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Bremsstrahlung of relativistic electrons in a plasma in a strong magnetic field

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The bremsstrahlung of relativistic electrons in a plasma in a strong magnetic field is considered, with effects of dynamic screening and of a new mechanism of transition bremsstrahlung [A. V. Akopyan and V. N. Tsytovich, Zh. Eksp. Teor. Fiz. **71**, 166 (1976) [Sov. Phys. JETP **44**, 87 (1977)] taken into account. It is shown that in a strong magnetic field the effects of dynamic screening strongly diminish the intensity of ordinary bremsstrahlung at high frequencies $\omega < \omega_{max}$; here $\omega_{max} = 2\delta^2\omega^*$, where $\omega^* = \omega_{pe}c/v_{Te}$, ω_{pe} is the electronic plasma frequency, $\delta/ = \epsilon/m_e c2$, ϵ is the energy of the relativistic electron, and v_{Te} is the mean thermal velocity of the plasma electrons ($v_{Te} \ll c$). The angular distribution of the bremsstrahlung spectral intensity and the total intensity are studied in detail, both for the usual bremsstrahlung mechanism and for transition bremsstrahlung.

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1. INTRODUCTION: GENERAL STATEMENT OF THE PROBLEM

Bremsstrahlung in a plasma which is in a very strong magnetic field has recently been intensively studied in connection with the problem of interpreting the radiation of pulsars.[1–6] Actually, as will be seen from what follows, bremsstrahlung is altered by a magnetic field even at relatively small field strengths (but for sufficiently small frequencies). Therefore bremsstrahlung in a magnetic field is also of immediate interest for laboratory experiments on the magnetic containment of a plasma.

In the papers already referred to,[1–6] and also in[7,8], the influence of a magnetic field on bremsstrahlung was not analyzed completely, attention being given mainly to the case of nonrelativistic particles in a quantizing magnetic field. At the same time, as we shall show, the effect of a plasma on bremsstrahlung is more important in a strong magnetic field than with no field. The results we shall give properly apply not only to plasmas, but to other media as well, namely in all cases in which the plasma approximation can be used for the dielectric constant.

In constructing a more or less complete theory of bremsstrahlung in a nonequilibrium magnetoactive plasma it is necessary to take into account at least four effects: 1) screening by the plasma of the fields of colliding particles, 2) effects of the plasma on the propa-

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gation of the radiation and the type of waves emitted, 3) allowance for the new mechanism of transition bremsstrahlung, and 4) allowance for the quantization of the magnetic field.

Earlier work[1–4,6] has largely included the quantization of the magnetic field and partially allowed for the influence of the plasma on the propagation of the waves, [items 2) and 4)] but has completely neglected other plasma effects [items 1) and 3)]. A general theory of bremsstrahlung in a plasma with no magnetic field has been constructed,[9–11] which has taken consistent account of items 1), 2), and 3) in the limit of the quasiclassical description. This description corresponds to frequencies $\hbar\omega \ll \varepsilon$ and does not involve any serious limitation, since the plasma effects are usually important for relatively small frequencies. The problem of the present paper is the extension of the results of[10,11] to the case of strong magnetic fields.

We shall assume that both the thermal electrons of the plasma and the superthermal relativistic electrons move strictly along the lines of force of the constant strong field. Furthermore these electrons are subject to an attractive force from the ions, which results in their having an accelerated motion and radiating without departing from the lines of force along which they move. In this connection it is appropriate to make the following remark about item 4), the quantization of the magnetic field.

