

because the main process is ferromagnetic rotation of the magnetic moment. With increasing temperature, the maximum decreases and shifts to the left. At a temperature corresponding to the critical value ( $\sigma_c^{-1} \approx 0.3$ ), the type of relaxation changes: the growth of the transverse components as a result of the rotation of the magnetic moment is offset in a certain time interval by their decrease as a result of the phase randomization; the character of the relaxation corresponds in this case to Fig. 1b. At temperatures higher than  $T_c$  the relaxation of the transverse components has a paramagnetic character—the decisive mechanism is not the phase randomization.

<sup>1</sup>Naturally, the precession direction is reversed when the

sign of  $H_z$  is reversed.

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## Optical and electro-optical properties of confocal cholesteric textures

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Textures with a period greatly exceeding the equilibrium pitch  $P_0$  of the helix were observed in a cholesteric liquid crystal layer with homotropic boundary conditions at thicknesses on the order of  $P_0$ . It is shown that the known methods of measuring the pitch on a confocal texture can be used only at thicknesses that exceed the equilibrium pitch by an order of magnitude. The appearance of peaks on the voltage-contrast and transition characteristics of the transition from the cholesteric to the nematic liquid crystal is attributed to changes in the electric field of the diffraction-reflection intensities.

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A cholesteric liquid crystal (CLC) placed in a sandwich cell between glasses prepared for planar orientation, forms the well known Grandjean texture. Such "planar" boundary conditions do not disturb the helical structure of the CLC (they are compatible with it), and their influence reduces to a change in the pitch of the helix, and only in thin cells (of thickness on the order of the pitch) does the helix become completely unbound.<sup>[1]</sup> At the same time, confocal textures formed by a CLC in a cell with glasses prepared for homotropic orientation of the molecules, are quite complicated (since these boundary conditions are not compatible with the helical character of the CLC) and have not been investigated to any extent. The need for a more detailed investigation of these textures at arbitrary layer thicknesses arises, in particular, when attempts are made to explain the observed difference<sup>[2,3]</sup> between the equilibrium helix pitch, determined microscopically and from the angle of diffraction of the laser beam by the confocal structure,<sup>[4,5]</sup> on the one hand, and the pitch measured by the wedge method, on the other.<sup>[6]</sup>

The untwisting of the cholesteric helix in an electric field (the CLC → NLC transition, where NLC stands for

nematic liquid crystal) in sandwich cells also goes through a stage of formation of confocal textures,<sup>[7,8]</sup> and in this case this is satisfactory explanation of the complicated behavior of the voltage-contrast curves of the CLC → NLC transition<sup>[9]</sup> or for the reasons for the appearance of peaks on the oscillograms of the relaxation photoresponse of the untwisted helix.<sup>[10,11]</sup>

The task of the present paper is to investigate in detail the optical properties of various types of confocal textures of CLC and their variations in an electric field, for the purpose of obtaining a correct approach to a procedure for measuring the pitch of a helix on a confocal structure and the investigation of the voltage-contrast curves of CLC → NLC effect.

### EXPERIMENTAL PROCEDURE

The textures produced in a CLC layer under homotropic boundary conditions were investigated in wedge-shaped sandwich cells, which are convenient for observation of the variation of the optical properties of the layer with increasing thickness. The wedge taper was set by means of Teflon liners of various thicknesses,

