Effects that hinder the observation of parity nonconservation in hydrogenlike and heliumlike ions

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The necessary conditions on the densities and external field strengths for the successful performance of certain previously proposed experiments for observing effects of parity nonconservation in hydrogenlike atoms and two-electron ions are discussed and the necessary measuring times are estimated. It is shown that the required conditions can actually be met only for the experiment with the Cu XXVIII ion.

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Effects of parity nonconservation in radiative atomic transitions due to the weak interaction between electrons and nucleons have been repeatedly discussed in recent years (see the review articles by Alekseev *et al.*,[1] Moskalev *et al.*,[2] and Gorshkov[3]). Specifically, it has been suggested[4,5] that parity violation might be observed in the radiation from the $2s_{1/2}\to 1s_{1/2}$ transition in the hydrogen atom and the $2^1S_0\to 1^1S_0$ transition in two-electron ions. However, the authors of Refs. 4 and 5 did not consider certain phenomena (or did not consider them in sufficient detail) that make such experiments more difficult or impossible. These phenomena will be discussed in this paper.

I. ALLOWANCE FOR COLLISIONS

As a result of collisions of the beam ions (atoms) with one another, with residual-gas ions (atoms),[1)] and with stray electrons, the $2s_{1/2}(2^1S_0)$ state may be de-excited with emission of photons in the continuous spectrum. The effect we are interested in—the radiation of an isolated ion—can be observed provided

$$\int\limits_{\omega_0-\omega_D/2}^{\omega_0+\omega_D/2}\frac{dW}{d\omega}d\omega\leqslant W_0, \tag{1}$$

where W and W_0 are the probabilities for one-photon de-excitation of the ion with and without allowance for collisions, $\omega_0=E_i-E_f$ where $E_{i,f}$ are the energies of the excited and ground states of the ion ($\hbar=c=1$), and ω_D is the Doppler width.

In estimating W it is sufficient to take the nearest intermediate states into account: the $2p_{1/2}$ state for the one-electron case, and the 2^1P_1 and 2^3P_1 states for the two-electron case. Thus we have the following expression for the probability for de-excitation of a hydrogen atom in collisions with residual-gas particles:

$$dW\sim v_0\rho\sigma W_p d\omega/[(\omega_0-\omega-L_p)^2+\Gamma_p^2/4], \tag{2}$$

where ρ is the residual-gas particle density, v_0 is the beam velocity, σ is the cross section for collisions accompanied by the $2s_{1/2}\to 2p_{1/2}$ transition, $L_p=E(2s_{1/2})-E(2p_{1/2})$, W_p is the probability for one-photon de-excitation of the $2p_{1/2}$ state, and Γ_p is the width of the $2p_{1/2}$ level. The quantity $n=\omega_D/\omega_0$ can be reduced[6] to values of the order of 10^{-5}.

We can write a similar formula for a two-electron ion:

$$dW\sim v_0\rho\sigma_k a_k^2 W_{pk}d\omega/[(\omega_0-\omega-L_{pk})^2+\Gamma_{pk}^2/4], \tag{3}$$

in which k assumes the two values 1 and 3 corresponding to the 2^1P_1 and 2^3P_1 intermediate states of the ion, $a_1=1$, and

$$a_3=\frac{\langle 2\,^3P_1|V_{LS}|2\,^1P_1\rangle}{E(2\,^3P_1)-E(2\,^1P_1)},$$

where V_{LS} is the spin-orbit interaction operator (Fig. 1).

For collisions of beam particles with neutral particles we can put $\sigma\sim\pi\eta^{-2}$, where $\eta=m\alpha Z$ is the average momentum of the ground state of an atom with nuclear charge Z. For collisions of atoms with charged particles we have $\sigma\sim\pi\eta^{-2}v^{-2}\alpha^2$ (see Ref. 7).

From formulas (1)–(3) we obtain upper bounds for the residual-gas particle density ρ and the atomic density ρ_0 of the beam. We note that there are no practical limitations on the density for ion beams. The wave function for the relative motion of the ions contains the factor $(1+e^{\pi\alpha Z/v})^{-1/2}$ on account of the Coulomb interaction. For the values $\sim 10^{-3}$–10^{-4} characteristic of v, the factor $(1+e^{\pi\alpha Z/v}$ that appears on the right in Eq. (3) is very small as compared with unity, and condition (1) is satisfied for all realistic values of ρ_0.