Beginning with the work of Chiu and Canuto,[1–4] bremsstrahlung in a strong magnetic field has been considered for the case in which all the particles are in the lowest Landau level (see[7,8,12]). Virtamo and Jauho[7] showed that the Chui-Canuto formulas hold only for the radiation exactly along the magnetic field, and that for an angle of the order of a radian there is an additional term in the radiated power which is much larger than the result of[1] and makes the main contribution to the total power radiated. In the case of a strong magnetic field the total radiated power as found in[1] is smaller than that found in[7] by a factor $\omega^2/\omega_{He}^2 \ll 1$ ($\omega_{He} = eH/m_e c$ is the precession frequency). Furthermore, the analysis shows that the result found by Virtamo and Jauho[7] is completely quasiclassical and does not contain Planck's constant, nor, by the way, the magnetic field intensity H. As we shall see, this result is easily obtained from the quasiclassical treatment we give here of the bremsstrahlung of particles which move exactly along the lines of force, if we neglect the influence of the plasma. Thus in a certain sense the quantization of the magnetic field plays no particular part in the bremsstrahlung, since the result found in[7] is in fact quasiclassical.

In the case in which the lowest Landau level is not the only one populated, the result involves the total density of electrons in all of the levels. A more essential and important assumption is that the motion of the electrons is one-dimensional during the emission process. This assumption is in this sense a more general one than that about the quantization and the lowest level alone being occupied. Therefore the present treatment includes all of the four points we have noted as necessary for the analysis of the bremsstrahlung in a plasma in a strong magnetic field. We remark also that whereas in the absence of a magnetic field the spectral intensity of the radiation depends logarithmically on the maximum momentum transfer, fixed by the limit of applicability of the quasiclassical treatment, with a strong magnetic field there is no such dependence, i.e., for $\hbar\omega \ll \varepsilon$ the effect is a purely classical one.

We shall here confine ourselves to the assumption that the superthermal electrons are ultrarelativistic, i.e.,

$$\gamma = \varepsilon/m_e c^2 \gg 1, \tag{1}$$

since it is precisely in this limit that the plasma effects influence the bremsstrahlung most strongly. The condition for the use of the assumption that the motion of the electrons is one-dimensional is $\omega \ll \omega_{He}/\gamma$ (see also[5]). For simplicity we shall also assume $\omega \gg \omega_{pe}$. In other words, we shall consider the bremsstrahlung in the plasma in the frequency range

$$\omega_{pe} \ll \omega \ll \omega_{He}/\gamma. \tag{2}$$

For relativistic particles the value of ω_{He} can reach the maximum frequency $m_e c^2\gamma/\hbar$ only for $H \sim 10^{14}$ G. If the inequality is not satisfied, in the frequency range $\omega > \omega_{He}/\gamma$ the bremsstrahlung will occur with a particle momentum change perpendicular to H. At such frequencies quantization can become significant.

2. FORMULATION OF THE GENERAL THEORY

The construction of the theory is similar to the procedure in our earlier paper.[11] Therefore we indicate here only the more important points of principle. Since the ions of the plasma can be regarded as motionless during the emission process, without loss of generality we can treat all of the particles of the plasma, which by hypothesis are nonrelativistic ($v_{T_e} \ll c$, where v_{T_e} is the mean velocity of the thermal electrons), and the relativistic emitting electrons as moving exactly along the lines of force. In[11] two parts of the Green's function were used, longitudinal and transverse, but in our present case there is a unique Green's function. Its structure can be seen easily if we write down from the Maxwell equations the connection between the Fourier components of the electric field and of the current.

Taking the z axis along **H**, we get

$$E_{k,z} = -\frac{4\pi i}{\omega}\frac{c^2k_z^2-\omega^2}{(c^2k_z^2-\omega^2)\varepsilon_{k_z}^{(0)}+k_\perp^2c^2}\,j_{k,z},$$
$$\mathbf{E}_{k,\perp} = -\frac{4\pi i}{\omega}\frac{c^2\mathbf{k}_\perp(\mathbf{k}\mathbf{j}_k)}{(c^2k_z^2-\omega^2)\varepsilon_{k_z}^{(0)}+k_\perp^2c^2}, \tag{3}$$

where the indices z and $\perp$ denote the components of vectors along and transverse to **H**, $\mathbf{j}_k$ is the total current density, and **k** is the wave vector, $k=\{k_z, \mathbf{k}_\perp, \omega/c\}$. The one-dimensional dielectric constant $\varepsilon_{k_z}^{(0)}$ which appears in Eq. (3) is determined from the kinetic equation and is given by

$$\varepsilon_{k_z}^{(0)} = 1 + \sum_{\alpha} \frac{m_\alpha \omega_{p_\alpha}^2}{k_z} \int \frac{1}{\omega - k_z v_z} \frac{\partial f_{p_z}^{(\alpha)}}{\partial p_z} dp_z, \tag{4}$$

where $f_{p_z}^{(\alpha)}$ is the distribution function of particles of type α and the summation is over all types of particles in the plasma.

