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Dynamic and stochastic oscillations of soliton lattices

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The dynamics of lattices (one-dimensional sequences) of solitons is investigated. It is shown that in the stable case the soliton motion is described by the Toda lattice equation. Approximate periodic and conditionally periodic solutions are constructed, corresponding to envelope waves for the initial sequence of the solitons. In the case of an unstable lattice made up of oscillating solitons, stochastization of the motion in the system was experimentally observed. It is shown that the effect is due to a nontrivial interaction of the oscillating solitons.

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It was shown that in a preceding paper that the interaction of solitary nonlinear waves—solitons—having close energies (velocities) is approximately described by the equations for classical particles. The same paper, with two solitons as an example, dealt with the possible types of their interaction, which are determined to a considerable degree by the character of the field far from the maxima. In this paper we regard solitons as classical particles and investigate the dynamics of larger ensembles of solitons, particularly infinite sequences (one-dimensional lattices) of solitons with nearly equal parameters. This approach makes it possible to solve the problem of the stability of the stationary soliton lattices (periodic waves) and to obtain, in the stable case, a system of nonstationary solutions corresponding to modulation waves propagating along the lattice. We indicate in this connection that recently Its and Matveev, and also Dubrovin and Novikov, [2] have obtained for the Korteweg–de Vries (KdV) equation exact solutions that are periodic and conditionally periodic analogs of multisoliton solutions (see the paper of Dubrovin *et al.* [3] concerning similar solutions for the Toda lattice equation). In all probability the solutions obtained below belong to this class. At the same time, these approximate solutions are more universal in the sense that they are valid for equations which cannot be integrated exactly.

A most interesting problem is that of the evolution of the wave motion in this case of an unstable soliton lattice. The presented results of experiments with lattices made up of oscillating solitons show that instability can lead to complicated, including stochastic, motions in the system. We note that at the present time there are several known examples of nonlinear conservative systems in which stochastization evolved from regular initial conditions (see, e.g., [4,5]). The distinguishing feature of the process observed by us is that the stochastization takes place in a traveling wave. It is important also that in this case the wave motion takes the form of a random sequence of pulses that are close in their parameters to oscillating solitons, and thus constitute an example of a strong wave turbulence.

THEORY

In the description of modulated nonlinear waves it is customary to use some variant of an averaging method. [6,7] A shortcoming of this approach is that in the first-order approximation one obtains for the parameters (amplitudes, frequencies, wave numbers) of the initial waves a system of equations of the hydrodynamic type, which leads in the general case to the appearance of physically inadmissible discontinuities, and consequently to the need for taking into account higher-order approximations. In addition, it is impossible to describe within the framework of the averaging method the evolution of the high-frequency perturbations (comparable with the period of the initial wave). The way out of the difficulties is simple for waves close to a sequence of solitons, if the investigated wave motion is regarded not as a modulated periodic wave, but as an aggregate of individual "particles"—solitons—which interact with another. These processes can be described in the most general form when the interacting solitons have nearly equal velocities. The equations for the soliton velocities, as shown in[1], then take the same form as the equations of motion for classical particles:

$$dv_n/dt=\alpha_0 \sum \operatorname{sign}(n-m) f(S_{n,m}), \tag{1}$$

where v_n is the velocity of the n-th soliton in the sequence, $S_{n,m}$ is the distance between the n-th and m-th solitons, and the function $f(S)$ is determined by the asymptotic behavior of the field of an individual soliton far from its maximum; the sign and magnitude of the interaction coefficient α_0 can be obtained in each concrete case with the aid of the asymptotic method for aperiodic waves.[8]

We consider the simplest case, when the intervals $S_{n,n+1}$ between the nearest solitons differ little from a certain average value Λ_0. Then, bearing in mind the rapid (usually exponential) decrease of the field of the individual soliton ($\sim f(S)$) we can confine ourselves to the interaction between the nearest neighbors only, and thus reduce the system (1) to equations of the simplest chain of coupled nonlinear oscillators

$$d^2S_n/dt^2=\alpha_0[f(S_n-S_{n-1})-f(S_{n+1}-S_n)], \tag{2}$$

where S_n is the coordinate of the n-th soliton. Equation (2), together with the system (1), is valid if the intervals between the solitons ($\sim \Lambda_0$) exceeds substantially their characteristic dimensions ($\sim \lambda_0$). Small deviations of $S_n - S_{n-1}$ from Λ_0 can be large in units of λ_0, so that the form of the function $f(S)$ becomes important.