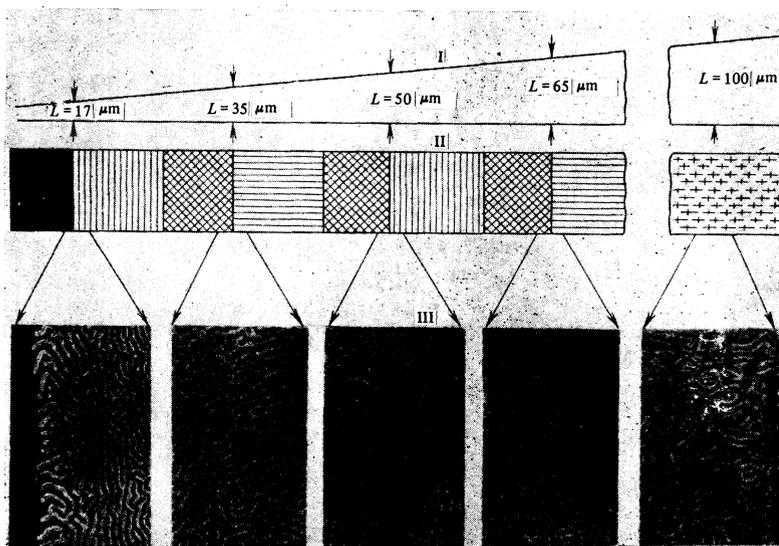


FIG. 1. Arbitrary representation (II) and photographs (III) of the periodic CLC structure observed under the microscope at different thicknesses  $L$  of a wedge-shaped layer ( $I$ ). The vertical and horizontal shadings of (II) correspond to the directions of the bands in the neighboring regions. The transition regions are shown cross hatched. The region with homotropic nematic structure is blackened. The crosses denote the region of the "fingerprint" texture. The arrows indicate the photograph texture of the wedge. The period of the textures ranges from 35 to 50  $\mu\text{m}$ ; the pitch of the helix is  $P_0 = 17 \mu\text{m}$ .

placed between the electrode glasses, and was of the order of  $10^{-3}$ . The homotropic orientation of the CLC molecules on the cell walls was ensured by a deep cleaning of the glasses.

The liquid crystals were mixtures of nematic matter with cholesteryl oleyl carbonate. The nematic component was a mixture of substance  $A^{[1]}$  (2 parts) and  $p$ -cyanophenyl ether of  $p$ - $n$ -heptyl benzoyl acid (1 part). The anisotropy of the dielectric constant of this mixture at the frequency 1 kHz was +7.6.

The investigated nematic-cholesteric mixtures had an equilibrium cholesteric-helix pitch  $P_0$  in the range from 60 to 1.5  $\mu\text{m}$ . The pitch was measured by the wedge method.<sup>[5]</sup>

Observation of the textures and their photography were carried out in a polarization microscope (with crossed polaroids). We used also a photometric setup comprising a laser ( $\lambda_0 = 0.633 \mu\text{m}$ ), a screen for the observations and measurements of the period of the structures from the position of the diffraction reflections, and a photodiode PD-2, which registered and transmitted light the intensity of the zeroth diffraction maximum.

The accuracy of the measurements of the pitch  $P_0$  and of the period of the diffraction structures was 10%.

## THE RESULTS AND DISCUSSION

The nemato-cholesteric mixture, as a function of the local thickness of the layer  $L$  and of the pitch  $P_0$  of the helix, formed the following textures:

a)  $L/P_0 \leq 1$  (left edge of Fig. 1). The "homotropic" walls have completely untwisted the cholesteric helix, and a homotropic nematic layer was produced. These observations agree with the experiments<sup>[12,13]</sup> and with the calculations.<sup>[14,15]</sup>

b)  $1 \leq L/P_0 \leq 5$  (Fig. 1). One can see regions that alternate with increasing thickness and have a characteristic texture in the form of bands whose period  $\lambda$  greatly exceeds the pitch  $P_0$  ( $\lambda \sim (3-5)P_0$ ). The bands

are seen also in unpolarized light, and their directions in neighboring regions are mutually perpendicular but are random with respect to the edge of the wedge. We observed altogether up to five such regions, separated by narrow intermediate regions having a texture either in the form of helices twisted in either direction, or in the form of a grid.

c)  $5 \leq L/P_0 \leq 10-20$  (right-hand edge of Fig. 1). Two-dimensional periodic formations are observed, similar to fingerprint textures, and the transition between the regions is smeared out. The very existence of the region is revealed by the regular oscillations of the period  $\lambda$  of the texture as a function of the local wedge thickness (see below).