According to Eq. (3), the zeroes of the Green's function determine the linear modes:

$$(k^2c^2\cos^2\vartheta - \omega^2)\varepsilon_{k_z}^{(0)} + k^2c^2\sin^2\vartheta = 0, \tag{5}$$

where ϑ is the angle between $\mathbf{k}$ and $\mathbf{H}$ (the angle of propagation of the waves). Substituting from Eq. (4) the value $\varepsilon_{k_z}^{(0)} \approx 1 - \omega_{pe}^2/\omega^2$ into Eq. (5), we get for $\omega \gg \omega_{pe}$

$$k^2c^2 = \frac{\omega^2 - \omega_{pe}^2}{1 - (\omega_{pe}^2/\omega^2)\cos^2\vartheta} \approx \omega^2 - \omega_{pe}^2 \sin^2\vartheta, \tag{6}$$

i.e., $\omega^2 = k^2c^2 + \omega_{pe}^2 \sin^2\vartheta$.

In the general case Eq. (5) gives two modes, one of which is almost longitudinal ($\omega \sim \omega_{pe}$), and the other, Eq. (6), is practically transverse for $\omega \gg \omega_{pe}$. Analysis shows that for the latter mode the deviation of the polarization vector from transversality gives only small corrections to the integral intensity of the radiation for $\omega \gg \omega_{pe}$.

The presence of the coefficient $\sin^2\vartheta$ with ω_{pe}^2 in the approximate expression (6) for the frequency is extremely important; it results in there being no Ter-Mikaelyan density effect[14] (see also[15]) in the present case; this effect for a plasma with no magnetic field was analyzed in detail in[9]. In fact, from (6) we have

$$\frac{1}{\omega - kv\cos\vartheta} \approx \frac{2}{\omega} \frac{1}{1/\gamma^2 + \vartheta^2(1 + \omega_{pe}^2/\omega^2)}.$$

This last relation is written for small angles of propagation ϑ and shows that the effect of the plasma reduces to a term in ω_{pe}^2/ω^2 which is small compared with unity; i.e., for $\omega \ll \gamma\omega_{pe}$ there is no density effect. Without a magnetic field the corresponding expression had a different denominator[9]

$$\frac{2}{\omega(1/\gamma^2 + \vartheta^2 + \omega_{pe}^2/\omega^2)}$$

and the ordinary bremsstrahlung is strongly suppressed for $\omega \ll \gamma\omega_{pe}$. However, the analysis which is to follow shows that in a strong magnetic field there is a more powerful suppression of the ordinary bremsstrahlung of an ultrarelativistic particle. Here the frequency $\gamma\omega_{pe}$ is no longer the critical value below which the bremsstrahlung falls off, as in the zero-field case. As the critical frequency we have the large quantity $2\gamma^2\omega^*$, where $\omega^* = \omega_{pe}c/v_{Te}$, which is associated with the Debye radius.

The two points we have discussed—the one-dimensionality of the Green's function and the absence of a density effect—give in principle the main differences in the bremsstrahlung processes when there is a strong magnetic field.