Most presently known solitons have an exponential asymptotic form $f(S) = \exp(-\lambda_0 S)$. Let us discuss the main consequences that follow from Eq. (2) in this case.

First, the periodic sequence of solitons, which is a stationary solution of the initial field equations, is stable if $\alpha_0 > 0$ and is unstable if $\alpha_0 < 0$, as can be easily verified by linearizing Eqs. (2) near $S_n = n\Lambda_0$. This fact is physically clear: the stable and unstable cases correspond respectively to lattices made up of mutually repelling and mutually attracting solitons. As expected, in the long-wave limit, when the difference operator in the right-hand side of (2) can be replaced by the differential operator $(\alpha_0 \partial^2/\partial n^2)$, Eq. (2) leads to Whitham's result,[6] whereby Eq. (2) is hyperbolic in the stable case for small perturbations, and elliptic in the unstable case. At the same time, owing to the discrete character of Eq. (2), short-wave perturbations are significantly affected by dispersion, which prevents singularities from appearing in the solution and ensures the existence of stationary waves.

In the stable case, Eq. (2) is the known equation of the Toda lattice, and admits of solutions in the form of periodic stationary waves (dependent on $\xi = n - vt$)[9]:

$$S_n-S_{n-1}=\Lambda_0+\lambda_0^{-1}\ln\{1+(2K\nu)^2[\operatorname{dn}^2(2K(\nu t_1-kn))-EK^{-1}]\}, \tag{3}$$

where K and E are complete elliptic integrals with modulus k; $t_1 = (\lambda_0\alpha_0)^{1/2}\exp\{-0.5\lambda_0\Lambda_0\}t$, while ν and k are connected by the dispersion relation

$$2K\nu=[\operatorname{sn}^{-2}(2Kk)-1+EK^{-1}]^{-1/2}.$$

The solutions (3) contain also solitary waves—solitons:

$$\begin{gathered} S_n-S_{n-1}=\Lambda_0+\lambda_0^{-1}\ln[1+\beta^2 \operatorname{sch}^2(\lambda_1(n-vt_1))], \\ \beta=\operatorname{sh}\lambda_1, \quad v=\lambda_1^{-1}\operatorname{sh}\lambda_1. \end{gathered} \tag{4}$$

(With respect to the initial sequence of solitons, the solutions (3) and (4) can be called stationary envelope waves). The most important properties of the Toda lattice equation is its complete integrability, which was proved by Manakov.[10] It follows therefore that the interaction of envelope solitons does not change their number and the parameters λ_1 (only phase shifts occur), and an arbitrary bounded perturbation breaks up into the asymptotically diverging solitons (4). Analogous properties of lattices of mutually repelling solitons follow also from the exact solution obtained by Kuznetsov and Mikhaĭlov[11] in the case when the initial field equation is the KdV equation.