In cases (b) and (c) the difference between the local thickness of the wedge, measured at the edges of any region, is of the order of the helix pitch  $P_0$ . The period of the texture in each region decreases with increasing layer thickness, reaching a minimal value  $\lambda_{\min} \sim 3P_0$  at the edge of the region facing the thick end of the wedge, and a maximum value  $\lambda_{\max} \sim (4-5)P_0$  on the edge. The value of the period averaged over the region increases slightly with increasing number of the region.

d)  $L/P_0 > 10-20$ . A strong decrease of the transparency of the CLC layer is observed, the transition from case (c) is abrupt (Fig. 2), and the confocal texture has



FIG. 2. Boundary between "fingerprint" textures ( $\lambda = 2.5 \mu\text{m}$ , left) and the confocal texture ( $\lambda = 15 \mu\text{m}$ );  $L = 55 \mu\text{m}$ ,  $P_0 = 5 \mu\text{m}$ .

a period equal to half the pitch of the cholesteric helix  $P_0/2$ .

At the present time there is no calculation of the distribution of the director of the CLC in a layer of arbitrary thickness with homotropic boundary conditions (the visualization of this distribution is in fact provided by the described textures), although it is shown in<sup>[16]</sup> that the free energy of the CLC decreases when the periodic textures are formed.

The results show that the determination of the helix pitch on the confocal texture (obtained with the aid of homotropic boundary conditions) can be effected microscopically<sup>[3]</sup> and by the diffraction angle of the laser beam only at CLC layer thicknesses greatly exceeding the equilibrium pitch ( $L > (10-20)P_0$ ), and in this case it is necessary to make independent measurements of the pitch by the wedge method<sup>[6]</sup> in order to identify the observed periods and diffraction reflections.

The above-described periodic textures vary as a function of the layer thickness and can be regarded as diffraction structures. This explains the irregular oscillations of the intensity of the transmitted light, observed by us in the case of scanning over the wedge, and having a form that depends on the apertures of the observation and illumination. These oscillations were not observed in white light because of the overlap of the diffraction spectra of the different wavelengths.

It is natural that the form of the observed textures is not influenced by the sign of the dielectric anisotropy  $\Delta\epsilon$  in the absence of an electric field. However, in accordance with the problem posed, we have investigated the electrooptics of CLC with only  $\Delta\epsilon > 0$  (a report of the behavior of CLC textures with  $\Delta\epsilon < 0$  in a field will be published later).

Superposition of an electric field on a wedge-shaped cell with CLC having  $\Delta\epsilon > 0$  causes the following texture changes: With increasing field, the boundary of the re-

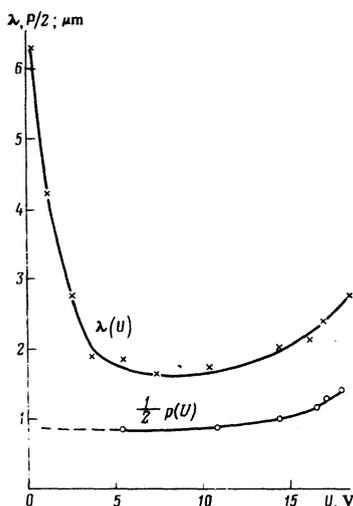


FIG. 3. Dependence of the texture periodicity  $\lambda$  and of the helix half-pitch  $P/2$  on the voltage  $U$ ;  $L = 15 \mu\text{m}$ ;  $P_0 = 1.7 \mu\text{m}$ . The total untwisting of the helix takes place at a voltage  $U_{c-N} = 25 \text{ V}$ .

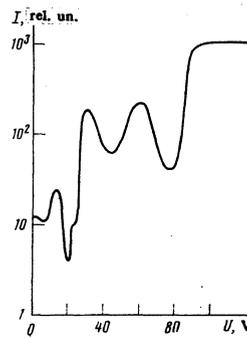


FIG. 4. Dependence of the intensity of the transmitted light  $I$  on the voltage  $U$  on the cell.  $L = 60 \mu\text{m}$ ;  $P_0 = 1.7 \mu\text{m}$ .

gion with homotropic nematic layer shifts towards the thicker part of the wedge because of the consecutive untwisting of the other regions. At thicknesses corresponding to cases (c) and (d), the boundaries between the regions in fields preceding the untwisting are practically obliterated. Both under the microscope and on the diffraction pattern one can see already two periodic structures: a texture structure with large periods, and a structure with a period equal to half the pitch of the cholesteric helix  $P/2$ . A typical plot of the period of these structures against the voltage on the cell is shown in Fig. 3, where an abrupt decrease of the texture period is observed in weak fields and an increase of the helix pitch  $P$  (and accordingly an increase of  $\lambda$ ) is observed in fields preceding the untwisting.<sup>[17]</sup>

An example of the dependence of the intensity of the transmitted light on the voltage, i. e., of the volt-contrast characteristics of the CLC-NLC transition is shown in Fig. 4. The oscillations of the measured intensity of the zeroth diffraction maximum are attributed to changes of the form of the diffraction structures with increasing cell voltage.