All calculations of matrix elements are made by the method of[11], and we present only the final results. As in[11], we represent the probability of bremsstrahlung emission of transverse electromagnetic waves in the form ($\hbar = 1$)

$$w = (2\pi)^6 |\mathbf{M}^{(c)} + \mathbf{M}^{(t)}|^2 \delta(\omega - (k_z + \varkappa_z)v), \quad v = \{0, 0, v_z\}, \tag{7}$$

where $\mathbf{M}^{(c)}$ and $\mathbf{M}^{(t)}$ are the matrix elements for ordinary and transverse bremsstrahlung. Starting from the equation of motion and the kinetic equation and using the electric field in the form (3) we get in the Born approximation

$$\mathbf{M}^{(c)} = \left[\pi^2 \frac{\partial}{\partial\omega}(\omega^2\varepsilon_{k_z}^{(0)})\Big|_{\omega=\omega_{\mathbf{k}}^t}\right]^{-1/2} \frac{\sqrt{2}e_e^2 e_i}{m_e\gamma^3} \frac{\omega\varkappa_z}{kv(\omega - k_z v)^2} \frac{[\mathbf{k}\times\mathbf{v}]}{\varkappa_z^2\varepsilon_{\varkappa_z}^{(0)} + \varkappa_\perp^2\gamma^2},$$

$$\mathbf{M}^{(t)} = \left[\pi^2 \frac{\partial}{\partial\omega}(\omega^2\varepsilon_{k_z}^{(0)})\Big|_{\omega=\omega_{\mathbf{k}}^t}\right]^{-1/2} \frac{\sqrt{2}e_e^2 e_i}{m_e\omega k} \frac{\varkappa_z}{\varkappa_z^2\varepsilon_{\varkappa_z}^{(0)} + \varkappa_\perp^2\gamma^2} \tag{8}$$

$$\times \frac{\varkappa_z + k_z}{(\varkappa_z + k_z)^2\varepsilon_{k_z}^{(0)} + (k_\perp + \varkappa_\perp)^2} [\mathbf{k}\times\varkappa_z](1 - \varepsilon_{\varkappa_z}^{(0)}) - [\mathbf{k}\times\varkappa_z + \mathbf{k}_z](1 - \varepsilon_{\varkappa_z+k_z}^{(0)(e)*})\}. \tag{9}$$

Here $\varkappa$ is the recoil momentum given by the electron to an ion with charge e_i and $\varepsilon_{\varkappa_z}^{(0)(e)}$ is the electronic component of the dielectric constant. With the approximation that the ion is infinitely heavy, $\varepsilon_{\varkappa_z}^{(0)(e)} \approx \varepsilon_{\varkappa_z}^{(0)}$. The diagrams corresponding to the matrix elements (8) and (9) have been presented in[11]. We shall hereafter consider separately the contributions made to the bremsstrahlung by each of the matrix elements (8) and (9).

3. EFFECT OF THE PLASMA SCREENING ON THE ORDINARY BREMSSTRAHLUNG IN A STRONG MAGNETIC FIELD

Although, as seen from Eq. (7), the two mechanisms of bremsstrahlung interfere with each other, by using their different frequency dependences we can in limiting cases consider them separately. From Eqs. (7) and (8) we have for the spectral intensity of the radiation given by ordinary bremsstrahlung, referred to the range $\sin\vartheta\, d\vartheta$,

$$\frac{dI_\omega^{(c)}(\vartheta)}{d\omega} = \frac{2r_e^2 e_i^2 c}{\gamma^6 v^3} \frac{\omega^4 \sin^2\vartheta}{(\omega - kv\cos\vartheta)^2} \times \left[\frac{1}{1/d^2 + (\omega - kv\cos\vartheta)^2/v^2} - \frac{1}{\varkappa_{\perp\, max}^2 + (\omega - kv\cos\vartheta)^2/v^2}\right], \tag{10}$$

where r_e is the classical electron radius, $\varkappa_{max}$ is the maximum momentum transfer, and $d = v_{Te}/\omega_{pe}$ is the Debye radius of the plasma.

It follows from Eq. (10) that the intensity of the radiation is symmetrical around v and vanishes in the directions along ($\vartheta = 0$) and opposite to ($\vartheta = \pi$) the velocity. In the ultrarelativistic case, according to Eq. (6), at frequencies $\omega \gg \omega_{pe}$ the radiation intensity (10) as a function of ϑ has a sharp maximum at angles near

$$\Delta\vartheta \leqslant \gamma^{-1}. \tag{11}$$

For weak screening of the field of the ion by the plasma, which holds for very high frequencies

$$\omega \gg 2\gamma^2\omega^*, \tag{12}$$

the ordinary bremsstrahlung of an ultrarelativistic particle occurs entirely within the angles given by Eq. (11).