The existence of periodic envelope waves (3) provides the key to the conception of a more extensive class of solutions. Indeed, the family of solutions (3) contains also some that are close to the sequence of envelope solitons (4). Since the envelope solitons are mutually repelling in this case, the Toda lattice equation is again valid for a sequence of such solitons. Repeating this reasoning, we arrive at a "hierarchy" of envelope waves of various orders, each of which constitutes an excitation of a soliton lattice of preceding order in the form (3). It is easy to write down an analytic expression for the envelope waves of M-th order:

$$\begin{gathered} u(x,t)=\sum U[x-v_0t-S_{n_1}(t_1)], \\ S_{n_m}-S_{n_{m}-1}=\Lambda_{m-1}+\lambda_{m-1}^{-1}\sum_{n_{m+1}}\ln\{1+\beta_m^2\operatorname{sch}^2(\lambda_m[n_m-v_m t_{m+1}-S_{n_{m+1}}(t_{m+1})])\}. \\ t_{m+1}=(\alpha_{PT}\lambda_m)^{1/2}\exp\{0.5\lambda_m\Lambda_m\}t_m, \quad m=1,2,\ldots,M. \end{gathered} \tag{5}$$

Here $u(x, t)$ is the variable of the initial field, $U(\xi = x - v_0 t)$ is the stationary soliton of the initial field, v_m and Λ_m are the average velocity and average distance for the solitons of order m in the corresponding lattice, and $\alpha_{PT}(v_m)$ is the interaction coefficient for solitons of the Toda lattice. The quantities β_m and λ_m in each order can be expressed in terms of $v_m + dS_{nm+1}/dt$ in accordance with formula (4).

Just like the periodic and conditionally periodic analogs of multisoliton solutions obtained for the KdV equation,[2] the solution (5) constitutes an M-periodic function with periods that are in general not multiples of one another. Unfortunately, in view of the complexity of the exact solutions obtained in[2], the question of a detailed comparison with the approximate solutions (5) is still open. It can be definitely stated, however, that the solutions (5) correspond to only a part of the class of the exact solutions. From the condition for the applicability of Eqs. (2) in each order, $dS_{nm+1}/dt \ll v_m$, it is clear that by letting all M periods tend to infinity, it is possible to obtain from (5) for the initial field equations only multisoliton solutions with close values of the velocities, whereas from the exact conditionally periodic solutions it is possible to obtain solutions with arbitrary velocity ratio. At the same time, for weakly modulated nonlinear waves that are close to a sequence of solitons, the results are more general, since they are not connected with exact integrability of the field equations and furthermore admit of a graphic physical interpretation. The Toda-lattice equation plays here the same role as the nonlinear Schrödinger equation for quasiharmonic waves.

A greater variety is exhibited in the dynamics of lattices made up of solitons with nonmonotonic variation of the asymptotic fields. Typical examples of this kind are the oscillating soliton (for which $f(S)\sim\exp(-\lambda_1^{-1}S)\times\cos(\lambda_2^{-1}S)$), first obtained by Kawahara[12] by numerically integrating the generalized KdV equation:

$$u_t+uu_x+\varepsilon u_x^{(3)}+u_x^{(5)}=0 \qquad (6)$$

and subsequently observed by us and Ostrovskiĭ in a line consisting of coupled nonlinear oscillators, [1] described by the equation

$$\frac{d^2}{dt^2}[q_n+\delta(q_{n+1}+q_{n-1})]=q_{n+1}+q_{n-1}-2q_n+(q_{n+1}-q_n)^2-(q_n-q_{n-1})^2. \qquad (7)$$

We note that Eq. (6) is a continual analog of (7) and can be obtained from the latter by a finite-difference expansion, with

$$\varepsilon=(12^{-1}-\varkappa)(360^{-1}-12^{-1}\varkappa)^{-1/2}, \quad \varkappa=\delta(1+2\delta)^{-1}.$$

It is clear that owing to the nonmonotonic character of the interaction potential, when the average distance between solitons changes the lattice enters successively in stable and unstable zones. The boundary between the zones is determined obviously by the zeros of the function $f(S)$. At small but finite lattice vibrations in the stable zones it is possible to obtain for the soliton sequence, just as above, solutions in the form of stationary envelope waves. The exact solution of the problem is unknown in this case, but if $f(S)$ for oscillations that are small in comparison with λ_1 and λ_2 is approximated by the polynomial $\tilde{f}(S)\sim\lambda S+(\lambda S)^2$, we can use the solutions (3) and (4), which correspond to $f\sim\tilde{f}$ in the case of small deviations of the solitons in the lattice. It is important that in this case we obtain envelope solitons with an exponential field profile in the asymptotic limit and it is possible to construct for them again a hierarchy of envelope waves of various orders.

