of rotation about the ω axis for $R>R_D$ is fixed by the dipole-dipole interaction, and is equal to 104°.

We are required to construct a mapping of a coordinate space with identical points at infinity, i.e., of a three-dimensional sphere S^3, onto the space of variation of the vector $\mathbf{V}$ (or ω), i.e., into a two-dimensional sphere S^2. Let us construct this mapping in the following manner. Let the field of the four-dimensional unit vector $n_\alpha(\mathbf{r})$ define an $S^3\to S^3$ mapping of degree N. Let us define the rotation matrix $R_{ik}\{n_\alpha\}$ according to the formula (2.3), and let this matrix act on a constant vector, e.g., $\hat{\mathbf{z}}$. The resulting vector will give the $S^3\to S^2$ mapping: $V_i(\mathbf{r})=R_{ik}\{n_\alpha(\mathbf{r})\}\hat{z}_k$, which is characterizable by the index N, called the Hopf invariant.

A similar mapping, called the Hopf stratification in topology, arose in the case of solitons of the first kind in the l-vector field (see (3.2)). Precisely because of this, the invariant (2.4) could be rewritten in the form (3.5), which depends only on l, a fact which can easily be verified with the aid of the relation(3.4) (see also Faddeev's lecture[11]).

The asymptotic forms of the solutions for the vectors n_α in a soliton corresponding to the Hopf invariant $N=1$ retain the form of (3.11) up to a scale transformation along the z axis, and, therefore, the spin current falls off at large distances from the soliton in the dipole fashion. Like solitons of the first kind in the B phase, solitons of the second kind are unstable, since the field of the spin variables $\mathbf{V}$ (and ω) possesses no momentum. The possibility of the production of similar states by a spin current requires a separate investigation.

In conclusion, we consider it our pleasant duty to thank S. P. Novikov for valuable consultations, as well as N. D. Mermin for sending us his preprint.

1) This circumstance, which is a consequence of the scaling invariance of Eq. (2.7), was pointed out to us by A. M. Polyakov, E. B. Bogomol'nyĭ, and V. A. Fateev.

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