In the frequency range

$$\omega_{pe} \ll \omega \ll 2\gamma^2\omega^*, \tag{13}$$

corresponding to the case of strong screening of the ion's field, Eq. (10) shows that there is also a contribution to the total bremsstrahlung from emission outside the cone of directions (12). In the nonrelativistic case, according to Eq. (10), the waves are emitted isotropically relative to the direction of motion of the electron.

For $v \sim c$, neglecting small terms, we get from Eq. (10) for the total spectral intensity of the radiation

$$\frac{dI_\omega^{(c)}}{d\omega} = \frac{2r_e^2 e_i^2}{\gamma^6}\frac{\omega}{\omega^*}\left[-2\frac{\omega}{\omega^*} + \left(\frac{\omega^2}{\gamma^4\omega^{*2}} - 1\right)\right.$$
$$\left.\times \operatorname{arctg}\frac{2\omega/\omega^*}{1+\omega^2/4\gamma^4\omega^{*2}} + \frac{\omega}{\omega^*}\ln\frac{1+4\gamma^4\omega^{*2}/\omega^2}{1+\omega^{*2}/4\omega^2}\right] \tag{14}$$

($\omega^* = \omega_{pe}c/v_{Te}$). At the high frequencies (12) we get from Eq. (14) the following simple expression:

$$dI_\omega^{(c)}/d\omega = 8r_e^2 e_i^2/3\gamma^2. \tag{15}$$

Accordingly, at frequencies $\omega \gg 2\gamma^2\omega^*$, corresponding to the vacuum limit, the intensity of ordinary bremsstrahlung decreases as the inverse square of the energy of the ultrarelativistic particle and is independent of the frequency. For the case of no magnetic field it was shown in[9] that the bremsstrahlung in a plasma at frequencies (12) is that given by the Bethe-Heitler formula, i.e., the intensity depends logarithmically on the frequency and on the energy of the particle.

The decrease with energy of the intensity of the bremsstrahlung of a one-dimensionally moving ultrarelativistic electron can be explained in the following way. In the presence of a strong magnetic field the acceleration of the electron in the Coulomb field of an ion occurs only in the longitudinal direction. A transverse displacement of the electron in its interaction with the decelerating quanta is forbidden. On the other hand, it follows from the equation of motion of the electron that the momentum change in the one-dimensional case is smaller than that in free motion by a factor γ^2. The bremsstrahlung of a free (unmagnetized) electron occurs through changes in both the longitudinal and the transverse momentum components. This is the basic difference between the bremsstrahlung of a magnetized fast particle and that of an unmagnetized fast particle.

The result (15) can be compared with the result of Virtamo and Jauho,[7] derived for the nonrelativistic case. In this case, according to Eq. (10), we must replace the factor γ^{-2} in Eq. (15) by c/v. This gives exactly the result of[7]. In precisely the same way, our result in this limit is one-half of the result found in[22]. This is due to the fact that Loskutov and Skobelev[12] allowed transverse displacements of the electron, which in the nonrelativistic limit make the same contribution to the bremsstrahlung as the longitudinal ones. The most important criterion is the inequality (12), which gives the limits of applicability of the results of[7,12], which neglect the influence of the plasma polarization on the bremsstrahlung. This criterion is very severe at large energies of the particles.