If the distance between the solitons corresponds to an unstable zone, then small perturbations lead to lattice vibrations that capture the closest stable zones. The approximation used above no longer holds here. It is interesting, however, that if the amplitude of the oscillations is of the order of λ_2, the potential energy of the interaction of the oscillating solitons duplicate qualitatively the profile of the Leonard–Jones (LJ) potential: the relatively steep section corresponding to repulsion (at short distances between the solitons) is replaced by a gently sloping section corresponding to attraction at large distances, so that on the whole a characteristic asymmetrical potential well is produced. Numerical experiments[4] have revealed stochastization of the motion of the particles in a lattice with an LJ interaction potential. Bearing in mind the analogy between solitons and classical particles, we can expect stochastization of the motion also in a lattice made up of oscillating solitons. The experimental results reported below confirm this assumption.

In concluding this section, we discuss briefly the case of strong modulation of soliton lattices. It must be borne in mind here that solitons and classical particles differ in principle: at a large velocity difference, even for mutually repelling solitons, an instant sets in when their fields overlap completely, after which the solitons diverge, and as a result the slow soliton is outdistanced by the fast one. We note that this process can also be described approximately.[1] If we now have not one but a sequence of slow solitons, then it is clear that the fast soliton will alternately interact individually with each lattice soliton, its motion is not uniform, and the average velocity changes by an amount proportional to the phase shift in each individual interaction. Thus, in the case of strong modulation, the motion of the lattice constitutes more readily propagation of a dislocation than oscillations of solitons relative to one another. If the fast soliton is also replaced by a sequence of such solitons, then we arrive at a particular case of a strong periodic modulation of the soliton lattice. Solutions of this kind were already discussed by Zakharov[13] and Zaslavskiĭ[14]; here we wish to indicate that dislocations represent the limiting type of motion into which the considered analog waves go over in the case of strong modulation. We emphasize also that cases of both strong and weak modulation of a sequence of solitons can be made to fit the approximate description. It must be borne in mind, however, that the solitons can interact inelastically. The radiation accompanying this interaction[15,16] can lead either to a finite lifetime of the excitations of the soliton lattice, or to a rapid destruction of the solitons, depending on the ratio of the radiation energy to the energy of the corresponding type of lattice motion.

EXPERIMENT

The experimental investigation of one-dimensional soliton lattices was carried out on transmission line consisting of nonlinear electromagnetic oscillators, described by Eq. (7), with a mutual inductance coupling between the neighboring elements. The use of this system is quite convenient, for by varying the coefficient of the mutual inductive coupling δ it is possible to obtain in this system solitons with different fields in the asymptotic result[1] and thus investigate soliton solutions of various types. To produce soliton lattices, a line of

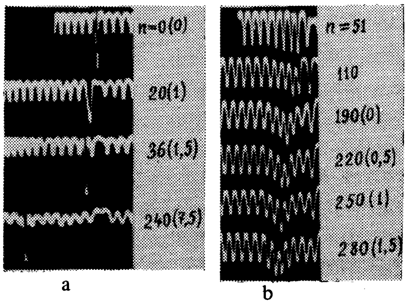

FIG. 1. Dislocation motion (a) and formation of an envelope soliton (b) in a stable soliton lattice. The numbers indicate the distance (the number of elements) traversed by the lattice along the line, while the numbers in the parentheses represent the distance (in terms of soliton-lattice cells) traversed by the dislocation and by the envelope soliton.

160–280 elements was excited on one end by a source of periodic oscillations, and was matched on the other end, so that a traveling-wave regime was established in the system (the reflection level did not exceed 10% of the wave amplitude). The evolution of the lattice motion could be easily traced by observing the successive (from element to element) changes in the time dependence of the oscillations of the voltage on the nonlinear capacitance of the elements.