These oscillations explain the peaks observed by us as well as in<sup>[10,11]</sup> on the oscillograms of the transition CLC-NLC. We add furthermore that the peaks on the voltage-contrast characteristic and on the oscillogram of the transition process appear also without polaroids, if registered in monochromatic light at small apertures. When observed with large aperture and in white light, no such peaks appear.

We have thus obtained the following principal results. In the CLC placed in a sandwich cell with homotropic boundary conditions, we observed regions that alternate with increasing thickness of the layer and whose texture has a period much larger than the equilibrium helix pitch. We have shown that the methods used to measure the pitch on the confocal texture can be used only at thicknesses that exceed the equilibrium pitch by one order of magnitude. The appearance of peaks on the voltage-contrast and transition characteristics of the CLC-NLC effect is attributed, in contrast to<sup>[9-11]</sup>, to field-induced changes of the intensities of the diffraction reflections.

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## Density of states in a one-dimensional random potential

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An exact expression is found for the density of energy states in a one-dimensional semiconductor having a narrow forbidden band in the presence of impurities. An equation of the Dirac type with a random Gaussian potential is chosen as a mathematical model for the description of such a system. It is shown that at low energies the density of states has the asymptotic form  $\rho(E) \sim E^\alpha$  where the exponent  $\alpha$  is a function of the impurity concentration. This dependence is used to explain the behavior of the paramagnetic susceptibility  $\chi(T)$  at low temperatures in organic crystals of the NMP-TCNQ type.

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### INTRODUCTION

The problem of the description of the energy spectrum in quantum one-dimensional (and quasi-one-dimensional) systems containing impurities is a timely problem. First of all because of the fact that even a small violation of strict periodicity in the one-dimensional case leads to an abrupt change in the nature of the system's spectrum, leads to localization of all eigenstates,<sup>[1,2]</sup> leads to the vanishing of the system's static conductivity,<sup>[3,6]</sup> and so forth. In this article we have investigated the energy spectrum of a system in the situation when the gap between two bands in a one-dimensional semiconductor is comparable in magnitude with a random field having a correlation function of the white-noise type. If the chemical potential of the system unperturbed by the random field is located in the middle of the forbidden band (this case occurs for example when the appearance of the gap is associated with the Peierls transition in a one-dimensional lattice), the density of energy states in the forbidden band determines such thermodynamic characteristics at low temperatures as the thermal conductivity, the magnetic susceptibility, etc. This connection is traced in detail in Ref. 7.

### 1. FORMULATION OF THE PROBLEM AND DERIVATION OF THE FUNDAMENTAL EQUATIONS

It is well known that a semiconductor with a narrow forbidden band can be described by an equation of the Dirac type<sup>[8]</sup>

$$\begin{aligned} -i \frac{\partial \psi_1}{\partial x} + \Delta(x) \psi_2 &= E \psi_1, \\ i \frac{\partial \psi_2}{\partial x} + \Delta(x) \psi_1 &= E \psi_2, \end{aligned} \quad (1)$$

where  $\psi_1$  and  $\psi_2$  represent the amplitudes of particles 1 and 2 moving to the right and to the left,  $\Delta(x)$  is the potential for the interaction between particles 1 and 2. The functions  $\psi_1(x)$  and  $\psi_2(x)$  satisfy the periodic boundary condition:

$$\psi_1(0) = \psi_1(L), \quad \psi_2(0) = \psi_2(L). \quad (2)$$

In our case  $\Delta(x)$  will be a random function of the form

$$\Delta(x) = \Delta_0 + \xi(x), \quad (3)$$

where  $\Delta_0$  is a constant and  $\xi(x)$  is a random field with correlation function  $\langle \dots \rangle$  denotes the operation of sta-