The limitations on ρ and ρ_0 are discussed in Gorshkov's review.[3]

II. EXTERNAL FIELDS

Let us suppose that there are stray external fields involved in the experiment: electric fields D and magnetic fields H. The de-excitation of the $2s_{1/2}$ and 2^1S_0 states are described in the nonrelativistic approximation by the graphs of Figs. 2 and 3, where HF and W

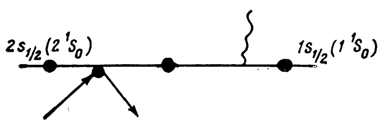

FIG. 1.

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represent the hyperfine and weak interactions of the electrons with the nucleus. The amplitude can be expressed in the form

$$F = F_a + F_b + F_c + F_d. \qquad (4)$$

When $H = D = 0$ we have $F_c = F_d = 0$; then the parity nonconservation effects arise from the interference of graphs a and b (see Figs. 2 and 3), and $F_b \ll F_a$. The fields D and H also stimulate de-excitation of the excited states (graphs c and d in Figs. 2 and 3). The corresponding terms in the amplitude are

$$F_{c(d)} = \langle \psi_f | A^* G V_{D(H)} | \psi_i \rangle, \qquad (5)$$

where $\psi_{i,f}$ are the wave functions for the initial and final states of the ions, A and $V_{D(H)}$ are the operators for the interaction of the electron with the photon field and with the electric and magnetic fields, and G is the Green's function. Neglecting the interaction between the electrons (which is small of the order of Z^{-1}) we use the wave function in the Coulomb field of the nucleus. For the operators $V_{D(H)}$ we have

$$V_D = -e(\mathbf{rD}), \qquad (6)$$

and

$$V_H = \frac{e}{2m}(\mathbf{L}+2\mathbf{S}, \mathbf{H}) + \frac{e^2}{8m}[\mathbf{H} \times \mathbf{r}]^2 . \qquad (7)$$

Formula (7) is obtained by substituting the vector potential

$$\frac{1}{2m}(\mathbf{p} - e\mathbf{A})^2 + eA_0 - \frac{e}{2m}(\boldsymbol{\sigma}\mathbf{H})$$

for a uniform magnetic field into the Pauli approximation

$$\mathbf{A} = {}^1\!/_2 [\mathbf{H} \times \mathbf{r}] \qquad (8)$$

to the Hamiltonian for the motion of the electron in the external field.

In the case of a two-electron ion the main contribution to G comes from the nearest intermediate states of parity opposite to that of the initial state, i.e., from the 2^1P_1 and 2^3P_1 states; then for G we can use the sum

$$\sum_{k=1,3} \frac{|\psi(2\,^kP_1)\rangle\langle\psi(2\,^kP_1)|}{E(2\,^kP_1) - E(2\,^1S_0)}$$

in which the contribution from the $k = 3$ term vanishes unless account is taken of the admixture of the 2^1P_1 state to the 2^3P_1 state as a result of the spin-orbit interaction:

$$\langle\psi(2\,^3P_1)|' = \frac{\langle\psi(2\,^3P_1)|V_{LS}|\psi(2\,^1P_1)\rangle}{E(2\,^3P_1) - E(2\,^1P_1)} \langle\psi(2\,^1P_1)|.$$

$$F_c = \frac{\alpha^{1/2}(\alpha Z)^3 W_{p3}{}^{1/2}(\mathbf{eD})}{4\cdot 2^{1/2} L_{p3}[E(2\,^3P_1) - E(2\,^1P_1)]} + \frac{3\alpha^{1/2} W_{p1}{}^{1/2}(\mathbf{eD})}{L_{p1}\eta} . \qquad (9)$$

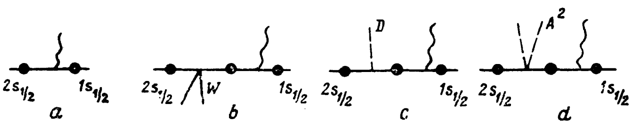

FIG. 2.

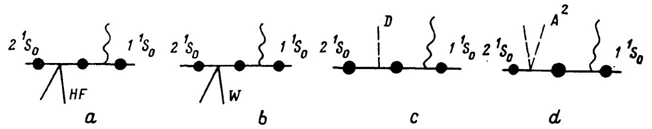

FIG. 3.

The expression for F_c for the one-electron case was obtained by Azimov *et al.* [4]

Turning to graph d, we see that the first term in formula (7) does not contribute to the amplitude since the spin of the initial state and the orbital angular momenta of the initial electrons are zero. The second term in (7) mixes states with orbital angular momentum $l = 2$ to the initial state without changing the spin. A calculation of F_d using the nonrelativistic Coulomb Green's function[8] gives

$$F_d = 8\pi^{1/2}\alpha^{9/2}\frac{H^2}{m}\frac{\omega_0}{\eta^4}(\mathbf{hn})(\mathbf{he}), \qquad (10)$$

where $\mathbf{h} = \mathbf{H}/H$ and $\mathbf{n}$ is a unit vector in the direction of the photon momentum.