At $\delta = 0$ the solitons for the line (7) have an exponential asymptotic form and are mutually repelling.[1] In the experiments, a line of such oscillators was excited by a parametric generator of a sequence of pulses[17] that were close to solitons in their parameters. In addition, a supplementary source synchronized with a first source produced different perturbations of the soliton lattice. The results of the two experiments are shown in Fig. 1. The first corresponds to the case of dislocation propagation. The perturbation represents in this case one lattice soliton magnified 3.5 times. It is known that at so large an amplitude ratio, one soliton outdistances the other. As seen from Fig. 1a, the evolution of the initial perturbation agrees with this process: the fast solitons absorb successively the lattice solitons and then emits them with the appropriate phase shift. The process is extremely localized—at each instant of time one lattice soliton interacts with a dislocation, and practically no dispersion of the soliton lattice appears.

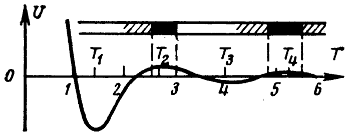

FIG. 3. Regions of stable (light gaps), unstable (shaded) and stochastisized (black) soliton lattices as functions of their period T. The profile of the potential energy of the interaction of the oscillating solitons is shown in the same T scale.

With decreasing perturbation of the soliton amplitudes, the time of the soliton interaction increases and others can enter into the interaction even before the completion of the collision of any particular pair. As a result, the process becomes essentially nonlocal, and we arrive in natural fashion to the case of soliton oscillations in the lattice. A process of this type is shown in Fig. 1b. The initial perturbation consists in this case of four lattice solitons that increase successively in amplitude. These solitons come initially closer together, since the velocity perturbation produces a displacement perturbation. Owing to the strong overlap of the field of the approaching solitons, a "pedestal" appears subsequently and duplicates the profile of the envelope of the crests of the solitons. Some increase of the slope of the envelope front, which takes place initially, then ceases because of the dispersion of the soliton lattice, and at a distance $n = 190$ there is formed an envelope soliton, the stationarity of which can be verified by observing the successive propagation of this wave. The envelope soliton profile, measured by determining the ratio of the amplitudes of the lattice solitons, agreed well with the theoretical value (4) (the differences did not exceed 12%).

We note that to observe the envelope soliton of next order it would be necessary to have a line of electromagnetic oscillators consisting of not less than 10^4 elements. This estimate follows from a comparison of the velocities of the lattice and the envelope solitons (Fig. 1b).

At $\delta = 0.175$, the solitons in the line (7) have an asymptotic limit of alternating sign: $f(S) = \exp(-\lambda_1^{-1}S) \times \cos(\lambda_2^{-1}S)$.[1] To obtain lattices of such solitons it was more convenient to use a harmonic source. As seen

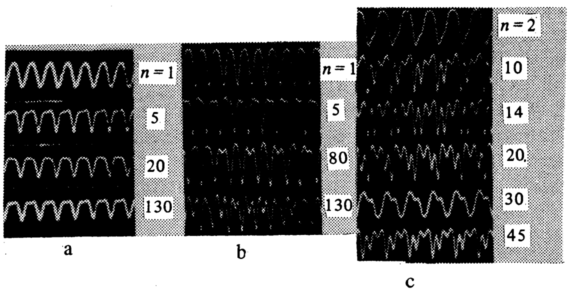

FIG. 2. Evolution of sinusoidal wave as a function of its initial period T: a) $T = 3.75$—onset of stationary lattice, b) $T = 4.5$—growth of oscillations in unstable lattice, c) $T = 7$—formation of complicated soliton lattice.