Let us now consider other frequencies (13) at which screening is important. For

r.p. 1829 (16)

we get from Eq. (14)

$$\frac{dI_\omega^{(c)}}{d\omega} = \frac{4}{3}\frac{r_e^2 e_i^2}{\gamma^6}\frac{\omega^2}{\omega^{*2}}\left(\ln\frac{2\gamma^2\omega}{\omega^*} - 1\right). \tag{17}$$

For the frequency range

$$\omega_{pe} \ll \omega \ll \omega^* \tag{18}$$

we get from (14)

$$\frac{dI_\omega^{(c)}}{d\omega} = \frac{8r_e^2 e_i^2}{\gamma^6}\frac{\omega^2}{\omega^{*2}}(\ln 2\gamma - 1). \tag{19}$$

Comparison of Eq. (15) with Eqs. (17) and (19) shows that the ordinary bremsstrahlung of a magnetized charged particle in a plasma goes most effectively at the high frequencies (12). At the lower frequencies the intensity decreases both with increasing energy ($\sim \gamma^{-6}$) and with decreasing frequency ($\sim \omega^2$). This is due to the increasing importance of screening of the virtual quanta by the plasma. We emphasize that when there is no strong magnetic field, as was shown in[9], the drop in the intensity of ordinary bremsstrahlung in a plasma begins not at frequencies $\omega \lesssim 2\gamma^2\omega^*$, as in the present case, but at frequencies $\omega \lesssim \omega_{pe}$.

4. TRANSITION BREMSSTRAHLUNG IN A STRONG MAGNETIC FIELD

As we have previously[9] emphasized, transition bremsstrahlung is due to the transitional scattering[16] of virtual quanta produced in collisions of the particles, into electromagnetic waves. The spectral distribution of intensity obtained from Eqs. (7) and (9) is rather cumbersome in the general case:

$$\frac{dI_\omega^{(t)}(\vartheta)}{d\omega} = \frac{2\gamma^2 r_e^2 e_i^2}{M^{3/2}}\frac{\omega^2}{\omega^{*2}}\sin^2\vartheta\left\{\left(1 - \frac{3}{2}\frac{(b-2pc)(\sqrt{a}-p^2\gamma)}{\gamma^2 M}\right)\right.$$
$$\times \ln(2\sqrt{RM} + 2Mt + b - 2pc) + \frac{\sqrt{M}}{(4ac-b^2)\sqrt{R}}\left[2c(b-2pc)\right.$$
$$-((4ac-b^2)-(b-2pc)^2)t + \frac{\sqrt{a}-\sqrt{c}\,p}{\sqrt{c}\,M}((4ac-b^2)Mt^2 + (b-2pc)$$
$$\left.\left.\times(10ac+2pbc-2p^2c^2-3b^2)t + c(8ac+4bpc-4p^2c^2-3b^2))\right]\right\}\Bigg|_{t_{min}}^{t_{max}}, \tag{20}$$

where

$$a = d^4(k^2+k_\perp^2\gamma^2)^2, \quad b = 2d^2\gamma^2(k^2+\gamma^2k_\perp^2), \quad c = \gamma^4,$$
$$p = 1 + \frac{(\omega - k_z v)^2 d^2}{v^2}, \quad M = a - bp + cp^2, \quad t = \frac{1}{\varkappa_\perp^2 d^2 + p}, \tag{21}$$
$$t_{min} = \frac{1}{\varkappa_{\perp max}^2 d^2 + p}, \quad t_{max} = \frac{1}{p}, \quad R = [c + (b-2pc)t + (a-bp+cp^2)t^2]^{1/2}.$$

It follows from Eqs. (10) and (20) that for one-dimensional motion of the ultrarelativistic electron and the thermal electrons of the plasma there is no bremsstrahlung in a strong magnetic field in the directions along and opposite to the velocity of the electron.

With weak screening of the virtual quanta we get from Eqs. (20) and (21), for frequencies (12), the formula

$$\frac{dI_\omega^{(t)}(\vartheta)}{d\omega}=\frac{2r_e^2e_i^2}{\gamma^2}\frac{\omega^{*4}}{\omega^4}\frac{\sin^2\vartheta(1+\gamma^2\sin^2\vartheta)}{(1-(v/c)\cos\vartheta)^2}. \tag{22}$$

It follows that for high frequencies (12) the bremstrahlung also has a sharp forward directionality inside a cone with the narrow aperture angle (11). The result of integrating the intensity (22) over all angles does not explicitly depend on the energy at high energies:

$$dI_\omega^t/d\omega={}^8/_3r_e^2e_i^2(\omega^*/\omega)^4. \tag{23}$$

A comparison of Eqs. (15) and (23) shows that under conditions of weak screening, i.e., in the high-frequency region (12), the bremsstrahlung is mainly ordinary bremsstrahlung.