In order to be able actually to perform the experiments proposed in Refs. 4 and 5, the fields D and H must be such that the following conditions are satisfied

$$F_c \leqslant \max(F_{a,b}), \qquad (11)$$

$$F_d \leqslant \max(F_{a,b}). \qquad (12)$$

In addition, account must be taken of the fact that an electric field D can be induced by the magnetic field:

$$\mathbf{D} = [\mathbf{v} \times \mathbf{H}] , \qquad (13)$$

where v is the particle velocity. An upper bound for D was calculated by Azimov *et al.* [4] and estimated by Gorshkov and Labzovskiĭ[5] from condition (11). For realistic velocities v, one obtains more rigid conditions on H from (13) than those obtained in Refs. 4 and 5 from the condition that the effects simulating parity nonconservation be small. Graph d was not taken into account in Refs. 4 and 5, although it substantially alters the situation for experiments in a strong magnetic field.[5] When H is large, Eq. (13) leads to a limitation on the transverse velocity component $v_\perp$.

III. MEASURING TIME

If the magnitude of the parity nonconservation effects is $\sim P$, the number N of recorded photons will be

$$N = \rho_0 V \nu P W_0 \tau, \qquad (14)$$

where V is the working volume, τ is the observation time, ρ_0 is the density of ions in the necessary state, and ν is a coefficient characterizing the efficiency of the experiment ($\nu \sim 10^{-3}$–10^{-6}). The recording of $P^{-1/2}$ photons must be interpreted as a fluctuation in the fundamental decay mode. If we require that $NP^{1/2} \sim 10$, we obtain

$$\tau \sim 10(\rho_0 V \nu W_0 P^{3/2})^{-1}. \qquad (15)$$

For most of the experiments proposed Gorshkov and Labzovskiĭ[5] τ turns out to be unrealistically long.

Now let us see what conditions (1), (11), (12), and (15) lead to for the cases discussed earlier in Refs. 4 and 5.

1. The H atom

A. For experiments in which no external magnetic field is applied and the characteristic values in $n \sim 10^{-5}$ and $v_0 \sim 10^{-3}$ obtain, we find: $\rho_0 \lesssim 10^5$ atoms/cm^3, $\rho \lesssim 10^3$ atoms/cm^3, $\rho \lesssim 10$ ions/cm^3, and $\rho \lesssim 10$ electrons/cm^3. Then the measuring time is $\tau \sim 10^7 \nu^{-1}$ sec (here and below we assume that $V \sim 1$ cm^3).

If the component of the velocity v perpendicular to the beam velocity is $v_\perp \sim 10^{-5}$, the limitation on D found in Ref. 4 leads to the following condition on the allowable magnetic field strength H, which is more stringent than the corresponding condition found in Ref. 4: $H \lesssim 10^{-4}$ G (in any case the component of H perpendicular to the beam velocity must satisfy the condition $H_\perp \lesssim 10^{-4}$ G; the condition on the component parallel to the beam velocity is $H_\parallel \lesssim 10^{-2}$ G).

B. If the experiment is performed in an external field of strength $H = 10^3$ G, [4] then $\rho_0 \lesssim 10^3$ atoms/cm^3, $\rho \lesssim 10$ atoms/cm^3, and $\rho \lesssim 100$ ions/cm^3; in this case, from the condition $D \lesssim 10^{-9}$ V/cm[4] there follows the condition $v_\perp \lesssim 10^{-14}$.

2. The He atom

A. For $H=0$, $n \sim 10^{-5}$, and $v_0 \sim 10^{-3}$, we obtain $\rho_0 \lesssim 10^7$ atoms/cm^3 for the beam density and $\rho \lesssim 10^4$ atoms/cm^3. The conditions on the stray fields are $D \lesssim 10^{-4}$ V/cm and $H \lesssim 5.10^{-4}$ G. Then from (15) we find $\tau \sim 10^{10}$ years.