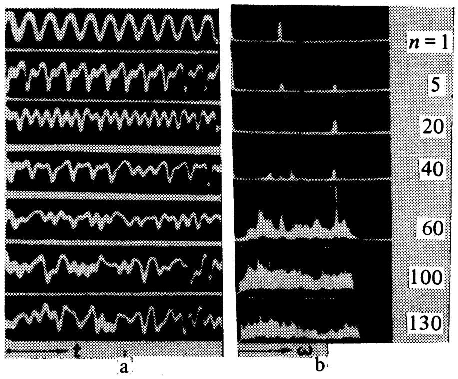

FIG. 4. Evolution of the profile (a) and of the spectrum (b) of a monochromatic wave at $T = 5$.

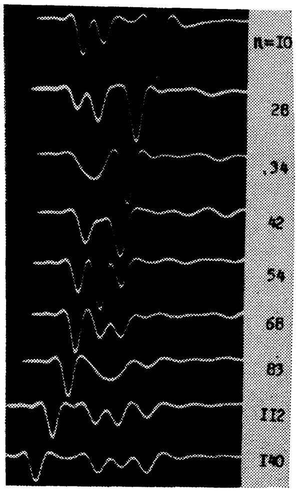

FIG. 5. Appearance of a third soliton in a pair after the latter interacts with a single soliton.

from Figs. 2a and 2b, even at a short distance from the start of the line ($n = 5$) the initially sinusoidal wave is transformed into a sequence of oscillating solitons having the same period. By varying the frequency of the oscillations of the external source it is possible to obtain soliton lattices with arbitrary periods T, up to values corresponding to the third stable zone reckoned from the maximum of the soliton (Fig. 3). At $T > T_4$, the sinusoidal wave is split into complicated lattices consisting of individual solitons and groups of bound solitons (Fig. 2c).

It can be seen from Fig. 3 that the regions of T where stable and unstable stationary lattices can exist do indeed correspond to zones of the mutual repulsion and attraction of the solitons. The propagation of a stable stationary lattice ($T = 3.75$ here and below in units of Eq. (7)) and the growth of the amplitude of the oscillations in the unstable lattice ($T = 4.5$) are illustrated in Figs. 2a and 2b.

The most remarkable feature of the unstable lattices is the possibility of their stochastization at excitation energies comparable with the depth of the potential well. The periods of the lattices occur then in the region of the edges of the potential wells (see Fig. 3). The very fact of stochastization, as already noted, agrees with the results of a numerical calculation[3] for a lattice with an LJ interaction potential. However, according to the numerical results of[3], the critical excitation energy, above which stochastic oscillations were noted, was only 1–2% of the depth of the potential well per particle. In our case this quantity is of the order of 100%. The apparent reason for the difference is that at excitation energies close to critical, the stochastization time is very large and greatly exceeds the time required for the soliton to travel over a line of 160 elements.

The development of stochastic motions in an unstable lattice is shown in Fig. 4. It is seen that at a distance $n > 60$ the spectrum of the wave motion becomes continuous and occupies practically the entire transparency band of the system. The very appearance of the continuous spectrum was in fact the criterion of the stochastization of the motion in the experiment.[1)] We note that in systems described by the KdV equation, the evolution of a sinusoidal wave is always reversible.[18,19]

The explanation of the stochastization must be sought in the elementary processes of the interaction of the oscillating solitons, inasmuch as after the regular lattice is destroyed the wave motion constitutes as before a sequence of solitons and of bound soliton groups, but now the sequence is random (see Fig. 4). To study the interaction processes, individual solitons and bound soliton pairs were applied to the line from synchronized sources. The initial amplitudes and the intervals between the solitons could be varied over a wide range. It was established in these experiments that, owing to the oscillatory structure of the field, bound states are possible of two or more solitons coupled by the first and second oscillations (counted from the maxima). In the case of solitons that diverge without limits after the interaction, or of groups of such solitons, we observed a redistribution of the number of solitons among the groups, production of new bound states, and also a redistribution of the energy among interacting formations whose composition did not change. One of the variants of a nontrivial collision of solitons is shown in Fig. 5.