If the screening is strong, for the frequency ranges (16) and (18) we have from Eq. (20) the respective results

$$\frac{dI_\omega^{(t)}(\vartheta)}{d\omega}=\frac{2r_e^2e_i^2}{\gamma^4}\left(\frac{\omega^*}{\omega}\right)^2\frac{\sin^2\vartheta}{1-(v/c)\cos\vartheta}, \tag{24}$$

$$\frac{dI_\omega^{(t)}(\vartheta)}{d\omega}=\frac{4r_e^2e_i^2}{\gamma^2}\frac{\sin^2\vartheta}{1+\gamma^2\sin^2\vartheta}. \tag{25}$$

Equation (24) shows that at frequencies (16) the waves are distributed over angles in the same way both inside and outside the directionality cone (11). In this case the main contribution to the total bremsstrahlung comes from the radiation propagated outside the range (11) of angles. Equation (25) shows that the transition bremsstrahlung at comparatively low frequencies (18) has a sharp maximum near the plane perpendicular to the direction of the velocity, in a range of angles

$$\Delta\vartheta^*=|\pi/2-\vartheta|\ll\gamma^{-1}. \tag{26}$$

For the total bremsstrahlung the main contribution in Eq. (25) is from emission occurring outside the range of angles (26).

Integrating over all angles, we have from Eqs. (24) and (25) the respective spectral intensities

$$\frac{dI_\omega^{(t)}}{d\omega}=\frac{4r_e^2e_i^2}{\gamma^4}\left(\frac{\omega^*}{\omega}\right)^2(\ln 2\gamma-1), \tag{27}$$

$$dI_\omega^{(t)}/d\omega=8r_e^2e_i^2/\gamma^4. \tag{28}$$

Thus, comparing Eqs. (27) and (28) with (17) and (19), applied under the respective conditions (16) and (18), we come to the conclusion that in a plasma in a strong magnetic field bremsstrahlung in the frequency range (13) occurs mainly as transition bremsstrahlung. In particular, in the spectral region (18) the emitted wavelength $\lambda\gg d$. Therefore the thermal electrons of the plasma, which give the dynamic polarization, radiate coherently when they scatter virtual quanta, and this gives the maximum intensity of the radiation.

As for the interference between ordinary and transition bremsstrahlung, it is unimportant because of the different dependences of the matrix elements (8) and (9) on frequency and on the energy of the particle.

5. DISCUSSION OF THE RESULTS

The bremsstrahlung of a fast particle in a plasma in a strong magnetic field depends on the frequency of the emitted wave and the energy of the particle. Only in the frequency region $\omega\gg 2\gamma^2\omega^*$ is the radiation governed by the ordinary bremsstrahlung mechanism; it is here that the intensity reaches its largest value, although it decreases with increasing energy of the particle [Eq. (16)]. In the frequency range $\omega_{pe}\ll\omega\ll 2\gamma^2\omega^*$ the main effect is already the transition bremsstrahlung arising from scattering of virtual quanta by the dynamic polarization. A comparison of Eqs. (15) and (28) shows that in this case the maximum intensity level is smaller by a factor γ^2 than that at frequencies $\omega\gg 2\gamma^2\omega^*$.

From a comparison of Eqs. (17) and (27) we find that the transition bremsstrahlung dominates for a frequency

$$\omega<\sqrt{\gamma}\omega^*, \tag{29}$$

which is in the range (16). In the other frequency range (18) the transition bremsstrahlung is always dominant.