B. The 2^3P_1 and 2^1S_0 levels cross at $H = 3.10^7$ G. If in this case $L_{p3} = (\beta/2)\Gamma_{p3}$(see Ref. 5), then $F_a \approx F_b$. In such an experiment, however, one cannot observe $P \sim 1$, as was proposed in Ref. 5. As is evident from (10), $F \sim F_d \sim 10^5 F_{a,b}$ for such values of H. Here the principal P-odd effect is interference between graphs b and d. The probability for the process is

$$dW = W_0\{1+(\mathbf{nh})[1-(\mathbf{nh})^2]P\}\frac{d\Omega}{4\pi}, \quad W_0 = \frac{2\alpha m H^4}{Z^2 m^8}. \tag{16}$$

and for $L_{p3} = \frac{1}{2}\Gamma_{p3}$, P is maximal ($P \approx \frac{1}{4}$ in Weinberg's model[9]). Then $\tau \sim 0.1$ sec and $\rho_0 \lesssim 10^8$ atoms/cm^3. Unfortunately, it follows from (11) that $D \lesssim 4 \cdot 10^{-8}$ V/cm, and this leads to the condition $v_\perp \lesssim 10^{-17}$, which cannot be satisfied.

The 2^3P_1 and 2^1S_0 levels cross at $H = 6 \cdot 10^7$ G (the second root of the secular equation in Ref. 5), and the 2^1P_1 and 2^1S_0 levels cross at $H = 10^8$ G. The interference between graphs b and d gives $P \sim 10^{-2}$ in the first case and $P \sim 10^{-6}$ in the second case. The respective conditions on the electric field are $D \lesssim 10^{-3}$ V/cm and $D \lesssim 10^{-1}$ V/cm, and these lead to the conditions $v_\perp \lesssim 10^{-13}$ and $v_\perp \lesssim 10^{-11}$.

3. The CV ion

A. For $H=0$ the conditions on the densities (when $v_0 \sim 10^{-3}$ and $n \sim 10^{-5}$) are $\rho \lesssim 10^9$ atoms/cm^3 and $\rho \lesssim 10^7$ electrons/cm^3. The limitations on the stray fields are $D \lesssim 0.2$ V/cm and $H_\perp \lesssim 1$ G ($H_\parallel \lesssim 100$ G). For the characteristic values $\rho_0 \sim 10^5$ ions/cm^3 and $\nu \sim 10^{-3}$, we find $\tau \sim 6 \cdot 10^6$ years.

B. The 2^1S_0 and 2^3P_1 levels cross at $H = 10^6$ G with $L_{p3} = (\beta_3/2)\Gamma_{p3}$; then $F_a \approx F_b \approx 4F_d$. The limitations on the densities are $\rho \lesssim 5 \cdot 10^3$ atoms/cm^3 and $\rho \lesssim 5 \cdot 10$ electrons/cm^3. Then the measuring time is $\tau \sim 1.3$ hours. The condition on the field is $D \lesssim 10^{-6}$ V/cm, from which follows the condition $v_\perp \lesssim 3 \cdot 10^{-15}$. Now if we make $L_{p3} \sim \frac{1}{2}\Gamma_{p3}$, the probability of the process increases by four orders of magnitude ($F \sim F_b$ and $P \sim F_b F_d / F_b^2 \sim 10^{-3}$); however, the conditions on D and $v_\perp$ remain as before.

4. The CuXXVIII ion

In this case, when $v_0 \sim 10^{-3}$ and $n \sim 10^{-5}$, the conditions on the densities are $\rho \lesssim 5 \cdot 10^9$ atoms/cm^3 and $\rho \lesssim 5 \cdot 10^7$ electrons/cm^3. For $\rho_0 \sim 10^5$ ions/cm^3 we have $\tau \sim 0.1 v^{-1}$ sec, i.e., $\tau \sim 2$ min if $v \sim 10^{-3}$. The conditions on the stray fields are $D \lesssim 2$ V/cm, $H_\perp \lesssim 7$ G, and $H_\parallel \lesssim 700$ G. At $H = 700$ G, the magnitude of the effects that imitate parity nonconservation is

$$Q \sim \frac{\mu H}{L_p}\frac{\Gamma_p}{L_p}\frac{\mu H}{L_s} \sim 10^{-10} \sim 10^{-6} P \ll P.$$

Formula (15) does not take into account the decrease in the density of ions in the 2^1S_0 state on traversing the distance x from the point of excitation to the point of observation: $\rho_0(x) = \rho_0(0)\exp(-x/v_0\tau_0)$. Under conditions (1) and (11) the lifetime τ_0 of the 2^1S_0 state is determined by two-photon decay: $\tau_0 \approx 1.5 \cdot 10^{-10}$ sec.[10] If $v_0 \sim 10^{-3}$, distances $x \lesssim 10^{-2}$ cm are allowable.