Thus, considerable relative displacement of the solitons, which occur in a strongly excited lattice, can lead to the collisions described above and as a result to a scrambled character of the motion as a whole. Under optimal conditions, the stochastization of the motion developed quite rapidly—within a time in which each solitons experienced on the average one or two collisions (in analogy with the case of thermalization of the gas of particles).

We indicate in conclusion that the excited lattices and stochastic ensembles of oscillating solitons, described above, can be realized for magnetosonic waves in a plasma, and also for capillary gravitational waves in shallow water, the propagation of which is described by the generalized KdV equation (7).[12]

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1) We note that in a system described by a KdV equation the evolution of a sinusoidal wave is always reversible.

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Fluctuations of plasmons localized in a plasma by an electromagnetic pump wave

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It is shown that spatially localized plasma waves that do not increase with time are produced in a spatially inhomogeneous plasma in the vicinity of the region where a pump wave decays into two plasmons. A theory of the stationary fluctuations of these spatially localized waves is formulated. The fluctuations are shown to undergo a critical growth when the instability threshold is approached. A collision integral that takes into account the influence of the localized plasma waves is obtained.

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A number of recent studies[1–4] have demonstrated a qualitative peculiarity of parametric resonance in a spatially inhomogeneous plasma, namely, it was shown that instabilities localized in a finite region of the inhomogeneous plasma can develop in the plasma. It must be assumed at the same time that the appearance of spatially localized plasma perturbations under the influence of a pump field is a rather common phenomenon that can take place in a stable plasma.

In this paper we investigate the conditions under which such waves are spatially localized in the case of an inhomogeneous plasma in which a pump wave decays into two high-frequency electron waves, henceforth dubbed plasmons.

In Sec. 1 we consider the dispersion properties of the plasmons and the region of their localization. It is shown that plasmon localization is possible at a pump-wave amplitude lower than the threshold obtained in[3] for plasma instability relative to two-plasmon decay.

In Sec. 2, following the method of[5,6], we formulate a theory for stationary fluctuations of plasmons localized in a parametrically stable plasma.[1)] As a consequence of the developed theory, we determine in Secs. 3 and 4 the energy of the thermal fluctuations of the plasmons and obtain the collision integral of a non-uniform plasma with allowance for its dynamic polarization.

1. In a non-uniform plasma, the electron Langmuir perturbations (plasmons) that are parametrically excited by a pump wave

$$E_y(x,t)=E_0\sin\left(\omega_0 t-\int k_0(x)\,dx\right),\quad B_z(x,t)=-c\int_{-\infty}^{t}dt'\frac{\partial}{\partial x}E_y(x,t') \tag{1}$$

are localized on the profile in the vicinity of a point x_q at which the plasma density is equal to one-quarter the critical density, i.e., $\omega_{Le}(x_q)=\omega_0/2$, where ω_{Le} is the electron Langmuir frequency. The dimension of the localization region is determined by the amplitude E_0 of the pump wave. Under conditions when the plasma is not uniform along the x axis, this region is bounded by the branch points $x_\pm$ of the plasmon wave vector; the formula for these points is

$$x_\pm=-6k_y^2r_D^2L_N\pm\left[k_0^2r_E^2-\left(4\frac{\gamma+\tilde\gamma}{\omega_0}+4\frac{\delta\omega}{i\omega_0}+3ik_yk_0r_D^2\right)^2\right]^{1/2}L_N. \tag{2}$$

Here r_D is the Debye radius of the electron, $r_E=eE_0/m\omega_0$ is the amplitude of the electron oscillations in the electric field of the pump wave, $\tilde\gamma$ is the plasmon damping decrement, and k_y is the projection of the plasmon wave vector on the y axis (it will be assumed for simplicity that the vector $\mathbf{k}$ lies in the xy plane). It is assumed that the plasma density profile depends linearly on the coordinate x, with a characteristic inhomogeneity length L_N, i.e.,

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