These results can be transferred easily to the case of a relativistic plasma, which is of interest for the interpretation of the radiation of pulsars.[17] Here we must use instead of ω_{pe} the value of the plasma frequency in a relativistic plasma, ω_p,[17] and set $v_{Te}\approx c$. The value of the plasma frequency varies over a wide range in the magnetosphere of a pulsar, and near the surface it can reach rather large values, so that the frequencies corresponding to the frequency (29) will be

$$\omega<\sqrt{\gamma}\omega_p,\quad \omega_p=(4\pi n_e e^2/\bar{\varepsilon})^{1/2}, \tag{30}$$

where $\bar{\varepsilon}$ is the mean energy of a plasma particle. The distributions of particles in pulsars are most probably nonthermal with tails extending far into the high-energy region. Under the condition (30) the total transition bremsstrahlung intensity of all the particles will be

$$\frac{dI_\omega^{(t)}}{d\omega}=\frac{4r_e^2e_i^2n_i}{\omega^2}\int\frac{\omega_p^2}{\gamma^4}(\ln 2\gamma+1)f_\gamma^e\,d\gamma \tag{31}$$

and can lie in the optical and x-ray regions. At large optical thicknesses induced processes are important, and give pumping of the electromagnetic waves. However, in the x-ray and optical regions the optical thickness is small.

Equation (31) gives a power-law spectrum of the radiation and allows determination of the density of particles for which the calculated radiation power corresponds to an observed value.

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Study of the magnetic structure of the system $Fe_{65}(Ni_{1-x}Mn_x)_{35}$ by methods of magnetic scattering of neutrons and the Mössbauer effect

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The magnetic structure of the system $Fe_{65}(Ni_{1-x}Mn_x)_{35}$ with $x = 0.14$, 0.28, and 0.37 was investigated by the methods of magnetic scattering of neutrons in the temperature interval 4.2–700 K and the Mössbauer effect in an external field 45 kOe at 4.2 K. A joint analysis of the results shows that in the case of a concentration antiferro-ferromagnetic transition there is observed a broad spectrum of magnetic inhomogeneities, constituting the interacting regions in the antiferro-, ferro-, and paramagnetic states, and possibly also regions of "spin glass." The parameters of the ferromagnetic regions are determined and their polarization character is demonstrated, the Neel temperatures and the average magnetic moment per sublattice of the regions with antiferromagnetic order are estimated.

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INTRODUCTION

The study of the magnetic structure of the system $Fe_{65}(Ni_{1-x}Mn_x)_{35}$ is of considerable interest from the point of view of the onset of the magnetically ordered state. As established by Shiga,[1] these alloys are ferromagnetic at $x<0.3$ and antiferromagnetic at $x>0.3$. It is still unclear, however, how the magnetic order changes in this system, although this question has been the subject of a large number of studies. Thus, the presence of exchange anisotropy in the system and the temperature dependence of the magnetization have given grounds for the authors of[2,3] to suggest the coexistence of paramagnetic and antiferromagnetic regions at critical concentrations. At the same time, investigations of the magnetization in strong fields and at high pressures and investigations of the low-temperature specific heat and of the spontaneous magnetostrictions are treated in[4–7] under the assumption that the alloys are weak homogeneous collectivized ferromagnets. The Mössbauer spectra of the Fe^{57} nuclei,[8,9] to the contrary, show that the magnetic structure of the system is inhomogeneous to a considerable degree. Thus, direct experimental investigations of the magnetic structure in the transition region of the concentrations are necessary.

In a preceding study,[10] magnetic scattering of neutrons was used to confirm the assumption that regions with ferromagnetic and antiferromagnetic types of order coexist in alloys with $x=0.28$ and 0.37, and the polarization character of the ferromagnetic regions was demonstrated. The present paper is devoted to a further study of the singularities of the magnetic structure of the system $Fe_{65}(NI_{1-x}MN_x)_{35}$ at critical concentrations, by the methods of magnetic scattering of neutrons and of the Mössbauer effect.

1. MEASUREMENT PROCEDURE AND SAMPLES

Neutron diffraction was investigated with a diffractometer with a wavelength $\lambda=1.07$ Å in the angle interval $10° \leqslant 2\theta \leqslant 55°$. Small-angle scattering of the neutrons was investigated with a diffractometer with $\lambda=1.59$ Å. The measurements were carried out on cylindrical polycrystalline samples of 8 mm diameter and