Thus, it is impossible to measure the effects of parity nonconservation in hydrogen atoms and CV ions by using an external magnetic field to bring the levels closer together because the necessary conditions on the transverse velocity $v_\perp$ of the beam particles and, in the case of hydrogen, on the densities, cannot be met. The corresponding experiment with helium atoms is impossible in principle because of the de-excitation of the atoms in the magnetic field; the modification of the helium experiment described above is also impossible because of limitations on $v_\perp$. Experiments in which no magnetic field is used to bring the levels together require unrealistically long measuring times in the case of hydrogen and helium atoms and CV ions, and for hydrogen atoms such experiments also impose severe conditions on the external fields and the densities. Only the conditions required for experiments with CuXXVIII ions turn out to be actually satisfiable.

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1) For simplicity we shall speak of ions.

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Bremsstrahlung in collisions between electrons and atoms

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General formulas have been obtained which express the contribution of atomic electrons to the bremsstrahlung and absorption cross sections in electron-atom collisions in terms of the dynamic atomic polarisibility. Application of these formulas enables experimental data on the optical breakdown threshold of alkali metal vapors to be brought into qualitative agreement with the cascade theory of breakdown.

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1. Bremsstrahlung and the absorption by electrons of photons when they are scattered by atoms play, as is well known, a determining role in the development of optical breakdown in gases.[1] The same mechanism to a significant extent determines the heating of plasma by laser radiation, and also manifests itself in a number of other phenomena.

In theoretical calculations of the bremsstrahlung effect it is usually assumed that the electromagnetic quantum is absorbed or emitted by the scattered electron as a result of which the bremsstrahlung cross section can be related to the transport cross section for elastic (nonradiative) scattering of electrons by atoms.[1,2] Although the question of the role played by atomic electrons in the emission and absorption of bremsstrahlung photons has been repeatedly discussed in the literature at the present time investigations of it do not exist. An exception is Ref. 3 in which the important role played by atomic electrons in bremsstrahlung was numerically demonstrated for the case of the scattering of electrons by the ground state of the hydrogen atom.

In the present paper the effect of atomic electrons on the brêmsstrahlung cross section is investigated by utilizing a number of approximations of the theory of nonradiative electron-atom collisions. As a result of this we succeed in obtaining physically transparent formulas for arbitrary atoms which contain a single characteristic—the dipole dynamic polarizability of an atom at the frequency under consideration.

2. We first state some elementary considerations which enable us to understand the principal result.[4] We consider induced bremsstrahlung which arises in the scattering of electrons by atoms in the presence of a strong electromagnetic wave of frequency ω. Under the action of the electric field of the wave the atom becomes polarized and long range forces $\sim 1/r^2$ of the "charge-dipole" type appear which significantly alter the cross section for the process.

For an atom in an S-state the polarizability is a scalar, and the scattering potential has the form

$$V(\mathbf{r},t)=V_A(\mathbf{r})+\frac{e\alpha(\omega)}{r^3}(\mathbf{E}_0\mathbf{r})\cos\omega t,$$

where

$$V_A(\mathbf{r})=-\frac{e^2Z}{r}+e^2\sum_{j=1}^{Z}\frac{1}{|\mathbf{r}-\mathbf{r}_j|}$$

is the interaction between an electron and an atom in the absence of the wave, Z is the nuclear charge, $\mathbf{r}_j$ are the coordinates of the atomic electrons, $\mathbf{E}_0$ is the amplitude of the electric field of the wave, which in the dipole approximation depends only on the time, α is the polarizability of the atom at the frequency ω.

We write the wave function for an electron in the field of the wave in the form $(c=\hbar=1)$

$$\psi_{\mathbf{p}}(\mathbf{r},t)=\exp\left\{i\mathbf{pr}-\frac{i}{2m}\int^t\left(\mathbf{p}+\frac{e\mathbf{E}_0}{\omega}\sin\omega t'\right)^2dt'\right\}.$$

The probability of scattering $\mathbf{p}\to\mathbf{p}'$ determined in the first Born approximation is by the expression

$$dw_{\mathbf{pp}'}=\left|\int d\mathbf{r}\,dt\exp\left\{i\mathbf{qr}+i\rho\cos\omega t-\frac{i}{2m}(p^2-p'^2)t\right\}\right.$$
$$\left.\times\left[V_A(\mathbf{r})+\frac{e\alpha(\omega)}{r^3}(\mathbf{E}_0\mathbf{r})\cos\omega t\right]\right|^2\frac{d^3\mathbf{p}'}{(2\pi)^3},$$

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